Math 101 Short Nswer Section

Short Answer Math 101 Final

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1. $\displaystyle\int_1^{\infty} \frac{1}{(1+x^{11})^{1/4}}\,dx$

· Comparison test $\rightarrow$ $0 \le a_n \le b_n$

$\displaystyle \rightarrow \frac{1}{(1+x^{11})^{1/4}} \le \frac{1}{x^{11/4}} = b_n$

$\displaystyle \lim_{t\to\infty} \int_1^t x^{-11/4} = \lim_{t\to\infty} \frac{-15}{4} x^{-15/4} \Big]_1^t$

$\displaystyle = \lim_{t\to\infty} \frac{-15}{4}\left[\frac{1}{t^{-15/4}} - \frac{1}{1}\right] = \frac{-15}{4}\left[0-1\right] = \frac{15}{4}$

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2. $\displaystyle\int_0^{5/2} \frac{dx}{\sqrt{25-x^2}} = \int_0^{5/2} \frac{5\cos\theta\,d\theta}{\sqrt{25-25\sin^2\theta}}$

$\displaystyle = \int_0^{5/2} \frac{5}{5}\,\frac{\cos\theta}{\sqrt{1-\sin^2\theta}} = \int_0^{5/2} \frac{\cos\theta\,d\theta}{\sqrt{\cos^2\theta}} = \int_0^{5/2} \frac{\cos\theta\,d\theta}{\cos\theta}$

$x = 5\sin\theta$

$\frac{5}{2} = 5\sin\theta$

$\frac{1}{2} = \sin\theta$

$\displaystyle = \int_0^{\pi/6} d\theta$

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3. $\int 2x \sin(x^2)\, dx$

let $u = x^4$ $v = x^2$

$du = 2x\, dx$ $dv = 2x$

$= x^2 x^3 - \int 4x^2 \cos(x^2)$

$u = x$ v

$du = dx$ $dv = \sin(x^2)$

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4. $\int x^n \cdot \ln|x| \, dx = x^{n+1}\left[\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right] + C$

$, n \neq -1$

$u = \ln|x|$ $\qquad$ $v = \frac{x^{n+1}}{n+1}$

$du = \frac{1}{x} \, dx$ $\qquad$ $dv = x^n \, dx$

$uv = \frac{\ln|x| \cdot x^{n+1}}{n+1}$

$\int v \, du = \int \frac{x^{n+1}}{x(n+1)} \, dx = \int \frac{x^n}{(n+1)} \, dx$

$= \frac{1}{n+1} \int x^n \, dx = \frac{1}{n+1}\left[\frac{x^{n+1}}{n+1}\right] + C$

$= \frac{\ln|x| \cdot x^{n+1}}{n+1} - \frac{1}{n+1}\left[\frac{x^{n+1}}{n+1}\right] + C$

$= x^{n+1}\left[\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right]$

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5. - 9% original remained
   - Half life carbon is 5731 years
   -

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6 Expand by partial

$$\frac{x+8}{(x+5)^2} = \frac{A}{(x+5)} + \frac{B}{(x+5)^2}$$

$$\frac{x+8}{\cancel{(x+5)^2}} \cdot (x+5)^2 = \frac{A(x+5)^{\cancel{2}}}{\cancel{(x+5)}} + \frac{B(x+5)^2}{(x+5)^2}$$

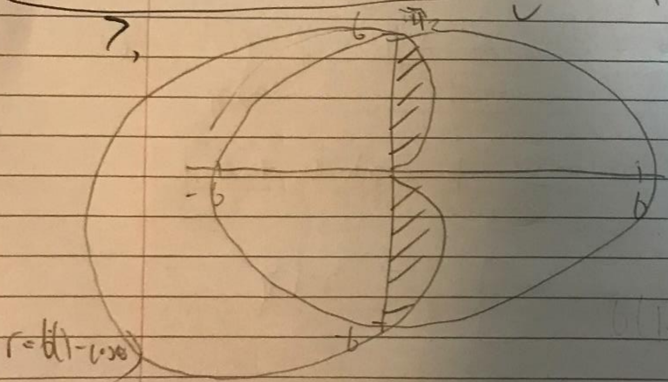
$$x+8 = A(x+5) + B$$

$$x+8 = Ax + 5A + B$$

- $A = 1$
- $5A + B = 8$
  - $\Rightarrow 5 + B = 8$
  - $B = 3$

$$\frac{x+8}{(x+5)^2} = \frac{1}{(x+5)} + \frac{3}{(x+5)^2}$$

7, $r = 6$,



$= 6 = 6(1-\cos\theta)$
$1 = 1 - \cos\theta$
$\cos\theta = 0$

$6(1-\cos\theta)^2$

$$A = \int_{\alpha}^{\beta} \frac{1}{2}\left(r_2^2 - r_1^2\right) d\theta$$

$$= \frac{1}{2}\int_0^{\pi} 1 - 2\cos\theta + \cos^2\theta - 36$$

$$= \frac{1}{2}\int_0^{\pi} \cos^2\theta - 2\cos\theta - 35$$

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$$= \cancel{\frac{1}{2}\left[\int_0^{\pi} \cos^2\theta - 2\cos\theta - 35 \, d\theta + \int_{\pi}^{2\pi} \cos^2\theta - 2\cos\theta - 35\right] d\theta}$$

