Assignment 1 – Written

CSC 225

Feb 3rd, 2017

Ryan Woodward

V00857268

1. Order the following functions by order of growth starting with the slowest.

- 5
- $(\log n)^5$
- $n^{0.1}$
- $4^{\lg n}$
- $5n$
- $2n \log(\log n)$
- n^5
- 5^n
- $n!$
- 2^{2^n}

2. Since $\log(a) + \log(b) = \log(ab)$ we see that

$$\sum_{i=1}^{n} \log i = \log(1) + \log(2) + \dots + \log(n)$$

$$= \log(1 \cdot 2 \cdot 3 \cdot \dots \cdot n)$$

$$= \log(n!)$$

$$\log(n!) = \log(1) + \log(2) + \dots + \log(n)$$

Case 1: $\geq$

$$\log(n) + \log(n) + \dots + \log(n) \geq \log(1) + \log(2) + \dots + \log(n)$$

$$n \log n \geq \log(n!)$$

Case 2: $\leq$

$$\lg(n/2) + \dots + \lg(n) \leq \lg(1) + \lg(2) + \dots + \lg(n)$$

$$\lg(n/2) + \dots + \lg(n/2) \leq \lg(1) + \lg(2) + \dots + \log(n)$$

$$\frac{n}{2} \log\left(\frac{n}{2}\right) \leq \log(n!)$$

$\therefore f(n) = n \log n$ and it follows

that $S(n)$ sum $= \Theta(f(n))$.

3. a) int sum = 0;

```
for (int n=N; n>0; n/=2)      will run log n times
   for (int i=0; i<n; i++)       "     n/2 times
      sum ++
```

$= O(N \log N)$

b) int sum = 0;

```
for (int i=1; i<N; i *=2)
   for (int j = 0; j<i; j++)  ← will run log N/2 times
```

$= O(N \log N)$

c) int sum = 0;

```
for (int i=1; i<N; i*=2)
   for (int j =0; j<N; j++)
      sum ++
```

$= O(N \log N)$

4. $$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \text{for all } n \geq 1$$

Base Case: $n = 1$

$$\frac{1}{1(1+1)} = \frac{1}{2} \qquad \frac{n}{n+1} = \frac{1}{1+1} = \frac{1}{2} \checkmark$$

I.H: $$\sum_{i=1}^{n+1} \frac{1}{i(i+1)} = \frac{(n+1)}{((n+1)+1)}$$

I.S.: $$\sum_{i=1}^{n+1} \frac{1}{i(i+1)}$$

$$= \sum_{i=1}^{n} \frac{1}{i(i+1)} + \frac{1}{(n+1)((n+1)+1)}$$

$$= \frac{n}{n+1} + \frac{1}{(n+1)(n+2)} = \frac{n(n+2)+1}{(n+1)(n+2)}$$

$$= \frac{n^2+2n+1}{(n+1)(n+2)} = \frac{(n+1)(n+1)}{(n+1)(n+2)} = \frac{(n+1)}{(n+2)}$$

$$= \frac{(n+1)}{((n+1)+1)}$$

Conclusion: so it's true that

$$\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1} \quad \text{for all } n \geq 1$$

5. Algorithm: FindUniqueNumber

Input: An $n-1$ element array A of integers

Output: An integer from the range $[0, n]$ that is missing from the input array A.

Let A be an array of n numbers.

```
sum <- 0
for i<-0 to n-1 do
    sum ≠ sum + A[i]
end
missingNumber <- (n(n+1)/2) - sum

return missingNumber
```