Notes

Singapore Physics Olympiad 2023

Topic 2: Dynamics of Motion

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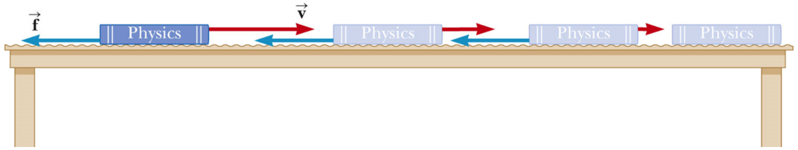
Latest update: April 20, 2023 10:33pm

Newton's first law

Notes

- **Newton's first law** states that an object moves at constant velocity unless acted on by a force
- The tendency for an object to maintain its *original* state of motion is called **inertia**
- **Mass** is the physical quantity that measures the *resistance* of an object to change in its velocity

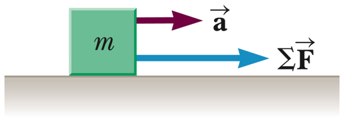


Newton's second law

- **Newton's second law** states that the acceleration of an object is
  - directly proportional to the *net force* acting on it; and
  - inversely proportional to its mass

- The proportional constant, in SI units, equals to 1

- The SI unit of force is the newton (N): $1\,\mathrm{N} \equiv 1\,\mathrm{kg}\cdot\mathrm{m/s^2}$



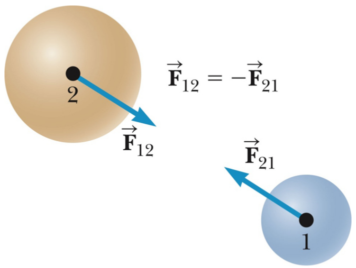
$$\boxed{\sum \vec{\mathbf{F}} = m\vec{\mathbf{a}}} \quad \Rightarrow \quad \begin{cases} \sum F_x = ma_x \\ \sum F_y = ma_y \\ \sum F_z = ma_z \end{cases}$$

Notes

Newton's third law

- **Newton's third law**: If object 1 and object 2 interact, the force $\vec{\mathbf{F}}_{12}$ exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force $\vec{\mathbf{F}}_{21}$ exerted by object 2 on object 1

- $\vec{\mathbf{F}}_{12}$ may be called the **action force** and $\vec{\mathbf{F}}_{21}$ the **reaction force**. Actually, either force can be the action or the reaction force



The action and reaction forces act on *different* objects.

Notes

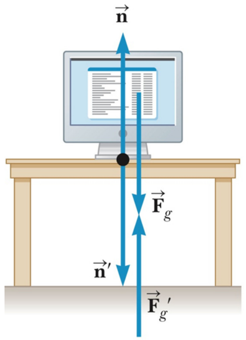
Weight and normal force

- The Earth exerts a force $\vec{\mathbf{F}}_g$ on any object called **weight**

$$\boxed{\left|\vec{\mathbf{F}}_g\right| = W = mg}$$

- If the object is at rest on a table, it does not accelerate downward because it is held up by the table. The table exerts an *upward* force $\vec{\mathbf{n}}$, called **normal force**, on the object

- The normal force balances the gravitational force acting on the object and can have *any value* needed up to the point of breaking the table



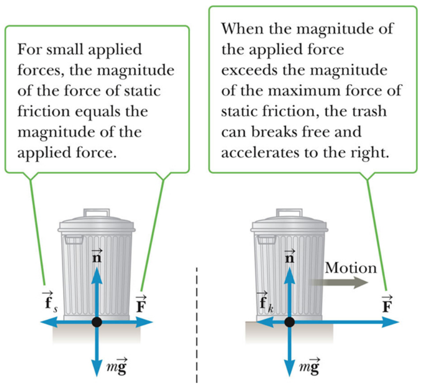
Notes

Normal and frictional forces

- There always have two types of forces at the **INTERFACE** between *any two surfaces in contact*: **normal force** (perpendicular to the interface) and **frictional force** (parallel to the interface)

- **Static friction** $\vec{\mathbf{f}}_s$ is always directed to oppose the *relative impending motion*

- **Kinetic friction** $\vec{\mathbf{f}}_k$ is always directed to oppose the *relative motion*



Notes

Static and kinetic frictional forces

Notes

- Magnitude of the static frictional force:

$$\boxed{f_s \leq \mu_s n}$$

μ_s is the coefficient of static frictional force

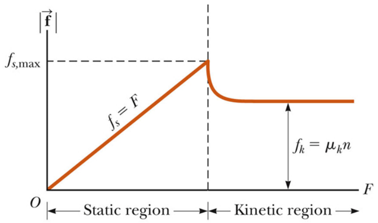
- Magnitude of the kinetic frictional force:

$$\boxed{f_k = \mu_k n}$$

μ_k is the coefficient of kinetic frictional force

- Values of μ_s and μ_k depend on the nature of the surfaces, but μ_k is generally less than μ_s

