

Homework 2

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Bayesian Statistical Modeling Winter 2023

Homework, Week 2

When is homework due? Homework is due on Mondays (Tuesday if Monday is a holiday). This assignment is due on Monday, 1/23/2022. Submit homework to gradescope:

Before submitting your work You should use this file as a template for completing the assignment. You should change the author to your name. You should use the features of Rmarkdown to write your text answers below and include R code and equations where necessary. You should turn in both the .Rmd file as well as the pdf file that you generate by using the Knit button.

Complete problems from the end of chapter 2.

2E1

The probability of rain on Monday = given it is Monday, what it is probability of raining. 2) and 4).

2E2

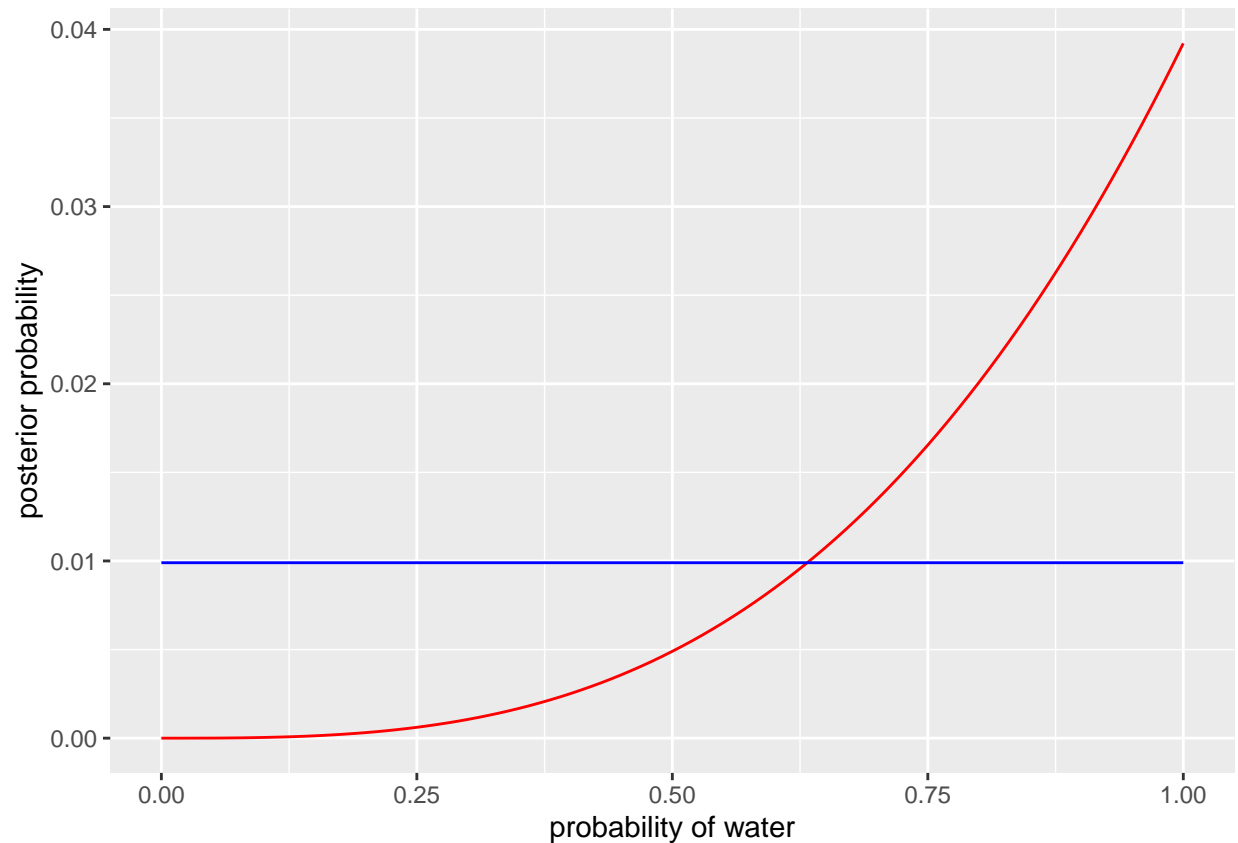
3).

