Q1)

- Normalized Dataset

GPA	GRE	Dummy	y
1	1	1	1
0.9	1	1	1
0.9	0.875	1	1
0.7	0.75	1	-1
0.6	0.875	1	-1

- Construct a Logistic Regression classifier using Gradient Descent for Maximum Likelihood.
- Start with all zero weight vector, what will the weight vector be after first iteration?

- $\nabla_E(w) = \frac{y \cdot x}{1 + e^{(y \cdot x \times w^T)}}$

- $y \cdot x = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0.9 & 1 \\ 1 & 0.9 & 0.875 \\ -1 & -0.7 & -0.75 \\ -1 & -0.6 & -0.875 \end{bmatrix}$ 5x3

- $w = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$

- $w^T = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 3x1

- $y \cdot x \times w^T = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0.9 & 1 \\ 1 & 0.9 & 0.875 \\ -1 & -0.7 & -0.75 \\ -1 & -0.6 & -0.875 \end{bmatrix} \times \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ 5x3 × 3x1 = 5x1

$$= \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

- $e^{y \cdot x \times w^T} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

- $1 + e^{y \cdot x \times w^T} = \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$

$$\begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

$$\nabla_E(w) = \frac{1}{n} \sum_{k=1}^{n} \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 0.5 & 0.45 & 0.5 \\ 0.5 & 0.45 & 0.4375 \\ -0.5 & -0.35 & -0.375 \\ -0.5 & -0.3 & -0.4375 \end{bmatrix}$$

$$\nabla_E(w) = [-0.1 \quad -0.15 \quad -0.125]$$

$$w = w - k \nabla_E(w)$$

$$= [0 \ 0 \ 0] - 2 \cdot [-0.1 \quad -0.15 \quad -0.125]$$

$$= [0 \ 0 \ 0] - [-0.2 \quad -0.3 \quad -0.25]$$

$$\boxed{w = [0.2 \quad 0.3 \quad 0.25]}$$

The weight vector w will be [0.2, 0.3, 0.25] after the first iteration with $k = 2$.