- μ_s and μ_k are *nearly* independent of the area of contact



Uniformly accelerated frames

Notes

- Newton's second law in a uniformly accelerated frame: $\vec{\mathbf{A}}$ is the acceleration of the frame with respect to an inertial frame and $\vec{\mathbf{F}}$ is the true force measured in the inertial frame

$$\boxed{\vec{\mathbf{F}} - m\vec{\mathbf{A}} = m\vec{\mathbf{a}}}$$

$\vec{\mathbf{F}}_{\text{fict}} \equiv -m\vec{\mathbf{A}}$ is known as **fictitious force**

- Angle of the string from the vertical and the tension in the string for a mass m hanging in a car accelerating at rate A:

Ground frame

$$\begin{cases} T\cos\theta - mg = 0 \\ T\sin\theta = mA \end{cases}$$

Car frame

$$\begin{cases} T\cos\theta - mg = 0 \\ T\sin\theta - mA = 0 \end{cases}$$

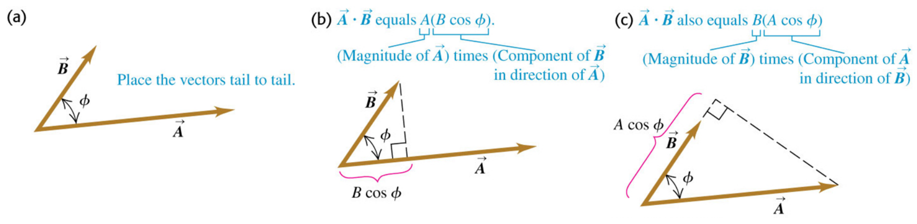
$$\Rightarrow \quad \tan\theta = \frac{A}{g}, \qquad T = m\sqrt{g^2 + A^2}$$

Scalar product of two vectors

Notes

- **Scalar product** of two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ is defined as a scalar quantity equal to the product of the magnitudes of the two vectors and the cosine of the angle ϕ between them:

$$\boxed{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} \equiv \left|\vec{\mathbf{A}}\right| \left|\vec{\mathbf{B}}\right| \cos\phi = AB\,\cos\phi}$$



Properties of scalar product

Notes

- If vector $\vec{\mathbf{A}}$ is parallel to vector $\vec{\mathbf{B}}$ ($\phi = 0$), then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = AB$

- If vector $\vec{\mathbf{A}}$ is perpendicular to vector $\vec{\mathbf{B}}$ ($\phi = 90°$), then $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = 0$

- Scalar product is **commutative**:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = \vec{\mathbf{B}} \cdot \vec{\mathbf{A}}$$

- Scalar product obeys the **distributive law of multiplication**:

$$\vec{\mathbf{A}} \cdot \left(\vec{\mathbf{B}} + \vec{\mathbf{C}}\right) = \vec{\mathbf{A}} \cdot \vec{\mathbf{B}} + \vec{\mathbf{A}} \cdot \vec{\mathbf{C}}$$

- Derivative of the scalar product with respect to some parameter such as t:

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}\right) = \vec{\mathbf{A}} \cdot \frac{\mathrm{d}\vec{\mathbf{B}}}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\mathbf{A}}}{\mathrm{d}t} \cdot \vec{\mathbf{B}}$$

Scalar product using Cartesian components

Notes

- Scalar products between Cartesian unit vectors:

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\imath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{k}} = 1$$

$$\hat{\boldsymbol{\imath}} \cdot \hat{\boldsymbol{\jmath}} = \hat{\boldsymbol{\jmath}} \cdot \hat{\boldsymbol{k}} = \hat{\boldsymbol{k}} \cdot \hat{\boldsymbol{\imath}} = 0$$

- Scalar products between two vectors:

$$\vec{\mathbf{A}} = A_x\,\hat{\boldsymbol{\imath}} + A_y\,\hat{\boldsymbol{\jmath}} + A_z\,\hat{\boldsymbol{k}}\,, \qquad \vec{\mathbf{B}} = B_x\,\hat{\boldsymbol{\imath}} + B_y\,\hat{\boldsymbol{\jmath}} + B_z\,\hat{\boldsymbol{k}}$$

$$\Rightarrow \quad \boxed{\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z}$$

Work done by a constant force

Notes

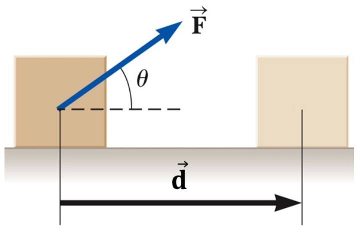
- **Work** $W_{a\to b}$ done by a constant force $\vec{\mathbf{F}}$ acting on a particle that moves from point a to point b with a displacement of $\vec{\mathbf{d}}$:

$$\boxed{W_{a\to b} = \vec{\mathbf{F}} \cdot \vec{\mathbf{d}} = Fd\cos\theta}$$

θ is the angle between $\vec{\mathbf{F}}$ and $\vec{\mathbf{d}}$

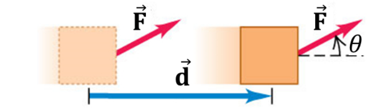
- Work is a scalar quantity
- SI unit of work is joule (J):

$$1\,\mathrm{J} \equiv 1\,\mathrm{kg}\cdot\mathrm{m}^2/\mathrm{s}^2$$

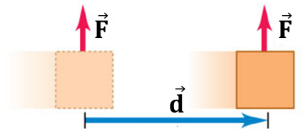


Work is a signed scalar quantity

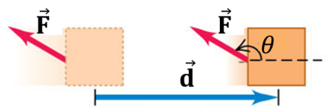
- Positive work $(0 \leq \theta < 90°)$: Force $\vec{\mathbf{F}}$ has a component in direction of displacement $\vec{\mathbf{d}}$



