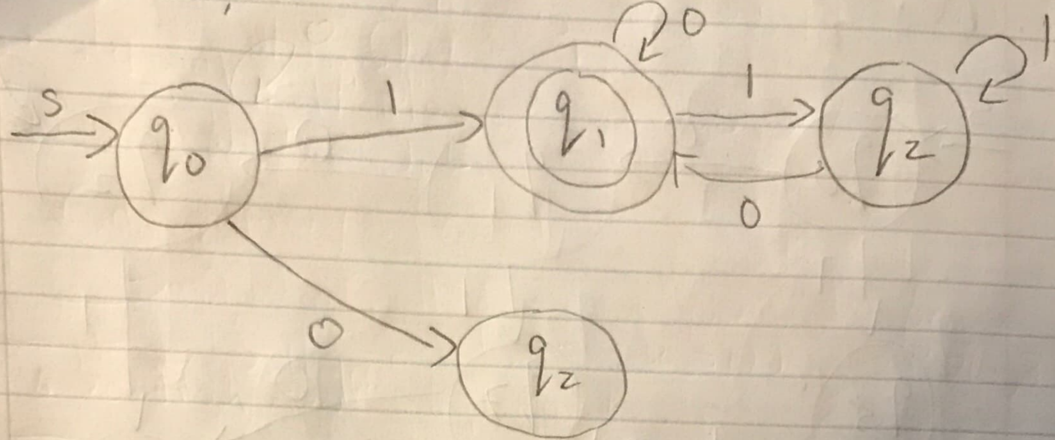
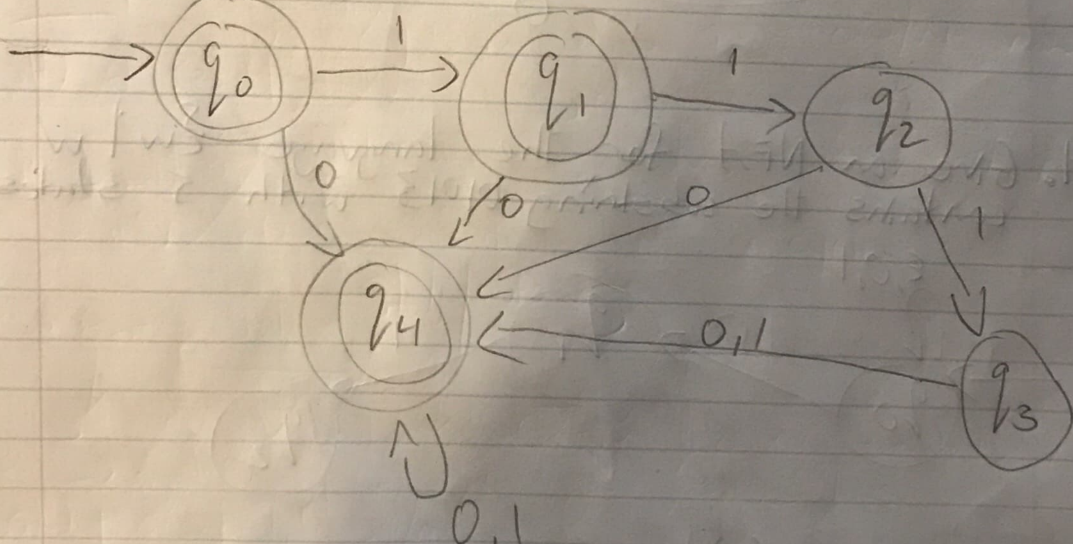
CSC 320 Assignment 1

