Instructions

This is a 3-hour paper with a maximum possible mark of 50, split across four problem tasks.

You will need to use a computer to view this question paper, and software to perform analysis on the provided data files. All answers are to be written in dark blue or black ink on the hardcopy answer sheets provided, with sufficiently detailed working shown.

At the end of the exam, you will be given time to upload a scan of your answers, and the hardcopy answer sheets should be submitted to your teacher. When uploading to LumiNUS, you should upload a single PDF file named *NNN*.pdf, where *NNN* is your SPhO 2021 index number.

Note: There is no need to indicate experimental uncertainty unless otherwise explicitly specified by the questions.

Introduction

Smartphones usually have an embedded accelerometer in them. By convention, the screen faces the positive z-direction, and the positive x- and y-directions are towards the “right” and “up” when looking at the phone in portrait mode, as indicated in the **Figure 1**.

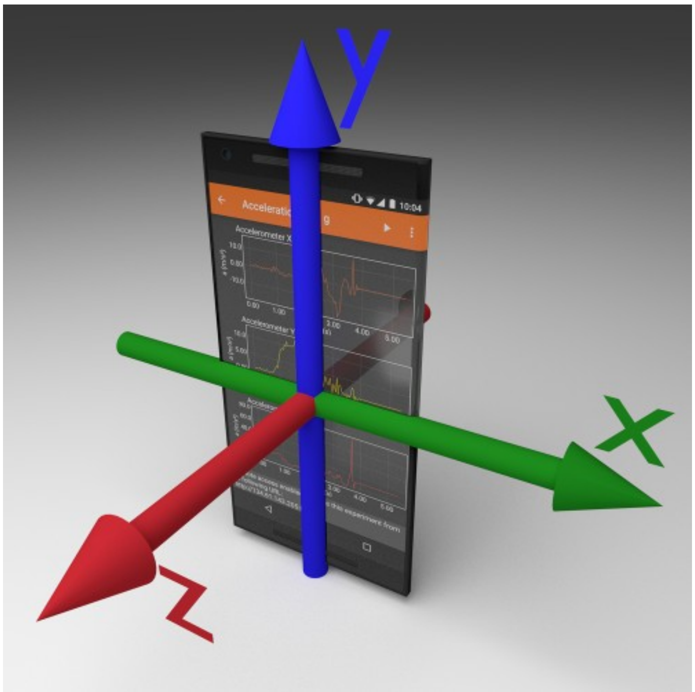


Figure 1: Conventional orientation axes for smartphone sensors

The accelerometer works by sensing the force on a mass (see **Figures 2–4** for further detail). For the purposes of this problem, assume that the accelerometer is perfectly calibrated.

Note that if the smartphone is placed, with its screen facing up, on a horizontal surface, the accelerometer would register an (upward) acceleration in the positive z-direction. This is because the downward force of gravity is indistinguishable from an upward acceleration.

