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N-body simulation of a Galaxy Cluster

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ABSTRACT

In this paper, the Barnes-Hut n-body approximation is used to evolve a galaxy cluster represented as a system of 908 point masses. In order to see a change in position on the order of a megaparsec the system must be evolved at least few hundred million years, if not a few billion. Evolving the system for 10 billion years, with a time step of 1 million years, results in a change in position of at least a few megaparsecs for most galaxies.

INTRODUCTION

A galaxy cluster is a relatively dense region of galaxies. Clusters are believed to be some of the largest objects bound by gravity in the universe. A galaxy cluster can contain anywhere from hundreds to thousands of galaxies, most of which lie in the dense core of the cluster. A poor cluster may have approximately 50 or more galaxies, while a rich cluster may have thousands of galaxies (Carroll & Ostlie (2018)). These clusters are typically dominated by redder elliptical galaxies, but may also contain spiral galaxies. Rich clusters can span as much as 6 Mpc or more and the galaxies in the cluster may have orbital speeds on the order of hundreds of km/s (Carroll & Ostlie (2018)). These two facts mean that any galaxy can have a crossing time, the time it take to cross the span of the cluster, on the order of billions of years. This means that the crossing time can as long as or greater than the age of the universe. The relaxation time is the amount of time it takes for the system to come to some sort of dynamical equilibrium and is important for deciding whether an n-body simulation should consider collisions or not. Clusters also have a large amount of hot intergalactic gas that emits light in the x-ray portion of the spectrum, but most of mass in a cluster is in the form of dark matter. Learning more about galaxy clusters can help us learn more about the basic structure of the universe (Carroll & Ostlie (2018)).

N-BODY SIMULATION

An n-body simulation can be used probe the dynamics of system composed of many particles. This usually involves using physical models, equations of motion, to evolve a system forward in time by calculating the forces on each particle due to every other particle and update their positions over time. For a gravitational system, such as a galaxy cluster, or a galaxy itself, this could involve calculating the force or acceleration of stars or galaxies using Newtonian gravity as seen in Equation 1.

$$\sum \vec{a} = \sum \vec{F}/m = \sum -\frac{Gm}{|\vec{r}|^2}\,\hat{r} \tag{1}$$

For an n-body simulation, one might use the method of direct summation, which involves calculating the force on one particle from every other particle and adding it up. When a system has many thousands of particles, this method can be quite slow even for efficient computers. A better method involves making some approximations that don't add too much error, but speed the process up significantly. One such method is the Barnes-Hut approximation.

BARNES-HUT N-BODY APPROXIMATION

Josh Barnes and Piet Hut developed a method to decrease the computational rigor of n-body simulations. The main advantage of the Barnes-Hut algorithm is that it approximates long range forces from the center of mass of many particles rather than calculating the force from each individually. The advantage come from doing fewer calculations than the number particles each iteration of the simulation. This is done by organizing the galaxies in a tree structure with nodes.

A tree is composed of individual nodes that represent regions of space. A cubic region is divided into 8 equally sized sub-cubes, which are children of the first parent node. This division is done until each galaxy is contained in single node, but no node has more than one galaxy in it. These final nodes are called leaves since they are the end of the tree.

In my method, I built the tree in python using three classes of objects: a tree, nodes, and galaxies. Then, from a list of galaxy objects, the nodes are created by dividing up each parent equally and taking into account whether a node actually has a galaxy in it. That being said, I did not create or keep track of empty nodes. The root node is a node encompassing the entire input space and is the first node in the tree. Once the tree is constructed, my method calculates the center of mass of each node, starting from the leaves and moving up the tree. This can be done since each node keeps track of its parent node and its own children.

