Asia Pacific Mathematical Olympiad for Primary Schools 2010

First Round
2 hours
(150 marks)

1. Find the value of
$$\left(1-\frac{1}{2}\right)\times\left(1-\frac{1}{3}\right)\times\left(1-\frac{1}{4}\right)\times\ldots\times\left(1-\frac{1}{98}\right)\times\left(1-\frac{1}{99}\right)\times\left(1-\frac{1}{100}\right)$$

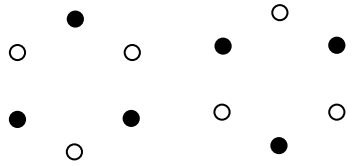
(SMOPS 2010 Q.1)

2. Find the value of $\frac{1}{2}+\frac{1}{2\times2}+\frac{1}{2\times2\times2}+\cdots+\frac{1}{\underbrace{2\times2\times2\times\ldots\times2}_{10\ of\ 2's}}$.

(SMOPS 2010 Q.2)

3. Find the total number of ways to arrange 3 identical white balls and 3 identical black balls in a circle on a plane. The two layouts below are considered as one way of arrangement.

(SMOPS 2010 Q.3)



4. Given that one and only one of the following statements is true, which one is true?
(a) All of the statements below are correct.
(b) None of the statements below is correct.
(c) One of the statements above is correct.
(d) All of the statements above are correct.
(e) None of the statements above is correct.

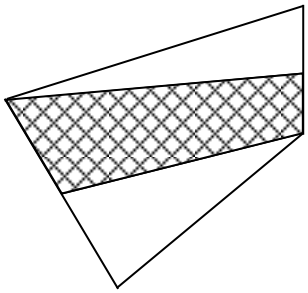
(SMOPS 2010 Q.4)

5. Let *n* be a whole number greater than 1. It leaves a remainder of 1 when divided by any single digit whole number greater than 1. Find the smallest possible value of *n*.

(SMOPS 2010 Q.5)

6. *M* and *N* are mid-points of the lines *AD* and *BC* respectively. Given that the area of *ABCD* is 2000cm^2, and the area of the shaded region *ANCM* = x cm^2, find the value of x.

(SMOPS 2010 Q.6)

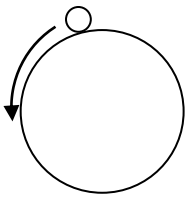


7. Your pocket money had previously been decreased by _____ %. To get back to the same amount of pocket money before the decrease, you need to have an increase of 25%.

(SMOPS 2010 Q.7)

8. A circle of diameter 2 cm rolls along the circumference of a circle of diameter 12 cm, without slipping, until it returns to its starting position. Given that the smaller circle turns $x°$ about its centre, find the value of x.

(SMOPS 2010 Q.8)



9. Find the last digit of the number $\underbrace{2 \times 2 \times 2 \times \ldots \times 2}_{859435\ of\ 2's}$

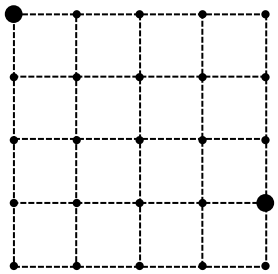
(SMOPS 2010 Q.9)

10. Three bus services operate from the same bus interchange. The first bus service leaves at 24 minute intervals, the second at 30 minute intervals and the third at 36 minute intervals. All three services leave the bus interchange together at 0900. Find the number of minutes that has passed when they next leave the interchange together.

(SMOPS 2010 Q.10)

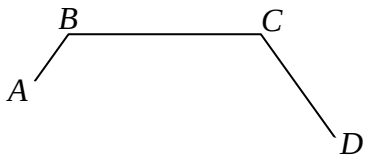
11. Twenty five boys position themselves in a 5 by 5 formation such that the distances between two adjacent boys in the same row or the same column are equal to 1 m. The two dark circles indicate a pair of boys whose distance is exactly 5 m. given that there are *n* pairs whose distance apart are exactly 5m, find the value of *n*.

(SMOPS 2010 Q.11)



12. When Albert begins walking up slope *AB* (1 km distance), across level ground *BC* (12 km distance), and down slope *CD* (3km distance), Daniel begins his journey in the opposite direction from D at the same time. Given that the speeds of both travelling up the slope, on level ground and down slope are 2 km/h, 4 km/h and 5 km/h respectively, find the number of hours that has passed when they meet.