- Zero work $(\theta = 90°)$: Force $\vec{\mathbf{F}}$ is perpendicular to direction of displacement $\vec{\mathbf{d}}$



- Negative work $(90 < \theta \leq 180°)$: Force $\vec{\mathbf{F}}$ has a component opposite to direction of displacement $\vec{\mathbf{d}}$



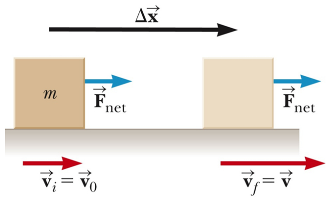
Notes

Work-Energy theorem and kinetic energy

$$v^2 = v_0^2 + 2a\,(\Delta x)$$

$$W_{\text{net}} = F_{\text{net}}\Delta x = (ma)\,\Delta x$$

$$\Rightarrow \quad W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$



- **Kinetic energy** K of a particle of mass m moving with a speed v:

$$\boxed{K = \frac{1}{2}mv^2}$$ SI unit: Joule (J)

- **Work-Energy theorem**: Net work done on a particle equals to the change in particle's kinetic energy

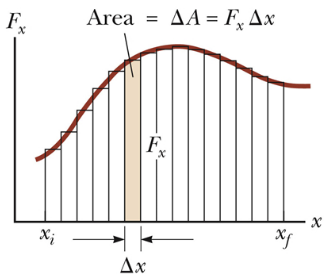
$$\boxed{W_{\text{net}} = \Delta K = K_f - K_i}$$

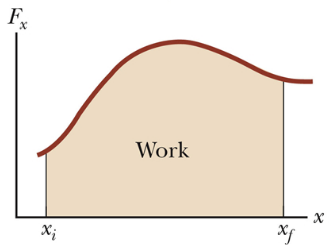
Notes

Work done by a varying force

- Work done by F_x on the object as it moves from x_i to x_f:

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F_x \, \Delta x \quad \Rightarrow \quad \boxed{W = \int_{x_i}^{x_f} F_x \, \mathrm{d}x}$$





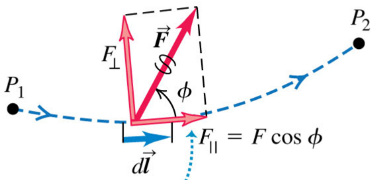
Notes

Work done by a varying force along an arbitrary path

- Total work done by $\vec{\mathbf{F}}$ on the particle as it moves from P_1 to P_2 is given by a **line integral**:

$$\boxed{W = \int_{P_1}^{P_2} \vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}}$$

$\mathrm{d}\vec{\mathbf{r}}$ is the infinitesimal displacement vector which is tangent to the path at its position.



Only the component of $\vec{F}$ parallel to the displacement, $F_{\parallel} = F \cos \phi$, contributes to the work done by $\vec{F}$.

To evaluate the line integral, we need a detailed description of the path from P_1 to P_2 and of the way in which $\vec{\mathbf{F}}$ varies along the path.

Notes

Conservative forces

- A force is **conservative** if the work it does on a particle moving between two points is *independent of the path* the particle takes between the points

- The work depends only upon the *initial* and *final* positions of the particle

- Any conservative force have a **potential energy function** associated with it

- Examples of conservative forces include:
  - gravity
  - spring force
  - electromagnetic force

Notes

Non-conservative forces

- A force is **non-conservative** if the work it does on a particle *depends on the path taken* by the particle between its final and starting points

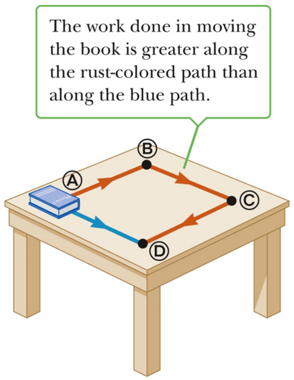
- Examples of non-conservative forces: kinetic friction, air drag, etc

- Work-Energy theorem:

$$\boxed{W_{\text{net}} = W_{\text{c}} + W_{\text{nc}} = K_f - K_i}$$

W_{c} : total work done by conservative forces

W_{nc} : total work done by non-conservative forces



Notes

Defining potential energy

Notes

- **Potential energy** U is associated with the *position* of the object within some system; it is a shared property between the system and the object

- When the object moves from a point where the potential energy is U_a to a point where it is U_b, the work done by the conservative force is

$$\boxed{W_{a\to b} = -\Delta U = U_a - U_b}$$

When the work done by the conservative force is positive, the potential energy decreases – the object moves from a point of higher potential energy to one of lower potential energy.

