Name of Participant : ______________________________ (Statutory Name)

Index No : ________ / _____ Name of School : ______________________________

Date Of Birth : ______________________________ (DD/MM/YY)

19 April 2008 **0900 h — 1100 h**

Hwa Chong Institution
Mathematics Learning And Research Centre

SMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPS
SMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPS
SMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPS
SMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPSSMOPS

Singapore Mathematical Olympiad for Primary Schools 2008

First Round
2 hours
(150 marks)

Instructions to Participants :

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Shade your answers on the OMR form.
Working may be done in the space below each question.
Marks are awarded for correct answers only.

This question paper consists of 16 printed pages (including this page)

Number of correct answers for Q1 to Q10 : ____________ *Marks (x 4) :* ____________
Number of correct answers for Q11 to Q20 : ____________ *Marks (x 5) :* ____________
Number of correct answers for Q21 to Q30 : ____________ *Marks (x 6) :* ____________

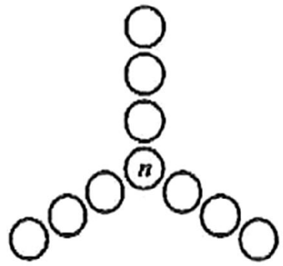
1 What is the difference between the sum of the first 2008 even numbers and the sum of the first 2008 odd numbers?

2 The sides of a triangle have lengths that are consecutive whole numbers and its perimeter is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

3 Find the value of 2008 x 20072007 – 2007 x 20082007.

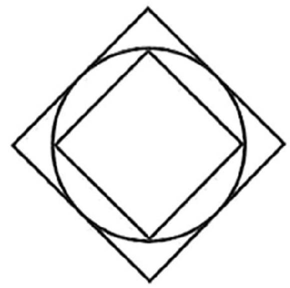
4 When a rectangular sheet of paper with length 8 cm is folded exactly into half, the ratio of its length to its width remains unchanged. If the square of the width of the original piece of paper = x cm^2, find the value of x.

5 The numbers 1 to 10 are arranged in the circles in such a way that the sum of the four numbers on each line is 21. What is the value if n?



6 Find the value of $(56789 + 67895 + 78956 + 89567 + 95678) \div 5$

7 The diagram shows a circle whose circumference touches the sides and the vertices of a large and a small square respectively. If the area of the small square is 9 cm^2 and the area of the large square is x cm^2, find the value of x.



8 One hundred numbers are placed along the circumference of a circle. When any five adjacent numbers are added, the total is always 40. Find the difference between the largest and the smallest of these numbers.

9 In triangle PQR, PQ = 6 cm, PR = 4 cm and QR = 6 cm. If sides PQ and PR are tripled while QR remains unchanged, then

(1) the area is tripled.

(2) the area increases by 9 times.

(3) the altitude is tripled.

(4) the area decreases to 0 cm^2.

(5) none of the above.

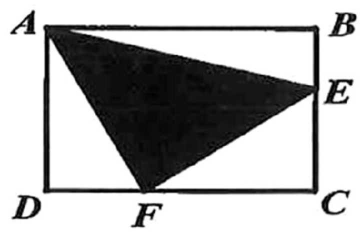
10 Find the last 5 digits of the sum

$1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777 + 88888888 + 999999999$.

11 If an arc of 80° on circle *A* has the same length as an arc of 60° on circle *B*, and that the ratio of the area of circle *A* to the area of circle *B* is a : b, find the smallest value of a + b.

12 A circle of circumference 1 m rolls around the equilateral triangle of perimeter 3 m. How many turns does the circle make as it rolls around the triangle once without slipping?

13 The diagram shows a rectangle *ABCD* with area 32 cm^2. Given that area of triangle *ADF* = 2 cm^2, area of triangle *ABE* = 8 cm^2 and area of the shaded region = x cm^2, find the value of x.



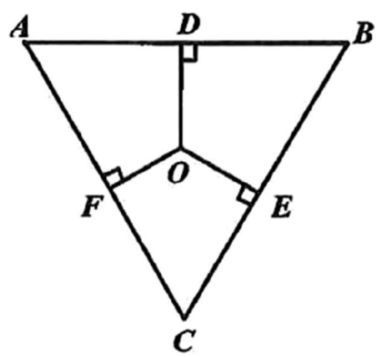
14 There are two containers A and B. Each of them contains 9 white marbles, 9 black marbles and 9 red marbles. If 10 marbles are removed from A and placed into B, how many marbles must be returned from B to A to make sure that there are at least 8 marbles of each colour in A?

15 9^{10} is a 10-digit number. If **A** is the sum of all digits of 9^{10}, **B** is the sum of all digits of **A** and **C** is the sum of all digits of **B**, find the value of **C**.