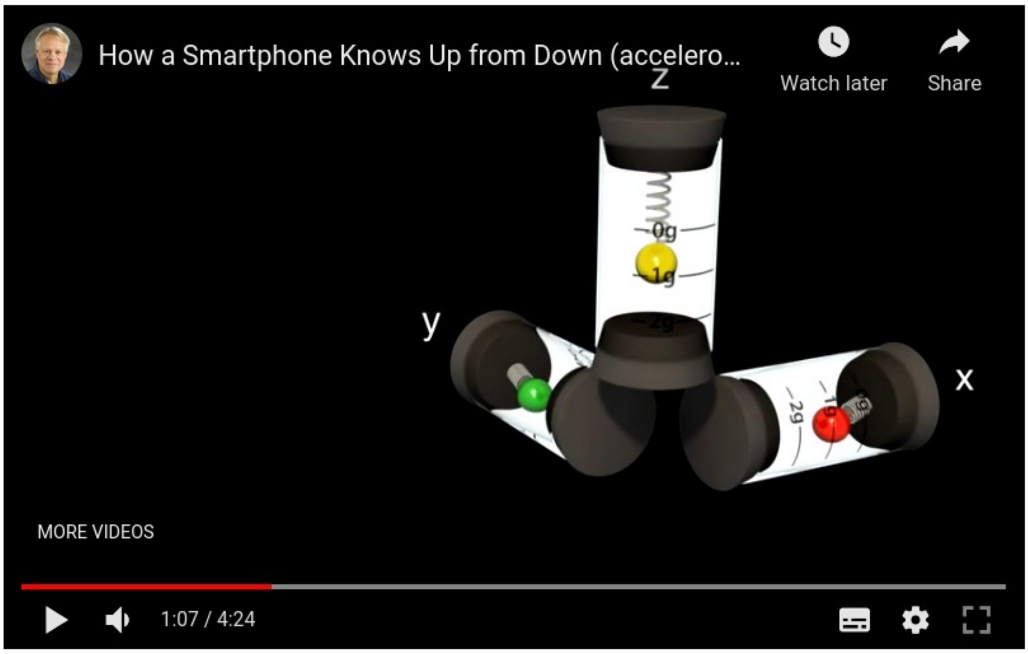


Figure 2: Mechanical spring-mass principle of an accelerometer

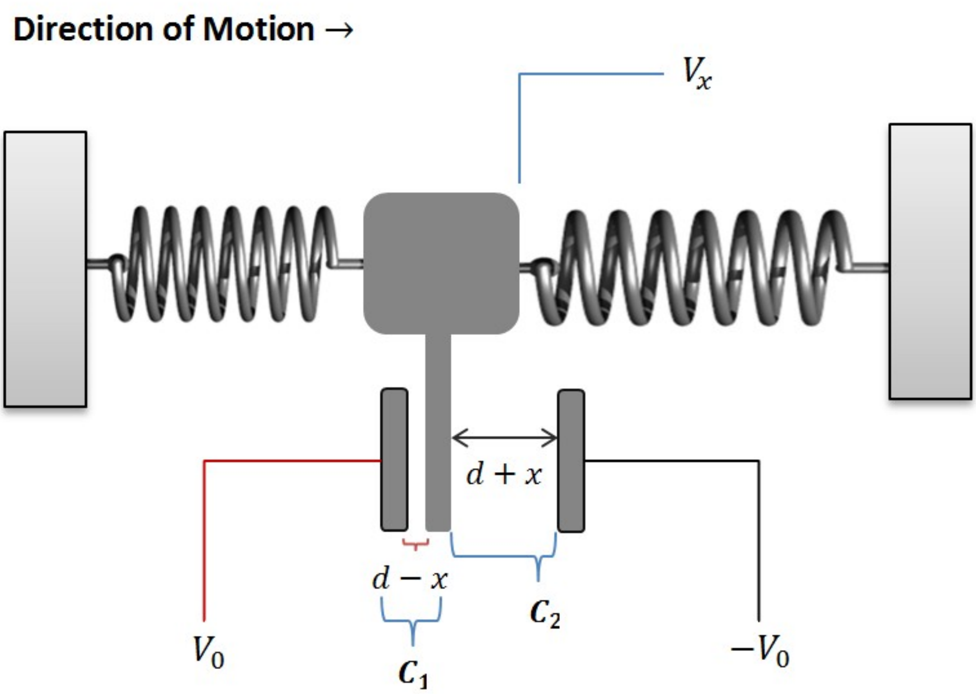


Figure 3: Concept of a capacitative micro-electromechanical system (MEMS) accelerometer