2M1

1) WWW

```
posterior_table1 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table1 <- mutate(posterior_table1 , prior=1/n())
posterior_table1 <- mutate(posterior_table1 , likelihood=dbinom(3,size=3,prob=p_w_proposed))
posterior_table1 <- mutate(posterior_table1 , raw_posterior = prior*likelihood)
posterior_table1 <- mutate(posterior_table1 , posterior = raw_posterior/sum(raw_posterior))

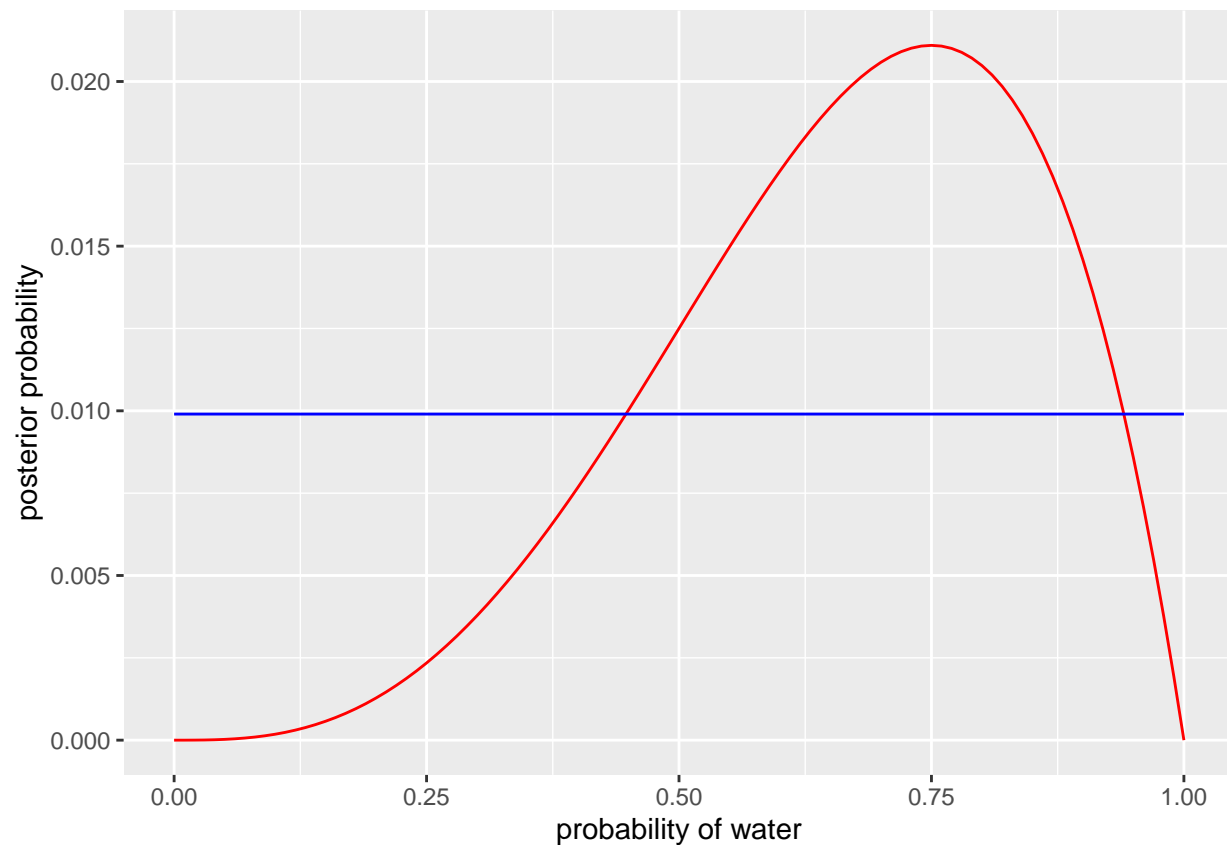