Gravitational potential energy

Notes

- Work done by gravitational force:

$$W_{i\to f} = -mg\,(y_f - y_i)$$

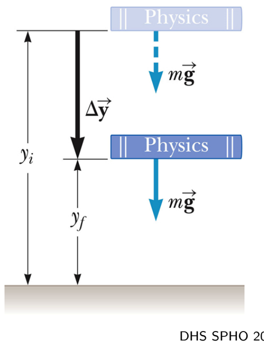
- Change in gravitational potential energy:

$$\Delta U \equiv U_f - U_i = -W_{i\to f} = mg\,(y_f - y_i)$$

- **Gravitational potential energy** of a system consisting of Earth and an object of mass m *near* Earth's surface:

$$\boxed{U_g = mgy}$$

y is the *vertical* distance of the mass relative to the surface of Earth (or some other reference point).



Zero reference for gravitational potential energy

Notes

- A location where the gravitational potential energy is zero must be chosen for each problem
- The choice is *arbitrary* since the change in the potential energy is the important quantity
- Once the position is chosen, it must remain fixed for the entire problem
- Choose a convenient location for the zero reference height
  - Often the Earth's surface
  - May be some other point suggested by the problem

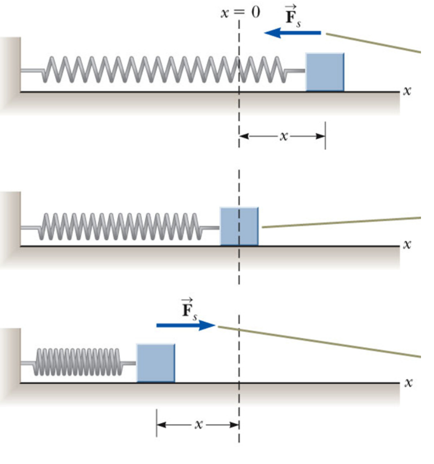
Elastic force (spring force)

Notes

- **Hooke's law**: Force exerted by the spring is proportional to the displacement from the equilibrium position

$$\boxed{F_s = kx}$$

k is called the **spring constant** and the force is always *directed towards* the equilibrium position.



Elastic potential energy

Notes

- Choose the equilibrium position to be $x = 0$, the work done by the spring force on the block when the block undergoes an arbitrary displacement from $x = x_1$ to $x = x_2$:

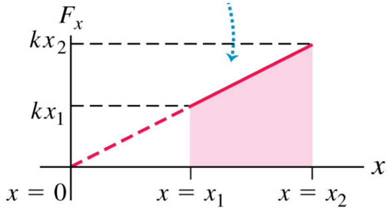
$$\vec{\mathbf{F}}_s = -kx\,\hat{\boldsymbol{\imath}}\,, \qquad \mathrm{d}\vec{\mathbf{r}} = \mathrm{d}x\,\hat{\boldsymbol{\imath}}$$

$$W_s = \int_{P_1}^{P_2} \vec{\mathbf{F}}_s \cdot \mathrm{d}\vec{\mathbf{r}} = -\int_{x=x_1}^{x_2} kx\,\mathrm{d}x = \frac{1}{2}kx_1^2 - \frac{1}{2}kx_2^2$$

- Elastic potential energy: $\Delta U_{1\to 2} \equiv U_2 - U_1$

$$\Delta U_{1\to 2} = -W_s = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$$\Rightarrow \quad \boxed{U_s = \frac{1}{2}kx^2}$$



Conservation of energy

Notes

- When a physical quantity is conserved, the numerical value of the quantity remains the same throughout the physical process

- **Conservation of energy**: Energy cannot be created or destroyed, only transferred from one form into another

- Work-Energy theorem:

$$W_{\mathsf{c}} + W_{\mathsf{nc}} = \Delta K \quad \Rightarrow \quad W_{\mathsf{nc}} = \Delta K + \Delta U_g + \Delta U_s + \Delta U_{\text{other c}}$$

- **Conservation of mechanical energy**: In the *absence* of non-conservative force, i.e. $W_{\mathsf{nc}} = 0$, total mechanical energy (kinetic energy + potential energies) is conserved

$$W_{\mathsf{nc}} = 0 \quad \Rightarrow \quad \Delta K + \Delta U_g + \Delta U_s + \Delta U_{\text{other c}} = 0$$