(SMOPS 2010 Q.12)

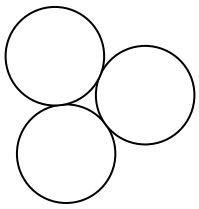


13. Find the value of $1^3 + 2^3 + 3^3 + 4^3 + \cdots + 20^3 + 21^3$.

(SMOPS 2010 Q.13)

14. Three identical circles have at most three points of contacts as shown below. Find the least number of identical circles required to have nine points of contact.

(SMOPS 2010 Q.14)



15. A goat in a horizontal ground is tied to one end of a 14 m long rope. The other end of the rope is attached to a ring which is free to slide along a fixed 20 m long horizontal rail. If the maximum possible area that the goat can graze is *x* m^2, find the value of x. [Ignore the dimension of the ring and take $\pi = \frac{22}{7}$]

(SMOPS 2010 Q.15)

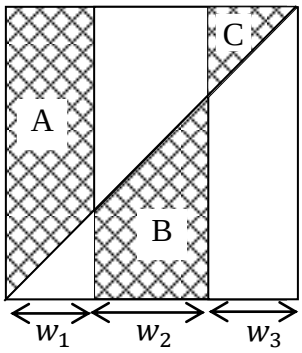
16. The 13 squares are to be filled with whole numbers. If the sum of any three adjacent numbers is 21, find the value of *x*.

(SMOPS 2010 Q.16)

				4				9				*x*

17. A square is divided into three rectangles of widths w_1 , w_2 and w_3, as shown. If $w_1 + w_3 = w_2$ and the areas of the shaded regions A, B and C are 8 cm^2, *x* cm^2 and 2 cm^2 respectively, find the value of *x*.

(SMOPS 2010 Q.17)

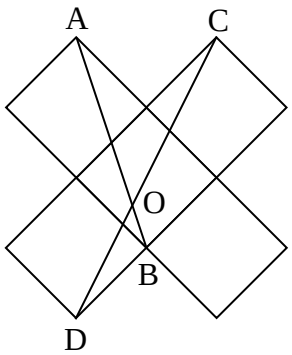


18. Given that $S = \frac{1}{\frac{1}{2001}+\frac{1}{2002}+\frac{1}{2003}+\cdots+\frac{1}{2009}+\frac{1}{2010}}$, find the largest whole number smaller than *S*.

(SMOPS 2010 Q.18)

19. The figure comprises five identical squares. A, B, C and D are vertices of the squares. AB cuts CD at O and angle $AOC = x°$, find the value of x.

(SMOPS 2010 Q.19)

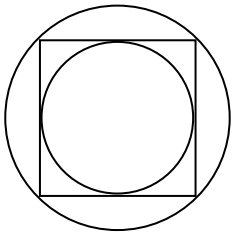


20. Find the smallest whole number that is not a factor of $1 \times 2 \times 3 \times ... \times 21 \times 22 \times 23$.

(SMOPS 2010 Q.20)

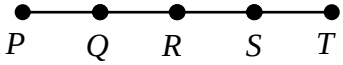
21. A square has its four vertices touching a circle and its four sides touching another smaller circle as shown below. If the area of the larger circle is x times that of the smaller one, find the value of x.

(SMOPS 2010 Q.21)



22. P, Q, R, S and T are equally spaced on a straight rod. If the rod is first rotated 180° about T, then 180° about S and finally 180° about P, which point's position remains unchanged?

(SMOPS 2010 Q.22)

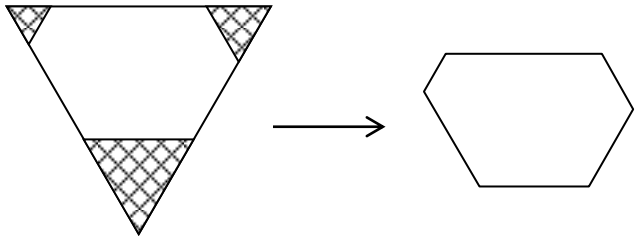


23. Given that the product of four different whole numbers is 10,000, find the greatest possible value of the sum of the four numbers.

(SMOPS 2010 Q.23)

24. An equilateral triangle PQR of side 32 cm has three equilateral triangles cut off from its corners to give rise to a hexagon $ABCDEF$. Another equilateral triangle LMN of side x cm gives rise to the same hexagon when subjected to the same treatment. If AB = 8 cm, BC = 15 cm, CD = 9 cm, DE = 10 cm, EF = 13 cm and FA = 11 cm, find the value of x.

