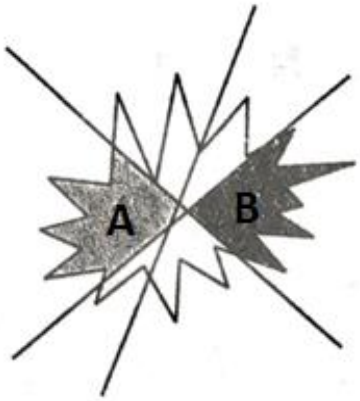
Asia Pacific Mathematical Olympiad for Primary Schools 2016

First Round
2 hours
(150 marks)

1. Find the value of
$$2016 + 2015 + 2014 - 2013 - 2012 - 2011 + 2010 + 2009 + 2008$$
$$- 2007 - 2006 - 2005 + \ldots + 6 + 5 + 4 - 3 - 2 - 1$$
(SMOPS 2016 Q. 1)

2. In the figure below, each of the three lines bisected the figure into two regions of equal areas. What is the relationship between shaded area A and shaded area B?

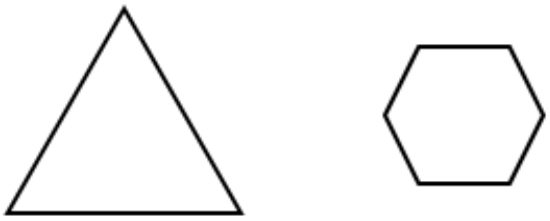


(A) $A > B$ (B) $A < B$ (C) $A = B$ (D) Insufficient information to deduce
(SMOPS 2016 Q. 2)

3. Find the number of fractions, among those listed below, which are in their simplest form.
$$\frac{1}{6}, \frac{2}{7}, \frac{3}{8}, \ldots, \frac{2010}{2015}, \frac{2011}{2016}$$
(SMOPS 2016 Q. 3)

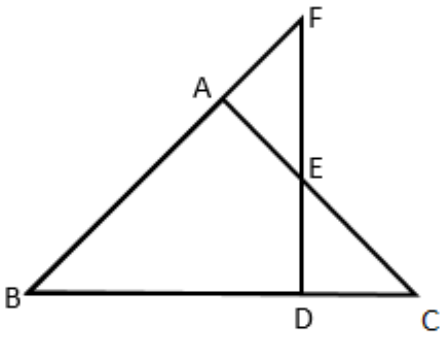
4. It is given that $a \oplus b = \frac{2}{a^2} + \frac{1}{b}$, which of the following equations is correct?
(A) $2 \oplus 4 = 4 \oplus 2$ (B) $3 \oplus 6 = 6 \oplus 3$
(C) $4 \oplus 8 = 8 \oplus 4$ (D) $1008 \oplus 2016 = 2016 \oplus 1008$
(SMOPS 2016 Q. 4)

5. In the diagram below, the equilateral triangle has a length of 3 cm and the hexagon has a length of 1 cm. Given that the ratio of the area of the equilateral triangle to the area of the hexagon is a : 2. Find the value of a.



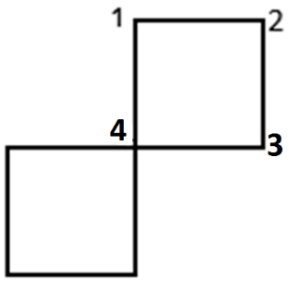
(SMOPS 2016 Q. 5)

6. The diagram shows an isosceles triangle ABC in which $\angle BAC = 90°$ and $BC = 6$ cm. F is a point on BA produced such that FD is perpendicular to BC and it intersects AC at E. D lies on BC. Find the total length of DE and DF.



(SMOPS 2016 Q. 6)

7. Daniel drives his car in the following manner:
He drives along for 20 m then turns left;
Then drives along for 20 m then turns right;
Then drives along for 20 m then turns right;
Then drives along for 20 m then turns right;
And he repeats the same cycle.
The diagram below shows the path that Daniel drove. From which point (1, 2, 3 or 4) did he begin his journey?

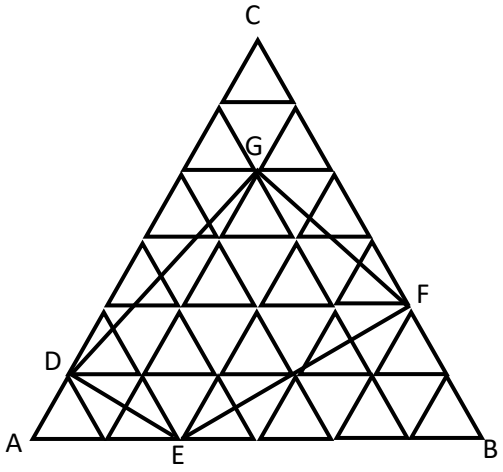


(SMOPS 2016 Q. 7)

8. Three cubes have different lengths of 3 cm, 4 cm and 5 cm respectively. All faces of the cubes were painted red. If all three cubes were cut up into smaller 1-cm cubes, find the number of 1-cm cubes that has at least one face painted red.