ggplot(data=posterior_table1, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



2) WWWL

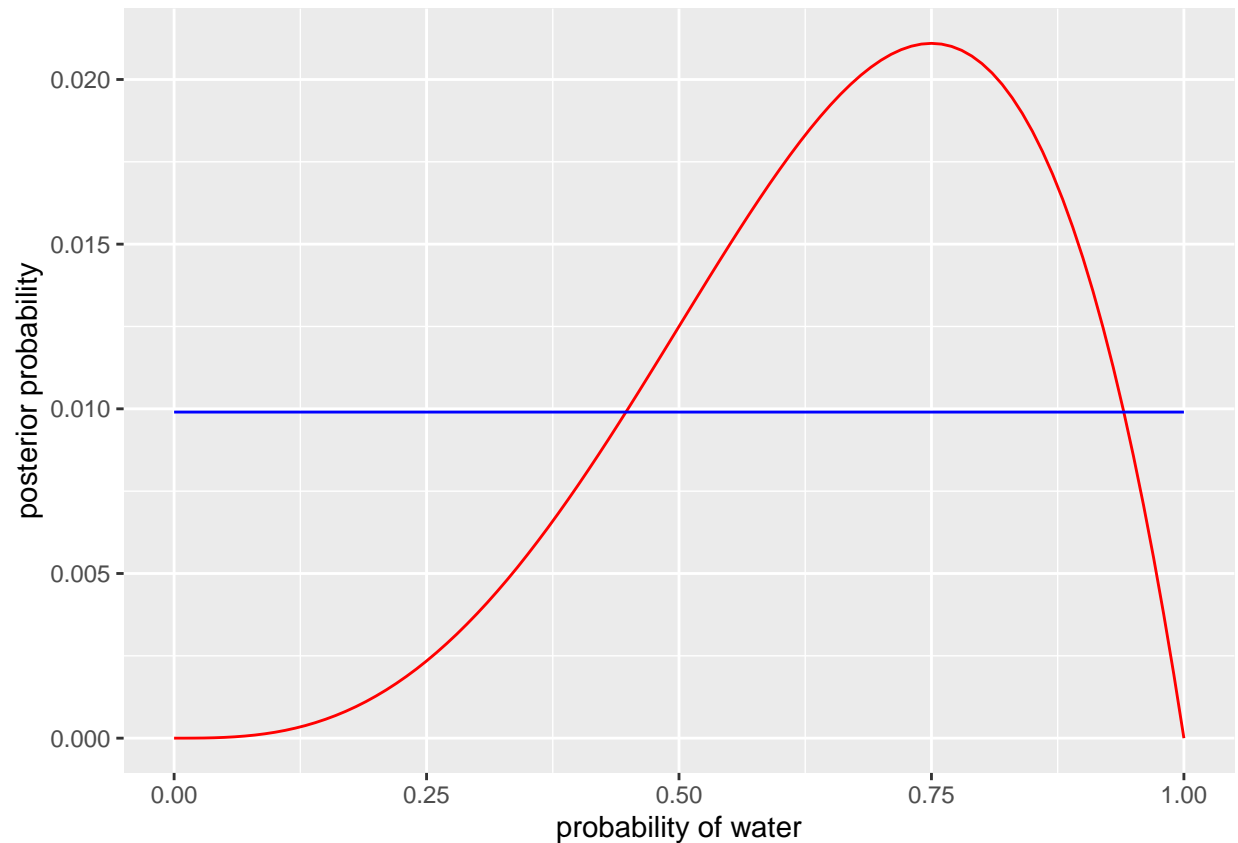
```
posterior_table2 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table2 <- mutate(posterior_table2 , prior=1/n())
posterior_table2 <- mutate(posterior_table2 , likelihood=dbinom(3,size=4,prob=p_w_proposed))
posterior_table2 <- mutate(posterior_table2 , raw_posterior = prior*likelihood)
posterior_table2 <- mutate(posterior_table2 , posterior = raw_posterior/sum(raw_posterior))