Power

Notes

- The **power** is the time rate at which work is done. When a quantity of work ΔW is done during a time interval Δt, the average work done per unit time or **average power** P_{av} is defined to be

$$\boxed{P_{\text{av}} = \frac{\Delta W}{\Delta t}}$$

SI unit: watt $(\mathrm{W} \equiv \mathrm{J/s})$

- The **instantaneous power** is the limiting value of the average power as Δt approaches zero:

$$\boxed{P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{\mathrm{d}W}{\mathrm{d}t}} \quad \Rightarrow \quad P = \frac{\vec{\mathbf{F}} \cdot \mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t} = \vec{\mathbf{F}} \cdot \frac{\mathrm{d}\vec{\mathbf{r}}}{\mathrm{d}t} = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}}$$

Linear momentum

Notes

- The **linear momentum** $\vec{\mathbf{p}}$ of a particle of mass m with a velocity $\vec{\mathbf{v}}$ is defined to be the product of the mass and velocity

$$\boxed{\vec{\mathbf{p}} = m\vec{\mathbf{v}}}$$

SI unit: $\mathrm{kg} \cdot \mathrm{m/s}$

- Linear momentum is a *vector* quantity and its direction is the same as the direction of the velocity

- Relationship between magnitude of linear momentum p and kinetic energy K of a particle with mass m:

$$\boxed{K = \frac{|\vec{\mathbf{p}}|^2}{2m} = \frac{p^2}{2m}}$$

This relationship is valid only for particles travelling at speeds *much less* than the speed of light.

Newton's second law and linear momentum

Notes

- Newton's second law can be used to relate the linear momentum of a particle with *constant* mass to the net force acting on it

$$\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}} = m\,\frac{\mathrm{d}\vec{\mathbf{v}}}{\mathrm{d}t} = \frac{\mathrm{d}\,(m\vec{\mathbf{v}})}{\mathrm{d}t} \quad \Rightarrow \quad \boxed{\vec{\mathbf{F}}_{\text{net}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t}}$$

- The time rate of change of linear momentum of a particle is equal to the net force acting on the particle

- This alternative statement of Newton's second law is more general than $\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$ and is also valid when the mass changes

Impulse

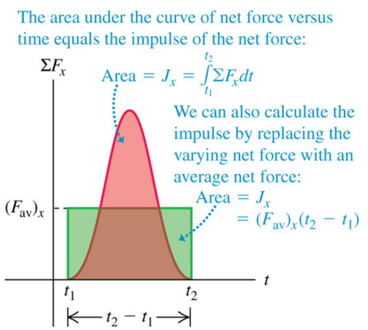
Notes

- The **impulse** $\vec{\mathbf{I}}$ of the net force $\vec{\mathbf{F}}_{\text{net}}$ acting on a particle is the integral of the net force over the time interval

$$\boxed{\vec{\mathbf{I}} = \int_{t_i}^{t_f} \vec{\mathbf{F}}_{\text{net}}\,\mathrm{d}t}$$

SI unit: $\mathrm{kg \cdot m/s}$

On a graph of the x-component of net force $\sum F_x$ versus time t, the x-component of impulse during the time interval from t_1 to t_2 is equal to the area under the curve.



Impulse-Momentum theorem

- Newton's second law:

$$\vec{\mathbf{F}}_{\text{net}} = \frac{\mathrm{d}\vec{\mathbf{p}}}{\mathrm{d}t} \quad \Rightarrow \quad \Delta\vec{\mathbf{p}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i = \int_{t_i}^{t_f} \vec{\mathbf{F}}_{\text{net}}\,\mathrm{d}t$$

$$\Rightarrow \quad \boxed{\vec{\mathbf{I}} = \vec{\mathbf{p}}_f - \vec{\mathbf{p}}_i}$$

- **Impulse-Momentum theorem**: the change in momentum of a particle during a time interval is equal to the impulse of the net force acting on the particle during that interval

Notes

Conservation of linear momentum

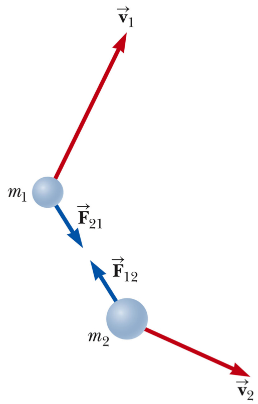
- **Principle of conservation of linear momentum**: If the vector sum of the external forces is zero, the total momentum of the system is constant

$$\vec{\mathbf{F}}_{21} + \vec{\mathbf{F}}_{12} = \vec{\mathbf{0}}$$

$$\Rightarrow \quad \frac{\mathrm{d}\vec{\mathbf{p}}_1}{\mathrm{d}t} + \frac{\mathrm{d}\vec{\mathbf{p}}_2}{\mathrm{d}t} = \vec{\mathbf{0}}$$

$$\Rightarrow \quad \frac{\mathrm{d}}{\mathrm{d}t}\left(\vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2\right) = \vec{\mathbf{0}}$$

$$\Rightarrow \quad \vec{\mathbf{p}}_{\text{tot}} = \vec{\mathbf{p}}_1 + \vec{\mathbf{p}}_2 = \text{constant}$$