(SMOPS 2016 Q. 8)

9. In the diagram below, each side of the equilateral triangle *ABC* is divided into six equal segments. Given that every small triangle in the diagram has an area of 1 cm^2, find the area of the quadrilateral *DEFG* in cm^2.



(SMOPS 2016 Q. 9)

10. Two rabbits, Harry and Bunny, leave from points *A* and *B* respectively and travel towards each other. Harry's speed is 1.5 times of Bunny's speed. If the two rabbits first meet at a point that is 12 m away from the mid-point of *AB*, find the distance of *AB*.

(SMOPS 2016 Q. 10)

11. It is known that the area of three different sides of a rectangular cuboid is in a ratio of 2 : 3 : 5. The total length of all its edges is 124 cm. Find the volume of the rectangular cuboid.

(SMOPS 2016 Q. 11)

12. There is a sequence of the first 300 positive integers 1, 2, 3, …, 299, 300. If the numbers that are divisible by 5 or 7 are removed from the sequence, find the 123rd number in the remaining sequence.

(SMOPS 2016 Q. 12)

13. 2016 distinct points are chosen within triangle *ABC* such that no three points among the 2019 points (inclusive of *A, B* and *C*) lie on a straight line. Using these 2019 points as vertices, how many smaller triangles can triangle *ABC* be cut into?

(SMOPS 2016 Q. 13)

14. Find the value of

$$84\times\left(\frac{1}{1\times3}-\frac{2}{3\times5}+\frac{3}{5\times7}-\frac{4}{7\times9}+...+\frac{9}{17\times19}-\frac{10}{19\times21}\right)$$

(SMOPS 2016 Q. 14)

15. Given that a, b and c are prime numbers, not necessarily distinct, and their product is the sum of 7 consecutive positive integers. Find the smallest possible value of the sum $a + b + c$.

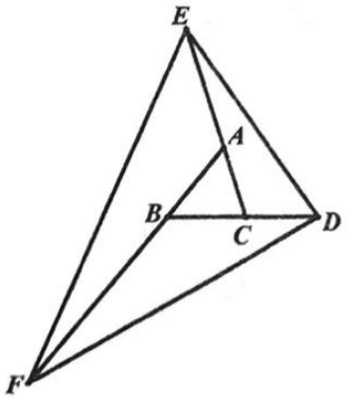
(SMOPS 2016 Q. 15)

16. Find the number of zeroes at the end of the resultant product

$$1\times4\times7\times10\times...\times697\times700$$

(SMOPS 2016 Q. 16)

17. In the figure below, not drawn to scale, the area of triangle ABC is 1 cm^2. The points D, E and F are on BC, CA and AB produced such that $BD = 2BC$, $CE = 3CA$ and $AF = 4AB$. Find the area of triangle DEF.



(SMOPS 2016 Q. 17)

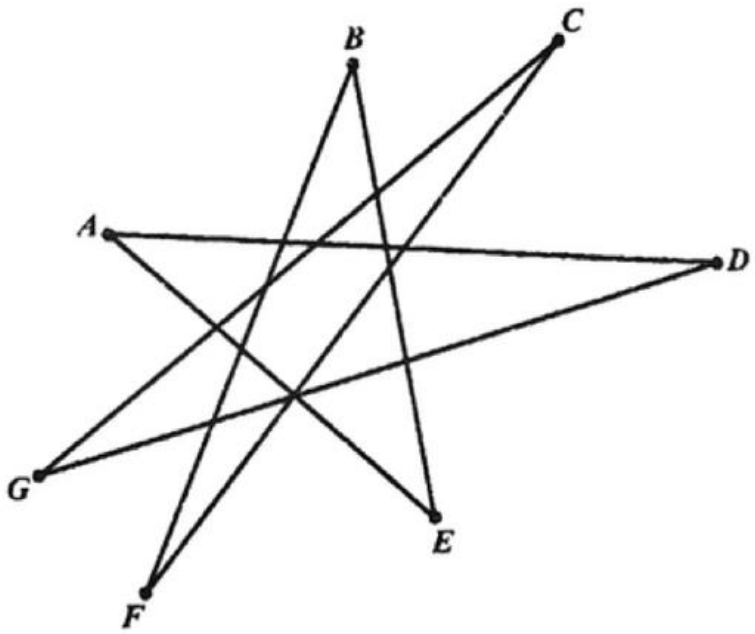
18. Six 10-cent coins have the same height as five 20-cent coins when stacked on top of each other. Four 10-cent coins have the same height as three 50-cent coins when stacked on top of each other. Using 10-cent, 20-cent and 50-cent coins to stack into three stacks of equal height, a total of 124 coins have been used. What is the total value of these coins?