ggplot(data=posterior_table2, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



```
# Or we can update what we have from posterior_table1
d_now = 0
posterior_table22 <- mutate(posterior_table1 , likelihood=(d_now==1)*p_w_proposed + (d_now==0)*(1-p_w_p
posterior_table22 <- mutate(posterior_table22 , raw_posterior = posterior*likelihood)
posterior_table22 <- mutate(posterior_table22 , posterior = raw_posterior/sum(raw_posterior))

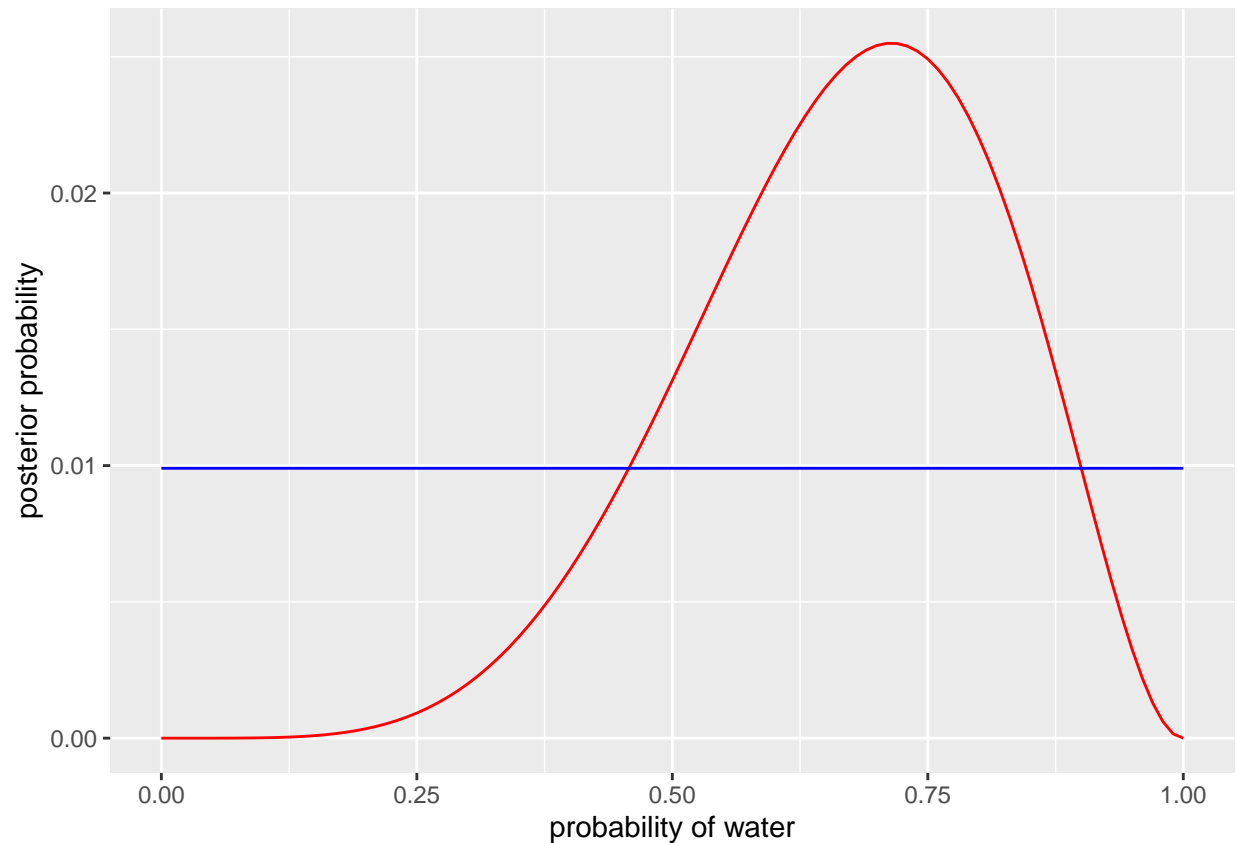
ggplot(data=posterior_table22, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



3) LWW LWWW