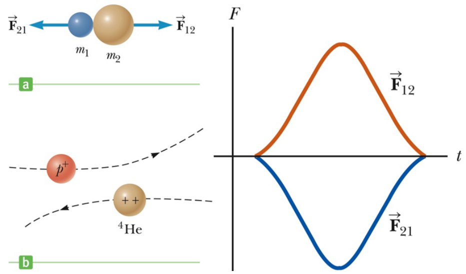


Notes

Collisions

- **Collision**: Two particles come together for a *short time* and thereby producing impulsive forces on each other

- Types of collisions:
  - **Elastic collision**: Both total momentum and total kinetic energy of the system are conserved
  - **Inelastic collision**: Total momentum of the system is conserved but total kinetic energy of the system is not conserved



Notes

Perfectly inelastic collision in one dimension

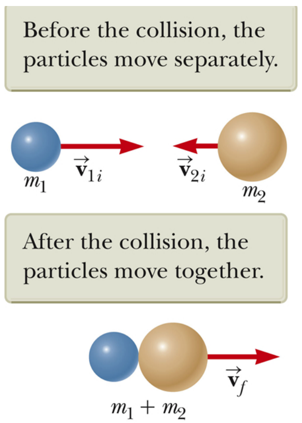
- In a perfectly inelastic collision, two particles collide and stick together after the collision

- Momentum:

$$m_1 v_{1i} - m_2 v_{2i} = (m_1 + m_2) v_f$$
$$\Rightarrow \quad v_f = \frac{m_1 v_{1i} - m_2 v_{2i}}{m_1 + m_2}$$

- Kinetic energy: $K_i > K_f$

$$K_i = \frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2$$
$$K_f = \frac{1}{2}(m_1 + m_2) v_f^2 = \frac{(m_1 v_{1i} - m_2 v_{2i})^2}{2(m_1 + m_2)}$$



Notes

Elastic collision in one dimension

Notes

- Momentum:

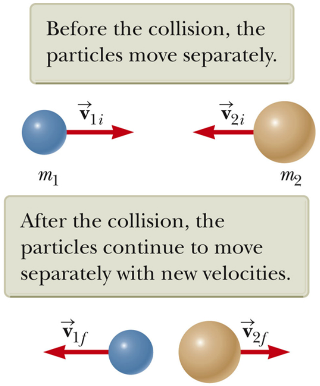
$$m_1 v_{1i} - m_2 v_{2i} = -m_1 v_{1f} + m_2 v_{2f}$$

$$\Rightarrow \quad m_1 \left(v_{1i} + v_{1f}\right) = m_2 \left(v_{2i} + v_{2f}\right)$$

- Kinetic energy:

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

$$\Rightarrow \quad m_1 \left(v_{1i}^2 - v_{1f}^2\right) = -m_2 \left(v_{2i}^2 - v_{2f}^2\right)$$



- Relative velocity of the two particles before the collision equals the negative of their relative velocity after the collision:

$$|\vec{\mathbf{v}}_{1i} - \vec{\mathbf{v}}_{2i}| = |\vec{\mathbf{v}}_{1f} - \vec{\mathbf{v}}_{2f}| \quad \Rightarrow \quad v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$$

Collision in two dimensions

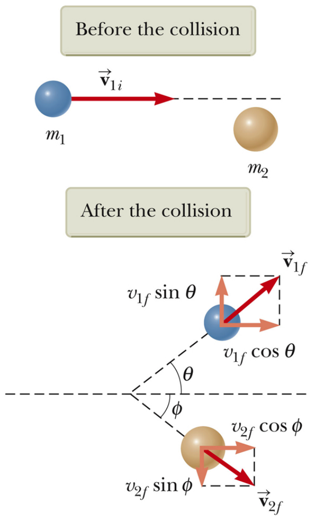
Notes

- The principle of conservation of linear momentum implies that the total momentum of the system in *each direction* is conserved respectively:

$$\begin{cases} m_1 v_{1i} = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi \\ 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\phi \end{cases}$$

- Elastic collision (if applicable):

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

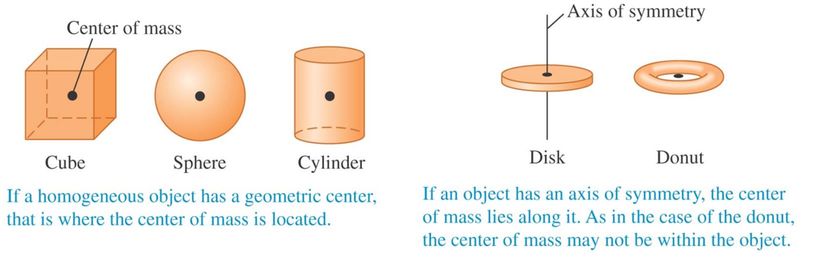


Center of mass

- There is a special point in a system, called the **center of mass**, that moves as if all of the mass of the system is concentrated at that point

- The system will move as if an external force were applied to a single particle of mass M (where M is the total mass of the system) located at the center of mass

- The center of mass of any symmetric object of uniform density lies on an axis of symmetry and on any plane of symmetry