$$\cancel{= \frac{1}{2}}$$

$$= \int_0^{\pi} \frac{1}{2}\left(6(1-\cos\theta)\right)^2 - 6^2 \, d\theta$$

$$= \int_0^{\pi} \frac{1}{2}\left(36(1-\cos\theta)^2 - 36\right) d\theta$$

$$= \frac{1}{2} \, 36 \int_0^{\pi} (1-\cos\theta)^2$$

$$= 18 \int_0^{\pi} (\cos^2\theta - 2\cos\theta + 1) \quad + \int_{\pi}^{2\pi} \quad (1)$$

$$= 18\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} + 2\sin\theta + \theta\right]$$

$$= 36 \cdot 2\left[\frac{3\pi}{2} + \frac{0}{2} + 2 \cdot 0\right] = 54\pi$$

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8. $y = 2x^{3/2}$ from $x=0$ to $x=5$.

$$L = \int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2}$$

$$= \int_0^5 \sqrt{\left(3x^{1/2}\right)}$$

$$= \int_0^5 (3x)^{1/4}$$

$$= \frac{3x^{5/4}}{1/4} = 12x^{5/4} \Big|_0^5$$

9. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ overestimate?

$$= \frac{-1}{1^4} + \frac{1}{2^4} - \frac{1}{3^4} + \text{------} + \frac{1}{100^4}$$

$\lim_{n\to\infty} \frac{(-1)^n}{+} = 0$ so converge?

= it will converge to 0 eventually so this can't be an overestimate.

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10. Taylor Polynomial

$f(x) = \frac{1}{x+4}$ $\qquad$ $f(0) = \frac{1}{4}$

$f'(x) = -(x+4)^{-2}$ $\qquad$ $f'(0) = \frac{1}{4^2}$

$f''(x) = -2(x+4)^{-3}$ $\qquad$ $f''(0) = \frac{-2}{4^3}$

$f'''(x) = 6(x+4)^{-4}$ $\qquad$ $f'''(0) = \frac{6}{4^4}$

<u>Math 101 Long Answer Section</u>

Math 101 Final - Long Answers

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Q1: $\int_2^{2\sqrt{2}} \sqrt{x^2-4}\ dx$

$= \text{let } x = 2\sec t \quad dt$

remember
$\sec^2 = \tan^2\theta - 1$

$\int_2^{2\sqrt{2}} 2\sec t\tan t \sqrt{(2\sec t)^2 - 4}\ dt$

$dx = 2\sec t\tan t \quad \cos^2 t + \sin^2 t = 1$

$= \int_2^{2\sqrt{2}} 2\sec t\tan t \sqrt{4\sec^2 t - 4}$

$= \int_2^{2\sqrt{2}} 2\sqrt{4}\sec t\tan t \sqrt{\sec^2 t - 1}\quad dt$

$= 4 \int_2^{2\sqrt{2}} \sec t\tan t \sqrt{\tan^2 t}\quad dt$

$= 4 \int_2^{2\sqrt{2}} \sec t\tan^2 t\quad dt$

$= 4 \int_2^{2\sqrt{2}} \sec t(\sec^2 t - 1)\quad dt$

1

$= 4 \int_2^{2\sqrt{2}} \sec^3 t + \sec t\quad dt$

$= 4 \int$ let

Math 101 - Final - Long Answer

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2. $\sum_{n=0}^{\infty} a_n x^n \qquad a_n = e^{an^2+bn}$

root test

$\lim_{n\to\infty} |a_n|^{1/n} = \lim_{n\to\infty} \left| e^{an^2/n} \cdot e^{bn/n} \right| \cdot x^{n/n}$

$= \lim_{n\to\infty} \left| e^{an} \cdot e^{b} \cdot x \right|$

· Convergent if $\lim_{n\to\infty} \left| e^{an+b} \right| x < 1$ if converges.

1) a if $an > 0$ $< \ln 1$

$an+b < 0$

then $\lim_{n\to\infty}$ can't converge since

e^{an+b} will always be greater than x.

