Ryan Woodward

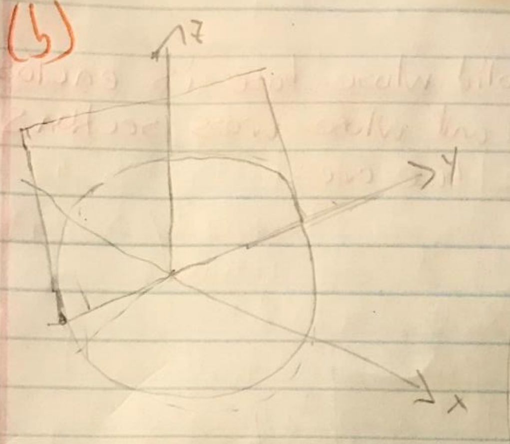
V00857268

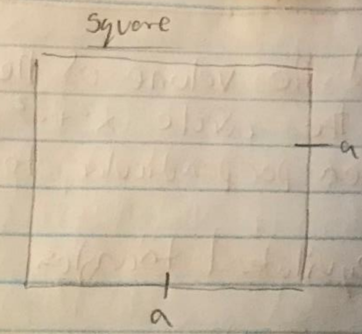
Math 101 – T04

Tutorial 10: 1b, 2b, 2e, 3b

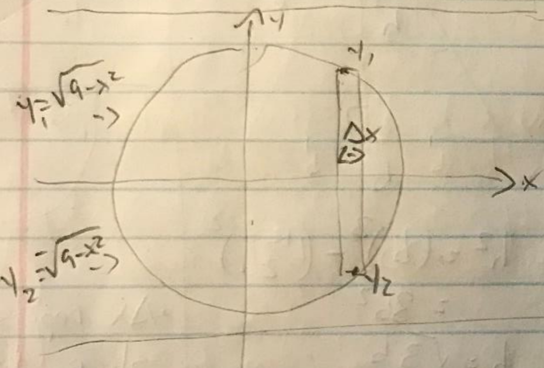
1b:

(b)





$A = a^2$



$y_1 = \sqrt{9-x^2}$ $\quad y_2 = -\sqrt{9-x^2}$

$y_1 - y_2 = 2\sqrt{9-x^2}$

↳ $y_1 - y_2 = a$, a side of the square

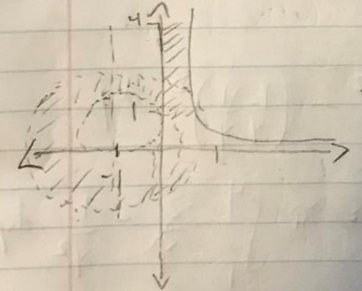
• Area of square $= a^2 = (2\sqrt{9-x^2})^2 = 4(9-x^2)$

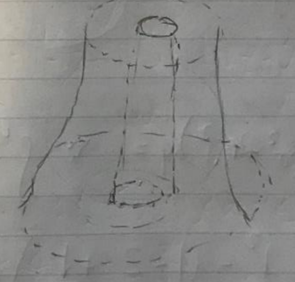
$$V = \int_{x=-3}^{x=3} A(x)\,dx = 2\int_0^3 4(9-x^2)\,dx = 8\int_0^3 9-x^2\,dx$$

$$= 8\left[9x - \frac{x^3}{3}\right]_0^3 = 8\left[27 - \frac{27}{3}\right] \boxed{= 144}$$

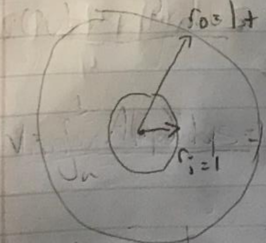
2b:

b) $x = -1$





Looks like a bell shape with a cylinder hole in middle, use washer method.



