Math 204, [A01]

Test 1 (Version A) — Part B, 06.11.20

Instructor: **Dr. S. IBRAHIM**

Student Name:	Ryan Woodward
Registration No.:	V00857268

1. OPTIONAL: **Print this .pdf**. If you do not have easy access to a printer, that is ok, but it is easier for us instructors to have a consistent format when marking so if you can print it out please do so. There are 3 questions and 4 pages (including this page).
2. For each problem, **write out a full solution**. Solutions should be clear, complete, and justified. *Final answers without supporting work will be graded as zero.*
3. **This exam is individual**. Communicating with anybody else during the test is a strict violation of Academic Integrity. Posting the test on the internet is a violation not just of academic integrity but of Canadian copyright law.
4. **This exam is open book**. You may consult your notes, the book, the videos, etc, but you must still write full solutions. We consider "googling" the problems to be unethical, and have written the problems aiming to minimize the usefulness of this.
5. **If you need help during the exam**, I will be available live at ibrahimis@uvic.ca
6. **Announcements** such as to report any discovered typos will be found at the TOP of Coursespaces highlighted in Yellow. Please refresh periodically.
7. The normal time for test is **30 minutes for writing and 15 minutes for scanning, and uploading**. If you have a time multiplier through CAL then the upload link will be available for that longer time period. Please do not leave this to the last few minutes to scan in case of a technological issue. Use your phone or other scanning device. Apps such as Adobe Scan can make a clean pdf file. Make sure all your pages are oriented correctly and in the right order. It's ok to insert your own pages if needed.
8. **If something goes wrong** with scanning and uploading let me know ASAP. Take a clean photo of each page and email them to mawais@uvic.ca by the end of the exam.
9. Please **keep your exam for at least two weeks** in case we need you to rescan.
10. Please **read and sign** the Confdentiality Agreement [1] before solving the quiz

[1] Confdentiality Agreement: I did not communicate with any other person or share this exam in any way. I followed all exam instructions.

SIGANUTURE: (MANDATORY) Ryan Woodward

1. [5 pts] Show that the vector field $\vec{F}(x,y) = (2xy^2+3)\vec{\imath} + (2x^2y+4)\vec{\jmath}$ is conservative, and compute the work done by the force $\vec{F}$ in moving a particle along the graph $y = x^3$ from $(1,1)$ to $(2,8)$.

- $\frac{\partial M}{\partial y} = 4xy$, $\frac{\partial N}{\partial x} = 4xy$

↳ since $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$ the field is conservative.

- $\vec{F} = (2xy^2+3)\hat{\imath} + (2x^2y+4)\hat{\jmath}$

$\frac{\partial f}{\partial x} = 2xy^2+3 \rightarrow \int (2xy^2+3)\,dx = x^2y^2+3x+g(y)$

- $f(x,y) = x^2y^2+3x+g(y)$

$\frac{\partial f}{\partial y} = 2x^2y + g'(y)$

$2x^2y+4 = 2x^2y + g'(y) \Rightarrow g'(y) = 4$

$\rightarrow \int 4\,dy = 4y + C$ (arbitrary) ; $g(y) = 4y+c$

- $f(x,y) = x^2y^2+3x+4y$

Work $= f(2,8) - f(1,1)$

$= [(2)^2(8)^2 + 3(2) + 4(8)] - [(1)^2(1)^2 + 3(1) + 4(1)]$

$= 106 - 8 = \boxed{98}$

Work done is $\boxed{98}$, path is independent since field is conservative.

2. [5pts] Use the divergence Theorem to evaluate the surface integral $\iint_S \vec{F}\cdot\vec{n}\, d\sigma$ for the vector field $\vec{F} = x^2\vec{i} + z\vec{j} - y\vec{k}$, and S is the unit sphere oriented so that the normal points outward (away from the centre)

$$\iint_S \vec{F}\cdot\vec{n} = \iiint_D \nabla\cdot\vec{F}\, dv$$

$$\operatorname{div}\vec{F} = \nabla\cdot\vec{F} = \frac{\partial M}{\partial x} + \frac{\partial N}{\partial y} + \frac{\partial P}{\partial z};$$

$$\frac{\partial M}{\partial x} = 2x, \quad \frac{\partial N}{\partial y} = 0, \quad \frac{\partial P}{\partial z} = 0$$

↳ Therefore $\operatorname{div}\vec{F} = 2x$

S = Unit Sphere

$x^2+y^2+z^2 = 1$

$x = r\sin(\phi)\cos(\theta)$

$y = r\sin(\phi)\sin(\theta)$

$z = r\cos(\phi)$

$r = 1$

$$\iiint \operatorname{div} F \cdot r^2 \cdot \sin(\phi)\, dr\, d\phi\, d\theta$$

$$= \iiint 2r\sin(\phi)\cos\theta) \cdot r^2 \sin(\phi)\, dr\, d\phi\, d\theta$$

$r \in [0,1]$

$\phi = [0,\pi)$

$\theta = [0,2\pi]$

$$= 2\int_0^{\pi}\int_0^{2\pi}\int_0^1 r^3 \sin^2(\phi)\cos\theta\, dr\, d\theta\, d\phi$$

$$= \cancel{2}\int_0^{\pi}\int_0^{2\pi} \frac{r^4}{\cancel{2}}\Big|_0^1 \sin^2(\phi)\cos\theta\, d\theta\, d\phi$$

$$= \int_0^{\pi} \sin^2(\phi)\, \sin\theta\Big|_0^{2\pi} = \int_0^{\pi} \sin^2(\phi)\,(0)$$

$$\boxed{= 0}$$

3. [10 pts] Let $\vec{F}(x,y,z) = (e^{xz} + e^{x+2y})\vec{\imath} + (\ln(2+y+z) + 2e^{x+2y})\vec{\jmath} + 3xyz\vec{k}$. Compute

$$\iint_S \text{curl}\vec{F} \cdot \vec{n}\, dA$$

where S is the portion of the surface $1 - x^2 - y^2$ above the xy-plane, oriented with the normal pointing upward.

$$\text{curl } \vec{F} = \nabla \times \vec{F} =$$

$$\left(\frac{\partial P}{\partial y} - \frac{\partial N}{\partial z}\right)\hat{\imath} + \left(\frac{\partial M}{\partial z} - \frac{\partial P}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}\right)\hat{k}$$

$$\left(3xz - \frac{1}{2+y+z}\right)\hat{\imath} + \left(3xy - (ze^{xz} - e^{x+2y}\right)\hat{\jmath}$$

$$+ \left(2e^{x+2y} - 2e^{x+2y}\right)\hat{k}$$