CSC 320 Final

1. S -> TU | UV | TW
   T -> TT | UT | WT | t | u
   U -> TU | VU | u
   V -> VW | r
   W -> w

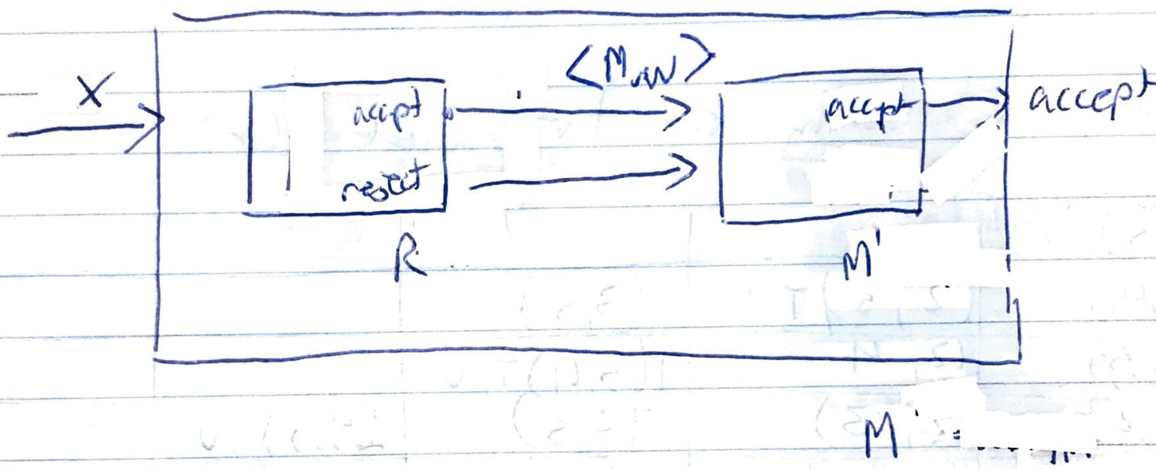
. Determine string s = trwuu ∈ L(G)

S, S, T, U, U, T				
(S, U	(U, U, T)			
∅	U	T, T, S, T, U		
∅	V	T	S, T, T, U	
T	V	W	T, U	T, U
t	r	w	u	u

a. T
b. V
c. W
d. T, U
e. T, U
f. empty
g. V
h. T
i. S, T, U
j. empty
k. U
l. S, T, U
m. S, U
n. T, U
p. S, T, U

2. (a) Give a mapping reducibility from A_{TM} to ALL_{TM}. Prove $A_{TM} \leq_m ALL_{TM}$

- Let us create the TM M' for this purpose.



M' = " On input x
1) Run x on R
- if x = w then accept
- if x ≠ w then accept

2) Run the output from R on M' as <M,w>
- if R accepted and M' halts then accept
- if R rejected and M' halts then accept
- else loop and thus reject <M,w>

3) If M' accepts then M accepts "

∴ Thus we have proven that if the string x is recognized as any string, then ALL_{TM} will accept any string. The results of this can be used to determine A_{TM} proving mapping reducibility.

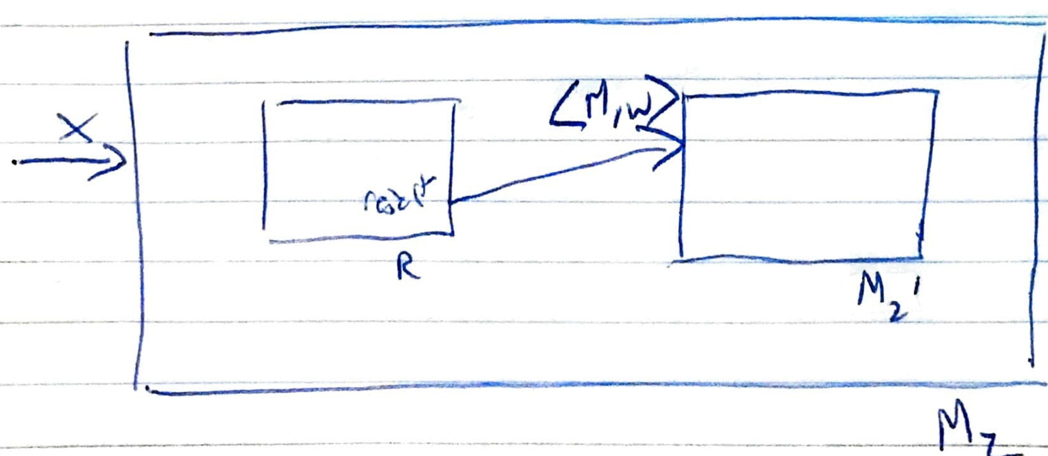
b) ALL_{TM} is not co-turing recognizable because there exists a case where we would be given a string in part a) that would loop on the verifier.

- Because R would have to have to accept all strings since Σ^* is the set of all languages including the empty string.

- However since M' is a recognizer for M we see that there is no point at which M' will recognize the input for $\overline{ALL_{TM}}$ as the recognizer sees and accepts all languages.

$\therefore$ Therefore since M' doesn't recognize that input, we can say that ALL_{TM} is not co-turing recognizable since $\overline{ALL_{TM}}$ isn't recognized.

c)



" On input x
1) Run x on R
if $x = w$ then reject
if $x \neq w$ then reject

2) Run the output from R on M_2' as $\langle M_2, w \rangle$
- If R accepted and M_2' halts then accept
  If R rejected then M_2' halts then accept
  else M_2' loops and thus rejects $\langle M_2, w \rangle$

3) If M_2' accepts then M_2 accepts.

$\therefore$ Thus we have shown a recognizer that has reduced A_{TM} to $\overline{ALL_{TM}}$. On input x we have shown that we can come to a conclusion for $\overline{ALL_{TM}}$

d) However b) and c) are a contradiction. If $\overline{ALL_{TM}}$ is recognizable in c) then by definition it would be co-recognizable in b).

- Thus by contradiction ALL_{TM} is not co-Turing recognizable.

3a) Prove that VERTEX-COVER $\in$ NP

1) Prove it is NP
2) Prove all NP problems are polynomial time reducible.

1) We use 3SAT reduced to VERTEX-COVER.

↳ Since we know the vertex cover has 3 nodes each.

↳ for vertex cover we know each edge is (u, v) where at least one of u, v is $\in E$.

↳ So a node can be x and $\bar{x}$ for true and false.

↳ we see by theorem 7.44 that $k = m + 2l$ and we can build ϕ which is a boolean formula that can be mapped to a graph G and a value k.

↳ Now for vertex cover since an edge touches each node we can say that ϕ can be reduced to G and k.

↳ Since ϕ has been reduced to a 3cnf formula which is NP we can say that VERTEX COVER $\in$ NP.