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Math 101 – T04

Tutorial 9: 2b, 3b, 4b, 5b

2b:

$$(b) \lim_{x\to 0} \frac{3\tan^{-1}x - 3x + x^3}{x^5}$$

$$= \lim_{x\to 0} \frac{3\left[x - \frac{x^3}{3} + \frac{x^5}{5} - \dots\right] - 3x + x^3}{x^5}$$

$$= \lim_{x\to 0}\left[\left(\frac{-3x + x^3}{x^5}\right) + \left[\frac{3x}{x^5} - \frac{x^3}{3x^5} + \frac{x^5}{5} - \dots\right]\right]$$

$$= \lim_{x\to 0}\left[\left(\frac{3x - 3x}{x^5}\right) + \left(\frac{x^3}{x^5}\left(1 - \frac{1}{3}\right)\right) + \frac{x^5}{5} - \dots\right]$$

$$= \lim_{x\to 0} \frac{2x^3}{x^5} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$= \lim_{x\to 0} \frac{2}{x^2} + \frac{x^5}{5} - \frac{x^7}{7} + \dots$$

$$= \lim_{x\to 0} \frac{2}{x^2} + \lim_{x\to 0}\left[\frac{x^5}{5} - \frac{x^7}{7} + \dots\right]$$

$$= \lim_{x\to 0} \frac{2}{x^2} + 0$$

$$\lim_{x\to 0^-} \frac{2}{x^2} = +\infty \qquad \lim_{x\to 0^+} \frac{2}{x^2} = +\infty$$

$\therefore$ Therefore since both approaching from left and right are both $+\infty$, we can say that

$$\boxed{\lim_{x\to 0} \frac{3\tan^{-1}x - 3x + x^3}{x^5} = +\infty}$$

3b:

(b) $(1+5x^2)^{-1/5} = (1+(5x^2))^{-1/5}$

$$\approx 1 + \left(\frac{-1}{5}\right)(5x^2) + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)(5x^2)^2}{2!} + \frac{\left(\frac{-1}{5}\right)\left(\frac{-6}{5}\right)\left(\frac{-11}{5}\right)(5x^2)^3}{3!}$$

$$= 1 - \frac{5x^2}{5} + \frac{6\cdot 5^2 x^4}{5^2\cdot 2} - \frac{66\cdot 5^3\cdot x^6}{5^3\cdot 6}$$

$$\boxed{= 1 - x^2 + 3x^4 - 11x^6}$$

4b:

(b) $1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \frac{x^4}{5} + \dots$

$$= \left(\frac{1}{1}\right)x^0 + \left(\frac{1}{2}\right)x^1 + \left(\frac{1}{3}\right)x^2 + \left(\frac{1}{4}\right)x^3 + \dots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n+1}$$

$$x + \frac{x^2}{2} + \frac{x^3}{3} + \dots + = \sum_{n=0}^{\infty} \frac{x^{n+1}}{n+1} = \sum_{n=1}^{\infty} \frac{x^n}{n}$$

$$-\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \dots +\right) = -\sum_{n=1}^{\infty} \frac{x^n}{n} = \ln(1-x)$$

$$-x\left(1 + \frac{x}{2} + \frac{x^2}{3} + \dots +\right) = \ln(1-x)$$

$$\boxed{1 + \frac{x}{2} + \frac{x^2}{3} + \frac{x^3}{4} + \frac{x^4}{5} = -\frac{\ln(1-x)}{x}}$$

5b:

(b) $\sum_{n=1}^{\infty} n e^{-2n}$

1. series is always positive
2. $(n e^{-2n})' = (1) e^{-2n} + n(e^{-2n}) \cdot (-2)$

$$= e^{-2n} + -2n e^{-2n} = e^{-2n}(1 + -2n)$$

↳ Since $n \geq 1$ e^{-2n} is positive

$1+2n$ is negative

making the function $(n e^{-2n})'$ negative

meaning that

$\sum_{n=1}^{\infty} n e^{-2n}$ is decreasing

3. $\sum_{n=1}^{\infty} n e^{-2n}$ is continuous.

· Integral Test $\int_1^{\infty} n e^{-2n}$

$$= \lim_{t\to\infty} \int_1^t \frac{n}{e^{-2n}}$$

let $u = x$ $\qquad v = -\frac{1}{2} e^{-2x}$

$du = dx$ $\qquad dv = e^{-2x}$

$$\lim_{t\to\infty} \left[x\left(\frac{-1}{2}\right)e^{-2x}\right]_1^t - \int_1^t -\frac{1}{2} e^{-2x}\,dx$$

$$\lim_{t\to\infty} \left[\frac{-1}{2} x e^{-2x}\right]_1^t + \frac{1}{2}\int_1^t e^{-2x}\,dx$$

$$= \lim_{t\to\infty} \left[\frac{-x}{2e^{2x}}\right]_1^t + \left[\frac{1}{2}\left(\frac{-1}{2}\right)e^{-2x}\right]_1^t$$

$$= \lim_{t\to\infty} \frac{-x}{2e^{2x}}\Big|_1^t - \frac{1}{4e^{2x}}\Big|_1^t$$

$$= \lim_{t\to\infty} \left[\left(\frac{-t}{2e^{2t}} + \frac{1}{2e^{2}}\right) - \left(\frac{1}{4e^{2t}} - \frac{1}{4e^{2}}\right)\right]$$

$$= \lim_{t\to\infty} \left[\frac{-t}{e^{2t}} + \frac{1}{2e^{2}} - \frac{1}{4e^{2t}} + \frac{1}{4e^{2}}\right]$$

$$= \lim_{t\to\infty} \frac{-t}{2e^{2t}} + \left[0 + \frac{1}{2e^{2}} - 0 + \frac{1}{4e^{2}}\right]$$

LH

$$= \lim_{t \to \infty} \frac{-1}{4e^{2t}} + \frac{3}{4e^2} \boxed{= \frac{3}{4e^2}} \approx 0.1015014624$$

- The series is confirmed to converge by the Integral Test.

$$S_3 = (1)e^{-2(1)} + (2)e^{-2(2)} + (3)e^{-2(3)}$$

$$\approx 0.17940028175$$

$$|R_3| \leq u_4 = (4)e^{-2(4)} \approx 0.001341850512$$

$$\boxed{\therefore \sum_{n=1}^{\infty} ne^{-2n} \approx 0.1794028175 \text{ with a maximum error of } 0.001341850512}$$