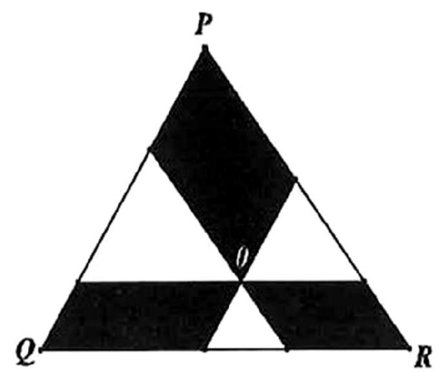
16 A car travels from point A to B at a constant speed of V km/h. If the car increases its speed by 20%, it will reach B one hour earlier. If the car increases its speed by 25% after traveling at V km/h for 120 km, it will reach B forty eight minutes earlier. If the distance between the two towns is x km, find the value of x.

17 After all the faces of a rectangular block are painted green, the block is cut into unit cubes each of volume 1 cm^3. It is found that 7 of the unit cubes have none of their faces painted green. How many of the unit cubes have exactly two faces painted green?

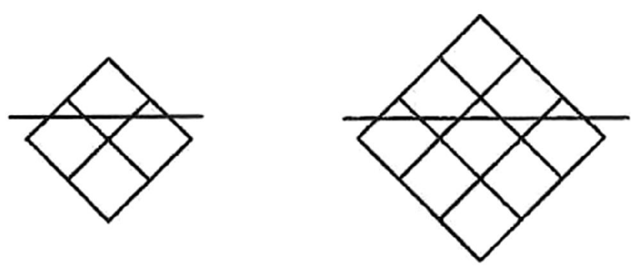
18 The diagram shows an equilateral triangle *ABC* with *OD, OE* and *OF* perpendicular to *AB, BC* and *CA* respectively. If $OD + OE + OF = 28$ cm and the height of triangle $ABC = x$ cm, find the value of x.



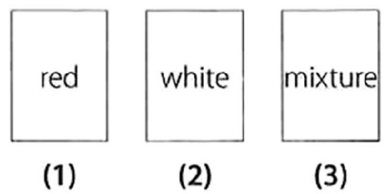
19 The diagram shows a triangle *PQR*. Three lines parallel to the sides of the triangle are drawn through a point *O*. Given that the areas of the three shaded triangles are 32 cm^2, 48 cm^2 and 96 cm^2 respectively, and the area of the triangle $PQR = x$ cm^2, find the value of x.



20 As shown in the diagram, a straight line can cut across at most 3 squares in a 2 by 2 square and at most 5 squares in a 3 by 3 square. What is the greatest number of squares that can be cut across by a straight line in a 2008 by 2008 square?

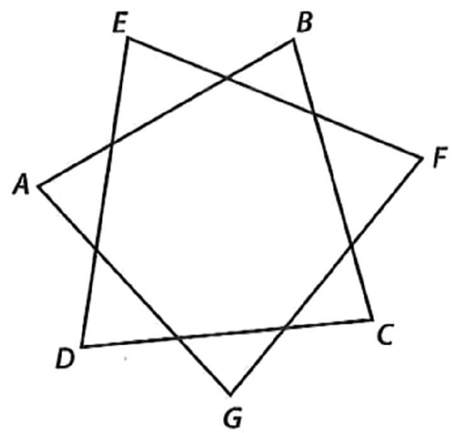


21 There are three containers. One contains red marbles, another white marbles and the third one a mixture of red and white marbles. Given that all of them are labelled wrongly and you are allowed to open only one of them to take out only one marble in order to state correctly where all the labels ought to go, which container should you open?



22 Given that $\underbrace{200820082008\ldots2008}_{n \text{ of } 2008}623$, find the smallest value of n such that the number is divisble by 11.

23 Given that $\angle ABC + \angle BCD + \angle CDE + \angle DEF + \angle EFG + \angle FGA + \angle GAB = x^0$, find the value of x.



24 Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner. If the whole number is 29, and Peter is the first player, what perfect number must he subtract in order for him to definitely win.

[*Note: 4, 9 and 16 are examples of perfect squares.*]

25 The inhabitants of an island are either gentlemen or liars. A gentleman always tells the truth and a liar always lies. A, B and C are three of the inhabitants. A sailor who landed on the island asked A: "Are you a gentleman or a liar?" A answered but the sailor could not hear clearly what he said. He then asked B, "What did A say?" B replied, "A said that he is a liar." At that instant, C immediately shouted "B is lying!"

I It is impossible to tell whether A is a gentlemen or a liar.
II B is a gentlemen and C is a liar.
III B is a liar and C is a gentlemen.

(1) Only I is true.
(2) Only II is true.
(3) Only III is true.
(4) Only I and II are true.
(5) Only I and III are true.