```
posterior_table3 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table3 <- mutate(posterior_table3 , prior=1/n())
posterior_table3 <- mutate(posterior_table3 , likelihood=dbinom(5,size=7,prob=p_w_proposed))
posterior_table3 <- mutate(posterior_table3 , raw_posterior = prior*likelihood)
posterior_table3 <- mutate(posterior_table3 , posterior = raw_posterior/sum(raw_posterior))

ggplot(data=posterior_table3, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



2M2

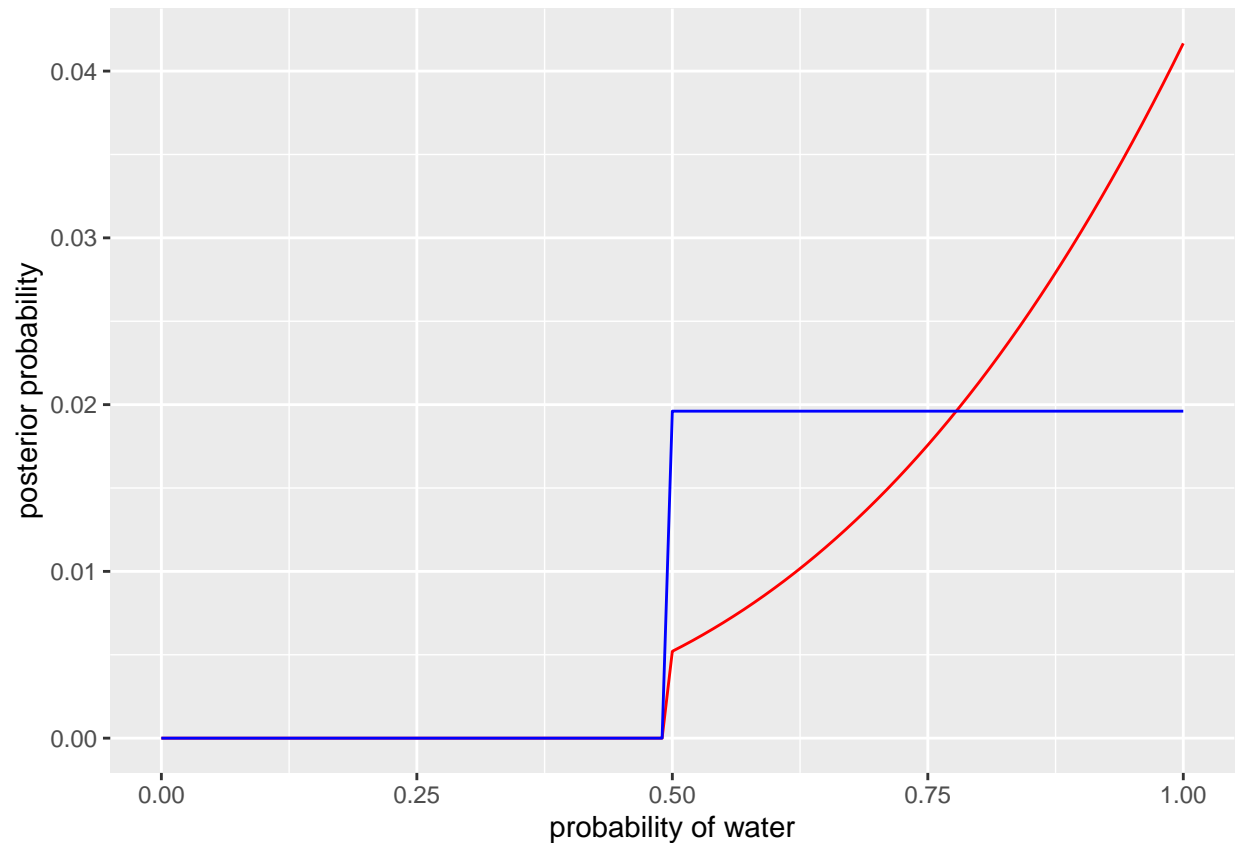
1) WWW

```
