Notes

Singapore Physics Olympiad 2023

Topic 5: Oscillatory Motion

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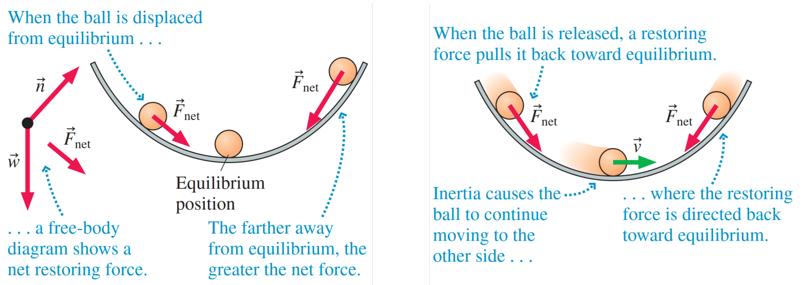
Oscillatory motion

Notes

- **Periodic motion** is the repeating motion of an object that regularly returns to a given position after a fixed time interval

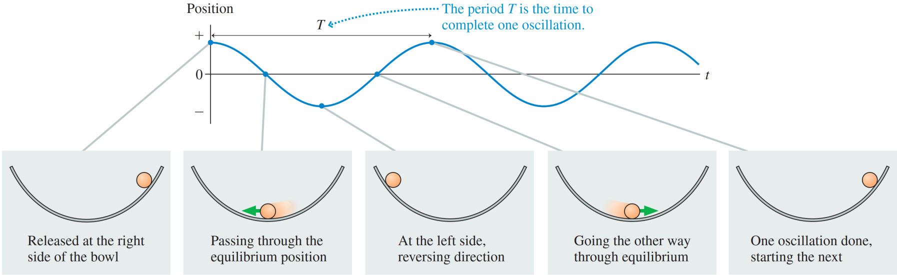
- **Oscillatory motion** is a periodic motion back and forth around an **equilibrium position** – a position at which no net force acting on the object

- An object locating away from the equilibrium position has a net force/torque – **restoring force/torque** – directed back toward the equilibrium position



Simple harmonic motion

- The most fundamental oscillation is the smooth *sinusoidal* oscillation – **simple harmonic motion** (SHM)
- Restoring force/torque is **linear** – the net force/torque is toward the equilibrium position and is proportional to the distance from equilibrium
- Oscillation about an equilibrium position with a linear restoring force/torque is always a simple harmonic motion



Notes

Describing oscillations

- **Amplitude** A is the maximum magnitude of displacement from equilibrium
- **Period** T is the time to complete one cycle; **frequency** f is the number of cycles per unit time

$$\boxed{T = \frac{1}{f}}$$

- **Angular frequency** ω is 2π times the frequency

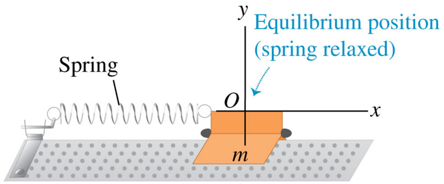
$$\boxed{\omega = 2\pi f = \frac{2\pi}{T}}$$

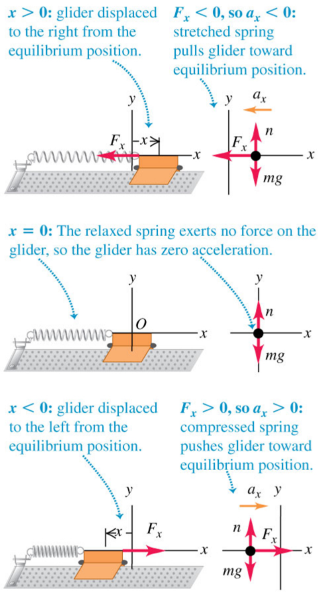
Notes

Motion of a spring-mass system

Notes

- When the spring is neither stretched nor compressed, the object is at rest at the equilibrium position of the system
- When the body is displaced from its equilibrium position, the spring force tends to restore it to the equilibrium position – restoring force