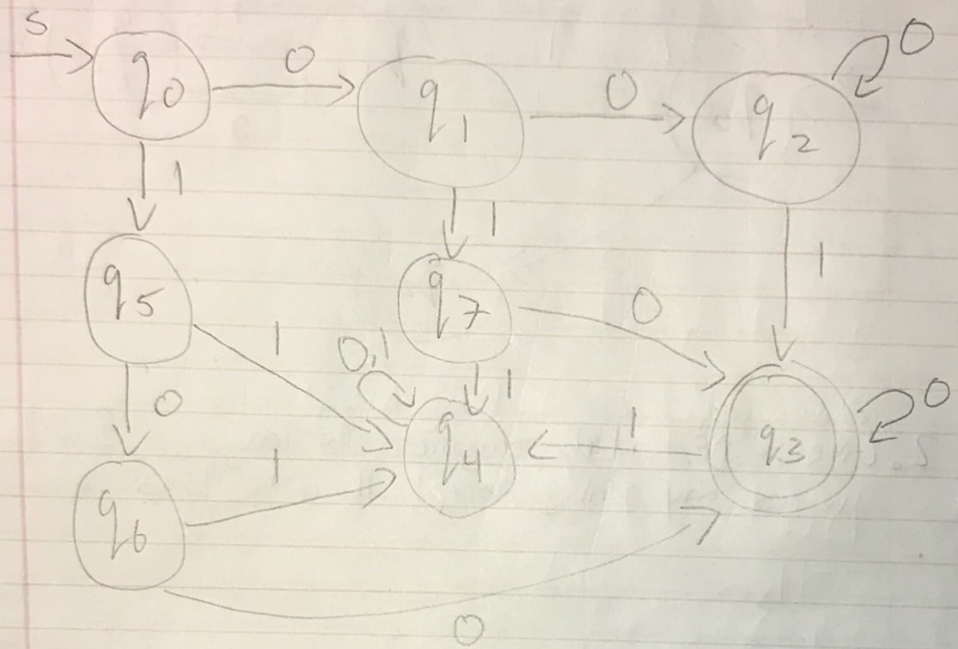
1. Give a DFA which recognizes the language $\{w \mid w$ begins with a 1 and ends with a $0\}$.



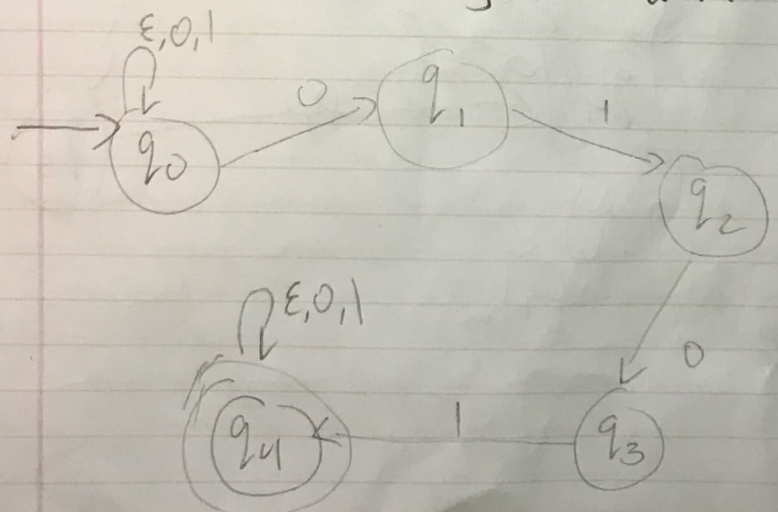
2. Give a DFA which recognizes the language $\{w \mid w$ is any string except 11 and $111\}$.