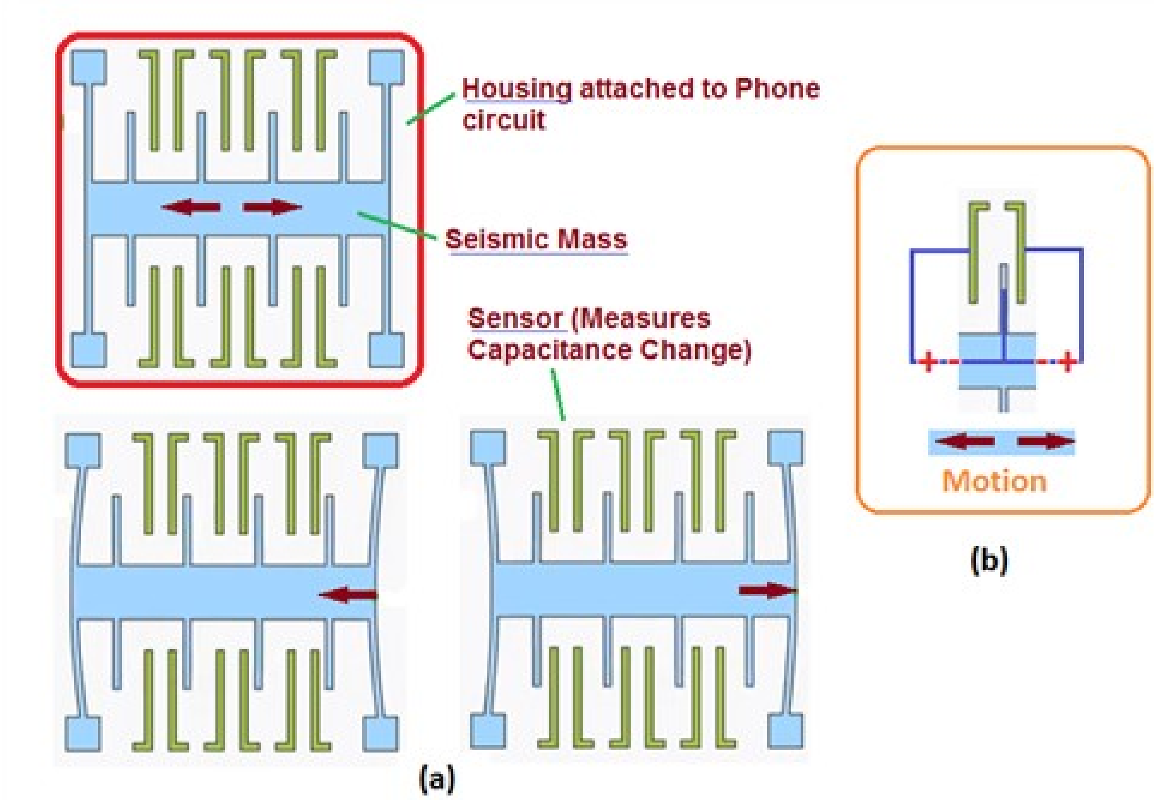


Figure 4: Schematic of a capacitative MEMS accelerometer

Task 1: Free-fall [7 marks]

A smartphone is held up and released to fall freely onto a bed.

Use the data file **task1.csv** for this problem.

(a) Choose one option: The **(+x / -x / +y / -y / +z / -z)** axis of the smartphone is initially pointing vertically upwards. **[1]**

(b) Write down t_0, the time when the phone was released, with its associated uncertainty. **[1]**

(c) Write down t_1, the time when the phone hit the bed, with its associated uncertainty. **[1]**

(d) Estimate the time of flight $T = t_1 - t_0$, with its associated uncertainty. **[1]**

(e) Estimate the height H which the phone fell through, with its associated uncertainty. **[3]**

Task 2: Ramp [13 marks]

A smartphone is placed with its screen up on a horizontal surface. The surface is then gradually inclined until the smartphone begins to slide down – the ramp is held approximately steady as the smartphone slides and then hits the ground.

Use the data files **task2-1.csv** and **task2-2.csv** for this problem.

(a) Using the data in **task2-1.csv**,
  (i) Estimate θ_1, the angle of the ramp (relative to the horizontal) at which the smartphone begins to slip. **[2]**
  (ii) Estimate μ_1, the coefficient of static friction between the ramp and the smartphone (i.e. assume static friction has a magnitude $F \leq \mu N$, where N is the magnitude of the normal contact force). **[2]**

(b) Using the data in **task2-2.csv**,
  (i) Estimate θ_2, the angle of the ramp (relative to the horizontal) at which the smartphone begins to slip. **[2]**
  (ii) Estimate μ_2, the coefficient of static friction between the ramp and the smartphone (i.e. assume static friction has a magnitude $F \leq \mu N$, where N is the magnitude of the normal contact force). **[2]**

(c) The two sets of data were obtained using the same smartphone and ramp setup, but the silicone case of the smartphone, which provides a degree of shock absorption, was put on in one instance and taken off in the other.
  (i) Which do you think is the dataset where the case was taken off, and why? **[2]**
  (ii) For which dataset was the phone travelling faster, and why? **[3]**

Task 3: Centre of gravity [13 marks]

A smartphone is freely suspended from each of its four corners in turn. Use the four sets of accelerometer data to estimate the location of the centre of gravity.

Use the data files **task3-TL.csv**, **task3-TR.csv**, **task3-BR.csv**, and **task3-BL.csv** for this problem. These were obtained when the smartphone was suspended from the respective corner, labelled with the screen facing towards us as top left (TL), top right (TR), bottom right (BR), and bottom left (BL).