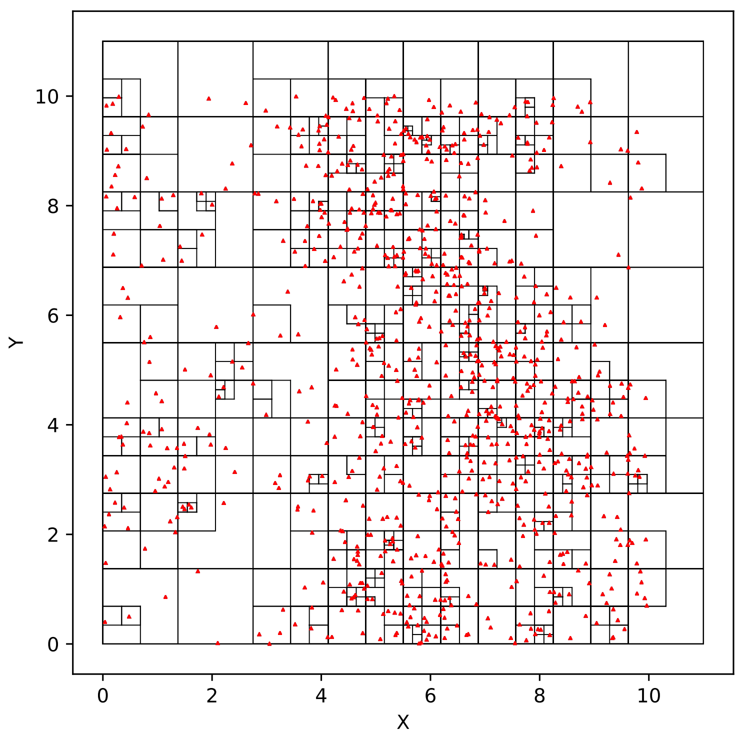

Figure 1. Plot of the whole tree, each individual box represents a node, while the points represent the galaxies. Some nodes appear to have multiple galaxies in the same node, but this is a result of projecting something in three dimensions into two. Those galaxies are in nodes with different ranges in the z-axis. The whole cube has side lengths of 11 Mpc. Some of the nodes become to small to distinguish in this plot.

Once all of the center of masses are calculated, the acceleration on each galaxy can be calculated. My method starts with a single galaxy and calculates the distance to each node starting at the root node. This is where the Barnes-Hut approximation comes in. For node size D, and distance between the galaxy and the node, r, if the value $\theta = D/r > 1$, then the acceleration on the galaxy is calculated using the node's center of mass, total mass within the node, and Equation 1. I used $\theta = 1$ as my limit, which means that if the node size is less than the separation distance, then the center of mass of a node is used.

Then I used the Verlet equation to update the galaxy position vector using the calculated acceleration vector. This is shown in Equation 2, where h is the time step in seconds, and $\vec{a}_i(\vec{r}_i)$ is the acceleration acting on the galaxy at the current time step, at position $\vec{r}_i$. In order to evolve the system, I used a simple loop, where each iteration is a new tree is constructed with the updated positions of the galaxies, some time step in the future.

$$\vec{r}_{i+1} = 2\vec{r}_i - \vec{r}_{i-1} + h^2\vec{a}_i(\vec{r}_i) \tag{2}$$

FORCE SOFTENING

To avoid problems with close encounters, I used force softening in the calculation for acceleration. Essentially, if the galaxies are within a certain close distance of each other, the acceleration calculation changes slightly reduce the force between the two. This is shown in Equation 3, where ϵ is sufficiently small that the denominator goes to $1/r^2$ for large r. I took ϵ to be 5 kpc, which is in the range of the typical effective radius of an elliptical galaxy.

$$\sum \vec{a} = \sum \vec{F}/m = \sum -\frac{Gm}{(|\vec{r}|^2 + \epsilon^2)}\,\hat{r} \tag{3}$$

RESULTS

In terms of results, my tree took the shape in Figure 1 for the initial positions of galaxies in the cluster. The nodes get to be pretty small where the galaxies have the highest density. Of the 908 galaxies, I plotted the trajectories of just 4 galaxies in Figure 2 to give a sense of what the trajectories look like as the cluster is evolved over 10 billion years. To give a sense of scale, the bounds of Figure 2 are a couple Mpc larger than the furthest extend of the cluster. In Figure 3, a zoomed in view of the galaxy trajectories is shown to get a more detailed view of the orbits. Two of the galaxies take interesting, but perhaps unphysical, paths through the cluster that seem to change direction and curve on somewhat short length scales. Figures 4 - 6 in the appendix show the approximate 2D projections of the galaxy trajectories.

