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Math 101 – T04

<u>Tutorial 11: 1c, 2b, 3b, 4b</u>

1c:

(c) $x = 1 - t^2$, $y = t\sqrt{2-t^2}$ $(-\sqrt{2} \le t \le \sqrt{2})$

→ $x+1 = 2-t^2$ $y^2 = t^2(2-t^2)$

$x + t^2 = -t^2$ $y^2 = (1-x)(x+1)$

→ $1+x = t^2$

$$\boxed{y = \pm\sqrt{(1-x)(x+1)}}$$

test 1: $y = -\sqrt{(1-x)(x+1)}$; $t = -\sqrt{2}$

$x = 1 - (-\sqrt{2})^2$ $y = -\sqrt{(1-x)(x+1)}$

$x = 1 - 2$ $= -\sqrt{0}$

$= -1$ $= 0$

$y = \sqrt{(1-x)(x+1)}$ $t = -\sqrt{2}$

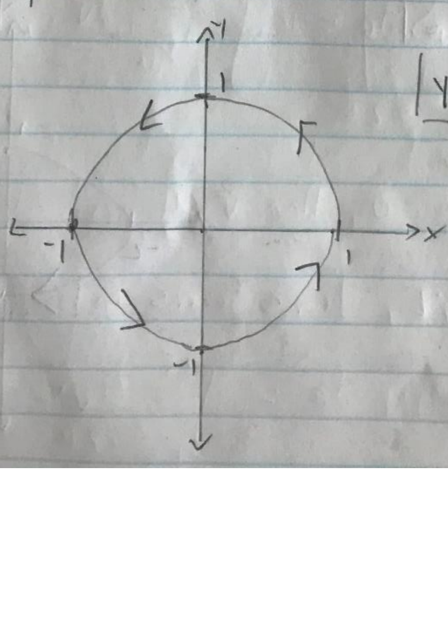
$x = -1$ $y = 0$

test 2: $y = -\sqrt{(1-x)(x+1)}$; $t = 0$

$x = 1$, $y = 0$ → same as $+\sqrt{(1-x)(x+1)}$

test 3: $t = -1$

$x = 0$, $y = -1$



$$\boxed{y^2 = (1-x)(x+1)}$$

2b:

(b) $x = \sin(2t)$, $y = \sin(t)$

• $\frac{dx}{dt} = \cos(2t) \cdot 2 = 2\cos(2t)$

• $\frac{dy}{dt} = \cos(t)$

• $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{\cos(t)}{2\cos(2t)}$

(i) • Horizontal tangent when $\frac{dy}{dt} = 0$; $\cos(t) = 0 \Rightarrow t = \pm \pi/2$

$x\left(\frac{-\pi}{2}\right) = \sin(-\pi) = 0$

$y\left(\frac{-\pi}{2}\right) = \sin\left(\frac{-\pi}{2}\right) = -1$

$\boxed{\text{• Point} = (0,-1)}$

$x\left(\frac{\pi}{2}\right) = \sin(\pi) = 0$

$y\left(\frac{\pi}{2}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$\boxed{\text{• Point} = (0,1)}$

(ii) • Vertical tangent when $\frac{dy}{dt}$ DNE ; $2\cos(2t) = 0 \Rightarrow \pm \pi/4$

$x\left(\frac{-\pi}{4}\right) = \sin\left(\frac{-\pi}{2}\right) = -1$

$y\left(\frac{-\pi}{4}\right) = \sin\left(-\frac{\pi}{4}\right) = \frac{-\sqrt{2}}{2}$

$\boxed{\text{• Point} = (-1, -\sqrt{2}/2)}$

$x = \left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{2}\right) = 1$

$y = \left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \sqrt{2}/2$

$\boxed{\text{• Point} = (1, \sqrt{2}/2)}$

3b:

(d) $x = \sin^2 t$, $y = 2\cos t$, $(0 \le t \le \pi/2)$

$\frac{dx}{dt} = 2\sin t \cos t$; $\frac{dy}{dt} = -2\sin t$

$$L = \int_0^{\pi/2} \sqrt{(2\sin t \cos t)^2 + (-2\sin t)^2}\, dt$$

$$L = \int_0^{\pi/2} \sqrt{4\sin^2 t \cos^2 t + 4\sin^2 t}\, dt$$

$$= \int_0^{\pi/2} \sqrt{4\sin^2 t\,(\cos^2 t + 1)}\, dt$$ Note sin

$$= \int_0^{\pi/2} 2\sin t \sqrt{\cos^2 t + 1}\, dt$$ • let $u = \cos t$, $du = -\sin t\, dt$

$$= -2\int_a^b \sqrt{u^2+1}\, dt$$ • let $u = \tan v$, $du = \sec^2 v \cdot dv$

$$= -2\int_a^b \sec^2 v \sqrt{\tan^2 v + 1}\, dt$$ • $\tan^2 v + 1 = \sec^2 v$

$$= -2\int_a^b \sec^2 v \sqrt{\sec^2 v}\, dt$$

$$= -2\int_a^b \sec^3 v\, dt$$

Note: $\int \sec^n v\, dv = \frac{n-2}{n-1}\int \sec^{n-2} v\, dv + \frac{\sec^{n-2} v \tan v}{n-1}$

• with $n=3$ $= \frac{1}{2}\int \sec v\, dv + \frac{\sec v \tan v}{2}$

$$= -2\frac{\sec v \tan v}{2} + \frac{-2}{2}\int \sec v\, dv$$

$$= -\sec v \tan v - \int \sec v\, dv$$

$$= -\sec v \tan v - \ln(\tan v + \sec v)$$

- Undo substitution so $v = \tan^{-1} u$

$\rightarrow \tan(\tan^{-1} u) = u \quad , \quad \sec(\tan^{-1} u) = \sqrt{u^2+1}$

$$= -\sqrt{u^2+1} \cdot u - \ln\left(u + \sqrt{u^2+1}\right)$$

$$= \left[-\sqrt{\cos^2 t+1} \cdot \cos t - \ln\left(\cos t + \sqrt{\cos^2 t+1}\right)\right]^{\pi/2}$$

$$= \left[-\sqrt{0+1} \cdot 0 - \ln(0+1)\right] - \left[-\sqrt{1+1} \cdot 1 - \ln\left(1+\sqrt{2}\right)\right]$$

$$= -\left[-\sqrt{2} - \ln(1+\sqrt{2})\right] \quad \boxed{= \sqrt{2} + \ln(1+\sqrt{2})}$$

4b:

(b) $r = \sec\theta \tan\theta$ $\qquad x = r\cos\theta \Rightarrow x/r = \cos\theta$

$r = \frac{1}{\cos\theta}\cdot\frac{\sin\theta}{\cos\theta}$ $\qquad y = r\sin\theta \Rightarrow y/r = \sin\theta$

$r = \frac{(y/r)}{(x/r)^2} \Rightarrow r = \frac{y}{r}\cdot\frac{r^2}{x^2} \Rightarrow r = \frac{ry}{x^2}$

$\boxed{y = x^2}$