(a) Mark out the centre of gravity in the scale drawing provided in the answer sheet. Make sure to explain your method and to show all necessary working. **[10]**

(b) Briefly discuss what other physical properties of the smartphone might be obtainable from these datasets (i.e. *only* using **task3-TL.csv**, **task3-TR.csv**, **task3-BR.csv**, and **task3-BL.csv**). You should provide mathematical expressions if possible but there is no need to determine actual values. **[3]**

Task 4: Locating the accelerometer [17 marks]

The location of the accelerometer within the smartphone is not important for characterising translational motion. However, for rotational motion, the accelerometer location (in the x-y plane) will affect the data obtained, especially since smartphones have large screens.

In this problem, we will use a geometrical method to determine this accelerometer position. A horizontal rectangular frame is mounted so that its centre coincides with the centre of a turntable underneath it, as shown in **Figure 5**.

The smartphone can be placed (screen facing upwards) against any of the four corners of the rectangle, which are labelled as quadrants 1, 2, 3, and 4 according to the usual convention in the x-y plane, as shown in **Figure 6**. In quadrant 1, the top right (TR) corner of the smartphone is flush with the top right corner of the rectangular frame; likewise for TL in quadrant 2, BL in quadrant 3, and BR in quadrant 4.

As shown in **Figure 7**, measured in the rectangular frame, with the origin at the centre of the rectangle, we let the (unknown) position of the accelerometer $\vec{R}$ be given by

$$\vec{R}=\vec{R}_d+\vec{r} \quad ,$$

where $\vec{R}_d$ is the position vector for the bottom left (BL) corner of the smartphone, and $\vec{r}$ is the position of the accelerometer measured relative to the BL corner of the smartphone.

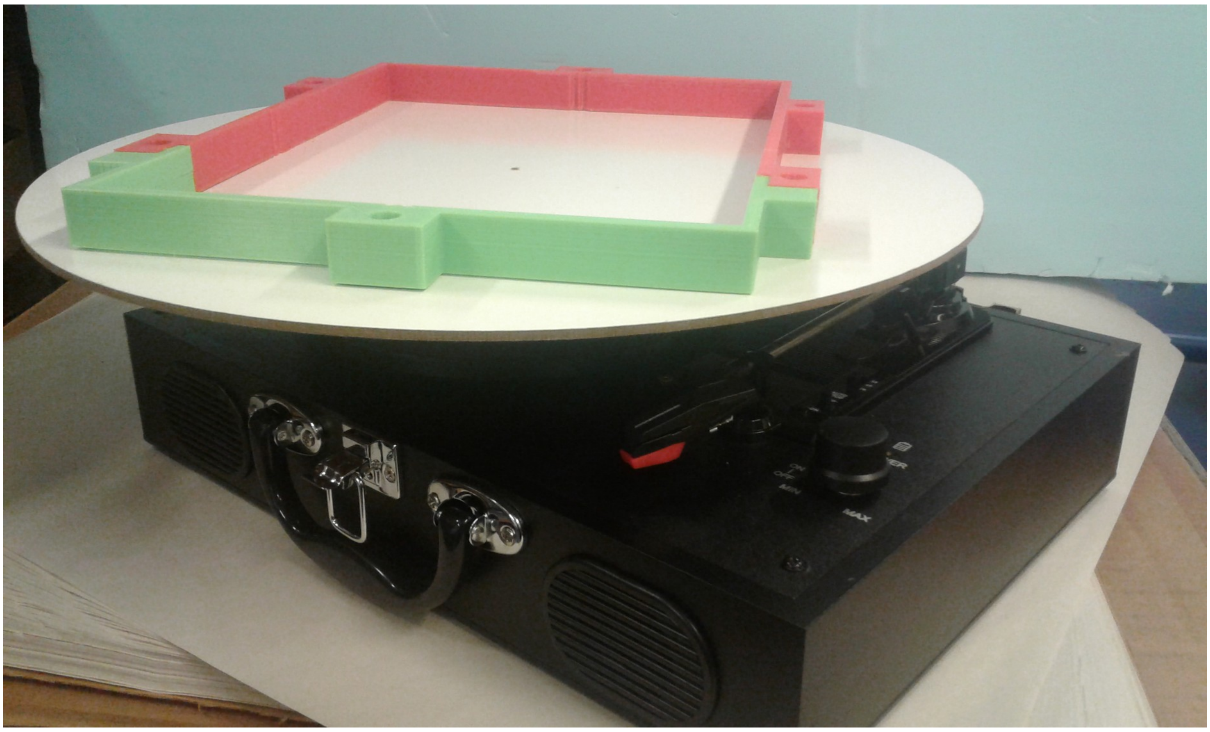


Figure 5: The “SpinFrame” apparatus: a 3D-printed frame connected to a standard turntable via a 3D-printed fitting (the fitting is not visible here).