$$V = \int_c^d A(y)\,dy = \int_1^4 \pi\left(r_o^2 - r_i^2\right) dy$$

$$= \pi \int_1^4 \left(1 + \tfrac{1}{y}\right)^2 - 1^2 \, dy = \pi \int_1^4 1 + \frac{2}{y} + \frac{1}{y^2} - 1 \, dy$$

$$= \pi \int_1^4 \frac{2}{y} + \frac{1}{y^2} \, dy$$

$$= \pi \left[2\ln(y) - \frac{1}{y} \right]_1^4$$

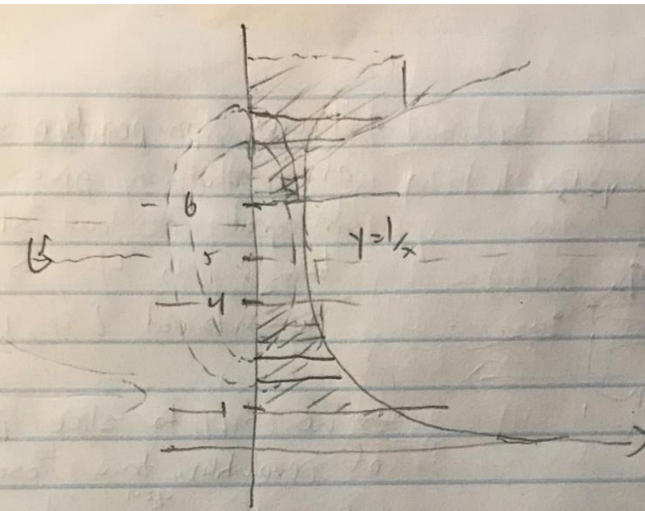
$$= \pi \left[\left(2\ln(4) - \frac{1}{4} \right) - \left(2\ln(1) - 1 \right) \right]$$

$$= \pi \left[2\ln(4) - \frac{1}{4} - 2\ln(1) + 1 \right]$$

$$= \pi \left[2\ln(4) + \frac{3}{4} \right] \quad \boxed{= 4\pi\left(8\ln(4) + 3\right)}$$

2e:

(e) $y = 5$



- radius $= 5 - y$, height $= x_2 - x_1 = \frac{1}{y} - 0 = \frac{1}{y}$

$$V = \int_1^4 2\pi r h \, dy = 2\pi \int_1^4 (5-y)\left(\frac{1}{y}\right) dy$$

$$= 2\pi \int_1^4 \frac{5}{y} - 1 = 2\pi \Big[5\ln|y| - y \Big]_1^4$$

$$= 2\pi \big((5\ln|4| - 4) - (5\ln|1| - 1)\big)$$

$$= 2\pi (5\ln|4| - 3)$$

3b:

(b) $y = \frac{x^3}{6} + \frac{1}{2x}$, $[1,3]$

$(x^4-1)(x^4-1)$
$= x^8 - x^4 - x^4 + 1$

- $f'(x) = \frac{x^2}{2} - \frac{1}{2x^2} = \frac{x^4-1}{2x^2}$

- $f'(x)^2 = \frac{(x^4-1)^2}{(2x^2)^2} = \frac{x^8-2x^4+1}{4x^4}$

- $1 + f'(x)^2 = 1 + \frac{x^8-2x^4+1}{4x^4} = \frac{x^8+2x^4+1}{4x^4}$

$$= \frac{(x^4+1)^2}{(2x^2)^2}$$

Arc Length $= \int_1^3 \frac{x^4+1}{2x^2} = \int_1^3 \frac{x^2}{2} + \frac{1}{2x^2}\,dx$

$$= \left[\frac{x^3}{6} + \frac{-1}{2x^{-1}}\right]_1^3 = \left[\frac{x^3}{6} - \frac{1}{2x}\right]_1^3$$

$$= \left[\left(\frac{9}{2} - \frac{1}{6}\right) - \left(\frac{1}{6} - \frac{1}{2}\right)\right] \boxed{= \frac{14}{3}}$$