Short Answer Math 101 Final

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1. $\int_1^{\infty} \frac{1}{(1+x^{11})^{1/4}} dx$

· Comparison test → $0 \le a_n \le b_n$

$\rightarrow \frac{1}{(1+x^{11})^{1/4}} \le \frac{1}{x^{11/4}} = b_n$

$\lim_{t \to \infty} \int_1^t x^{-11/4} = \lim_{t \to \infty} \frac{-15}{4} x^{-15/4} \Big]_1^t$

$= \lim_{t \to \infty} \frac{-15}{4} \left[\frac{1}{t^{-15/4}} - \frac{1}{1} \right] = \frac{-15}{4} \left[0 - 1 \right] = \frac{15}{4}$

2. $\int_0^{5/2} \frac{dx}{\sqrt{25-x^2}} = \int_0^{5/2} \frac{5\cos\theta^{5/2} d\theta}{\sqrt{25-25\sin^2\theta}}$

$= \int_0^{5/2} \frac{5}{5} \frac{\cos\theta}{\sqrt{1-\sin^2\theta}} = \int_0^{5/2} \frac{\cos\theta \, d\theta}{\sqrt{\cos^2\theta}} = \int_0^{5/2} \frac{\cos\theta \, d\theta}{\cos\theta}$

$x = 5\sin\theta$

$\frac{5}{2} = 5\sin\theta$

$\frac{1}{2} = \sin\theta$

$= \int_0^{\pi/6} d\theta$

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3. $\int 2x \sin(x^2)\, dx$

let $u = x^{4}(x^2)$ $\qquad v = x^2$

$du = 2x\, dx + 2x$ $\qquad dv = dx$

$= x^2 x^3 (x^2) - \int 4x^2 \cos(x^2)$

$u = x$ $\qquad v$

$du = dx$ $\qquad dv = \sin(x^2)$

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4. $\int x^n \cdot \ln|x| \, dx = x^{n+1}\left[\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right] + C$

$, n \neq -1$

$u = \ln|x|$ $\quad$ $v = \frac{x^{n+1}}{n+1}$

$du = \frac{1}{x}\,dx$ $\quad$ $dv = x^n\,dx$

$uv = \frac{\ln|x| \cdot x^{n+1}}{n+1}$

$\int v\,du = \int \frac{x^{n+1}}{x(n+1)}\,dx = \int \frac{x^n}{(n+1)}\,dx$

$= \frac{1}{n+1}\int x^n\,dx = \frac{1}{n+1}\left[\frac{x^{n+1}}{n+1}\right] + C$

$= \frac{\ln|x| \cdot x^{n+1}}{n+1} - \frac{1}{n+1}\left[\frac{x^{n+1}}{n+1}\right] + C$

$= x^{n+1}\left[\frac{\ln|x|}{n+1} - \frac{1}{(n+1)^2}\right]$

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5. - 9% original remained
   - Half life carbon is 5731 years
   -

6. Expand by partial

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$$\frac{x+8}{(x+5)^2} = \frac{A}{(x+5)} + \frac{B}{(x+5)^2}$$

$$\frac{x+8}{\cancel{(x+5)^2}} \cdot \cancel{(x+5)^2} = \frac{A(x+5)^{\cancel{2}}}{\cancel{(x+5)}} + \frac{B \cancel{(x+5)^2}}{\cancel{(x+5)^2}}$$

$$x+8 = A(x+5) + B$$

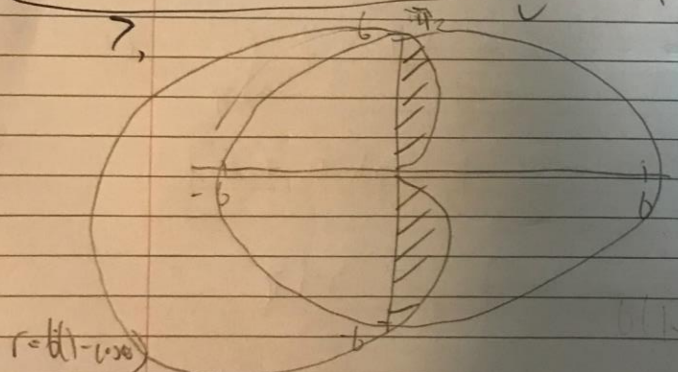
$$x+8 = Ax + 5A + B$$

- $A = 1$
- $5A + B = 8$
  $\Rightarrow 5 + B = 8$
  $B = 3$

$$\frac{x+8}{(x+5)^2} = \frac{1}{(x+5)} + \frac{3}{(x+5)^2}$$

7.

$r = 6$



$6 = 6(1-\cos\theta)$
$1 = 1 - \cos\theta$
$\cos\theta = 0$

$(1-\cos\theta)^2$

$$A = \int_\alpha^\beta \frac{1}{2}\left(r_2^2 - r_1^2\right) d\theta$$

$$= \frac{1}{2}\int_0^\pi 1 - 2\cos\theta + \cos^2\theta - 36$$

$$= \frac{1}{2}\int_0^\pi \cos^2\theta - 2\cos\theta - 35$$

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$\cancel{= \frac{1}{2}\left[\int_0^\pi \cos^2\theta - 2\cos\theta - 35 \, d\theta + \int_\pi^{2\pi} \cos^2\theta - 2\cos\theta - 35\right] d\theta}$

$\cancel{= \frac{1}{2}}$

$$= \int_0^\pi \frac{1}{2}\left(6(1-\cos\theta)\right)^2 - 6^2 \, d\theta$$

$$= \int_0^\pi \frac{1}{2}\left(36(1-\cos\theta)^2 - 36\right) d\theta$$

$$= \frac{1}{2}\, 36\int_0^\pi (1-\cos\theta)^2$$

$$= 18\int_0^\pi (\cos^2\theta - 2\cos\theta + 1) \quad ① \quad + \int_\pi^{2\pi} \quad ①$$

$$= 18\left[\frac{\sin\theta\cos\theta}{2} + \frac{\theta}{2} + 2\sin\theta + \theta\right]$$

$$= 36\cdot 2\left[\frac{3\pi}{2} + \frac{0}{2} + 2\cdot 0\right] = 54\pi$$

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8, $y = 2x^{3/2}$ from $x=0$ to $x=5$.

$$L = \int_a^b \sqrt{\left(\frac{dy}{dx}\right)^2}$$

$$= \int_0^5 \sqrt{(3x^{1/2})}$$

$$= \int_0^5 (3x)^{1/4}$$

$$= \frac{3x^{5/4}}{1/4} = 12x^{5/4} \Big|_0^5$$

9, $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^4}$ overestimate?

$$= \frac{-1}{1^4} + \frac{1}{2^4} - \frac{1}{3^4} + \cdots + \frac{1}{100^4}$$

$\lim_{n \to \infty} \frac{(-1)^n}{1} = 0$ so converge?

= it will converge to 0 eventually
so this can't be an overestimate.

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10. Taylor Polynomial

$f(x) = \frac{1}{x+4}$ $\qquad$ $f(0) = \frac{1}{4}$

$f'(x) = -(x+4)^{-2}$ $\qquad$ $f'(0) = \frac{1}{4^2}$

$f''(x) = -2(x+4)^{-3}$ $\qquad$ $f''(0) = \frac{-2}{4^3}$

$f'''(x) = 6(x+4)^{-4}$ $\qquad$ $f'''(0) = \frac{6}{4^4}$