(SMOPS 2010 Q.24)



25. The order of the following three numbers

$$\underbrace{3 \times 3 \times 3 \times \ldots \times 3}_{40\ of\ 3's} \qquad \underbrace{5 \times 5 \times 5 \times \ldots \times 5}_{30\ of\ 5's} \qquad \underbrace{7 \times 7 \times 7 \times \ldots \times 7}_{20\ of\ 7's}$$

A ***B*** ***C***

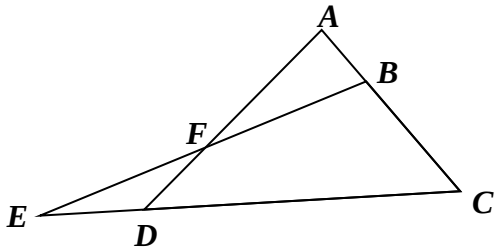
from largest to smallest is _____.

(1) *A, B, C* (2) *A, C, B* (3) *B, C, A* (4) *B, A, C* (5) *C, A, B* (6) *C, B, A*

(SMOPS 2010 Q.25)

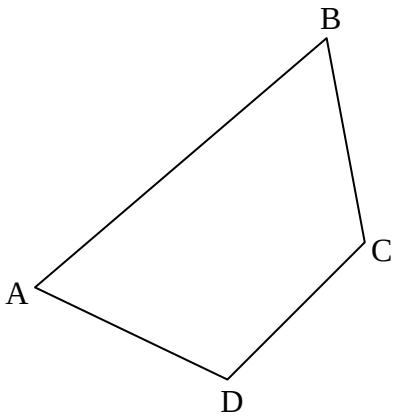
26. *B* and *D* lie on *AC* and *CE* respectively and *AD* cuts *BE* at *F*. If B*C* = 2*AB*, *AF* = *2FD*, area of *EFD* = *x* cm^2 and area of *BCDF* = 1750 cm^2, find the value of *x*.

(SMOPS 2010 Q.26)



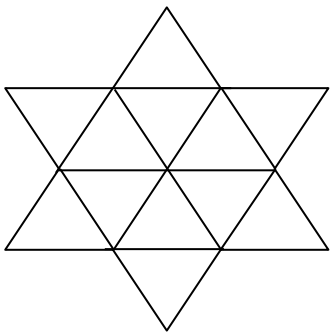
27. In the diagram, *AD* = *DC* = *CB*, angle *ADC* = 110°, angle *DCB* = 130°, and angle *ABC* = *x*°, find the value of *x*.

(SMOPS 2010 Q.27)



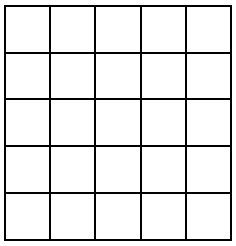
28. The figure comprises twelve equilateral triangles. Find the total number of trapezia in the figure. Here we define a trapezium to be a 4-sided figure with exactly one pair of parallel sides.

(SMOPS 2010 Q.28)



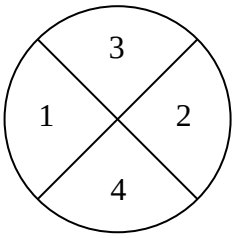
29. The following 5 by 5 grid consists of 25 unit squares. Find the largest number of unit squares to be shaded so that each row, each column and each of the two main diagonal lines has at most 2 unit squares that are shaded.

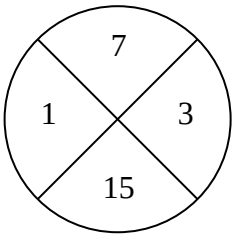
(SMOPS 2010 Q.29)

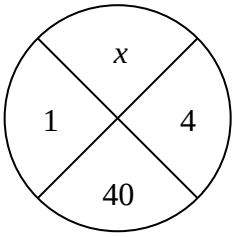


30. Find the value of x.

(SMOPS 2010 Q.30)







Number of correct answers for Q1 to Q10 : _____ Marks (×4) : _____
Number of correct answers for Q11 to Q20 : _____ Marks (×5) : _____
Number of correct answers for Q21 to Q30 : _____ Marks (×6) : _____

Answers:

SMOPS 2010			
1	$\frac{1}{100}$	16	8
2	$\frac{1023}{1024}$	17	10
3	4	18	200
4	(e)	19	45
5	2521	20	29
6	1000	21	2
7	20	22	Q
8	2520	23	1257
9	8	24	34
10	360	25	(4)
11	8	26	250
12	$2\frac{1}{2}$	27	55
13	53361	28	30
14	6	29	10
15	1176	30	13