14.3.1 Iterated Extended Kalman Filter

The traditional EKF takes only one Gauss-Newton step; the iterated EKF takes multiple Gauss-Newton steps, re-linearizing only the measurement equation at each step. One could also imagine re-linearizing the f^{-1} dynamics using $\hat{x}(k+1)$ *and* $\hat{v}(k)$. In fact, one can imagine re-linearizing over the past N steps. This is the idea behind the moving horizon estimator, also known as the backward-smoothing EKF (BSEKF) [8, **?**].

Consider

$$\begin{aligned} J_c[k+1, x(k+1)] &= J_b[k, v_{\text{OPT}}(k), x(k+1)] \\ &\approx \tfrac{1}{2}[x(k+1) - \bar{x}(k+1)]^T \bar{P}^{-1}(k)[\ldots] \\ &\quad + \tfrac{1}{2}\{z(k+1) - h[k+1, x(k+1)]\}^T R^{-1}(k+1)\{\ldots\} \end{aligned}$$

where v_{OPT} is the value of $v(k)$ that minimizes J_b. The approximation in the above comes from our having linearized the dynamics about $\bar{x}(k+1)$. The measurement equation remains nonlinear.

Strategy: Start with a guess of $x(k+1)$; solve the linearized LS problem for a new guess; iterate.

Let $\hat{x}^i(k+1)$ be the estimated $x(k+1)$ after the ith Gauss-Newton step; note that $\hat{x}^0(k+1) \triangleq \bar{x}(k+1)$. Also let

$$H^i(k+1) \triangleq \left[\left.\frac{\partial h}{\partial x}\right|_{k+1, \hat{x}^i(k+1)} \right]$$

The linearized measurement equation after the ith step becomes

$$0 = z(k+1) - h\left[k+1,\ \hat{x}^i(k+1)\right] - H^i(k+1)\left[x(k+1) - \hat{x}^i(k+1)\right]$$

Letting J_c^i be the cost function with the ith linearized form of the measurement equation, the such iterations can be related back to the standard KF equations as follows:

$$\begin{aligned} 0 &= \left[\frac{\partial J_c^i}{\partial x(k+1)}\right]^T \\ &= \bar{P}^{-1}(k+1)[x(k+1) - \bar{x}(k+1)] \\ &\quad - H^{i^T}(k+1)R^{-1}(k+1)\left\{z(k+1) - h\left[k+1,\ \hat{x}^i(k+1)\right] - H^i(k+1)\left[x(k+1) - \hat{x}^i(k+1)\right]\right\} \end{aligned}$$

Note that this equation is linear in $x(k+1)$.

Let

$$P^{i+1}(k+1) = [\bar{P}^{-1}(k+1) + H^{i^T}(k+1)R^{-1}(k+1)H^i(k+1)]^{-1}$$

Then solving for $x(k+1)$ yields

$$\begin{aligned} \hat{x}^{i+1}(k+1) &= \hat{x}^i(k+1) + P^i(k+1)H^{i^T}(k+1)\bar{P}^{-1}(k+1)\left\{z(k+1) - h\left[k+1,\ \hat{x}^i(k+1)\right]\right\} \\ &\quad + P^i(k+1)\bar{P}^{-1}(k+1)[\bar{x}(k+1) - \hat{x}^i(k+1)] \end{aligned}$$

with

$$\hat{P}^0(k+1) = \bar{P}(k+1) \quad \text{and} \quad \hat{x}^0(k+1) = \bar{x}(k+1)$$

Note that $\hat{x}^1(k+1)$ is the traditional (non-iterated) EKF estimate. Stop iterating when

$$\left\|\hat{x}^{i+1}(k+1) - \hat{x}^i(k+1)\right\|$$

gets very small. Use step size adjustment as before there is reason to worry about divergence.