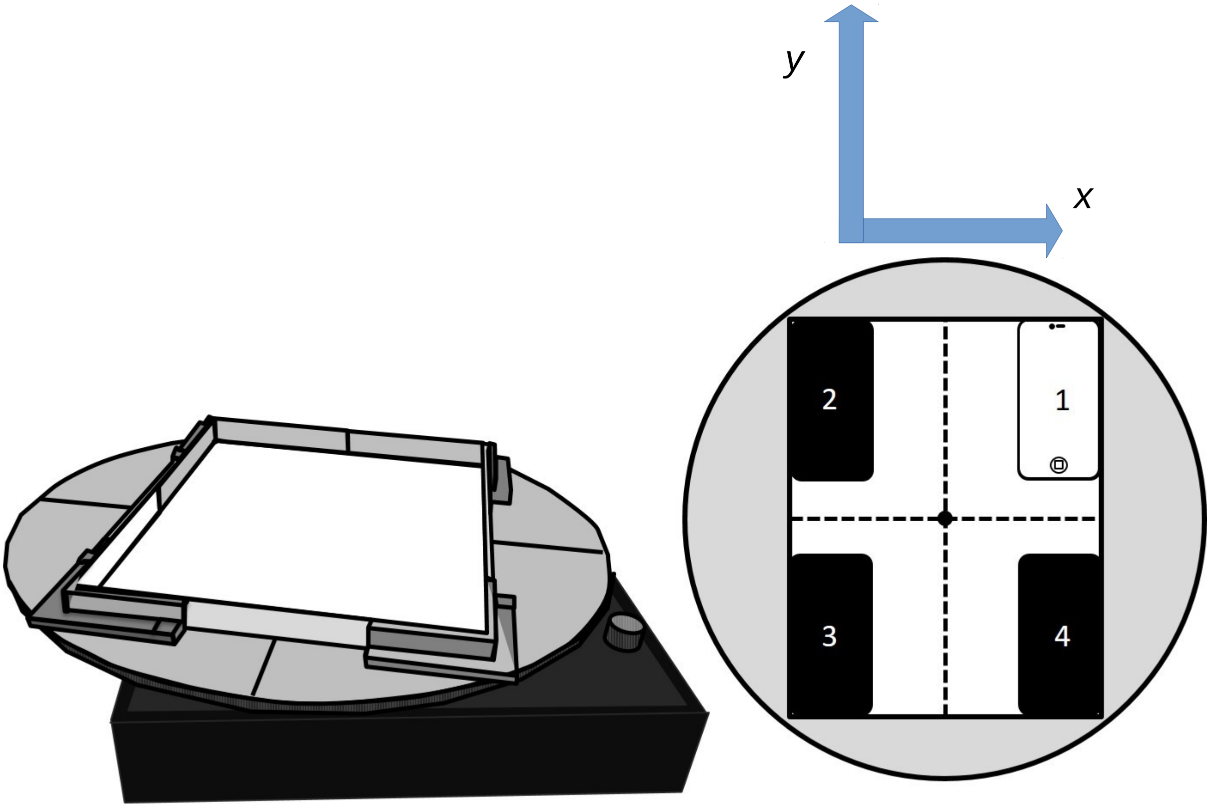


Figure 6: A 16-inch circular disk (left) with a 8.5-inch by 11-inch frame centered upon it. The disk is fitted to a conventional turntable configured to rotate quickly. The four corner positions of the frame (right) used for obtaining estimates of the intra-device accelerometer position are indicated. The device is always placed flushed against the corners in a consistent orientation regardless of the quadrant.