posterior_table4 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table4 <- mutate(posterior_table4 , prior=ifelse(p_w_proposed < 0.5, 0, 2/(n()+1)))
sum(posterior_table4$prior)
```

```
## [1] 1
```

```
posterior_table4 <- mutate(posterior_table4 , likelihood=dbinom(3,size=3,prob=p_w_proposed))
posterior_table4 <- mutate(posterior_table4 , raw_posterior = prior*likelihood)
posterior_table4 <- mutate(posterior_table4 , posterior = raw_posterior/sum(raw_posterior))

ggplot(data=posterior_table4, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



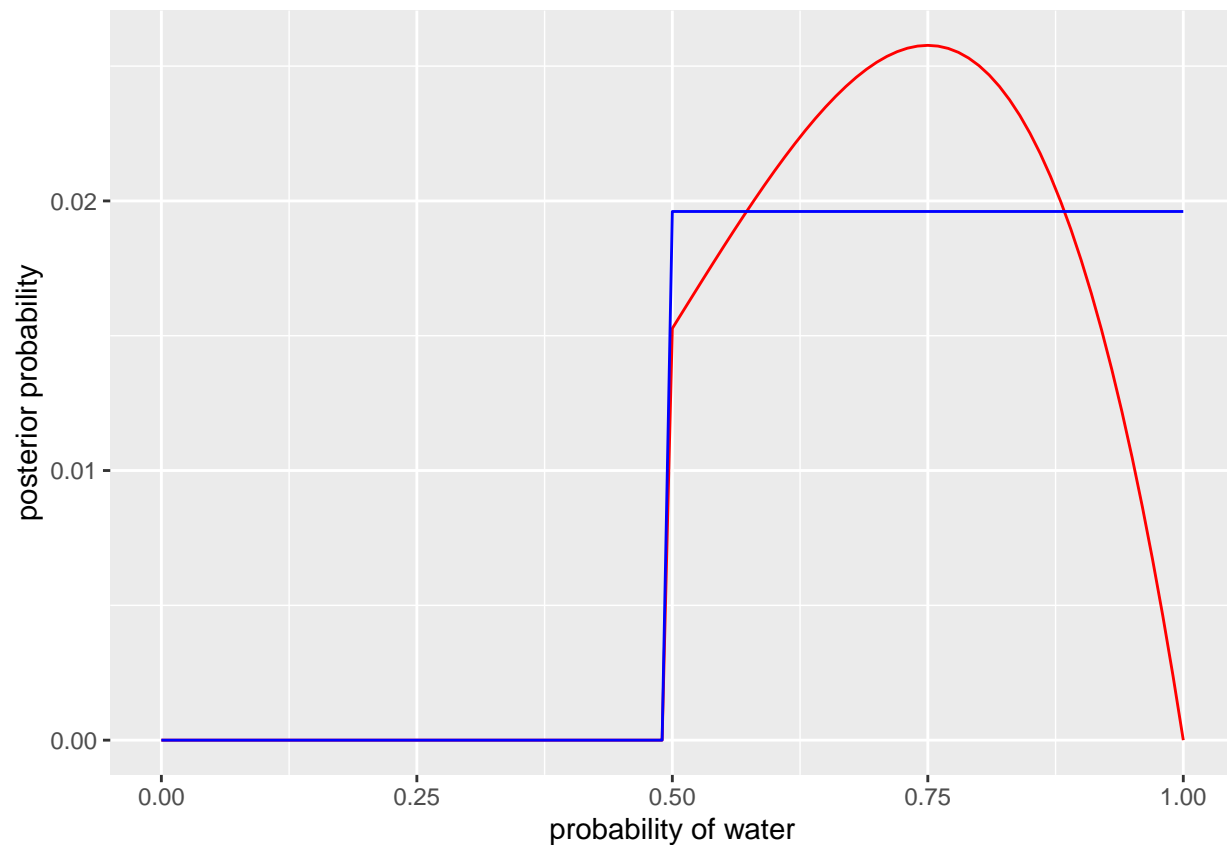
2) WWWL

