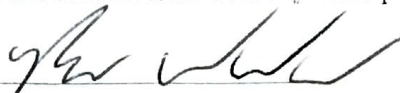


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- **OPTIONAL: Print this .pdf.** If you do not have easy access to a printer, that is ok, but it is easier for us instructors to have a consistent format when marking so if you can print it out please do so. There are 3 questions and 4 pages (including this page).
- For each problem, **write out a full solution.** Solutions should be clear, complete, and justified. *Final answers without supporting work will be graded as zero.*
- **This exam is individual.** Communicating with anybody else during the test is a strict violation of Academic Integrity. Posting the test on the internet is a violation not just of academic integrity but of Canadian copyright law.
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- Please **keep your exam for at least two weeks** in case we need you to rescan.
- Please **read and sign** the Confidentiality Agreement ¹ before solving the quiz

¹Confidentiality Agreement: I did not communicate with any other person or share this exam in any way. I followed all exam instructions.

SIGANUTURE: (MANDATORY)



1. [5 pts]

- Find a particular solution to

$$y_p(x) = -\frac{5}{4}e^{2x} + \frac{5}{4}xe^{2x}$$

$$y'' + y = 5xe^{2x} \quad (1)$$

$$r^2 + 1 = 0$$

$$r = \pm i$$

$$y = c_1 \cos x + c_2 \sin x$$

$$4B = 5$$

$$B = 5/4$$

$$4A + 4B = 0$$

$$4A + 4(5/4) = 0 \Rightarrow 4A + 5 = 0 \Rightarrow A = -5/4$$

- Use the above solution to solve the initial value problem

$$y'' + y = 5xe^{2x}, y(0) = 1, y'(0) = 2. \quad (2)$$

$$y(x) = c_1 \cos x + c_2 \sin x - \frac{5}{4}e^{2x} + \frac{5}{4}xe^{2x}$$

$$y(0) = 1 = c_1 + 0 + (-5/4) \Rightarrow 1 + 5/4 = c_1 \Rightarrow c_1 = 9/4$$

$$y'(x) = -c_1 \sin x + c_2 \cos x - \frac{5}{2}e^{2x} + \frac{5}{4}e^{2x} + \frac{5}{2}xe^{2x}$$

$$y'(0) = 2 = 0 + c_2 - \frac{5}{2} + \frac{5}{2} \Rightarrow c_2 = 2$$

$$2 = c_2 - 5/2 + 5/2$$

$$c_2 = 13/4$$

$$y(x) = \frac{9}{4} \cos x + \frac{13}{4} \sin x - \frac{5}{4}e^{2x} + \frac{5}{4}xe^{2x}$$

2. [5 pts] Find the value of the parameter $m \geq 0$ so that the solution to the forced ODE

$$\frac{d^2y}{dt^2} + m \frac{dy}{dt} + 25y = 10 \cos(5t)$$

will exhibit a **resonant** behaviour.

resonant.



$$y'' + my + 25y = 10 \cos(5t) \rightarrow R_0 = 10 \quad \omega = 5 = \omega_0$$

$k =$

$$x(t) =$$

3. [5 pts] Consider the differential equation

$$\sin xy'' - 2 \cos xy' - \sin xy = 0, \quad 0 < x < \pi. \quad (3)$$

- Verify that $y_1(x) = \sin x$ solves the above ODE.

$$\begin{aligned} y_1 &= \sin x \\ y_1' &= \cos x \\ y_1'' &= -\sin x \end{aligned}$$

$$\begin{aligned} \sin x (-\sin x) - 2 \cos x \cos x - \sin x \sin x &= 0 \\ -\sin^2 x - 2 \cos^2 x - \sin^2 x &= 0 \\ -2(\sin^2 x + \cos^2 x) &= 0 \\ -2 &\neq 0 \end{aligned}$$

$y_1(x) = \sin x$ doesn't solve the ODE

- By varying the parameter $u(x)$, find a second linearly independent solution $y_2(x) = u(x)y_1(x)$ of the differential equation (3).