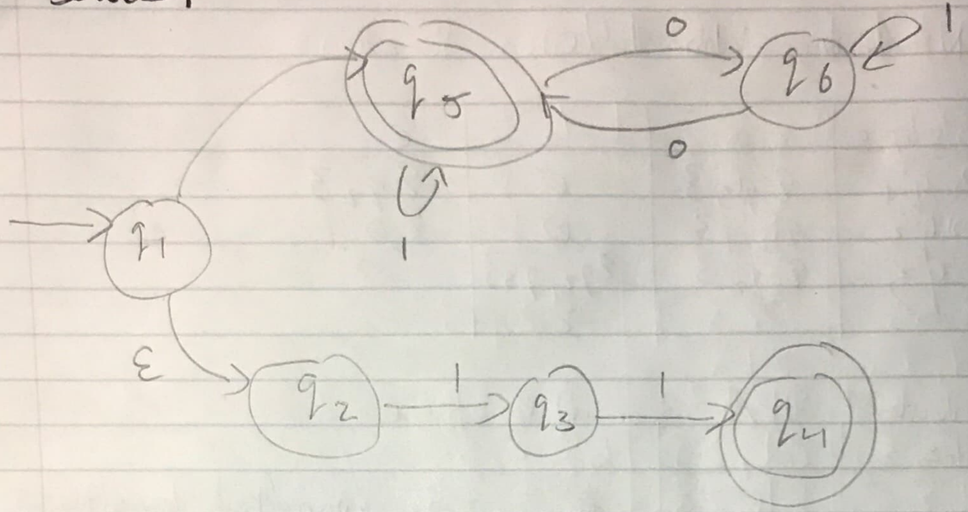
3. Give a DFA which recognizes the language, $\{ w \mid w$ contains at least two 0s and at most one 1$\}$.



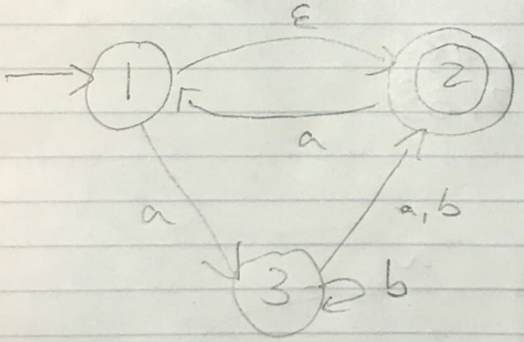
4. Give an NFA for the language $\{w \mid w$ contains the substring 0101$\}$ with 5 states.



5. Give an NFA for the language {w | w contains an even number of 0s or contains exactly two 1s} with six states.



6. Convert the NFA below to a DFA using the procedure described in class.



1) We know that the DFA will have 2^k states as NFA, so states = 8, with the states being: $\{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$.

2) Start state is $E(\{1\})$ and all states reachable from $E(\{1\})$ by ε-arrows.

$E(\{1\}) = \{1, 2\}$

- the new accept states containing the NFA's accept state {2} are then
  {{2}, {1,2}, {2,3}, {1,2,3}}

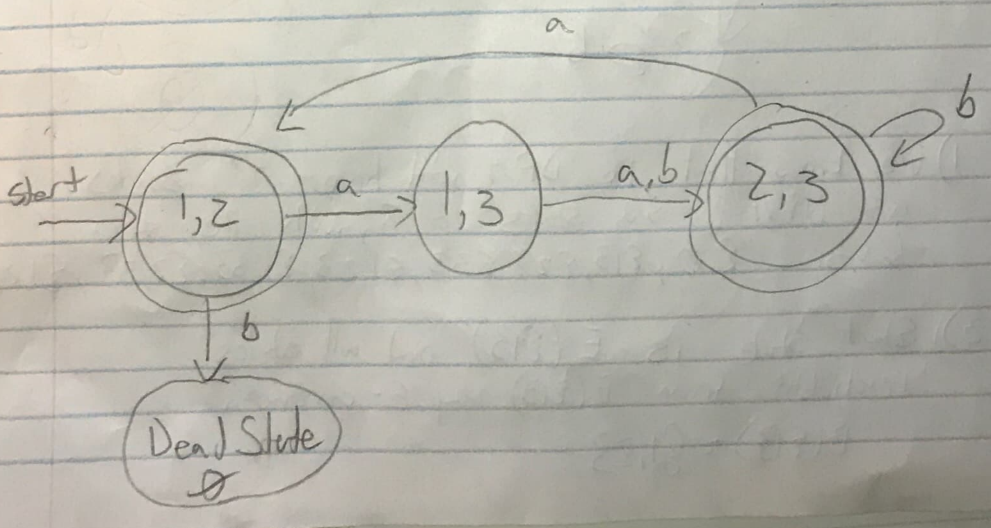
3) NFA transition table

q	a	b	ε
start → q_1	$\{q_3\}$	$\emptyset$	$\{q_2\}$
accept → q_2	$\{q_1\}$	$\emptyset$	$\emptyset$
q_3	$\{q_2\}$	$\{q_2, q_3\}$	$\emptyset$