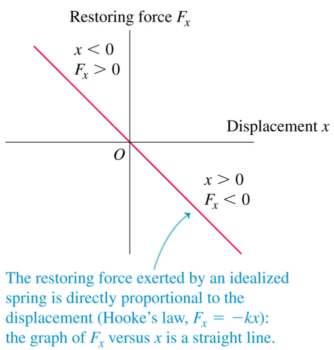
Force exerted by a spring

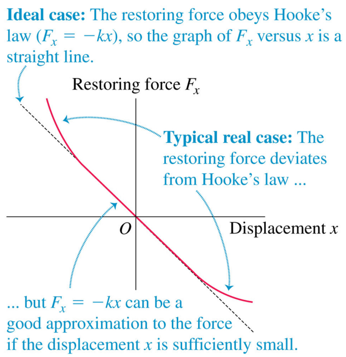
Notes

- **Hooke's law**: the spring exerts a force that is directly proportional to the displacement from equilibrium position and is always directed towards the equilibrium position

$$\boxed{F_x = -kx}$$

k : spring constant





Kinematics of simple harmonic motion

Notes

- Newton's second law: second order ordinary differential equation

$$F_x = -kx_x = ma_x \quad \Rightarrow \quad a_x = -\frac{k}{m}x \quad \Rightarrow \quad \frac{\mathrm{d}^2x(t)}{\mathrm{d}t^2} = -\frac{k}{m}x(t) \equiv -\omega^2 x(t)$$

- General solution:

$$x(t) = A\cos(\omega t + \phi_0) \qquad \phi_0: \text{ phase constant}$$

- A and ϕ_0 are constants to be determined by initial conditions $x(t=0)$ and $v_x(t=0)$

Kinematics of simple harmonic motion – cont'd

Notes

- Displacement as a function of time:

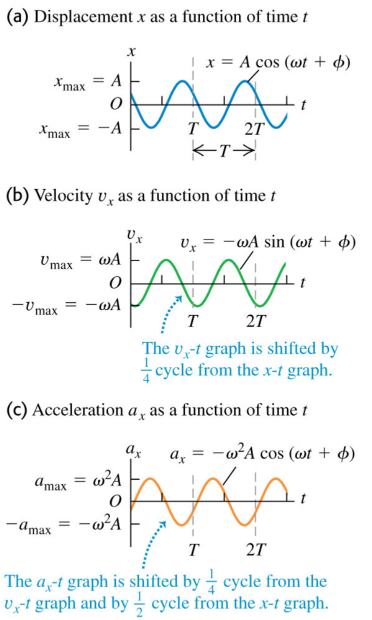
$$x(t) = A\cos(\omega t + \phi_0)$$

- Velocity as a function of time:

$$v_x(t) = \frac{\mathrm{d}x(t)}{\mathrm{d}t} = -\omega A\sin(\omega t + \phi_0)$$

- Acceleration as a function of time:

$$a_x(t) = \frac{\mathrm{d}v_x(t)}{\mathrm{d}t} = -\omega^2 A\cos(\omega t + \phi_0)$$



Energy in simple harmonic motion

- Kinetic energy:

$$K(t) = \frac{1}{2}mv_x^2(t) = \frac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \phi_0)$$

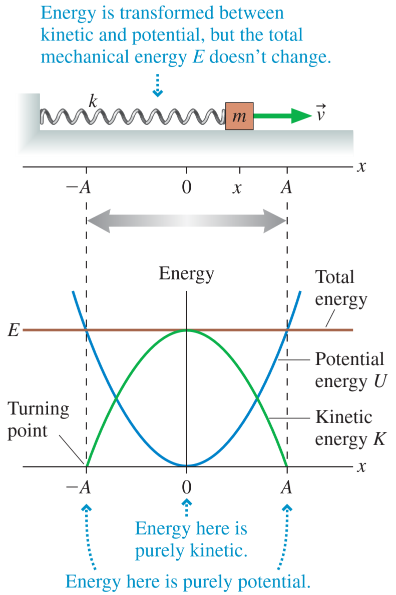
- Potential energy:

$$U(t) = \frac{1}{2}kx^2(t) = \frac{1}{2}kA^2 \cos^2(\omega t + \phi_0)$$

- Total mechanical energy:

$$E(t) = K(t) + U(t) = \frac{1}{2}kA^2$$

$$\Rightarrow \quad v_x(t) = \sqrt{\frac{k}{m}\left[A^2 - x^2(t)\right]}$$

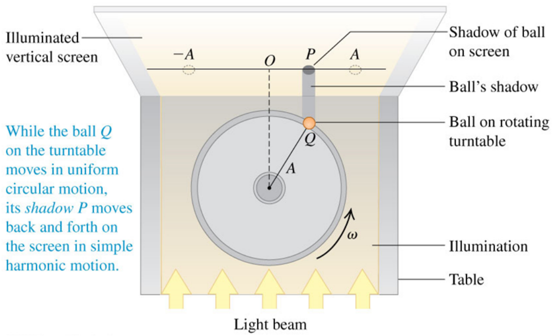


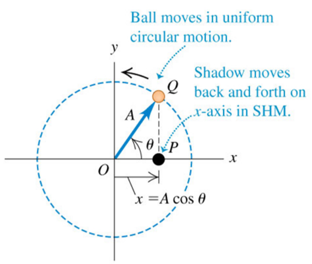
Notes

SHM and uniform circular motion

Notes

- Uniform circular motion projected onto one dimension is a simple harmonic motion
- As the point Q moves around the reference circle with constant angular speed ω, the vector OQ rotates with the same angular speed. Such a rotating vector is called a **phasor**





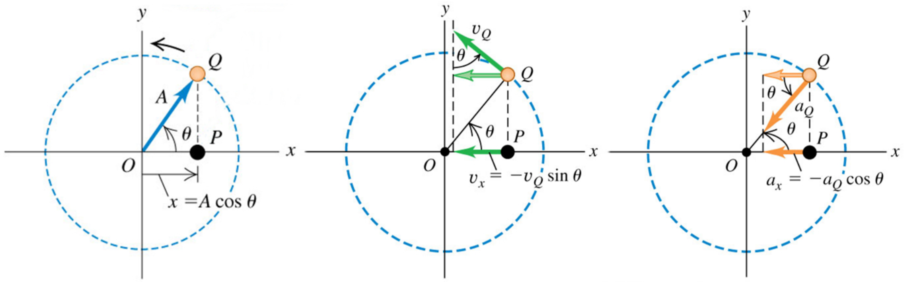
SHM and uniform circular motion – cont'd

- x-component of the phasor is the x-coordinate of the point P:

$$x(t) = A\cos\theta(t) = A\cos(\omega t)$$

- x-component of the velocity and acceleration of point P:

$$v_x(t) = -\omega A \sin(\omega t)\ , \qquad a_x(t) = -\omega^2 A\cos(\omega t)$$



Notes

Period and frequency for SHM

- Periodicity of the oscillation:

$$x(t+T) = x(t) \quad \Rightarrow \quad A\cos[\omega(t+T)+\phi] = A\cos(\omega t+\phi) \quad \Rightarrow \quad \omega T = 2\pi$$

- Period and frequency:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}\,, \qquad f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

- Frequency and period of SHM are determined by the physical properties of the oscillator

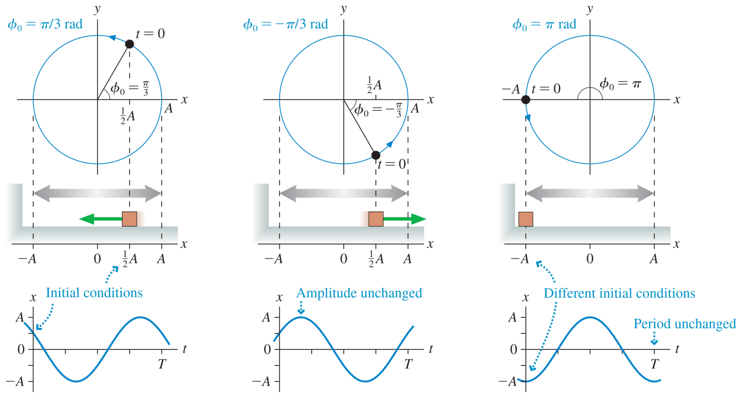
- Frequency and period of SHM do not depend on the amplitude A

Notes

Phase and phase constant

- The quantity $\theta(t) = \omega t + \phi_0$ is called the **phase** of the oscillation and it describes the stage of the oscillation with respect to one complete oscillation

- Different values of the phase constant correspond to different starting points on the circle and thus to different initial conditions



Notes

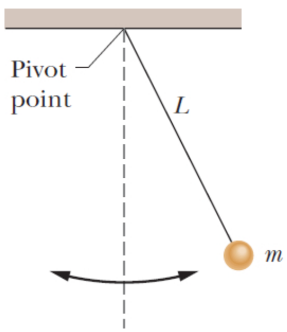
Simple pendulum

- The **simple pendulum** is a mechanical system exhibiting oscillatory motion

- It consists of a particle of mass m suspended from one end of an unstretchable, massless string of length L that is fixed at the other end