26 The product of n whole numbers $1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times (n-1) \times n$ has twenty eight consecutive zeros. Find the largest value of n.

27 Find the largest number n such that there is only one whole number k that satisfies

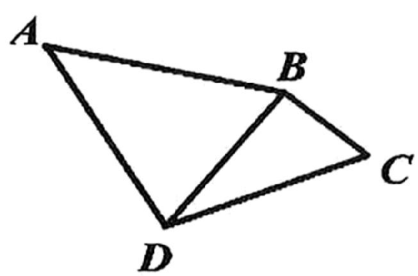
$$\frac{8}{21} < \frac{n}{n+k} < \frac{5}{13}$$

[Note: A < C < B means that value of C is between A and B, example 4 < 9 < 16]

28 How many ways are there to distribute 28 identical marbles into 3 different boxes such that no box is empty?

29 If Peter walks up an up-going escalator at the rate of 1 step per second, he is able to reach the top in 10 steps. If he increases his rate to 2 steps per second, he can reach the top in 16 steps. Find the number of steps of the escalator.

30 The diagram shows a quadrilateral ABCD. If AB = CD, angle ADB + angle CBD = 180°, angle BCD = 55° and angle BAD = x°, find the value of x.



END OF PAPER

Singapore Mathematical Olympiad for Primary Schools 2008 First Round – Answers Keys					Total Marks 150
Question	**Answers**	**For markers' use only**	**Question**	**Answers**	**For markers' use only**
	Questions 1 to 10 Each carries 4 marks		16	360	
1	2008		17	36	
2	2010		18	28	
3	2007		19	288	
4	32		20	4015	
5	4			**Questions 21 to 30 Each carries 6 marks**	
6	77777		21	3	
7	18		22	3	
8	0		23	540	
9	4		24	9	
10	93685		25	5	
	Questions 11 to 20 Each carries 5 marks		26	124	
11	25		27	80	
12	4		28	351	
13	15		29	40	
14	27		30	55	
15	9				

FULL SOLUTIONS TO SELECTED QUESTIONS (FIRST ROUND)

1 2008
Hint : Consider the difference between the sum of the first 4 even numbers and the sum of the first 4 odd numbers.

2 $(a) + (a + 1) + (a + 2) > 2008 \Rightarrow 3a + 3 > 2008 \Rightarrow a > 668\frac{1}{3}$

The three sides are 669, 670,671, the least possible perimeter = 2010.

3 Let $x = 2007$
$2008 \times 20072007 - 2007 \times 20082007$
$= (x + 1) \times (10001x) - x\,(1000\,(x + 1) + x)$
$= x$
$= 2007$

4 *Let length* = 8 = 2*L*, *width* = *w*
$\frac{length}{width} = \frac{2L}{w} = \frac{w}{L} \Rightarrow w^2 = 2L^2 = 2(4)^2 = 32$

5 4

6 $(56789 + 67895 + 78956 + 89567 + 95678) \div 5$
$= (350000 + 35000 + 3500 + 350 + 35) \div 5$
$= 77777$

7 Rotate the inner smaller square about the centre by 90 degrees, you will notice that the size of the small square is exactly half that of the large square.
Therefore area of the large square is 18 cm^2.

8 $n_1 + n_2 + n_3 + n_4 + n_5 = 40$

$n_2 + n_3 + n_4 + n_5 + n_6 = 40$

$\Rightarrow n_1 = n_6 = n_{11} = \ldots = n_{886}$

$n_{885} + n_{886} + n_{887} + n_{888} + n_1 = 40$

$n_{886} + n_{887} + n_{888} + n_1 + n_2 = 40$

$\therefore n_1 = n_2$

They are all 8s. Therefore answer is 0.

9 Option (4).

In the new triangle, PQ = PR + QR, that is R lies on the line PQ.

The height from R is zero, therefore area of the triangle is zero.

10 1,000,000,000 (= 1 + 999,999,999)

88, 888,910 (= 22 + 88,888,888)

7,778,110 (= 333 + 7,777,777)

671,110 (= 4,444 + 666,666)

55,555

1097393685 (Total)

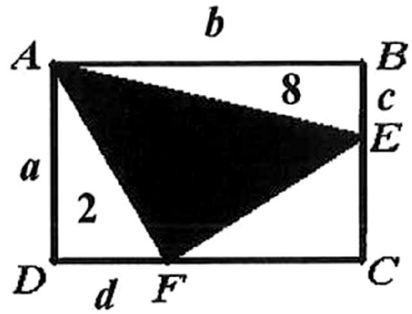
11 $\frac{80}{360} \times 2 \times \pi \times r_A = \frac{60}{360} \times 2 \times \pi \times r_B \Rightarrow \frac{r_A}{r_B} = \frac{60}{80} = \frac{3}{4}$

