

First of all, calculate the back propagation formula:

- ~~1st~~ 2nd Hidden layer:

$$\frac{\partial \mathcal{LSS}}{\partial W_{i1}^{(3)}} = \frac{\partial \mathcal{LSS}}{\partial a_i^{(4)}} \cdot \frac{\partial a_i^{(4)}}{\partial z_i^{(4)}} \cdot \frac{\partial z_i^{(4)}}{\partial W_{i1}^{(3)}} = -2(y_i - a_i) z_i^{(4)} (1 - z_i^{(4)}) a_i^{(3)}$$

$$\Rightarrow W_{i1}^{(3)} = W_{i1}^{(3)} + 2(y_i - a_i^{(4)}) z_i^{(4)} (1 - z_i^{(4)}) a_i^{(3)} \cdot \eta$$

- 1st layer:

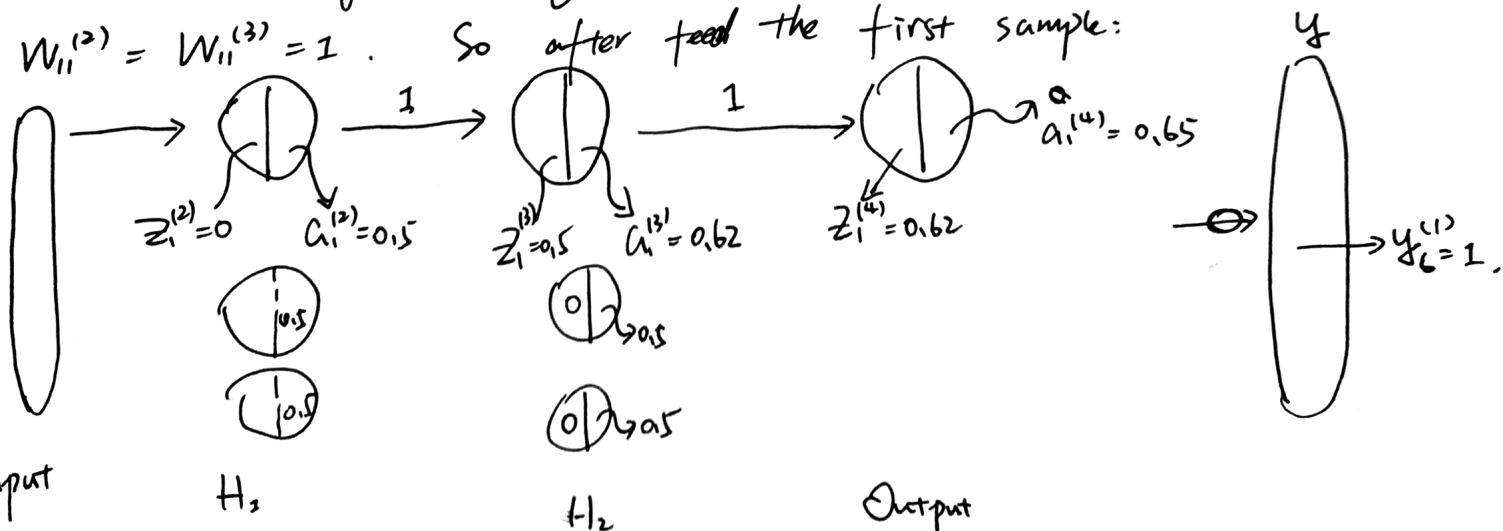
$$\frac{\partial \mathcal{LSS}}{\partial W_{i1}^{(2)}} = \sum_{k=1}^{10} \frac{\partial \mathcal{LSS}}{\partial a_k^{(4)}} \cdot \frac{\partial a_k^{(4)}}{\partial z_k^{(4)}} \cdot \frac{\partial z_k^{(4)}}{\partial a_i^{(3)}} \cdot \frac{\partial a_i^{(3)}}{\partial z_i^{(3)}} \cdot \frac{\partial z_i^{(3)}}{\partial W_{i1}^{(2)}}$$

$$= -2z_i^{(3)} (1 - z_i^{(3)}) a_i^{(2)} \sum_{k=1}^{10} (y_k - a_k^{(4)}) z_k^{(4)} (1 - z_k^{(4)}) W_{ik}^{(3)}$$

$$\Rightarrow W_{i1}^{(2)} = W_{i1}^{(2)} + 2z_i^{(3)} (1 - z_i^{(3)}) a_i^{(2)} \sum_{k=1}^{10} (y_k - a_k^{(4)}) z_k^{(4)} (1 - z_k^{(4)}) W_{ik}^{(3)} \cdot \eta$$

Then set learning rate: $\eta = 1$ and all the weights to be zero except:

$W_{i1}^{(2)} = W_{i1}^{(3)} = 1$. So after feed the first sample:



1st layer:

$$\Rightarrow W_{i1}^{(3)} = W_{i1}^{(3)} + 2(0 - 0.65) \cdot 0.62 (1 - 0.62) \cdot 0.62 = W_{i1}^{(3)} - 0.19$$

$$\Rightarrow W_{i1}^{(3)} = \underline{0.81}, \quad W_{i1}^{(2)} = -0.19 \quad (i=0, 2, 3)$$

2nd layer:

$$W_{i1}^{(2)} = W_{i1}^{(2)} + 2 \cdot 0.5 (1 - 0.5) \cdot 0.5 \sum_{k=1}^{10} (y_k - a_k^{(4)}) z_k^{(4)} (1 - z_k^{(4)}) W_{ik}^{(3)}$$

Only $W_{i1}^{(3)} = 1$:

$$W_{i1}^{(2)} = W_{i1}^{(2)} + 0.25 (y_i - a_i^{(4)}) z_i^{(4)} (1 - z_i^{(4)}) W_{i1}^{(3)} = W_{i1}^{(2)} + 0.25(0 - 0.65) \cdot 0.62 \cdot 0.38$$

$$= W_{i1}^{(2)} - 0.021 \Rightarrow W_{i1}^{(2)} = 0.979, \quad W_{i1}^{(2)} = -0.021 \quad (i=0, 2, 3)$$