Equivalent DFA transition table

states:	a	b
$\emptyset$	$\emptyset$	$\emptyset$
{1}		
{2}		
{3}		
start → {1,2}	{1,3}	$\emptyset$
{1,3}	{2,3}	{2,3}
{2,3}	{1,2}	{2,3}
{1,2,3}		

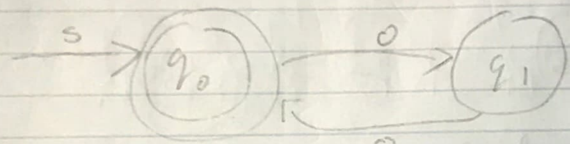
- unmarked transitions are unnecessary in the table.



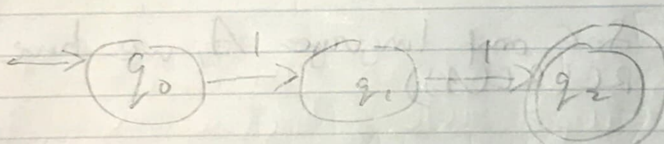
7. Convert the following regular expression to an NFA:

$$(((00)^*(11)) \cup 01)$$

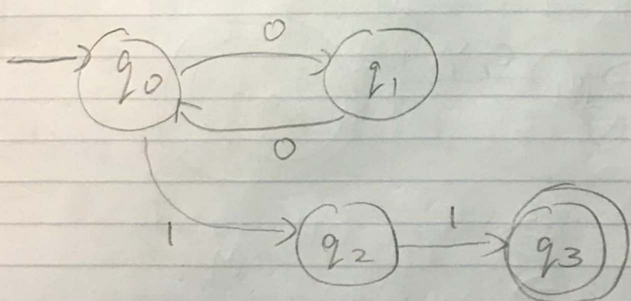
NFA: $(00)^*$



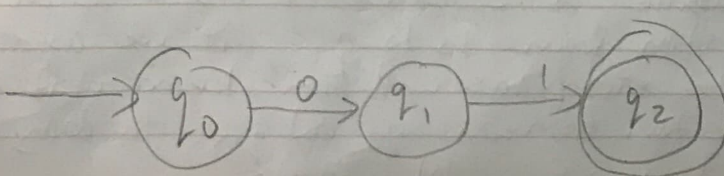
NFA: (11)