CONCLUSION

Ultimately this is an interesting simulation, but it is quite basic since it doesn't take into account many observed aspect of cluster such as the intracluster medium, which accounts for a not-insignificant fraction of the mass in a cluster, or the dark matter mass of the system, which contributes a very large portion of the mass. Both of these components would bring the simulation closer to reality and could likely be modelled as a continuous mass distribution over the extent of the cluster. A truly full treatment of the simulation would likely take into account cosmological factors over the many Mpc distances as well

REFERENCES

Carroll, B. W., & Ostlie, D. A. 2018, An Introduction to Modern Astrophysics (Cambridge University Press)

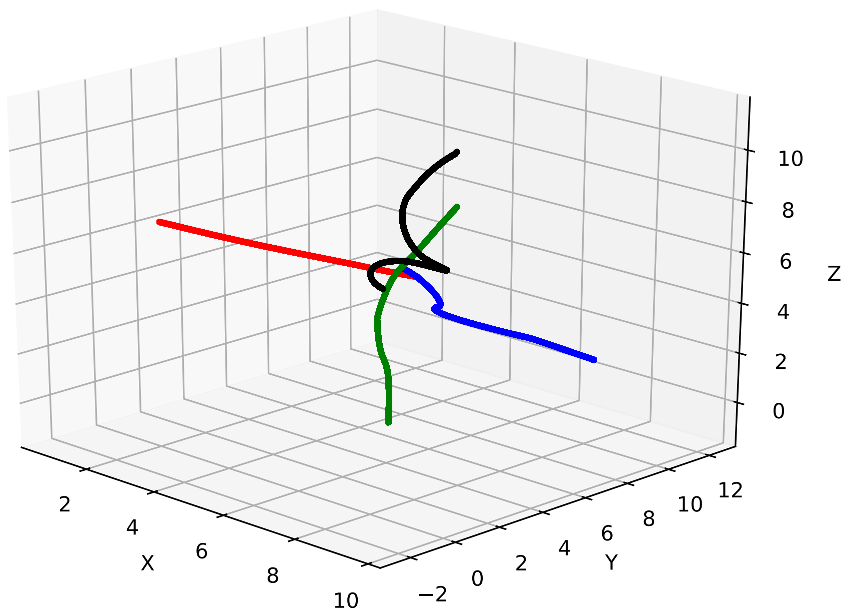

Figure 2. This figures shows a 3D view of the trajectories that 4 galaxies take through the cluster over the course of 10 billion years. The time step used for this plot is intervals of 1 million years.

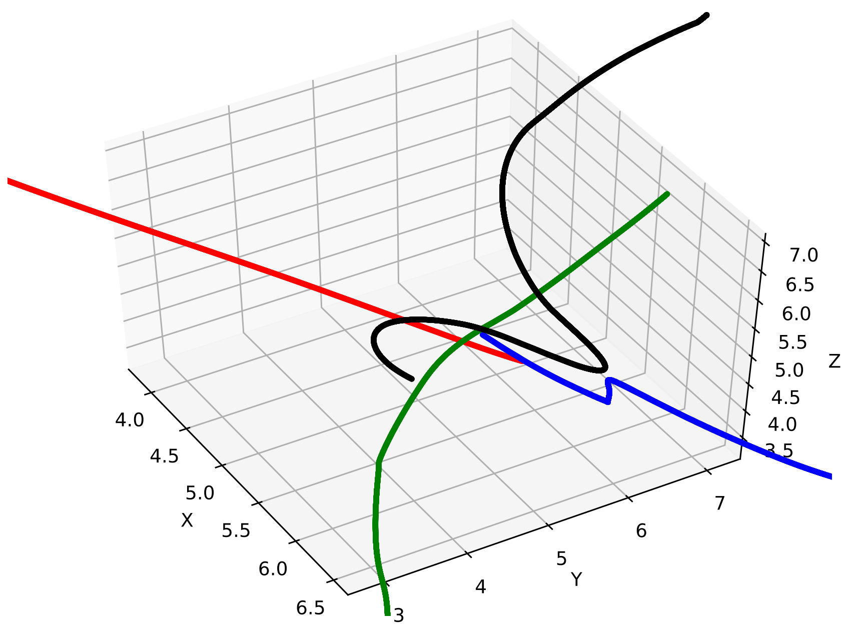

Figure 3. A closer view of the galaxy trajectories. Some of the paths change over quite small length scales that may be unrealistic.