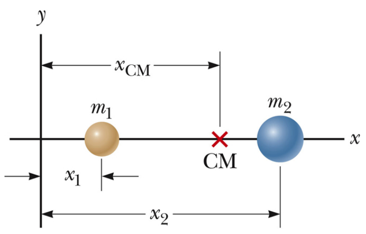


Notes

Center of mass for a system of point particles

- Two point particles:

$$\boxed{x_{\text{CM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}}$$



- System of point particles:

$$\boxed{x_{\text{CM}} = \frac{\sum_i m_i x_i}{\sum_i m_i}} \quad \boxed{y_{\text{CM}} = \frac{\sum_i m_i y_i}{\sum_i m_i}} \quad \boxed{z_{\text{CM}} = \frac{\sum_i m_i z_i}{\sum_i m_i}}$$

Notes

Velocity and momentum of a system of particles

Notes

- Assume the total mass M of the system remains constant. We can describe the motion of the system in terms of the velocity and acceleration of the center of mass of the system

- Velocity of the center of mass of a system of particle:

$$\vec{\mathbf{v}}_{\mathsf{CM}} = \frac{\mathrm{d}\vec{\mathbf{r}}_{\mathsf{CM}}}{\mathrm{d}t} = \frac{1}{M}\sum_i m_i \vec{\mathbf{v}}_i$$

- Total momentum of the system equals the total mass multiplied by the velocity of the center of mass:

$$M\vec{\mathbf{v}}_{\mathsf{CM}} = \sum_i m_i \vec{\mathbf{v}}_i = \sum_i \vec{\mathbf{p}}_i = \vec{\mathbf{p}}_{\mathsf{tot}}$$

Newton's second law for a system of particles

Notes

- Acceleration of the center of mass:

$$\vec{\mathbf{a}}_{\mathsf{CM}} = \frac{\mathrm{d}\vec{\mathbf{v}}_{\mathsf{CM}}}{\mathrm{d}t} = \frac{1}{M}\sum_i m_i \vec{\mathbf{a}}_i \quad \Rightarrow \quad M\vec{\mathbf{a}}_{\mathsf{CM}} = \sum_i \vec{\mathbf{F}}_i$$

- Summing over all the internal forces, they cancel in pairs and the net force on the system is caused only by the external forces:

$$\sum \vec{\mathbf{F}}_{\mathsf{ext}} = M\vec{\mathbf{a}}_{\mathsf{CM}}$$

- The center of mass of a system of particles of total mass M moves likes an equivalent particle of mass M would move under the influence of the net external force on the system

Impulse and momentum of a system of particles

Notes

- Impulse imparted to the system by the external force:

$$\sum \vec{\mathbf{F}}_{\text{ext}} = M\vec{\mathbf{a}}_{\text{CM}} \quad \Rightarrow \int \sum \vec{\mathbf{F}}_{\text{ext}}\,\mathrm{d}t = M \int \mathrm{d}\vec{\mathbf{v}}_{\text{CM}} \quad \Rightarrow \quad \Delta\vec{\mathbf{p}}_{\text{tot}} = \vec{\mathbf{I}}$$

- The total linear momentum of a system of particles is conserved if no net external force is acting on the system:

$$M\vec{\mathbf{v}}_{\text{CM}} = \vec{\mathbf{p}}_{\text{tot}} = \text{constant} \qquad \text{when} \qquad \sum \vec{\mathbf{F}}_{\text{ext}} = \vec{\mathbf{0}}$$

- For an isolated system of particles, both the total linear momentum and the velocity of the center of mass are constant in time

Center of mass of an extended object

Notes

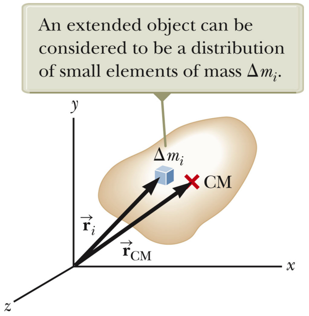
- Think of an extended object as a system containing a large number of small mass elements, the x-coordinate of the center of mass is

$$x_{\text{CM}} = \lim_{\Delta m_i \to 0} \frac{1}{M} \sum_i x_i\,\Delta m_i \qquad \Rightarrow \qquad \boxed{x_{\text{CM}} = \frac{1}{M}\int x\,\mathrm{d}m}$$

- Likewise, for y_{CM} and z_{CM} we obtain

$$\boxed{y_{\text{CM}} = \frac{1}{M}\int y\,\mathrm{d}m} \qquad \boxed{z_{\text{CM}} = \frac{1}{M}\int z\,\mathrm{d}m}$$

The center of mass needs not be within the object itself.



Problem 1

Notes

A rock with mass $m = 3.00\,\mathrm{kg}$ falls from rest in a viscous medium. The rock is acted on by a net constant downward force F of $18.0\,\mathrm{N}$ (a combination of gravity and the buoyant force exerted by the medium) and by a fluid resistance force $f = kv$ where v is the speed in $\mathrm{m/s}$ and $k = 2.20\,\mathrm{N \cdot s/m}$.

