

# A tumbling T-handle in space: the Dzhanibekov effect

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## §1 Introduction

In the 1980s, when cosmonaut Vladimir Dzhanibekov uncrewed a T-shaped nut which has three axes of rotation, he noticed a peculiar behavior: the nut spun and then flipped. An animation of this phenomenon can be found [here](#) (if the link is still accessible).

A team of mathematicians also independently noticed the effect in a tennis racket. They published their paper, named “The Twisting Tennis Racket”.

Formally, this is known as the intermediate axis theorem, which goes as follows.

### Theorem 1.1: Intermediate Axis Theorem

An object rotating about its intermediate axis with a very slight perturbation will undergo periodic flips in its orientation in the absence of external forces.

## §2 Proof

We first identify the three axes of rotation, as shown in fig. 1.

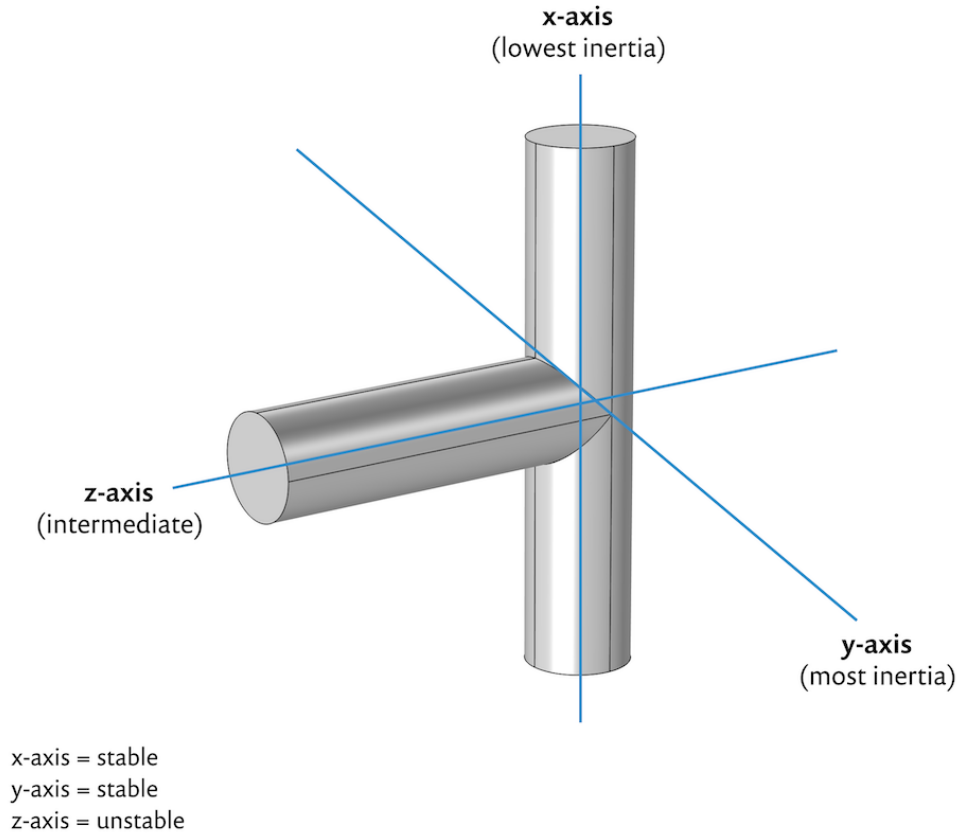


Figure 1: Axes of rotation [1]

The rotation of rigid bodies are described by **Euler's laws of motion**<sup>1</sup> [2], given by

$$\begin{aligned} I_1 \alpha_1 - (I_2 - I_3) \omega_2 \omega_3 &= \tau_1 \\ I_2 \alpha_2 - (I_3 - I_1) \omega_3 \omega_1 &= \tau_2 \\ I_3 \alpha_3 - (I_1 - I_2) \omega_1 \omega_2 &= \tau_3 \end{aligned} \tag{1}$$

where  $I_i$  denotes the moment of inertia along each of the three axes,  $\alpha_i$  denotes the component of angular acceleration along the axes,  $\omega_i$  denotes the component of angular velocity along the axes,  $\tau_i$  denotes the torque. For our rigid body, we have  $I_1 \neq I_2 \neq I_3$ .

**Remark.** The proof of the Euler's laws of motion is rather complicated to be discussed in this text. It can be found [here](#).

From  $\tau = I\alpha$ , the value for the moment of inertia  $I$  tells you how much torque you need to produce a given angular acceleration about that axis. The highest moment of inertia needs the most torque, while the lowest moment needs the least torque.

<sup>1</sup>These laws are an extension of Newton's laws, and extends them from Newton's point particles to rigid bodies.

If we consider the case where a rigid body is rotating freely around the 3-axis, we can examine under what conditions this rotation is stable or unstable. This is done by assuming small perturbations to the angular velocities around the 1- and 2-axes. After some manipulations of Euler's equations in eq. (1), we arrive at the following equation:

$$\dot{\alpha}_1 = - \left[ \frac{(I_3 - I_2)(I_3 - I_1)}{I_1 I_2} \omega_3^2 \right] \omega_1 \quad (2)$$

Since the part within the brackets is just a constant, let's call it  $k$ .

We have the following three cases:

1.  $I_3$  is the largest moment of inertia.
2.  $I_3$  is the smallest moment of inertia.
3.  $I_3$  is neither, i.e.  $I_3$  is the *intermediate axis*.

### Cases 1 and 2:

It is easy to see that  $k$  is positive. This gives us

$$\ddot{\omega}_1 = -k\omega_1 \quad (3)$$

using the fact that  $\alpha$  is the first time derivative of  $\omega$ .

Looks familiar? This is the equation for simple harmonic motion. This is a stable motion, meaning that a small perturbation will not bring the body out of its equilibrium.

### Case 3:

$k$  is negative, giving us a positive overall constant. This equation is unstable.

# References

- [1] Julia Abrams. <https://www.comsol.com/blogs/why-do-tennis-rackets-tumble-the-dzhanibekov-effect-explained/>, 2020.
- [2] Physics LibreTexts. [https://phys.libretexts.org/Bookshelves/Classical\\_Mechanics/Classical\\_Mechanics\\_\(Tatum\)/04%3A\\_Rigid\\_Body\\_Rotation/4.05%3A\\_Euler's\\_Equations\\_of\\_Motion](https://phys.libretexts.org/Bookshelves/Classical_Mechanics/Classical_Mechanics_(Tatum)/04%3A_Rigid_Body_Rotation/4.05%3A_Euler's_Equations_of_Motion).