```
posterior_table4 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table4 <- mutate(posterior_table4 , prior=ifelse(p_w_proposed < 0.5, 0, 2/(n()+1)))
sum(posterior_table4$prior)
```

```
## [1] 1
```

```
posterior_table4 <- mutate(posterior_table4 , likelihood=dbinom(3,size=4,prob=p_w_proposed))
posterior_table4 <- mutate(posterior_table4 , raw_posterior = prior*likelihood)
posterior_table4 <- mutate(posterior_table4 , posterior = raw_posterior/sum(raw_posterior))

ggplot(data=posterior_table4, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



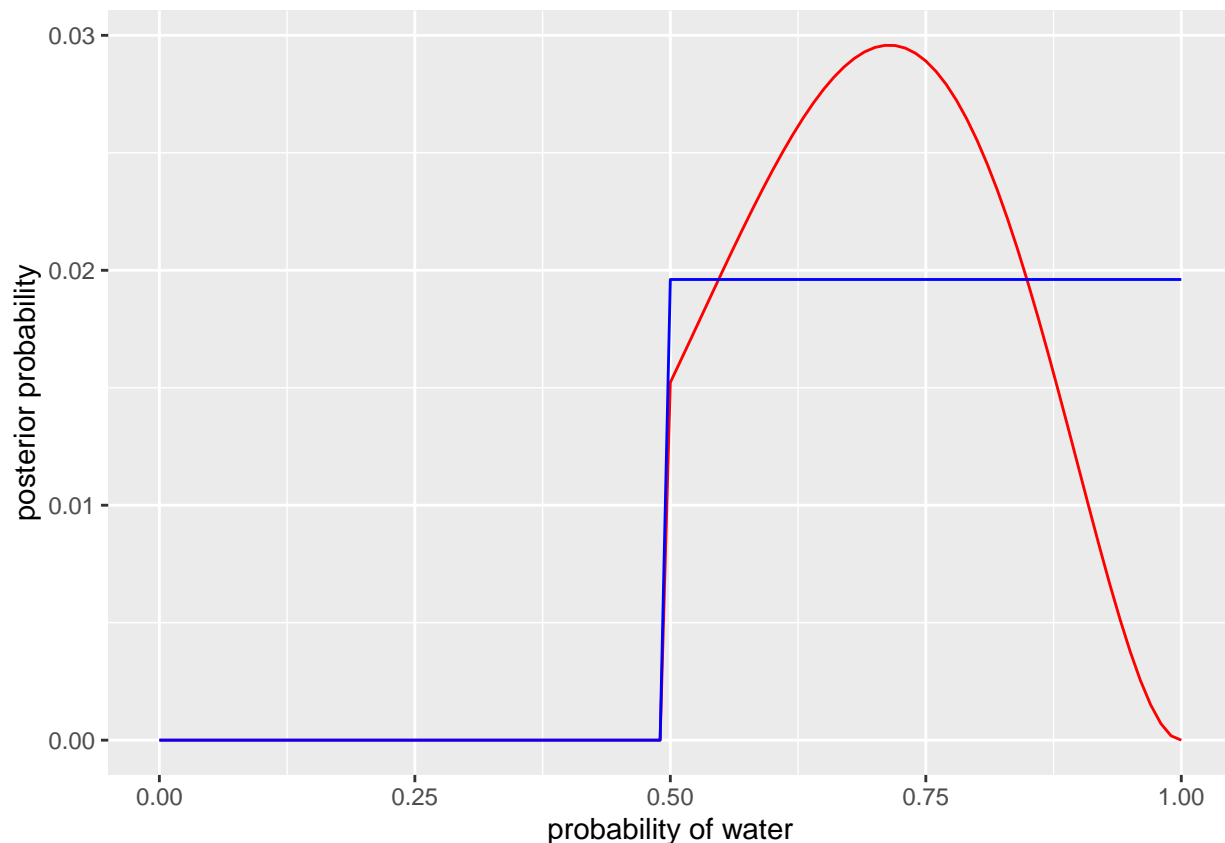
3) LWW LWWW

```
posterior_table5 =tibble(p_w_proposed = seq(from=0, to = 1, by = 0.01) )
posterior_table5 <- mutate(posterior_table5 , prior=ifelse(p_w_proposed < 0.5, 0, 2/(n()+1)))
sum(posterior_table5$prior)
```

```
## [1] 1
```

```
posterior_table5 <- mutate(posterior_table5 , likelihood=dbinom(5,size=7,prob=p_w_proposed))
posterior_table5 <- mutate(posterior_table5 , raw_posterior = prior*likelihood)
posterior_table5 <- mutate(posterior_table5 , posterior = raw_posterior/sum(raw_posterior))

ggplot(data=posterior_table5, aes(x=p_w_proposed, y=posterior))+
  geom_line(color="red")+
  geom_line(aes(x=p_w_proposed, y=prior) , color="blue")+
  labs(x = "probability of water", y = "posterior probability")
```