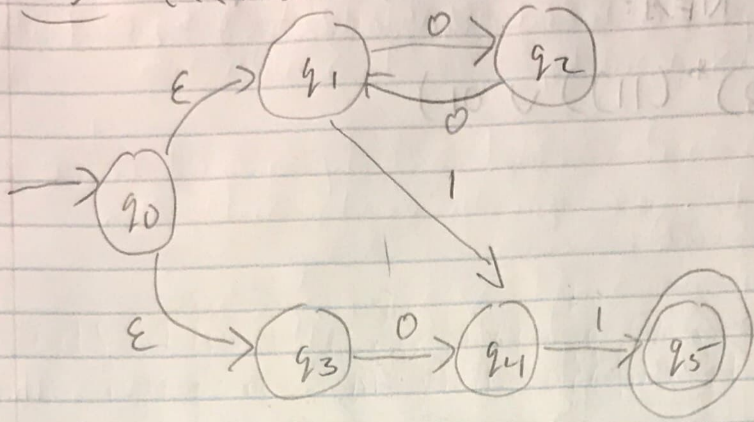
NFA: $(00)^*(11)$



NFA: 01



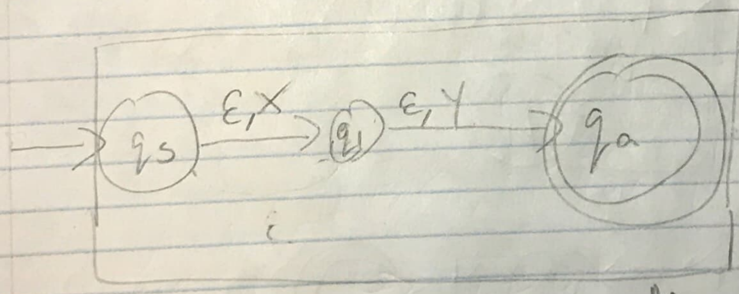
NFA: $(((00)^*(11)) \cup 01)$



8. Let the rotational closure of language A be $RC(A) = \{yx \mid xy \in A\}$

a) Show that for any language A, we have $RC(A) = RC(RC(A))$

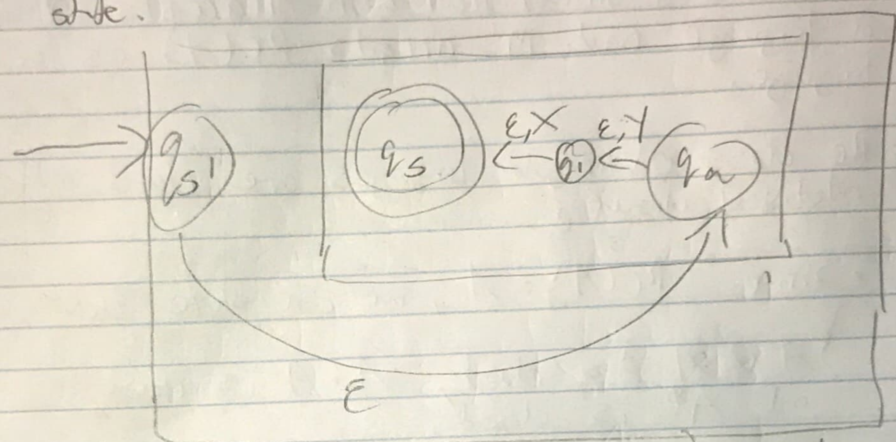
- We assume M is an NFA that recognizes $w = xy$.



$M = (Q_M, \Sigma_M, \delta_M, q_s, q_a)$

↳ If we assume $X = \varepsilon$ then there has to be an ε-transition from q_s to q_1, if we assume y to be empty as well then there must be a transition between q_1 and q_a. In doing so if $w = \varepsilon$, M accepts

① To build M' we create a new start state that has an ε-transition to the accept state of M, it now becomes a non-accepting state.



$M' = (Q_M \cup \{q_s'\}, \Sigma_M, \delta_{M'}, q_s', q_s)$

② Then the start state of M now becomes the accepting state of M', and the directional arrows get changed.

③ Now M' will accept all languages that are $RC(A)$.

↳ If we were to do $RC(A) = RC(RC(A))$

↳ If A is the language that accepts any language $w = xy$ where $x = \varepsilon$. therefore $w = y$.

↳ If we substitute this into $RC(A) = RC(RC(A))$ we notice the $RC(A) = A$ when $x = \varepsilon$, therefore $RC(A) = RC(A)$

b) Show that the class of languages is closed under rotational closure.

- As above I showed that both M accepts A as a language and M' accepts $RC(A)$ as a language.

- The class of regular languages are closed under rotational closure,

  - Since $RC(A) = A$, we can state that

    $A \cup A$ is closed, $A \circ A$ is closed, and A^* is closed.

↳ Since M' recognizes the language $RC(A)$ and A is closed, we can determine that any regular language will be closed under rotational closure.