2) if $a = 0$

$\lim_{n\to\infty} \left| e^{0+b} \right| = e^{b} < 1$ so as long

as $e^{b} \cdot x < 1 \quad = \left(-\infty, \frac{1}{e^{b}}\right)$

3) if $a < 0$

$\lim_{n\to\infty} \left| e^{-n+b} \right| \quad e^{-n+b}\, x^n < 1$

$=$

Final Long Answer Question 3

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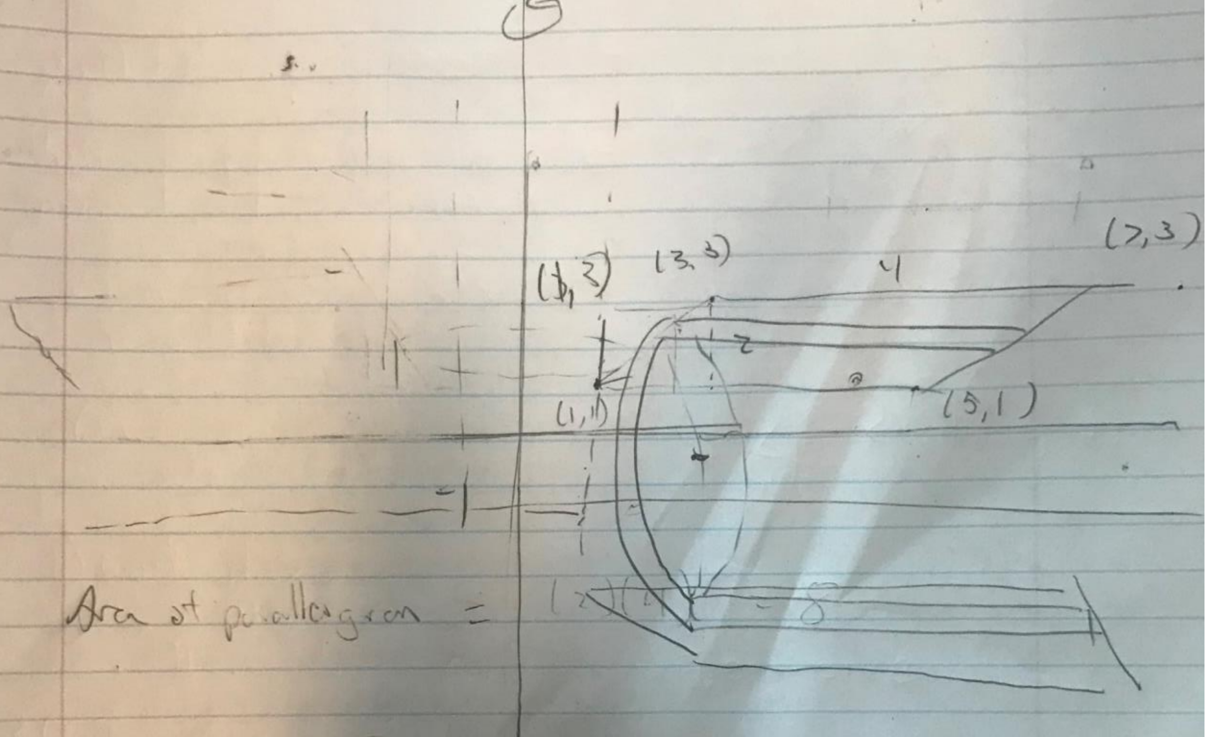
3. $(1,1), (5,1), (3,3), (7,3)$ about line $y=-1$



Area of parallelogram $= (2)(4) = 8$

radius $= 1 + y = 3$ height $= 4$

$$V = \int_1^3 2\pi r h \, dy = 2\pi \int_1^3 (4) + 4y \, dy$$

$$= 8\pi \int_1^3 4y \, dy = 8\pi \left[2y^2\right]_1^3$$

$$= 16\pi \left[3^2 - 1^2\right] = 16\pi \cdot 8 = 128\pi$$

Math 101 - Long Answer - Q4

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4. $y = t - 4/t \qquad x = t + 4/t$

$\frac{dy}{dt} = 1 + \frac{4}{t^2} \qquad \frac{dx}{dt} = 1 - \frac{4}{t^2}$

a) $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1 + 4/t^2}{1 - 4/t^2} = \frac{\frac{t^2+4}{t^2}}{\frac{t^2-4}{t^2}} = \frac{t^2+4}{t^2} \cdot \frac{t^2}{t^2-4}$

$\boxed{\frac{dy}{dx} = \frac{t^2+4}{t^2-4}}$

1) Horizontal tangent if $t^2+4 = 0$

$\boxed{\text{↳ these don't exist}}$

2) Vertical tangents if $t^2-4 = 0$

$\boxed{\text{↳ vertical tangents at } t = \pm 2}$

b) $\frac{d^2y}{dx^2} = \frac{\left(\frac{dy}{dx} \cdot \frac{d}{dt}\right)}{dx/dt} = \frac{\frac{(2t)(t^2-4) - (2t)(t^2+4)}{(t^2-4)^2}}{t^2 - 4/t^2}$

$\boxed{\frac{d^2y}{dx^2} = \frac{2t^3 - 8t - 2t^3 + 8t}{(t^2-4)^3} \cdot t^2 = \frac{-16t^3}{(t^2-4)^3}}$

ve intersects t-axis,
limit of integration is 0.

concave up when if $\frac{d^2y}{dt^2} > 0$

- $\frac{-16t^3}{(t^2-4)^3} > 0$

↳ If $t > 2$ then function remains positive and is concave up

↳ If $t < 2$ then concave down