2M3

$$\Pr(\text{water}|\text{Earth}) = 70\% \quad \Pr(\text{land}|\text{Earth}) = 1-70\% = 30\%$$

$$\Pr(\text{land}|\text{Mars}) = 1 \quad \Pr(\text{Earth}) = \Pr(\text{Mars}) = 50\%$$

$$\Pr(\text{Earth}|\text{land}) = \Pr(\text{Earth and land}) / \Pr(\text{land}) = [\Pr(\text{land}|\text{Earth}) * \Pr(\text{Earth})] / [\Pr(\text{land}|\text{Mars}) * \Pr(\text{Mars}) + \Pr(\text{land}|\text{Earth}) * \Pr(\text{Earth})] = (0.3 * 0.5) / (0.3 * 0.5 + 1 * 0.5) = 3/13 = 0.23$$

2H1

$$\Pr(\text{twins}|\text{A}) = 0.1 \quad \Pr(\text{twins}|\text{B}) = 0.2$$

$$\Pr(\text{A})=0.5 \quad \Pr(\text{B})=0.5$$

$$\Pr(\text{twins and A}) = \Pr(\text{twins}|\text{A}) * \Pr(\text{A}) = 0.1 * 0.5 = 0.05 \quad \Pr(\text{twins and B}) = \Pr(\text{twins}|\text{B}) * \Pr(\text{B}) = 0.2 * 0.5 = 0.1$$

$$\Pr(\text{twins}) = \Pr(\text{twins and B}) + \Pr(\text{twins and A}) = 0.15$$

$$\Pr(\text{A}|\text{twins}) = \Pr(\text{twins}|\text{A}) * \Pr(\text{A}) / \Pr(\text{twins}) = 0.1 * 0.5 / 0.15 = 1/3 \quad \Pr(\text{B}|\text{twins}) = \Pr(\text{twins}|\text{B}) * \Pr(\text{B}) / \Pr(\text{twins}) = 0.2 * 0.5 / 0.15 = 2/3$$

$$\Pr(\text{Next_twins}) = \Pr(\text{A}|\text{twins}) * \Pr(\text{twins}|\text{A}) + \Pr(\text{B}|\text{twins}) * \Pr(\text{twins}|\text{B}) = 1/3 * 0.1 + 2/3 * 0.2 = 1/6$$