(SMOPS 2016 Q. 18)

19. What is the remainder when $3^{2016}+2$ is divided by 11?

(SMOPS 2016 Q. 19)

20. In the figure below, find the value of $\angle A+\angle B+\angle C+\angle D+\angle E+\angle F+\angle G$.



(SMOPS 2016 Q. 20)

21. There are three football teams *A*, *B* and *C*.
A scored 2 goals and had 1 goal against them.
B scored 1 goal and had 2 goals against them.
C scored 3 goals and had 3 goals against them.
What was the result of the game between team *A* and team *C*?
(A) 1 – 0 (B) 0 – 0 (C) 2 – 0 (D) 2 – 1 (E) 1 – 1

(SMOPS 2016 Q. 21)

22. Let *N* be a whole number $\overline{abc}$.
Given that $N=\overline{abc}=\overline{ac}+\overline{ba}+\overline{ca}+\overline{ab}+\overline{cb}+\overline{bc}$, find the largest possible value of *N*.

(SMOPS 2016 Q. 22)

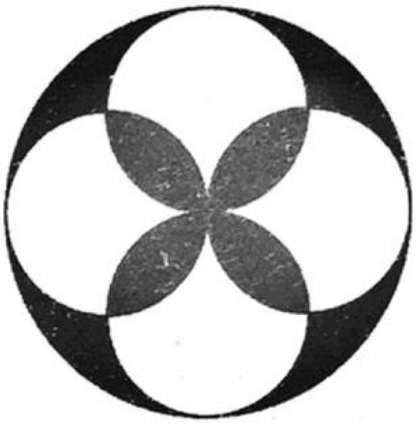
23. Steve lives in Town *A* and has to travel by bus to Town *B* which is 60 km away. There are two types of bus services. The small bus will stop at each 2 km mark along the route and charges \$3 for every 2 km travelled. The big bus will stop at each 3 km mark along the route and charges \$4 for every 3 km travelled. If Steve had to alight for a short while at the stops 4 km, 21 km, 33 km and 44 km away from Town *A*, what is the minimum amount of money, in dollars, he has to spend on the bus fare for the entire trip from Town *A* to Town *B*?

(SMOPS 2016 Q. 23)

24. Find the smallest positive whole number whose sum of digits is equal to 35, last two digits is 35 and is divisible by 35.

(SMOPS 2016 Q. 24)

25. In the diagram below, the radius of the largest circle is 7 cm. Find the area of the shaded region. Take $\pi = \frac{22}{7}$.

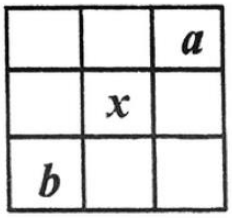


(SMOPS 2016 Q. 25)

26. A computer programme arranges all six letter words formed from the letters *A*, *P*, *M*, *O*, *P*, *S* in alphabetical order, resulting in the following sequence: *AMOPPS*, *AMOPSP*, *AMOSPP*, *AMPOPS*, …, *SPPMOA*, *SPPOAM*, *SPPOMA*. Find the position of *POAMSP*.

(SMOPS 2016 Q. 26)

27. In the 3×3 grid below, each box is occupied by the numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 respectively, such that any two consecutive numbers must be arranged in adjacent boxes. i.e. 1 and 2 are adjacent, 2 and 3 are adjacent, …, 8 and 9 are adjacent.



Given that the sum of *a* and *b*, as shown in the diagram, is equal to 12, find the value of *x*.

(SMOPS 2016 Q. 27)

28. The ticket prices for a charity concert are as such: $23 for adults, $15 for students and $7 for children. A total of 136 people bought tickets and $2016 was collected in all. Given that among the different categories, the number of children are the most and the number of adults are the least. Find the number of student tickets sold.

(SMOPS 2016 Q. 28)

29. There are five consecutive two digit numbers. Three of these numbers have a sum that is divisible by 37. Three of these numbers have a sum that is divisible by 60. Find the largest two digit number.

(SMOPS 2016 Q. 29)

30. There are 600 pupils sitting for an examination. The average score for a test is 84. The average score for the boys is 80. The average score for the girls is 92. How many girls participated in the examination?

(SMOPS 2016 Q. 30)

Number of correct answers for Q1 to Q10 : _____ Marks (×4) : _____
Number of correct answers for Q11 to Q20 : _____ Marks (×5) : _____
Number of correct answers for Q21 to Q30 : _____ Marks (×6) : _____

Answers:

SMOPS 2016					
1	3024	11	900	21	(D)
2	(B)	12	178	22	396
3	1609	13	4033	23	82
4	(B)	14	20	24	289835
5	3	15	11	25	56
6	6	16	60	26	230
7	3	17	18	27	7
8	180	18	$30.80	28	45
9	17	19	5	29	63
10	120	20	180	30	200