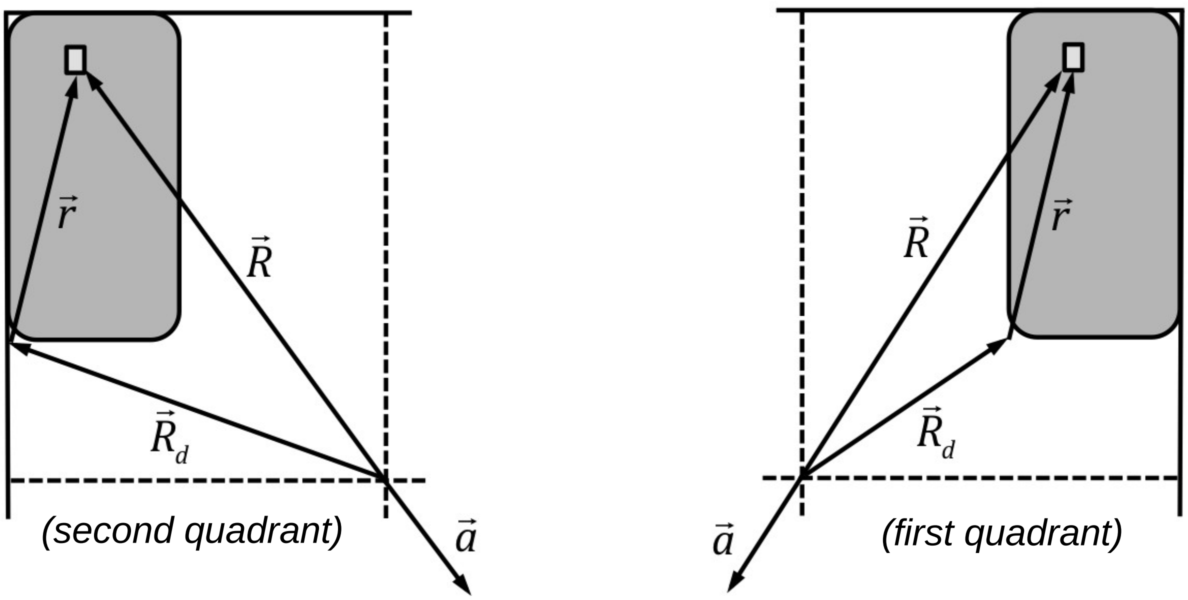


Figure 7: Vector relationships for a host device positioned in the first and second quadrants of the rotating coordinate system.

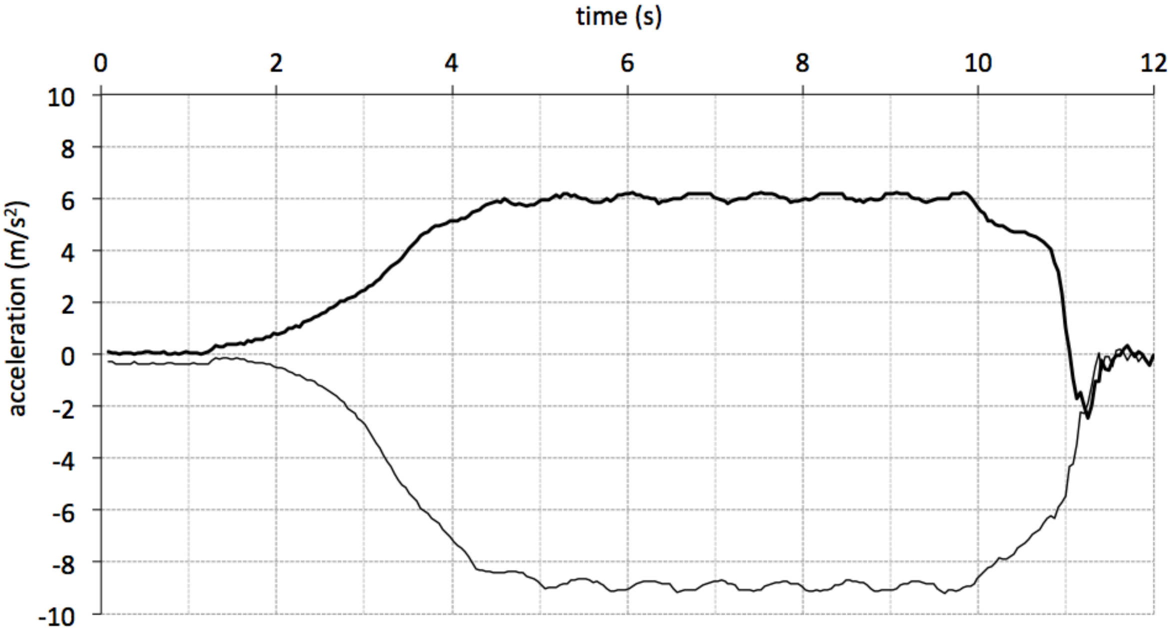


Figure 8: Acceleration components as a function of time for the device. The heavier upper line is a_x and the lower thinner line is a_y. The device was placed in the second quadrant (see Fig. 6), for which a centrally-pointing acceleration vector would necessarily have a positive x-component and a negative y-component.