- The oscillatory motion occurs in the vertical plane and is driven by the gravitational force

- When the point particle is pulled to one side of its straight-down equilibrium position and released, it oscillates about the equilibrium position



Notes

Simple pendulum motion as SHM

Notes

- Tangential component of the net force: $s = L\theta$

$$F_t = ma_t \quad \Rightarrow \quad -mg\sin\theta(t) = m\,\frac{\mathrm{d}^2 s(t)}{\mathrm{d}t^2}$$

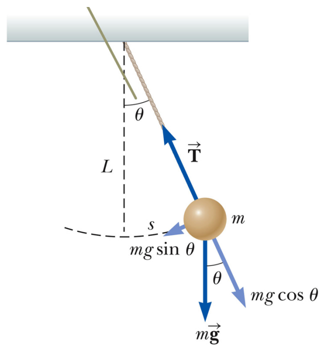
- Small angle approximation: $\sin\theta \approx \theta$

$$\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = -\frac{g}{L}\sin\theta(t)$$

$$\Rightarrow \quad \frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} \approx -\frac{g}{L}\,\theta(t) \equiv -\omega^2\theta(t)$$

- Period of the motion:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{L}{g}}$$



Physical pendulum motion as SHM

Notes

- **Physical pendulum** consists of an *extended* object swinging back and forth on a pivot under the influence of gravity

- Net torque with respect to the pivot:

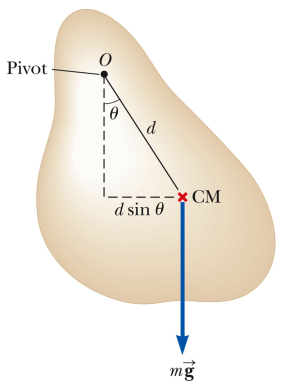
$$-mgd\sin\theta(t) = I\,\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} \Rightarrow \frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = -\frac{mgd}{I}\sin\theta(t)$$

- Small angle approximation: $\sin\theta \approx \theta$

$$\frac{\mathrm{d}^2\theta(t)}{\mathrm{d}t^2} = -\frac{mgd}{I}\,\theta(t) \equiv -\omega^2\theta(t)$$

- Period of the motion:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{mgd}}$$



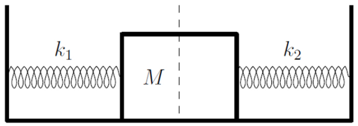
Problem 1

Notes

A mass $M = 1\,\mathrm{kg}$ is connected to two springs 1, 2 of spring constants $k_1 = 1\,\mathrm{N/m}$, $k_2 = 2\,\mathrm{N/m}$ and slides on a smooth horizontal table. In the equilibrium position it is given a velocity $v_0 = 0.5\,\mathrm{m/s}$ towards spring 2.

(a) How long will it take to reach its maximum compression of spring 1? What will this be?

(b) How long does it take for it take to reach the point where it compresses spring 1 by $x = -0.1\,\mathrm{m}$ for the first time?



Problem 2

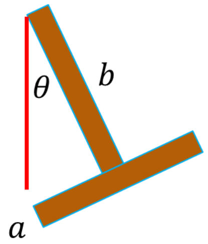
Notes

A block of mass m is attached to a spring of mass $M \ll m$ and oscillates in simple harmonic motion on a frictionless horizontal surface. The force constant of the spring is k and the equilibrium length is L. Assume all portions of the spring oscillate in phase and the segments of the spring undergo displacements proportional to their distances from the fixed end. Find the period of the motion.

Problem 3

Notes

A pendulum is constructed from two identical uniform thin rods of a and b each of length L and mass m, connected at right angles to form a 'T' by joining the center of rod a to one end of rod b. The 'T' is then suspended from the free end of rod b and the pendulum swings in the plane of the 'T'. Show that the pendulum exhibits simple harmonic motion if the angle θ of inclination to the vertical of the pendulum is small. Hence or otherwise, determine the period of the simple harmonic motion.



Problem 4

Notes

A solid sphere of radius R rolls without slipping in a cylindrical through of radius $5R$. Show that, for small displacements from equilibrium perpendicular to the length of the trough, the sphere executes simple harmonic motion. Hence, or otherwise, determine the period of the oscillation.