APPENDIX

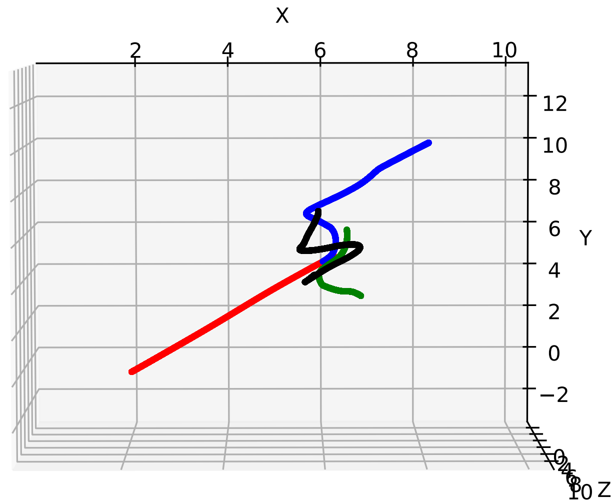

Figure 4. XY projection of the trajectories of 4 galaxies through the cluster over 10 billion years. 1 million year timesteps.

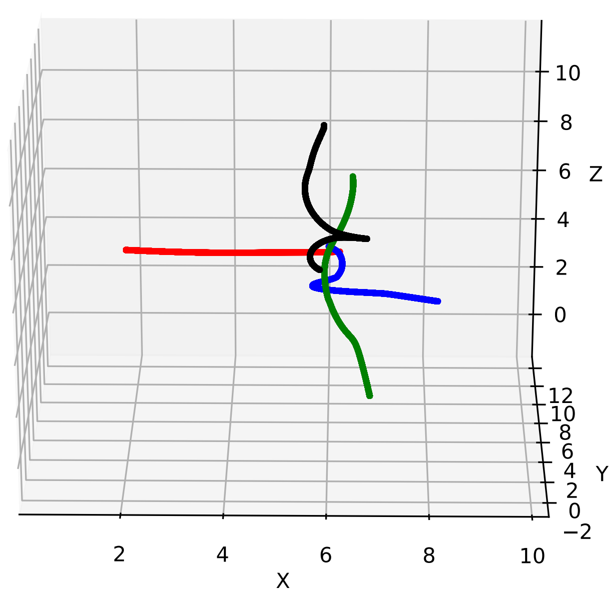

Figure 5. XZ projection of the trajectories of 4 galaxies through the cluster over 10 billion years. 1 million year timesteps.

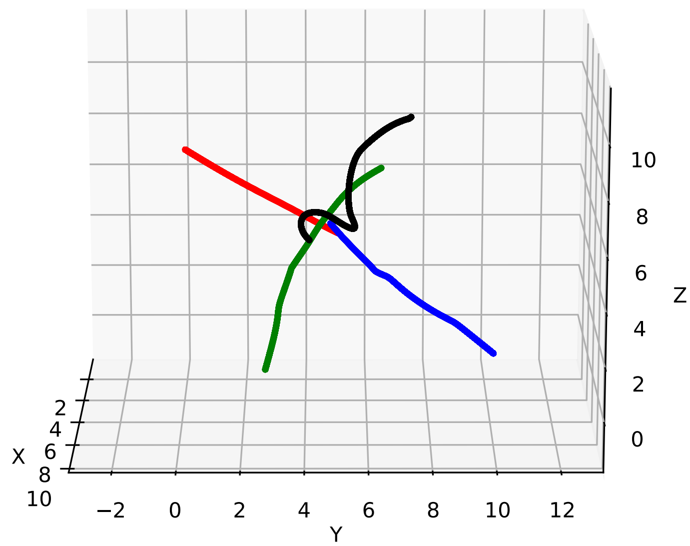

Figure 6. YZ projection of the trajectories of 4 galaxies through the cluster over 10 billion years. 1 million year timesteps.