(a) Denoting the angular speed of the turntable and rectangular frame as Ω,

(i) Write down an expression for the acceleration $\vec{a}$ measured by the accelerometer in terms of $\vec{R}$. **[1]**

(ii) Using the data in **Figure 8** for the device placed in quadrant 2, deduce that Ω is a rate of approximately 78 revolutions per minute. **[2]**

(iii) Hence, use the data in **Figure 8** to find the radial distance $R = |\vec{R}|$ of the accelerometer when placed in quadrant 2. **[2]**

(b) Some data at the same angular speed for the same device placed in various quadrants is provided in **Table I**. The inner width of the rectangular frame (x-direction) is 21.6 cm and the inner height of the rectangular frame (y-direction) is 27.9 cm.

(i) Deduce the device dimensions in the x-direction and the y-direction. **[2]**

(ii) Complete the missing cells in the copy of **Table I** In the answer sheet. **[8]**

(iii) Write down the position of the accelerometer (measured from the BL corner of the device) along with its associated uncertainty. **[2]**

quadrant	$\vec{R}_d$ **(cm, cm)**	$\vec{a}$ **(ms^{-2}, ms^{-2})**	$\vec{R}$ **(cm, cm)**	$\vec{r}$ **(cm, cm)**
1	(3.2, 11.0)	(-3.8, -9.0)		
2	(-10.8, 11.0)			
3	(-10.8, -14.0)	(5.9, 8.3)		
4	(3.2, -14.0)	(-3.8, 8.5)		

Table I: Results for the same device. Vectors are presented as ordered pairs (*x*, *y*).

Name: **SPhO 2021 Index Number:** ________

School:

Answer Sheets

Task 1

1(a) Vertically upwards: __________________

[1]

1(b) t_0 = (±) s

[1]

1(c) t_1 = (±) s

[1]

1(d) T = (±) s

[1]

1(e) H = (±) m

[3]

working for 1(e):

Task 2

2(a)(i) θ_1 =

[2]

2(a)(ii) μ_1 =

[2]

working for 2(a)(i) and 2(a)(ii):

2(b)(i) θ_2 =

[2]

2(b)(ii) μ_2 =

[2]

working for 2(b)(i) and 2(b)(ii):

2(c)(i) Dataset where phone case was *off*: ______________________________ [2]

explanation for 2(c)(i):

2(c)(ii) Dataset where phone travelled *faster*: ______________________________ [3]

explanation for 2(c)(ii):

Task 3

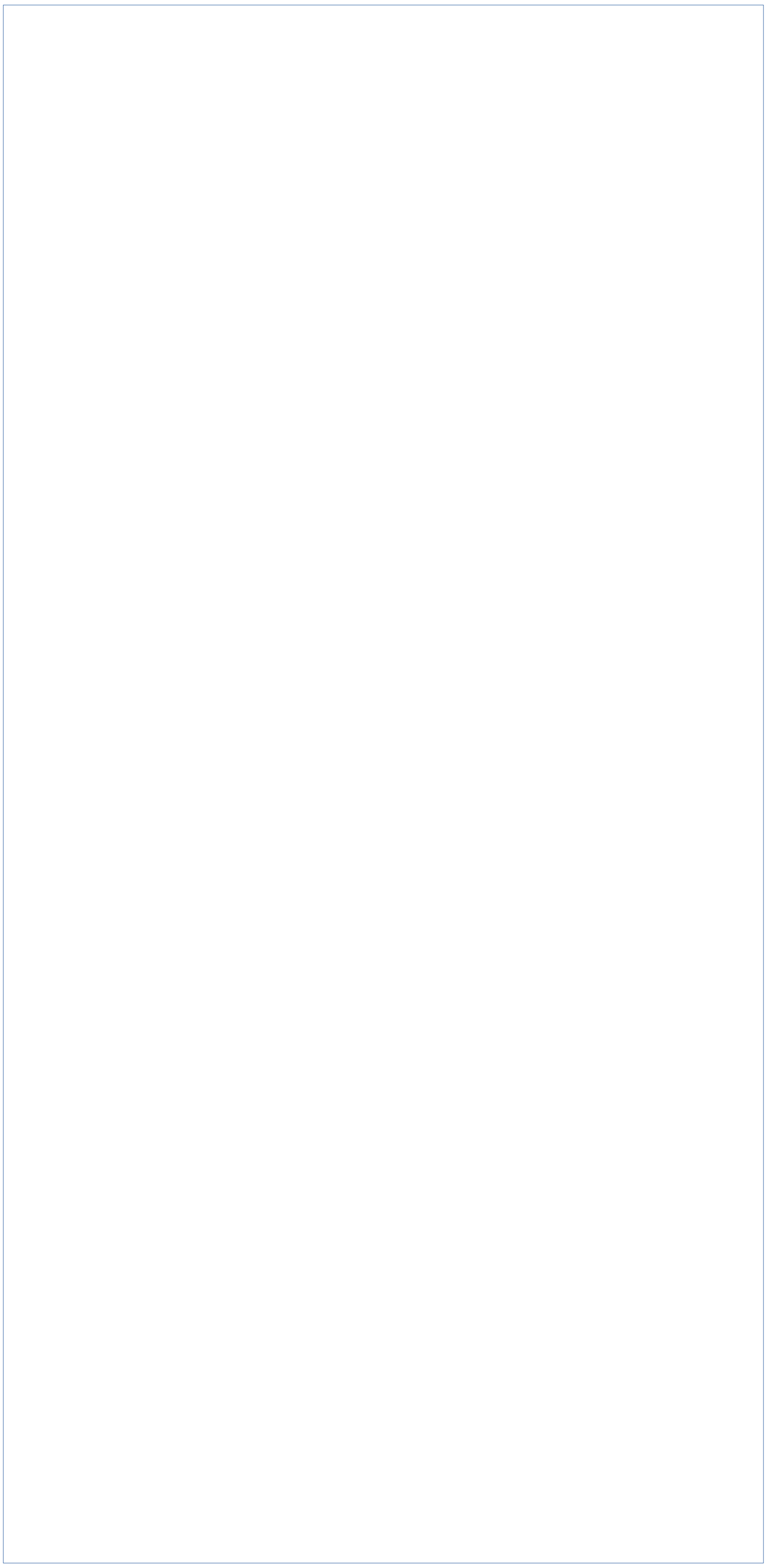
3(a) Explain and describe your method, record detailed working, and make markings on the next page.