$\frac{a}{b} = \frac{\pi r_A^2}{\pi r_A^2} = \frac{9}{16}$

$\therefore a + b = 25$

12 Four turns. One -third turn at each of the vertices plus one turn for each side of the triangle.

13



Area of triangle CEF $= \frac{1}{2}(a-c)(b-d)$

$= \frac{1}{2}(ab - ad - bc + cd)$

$= \frac{1}{2}(32 - 4 - 16 + \frac{4}{a} \times \frac{16}{b})$

$= 7$

Area of shaded region $= 32 - 2 - 8 - 7 = 15$ cm^2

14 27

15 The sum of all the digits of a multiple of 9 is also a multiple of 9. Therefore **A** must be equal or less than 9 x 10 = 90, **B** must be equal or less than 8 + 9 = 17, **C** must be 9.

16 360

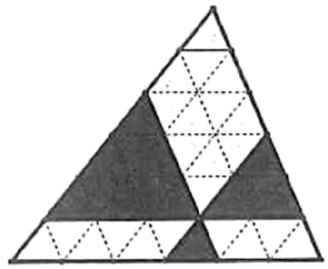
17 36

18 $\frac{1}{2} \times b \times h = \frac{1}{2} \times b \times OD + \frac{1}{2} \times b \times OE + \frac{1}{2} \times b \times OF$

$= \frac{1}{2} \times b \times (OD + OE + OF)$

Therefore $h = (OD + OE + OF) = 28$.

19



Therefore area of the triangle $PQR = 288\text{ cm}^2$.

20 For a 1 by 1 square, maximum number of squares that can be crossed
$= 1 \times 2 - 1 = 1$

For a 2 by 2 square, maximum number of squares that can be crossed
$= 2 \times 2 - 1 = 3$

For a 3 by 3 square, maximum number of squares that can be crossed
$= 3 \times 2 - 1 = 5$

For a 2008 by 2008 square, maximum number of squares that can be crossed
$= 2008 \times 2 - 1 = 4015$

21 The container labeled "mixture", option (3).
If a red marble is selected from the "mixture" container, since it is wrongly labelled, this container must contain only red marbles. The one labelled "white" must contain a mixture of red and white marbles while the one labelled "red" must contain only white marbles.
The same argument applies if a white marble is selected from the container labelled "mixture".

22 $n = 3$
Sum of even digits $= 8n + 2$
Sum of odd digits $= 2n + 9$, Difference $= 6n - 7$,
smallest $n = 3$ for the difference to be divisible by 11.

23 $7 \times 180^0 - 2 \times 360^0 = 540^0$

24 9

25 Option (5).

If <u>A</u> is a gentleman, <u>A</u> would tell the truth and say "I am a gentleman".
If <u>A</u> is a liar, <u>A</u> would lie and say " I am a gentleman".
Regardless of what <u>A</u> is, he could not have said "I am a liar."
Hence <u>B</u> is lying and <u>C</u> spoke the truth.
Lastly, it is impossible to tell what <u>A</u> is.

26 When 124 is divided by 5, we get quotient 24, remainder 4.
When 124 is divided by 25, we get quotient 4, remainder 24.
The largest value of n is 124.

27 Can simplify the inequality into $64n < 40k < 65n$.
The interval $64n$ to $65n$ contains $(n - 1)$ whole numbers.
If $n - 1 \geq 2 \times 40 = 80$, it will contain at least two multiples of 40.
Therefore if $n = 80$, we will get largest value of n for which there is only one multiple of 40.
Therefore $64 \times 80 < 40k < 65 \times 80$ which implies $128 \times 40 < 129 \times 40 < 130 \times 40$.
Hence $k = 129$.

28 Consider

$$1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=28$$

If we mark two of the positive signs, we can divide the left hand side
into 3 parts. For example

$$1+1+1+1 \ast 1+1+1+1+1+1+1 \ast 1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1+1=28$$

means that there are 4 , 7 and 17 marbles in boxes A, B and C respectively.

Now there are 27 ways to choose the first positive sign to mark.
Then we are left with 26 ways to choose the second positive sign to mark.
Since order is not important, there are $(27 \times 26) \div 2 = 351$ ways

29 Let the rate of the escalator be r steps per second..
In the 1st scenario, Peter covers **10 steps** in 10 seconds while the escalator covers **$10r$ steps**.
In the 2nd scenario, Peter covers **16 steps** in 8 seconds while the escalator covers **$8r$ steps**.
Number of steps of the escalator $= 10 + 10r = 16 + 8r$, we get $r = 3$.
Therefore the number of steps of the escalator $= 10 + 10(3) = 40$.

30 Reflecting triangle BCD in the line perpendicular to BD will give rise to the figure below. It is clear that triangle ABC' is isosceles and hence $\angle BAD$ is 55°.