[10]

answer for 3(a):

Mark out the location of the centre of gravity using this outline of the smartphone (screen facing up). Corners are labelled top left (TL), top right (TR), bottom right (BR), and bottom left (BL).

TL TR



BL BR

3(b) Discuss with suitable equations and notation. There is no need to determine actual values.

[3]

answer for 3(b):

Task 4

4(a)(i) $\vec{a}$ =

[1]

4(a)(ii) Show method and working:

[2]

4(a)(iii) $R = |\vec{R}|$ =

[2]

working for 4(a)(iii):

4(b)(i) device dimension (x-direction) =

device dimension (y-direction) =

[2]

working for 4(b)(i):

4(b)(ii) Complete the table and show working below.

[8]

quadrant	$\vec{R}_d$ **(cm, cm)**	$\vec{a}$ **(ms^{-2}, ms^{-2})**	$\vec{R}$ **(cm, cm)**	$\vec{r}$ **(cm, cm)**
1	(3.2, 11.0)	(-3.8, -9.0)		
2	(-10.8, 11.0)			
3	(-10.8, -14.0)	(5.9, 8.3)		
4	(3.2, -14.0)	(-3.8, 8.5)		

Table I: Results for the same device. Vectors are presented as ordered pairs (*x*, *y*).

working for 4(b)(ii):

4(b)(iii) $\vec{r} = (r_x, r_y)$

r_x = (±) cm

r_y = (±) cm

[2]

working for 4(b)(iii):

References

Task 4 is based on a couple of papers from physics education journals[1,2].

Figure 1 is taken from https://physique.ensc-rennes.fr/tp_pendules_texteV2.php

Figure 2 is a screengrab taken from Bill Hammack's (engineerguy) YouTube video https://www.youtube.com/watch?v=KZVgKu6v808

Figure 3 is taken from https://makersportal.com/blog/2017/9/25/accelerometer-on-an-elevator

Figure 4 is taken from https://www.quora.com/How-does-gravity-sensor-work-in-mobile-hardware-support

Figure 5 is taken from https://arxiv.org/abs/1903.11516

Figure 6 is taken from https://arxiv.org/abs/1903.10284

Figure 7 is taken from https://arxiv.org/abs/1903.10284

1 Christopher Isaac Larnder & Brian Larade, "On the determination of accelerometer positions within host devices", American Journal of Physics 87, 130-135 (2019) https://doi.org/10.1119/1.5082536
2 Chris Isaac Larnder, "A Purely Geometrical Method of Locating a Smartphone Accelerometer", The Physics Teacher 58, 52-54 (2020) https://doi.org/10.1119/1.5141974