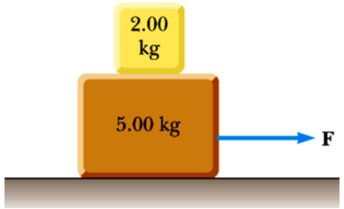
(a) Find the initial acceleration a_0.
(b) Find the acceleration when the speed is $3.00\,\mathrm{m/s}$.
(c) Find the speed when the acceleration equals to $0.1a_0$.
(d) Find the terminal speed v_t.
(e) Find the coordinate, speed and acceleration $2.00\,\mathrm{s}$ after the start of the motion.
(f) Find the time required to reach a speed of $0.9v_t$.

Problem 2

Notes

A 2.00-kg block is placed on top of a 5.00-kg block. The coefficient of kinetic friction between the 5.00-kg block and the surface is 0.200.

(a) Calculate the force needed to pull both blocks with an acceleration of $3.00\,\mathrm{m/s^2}$.
(b) Find the minimum coefficient of static friction between the blocks such that the upper block does not slip under this acceleration.



Problem 3

Notes

A mass m is placed on a rough inclined plane and attached by a string to a hanging mass M over a frictionless pulley. The angle α of the slope is such that $\sin\alpha = 0.6$. The coefficient of static friction between the mass m and the plane is $\mu_s = 0.2$. Show that equilibrium is possible only if M lies between two values M_1, M_2 and find the values of M_1, M_2 in terms of m.

Problem 4

Notes

At the moment $t = 0$, the force $F = \alpha t$ (where α is a constant) is applied to a small block of mass m resting on a smooth horizontal plane. The force always forms an angle θ with the horizontal. Find

(a) the velocity of the block at the moment of its breaking off the plane.

(b) the distance travelled by the block up to this moment.

Problem 5

Notes

Atwood's machine consists of a smooth pulley with two masses (m_1 and m_2 where $m_1 > m_2$) suspended from a light string at each end. Find the acceleration of the masses and the tension of the string

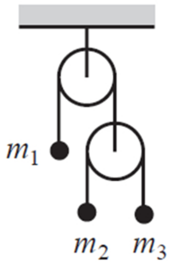
(a) when the pulley center is at rest; and

(b) when the pulley is descending in an elevator with constant acceleration a_0.

Problem 6

Notes

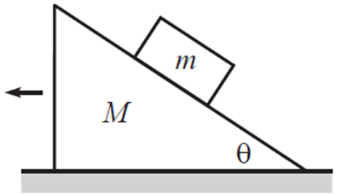
A double Atwood's machine is as shown with masses m_1, m_2 and m_3. Find the accelerations of the masses and the tensions of the strings. You may assume that all strings are light and both pulleys are light and frictionless.



Problem 7

Notes

A wedge of mass M is placed on a horizontal floor. Another mass m is placed on the incline of the wedge. Assume that all surfaces are frictionless and the incline makes an angle θ with the horizontal. The mass m is released from rest on mass M which is also initially at rest. Find the acceleration of M with respect to ground and the acceleration of m with respect the wedge.



Problem 8

Notes

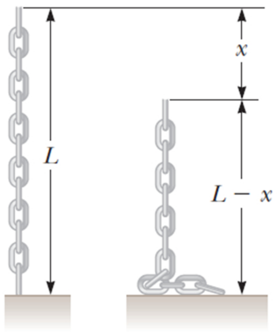
Consider a rocket with mass M_i (including fuel) fired in outer space where there is no gravitational force and no air resistance. It starts from rest at the position $x = 0$, turns on its engine at time $t = 0$ and puts out exhaust with a speed v_e relative to the rocket at a constant rate k.

(a) Find the velocity $v(t)$ of the rocket as a function of time during the burn.
(b) Find the acceleration $a(t)$ of the rocket as a function of time.
(c) Find the position of the rocket $x(t)$ as a function of time.

Problem 9

Notes

A chain of length L and total mass M is released from rest with its lower end just touching the top of a table as shown. Find the force exerted by the table on the chain after the chain has fallen through a distance x as shown. Assume each link comes to rest the instant it reaches the table.



Problem 10

Notes

A railway flatcar of mass M can roll without friction along a horizontal track. N men, each with mass m, are initially standing on the car which is at rest.

(a) The N men run to one end of the car and jump off the car all at the same time with a speed u relative to the car. Find the final speed of the car after the men have jumped off.

(b) Find the final speed of the car if they jump off one at a time.

Problem 11

Notes

Two particles with masses m and $3m$ are moving toward each other along the x axis with the same initial speeds v_i. Particle m is travelling to the left and particle $3m$ is travelling to the right. They undergo an elastic glancing collision such that particle m is moving in the negative y direction after the collision at a right angle from its initial direction.

(a) Find the final speeds of the two particles.
(b) What is the angle θ at which the particle $3m$ is scattered?

Problem 12

Notes

Calculate the x- and y-coordinates of the center of mass of a semi-circular metal plate with uniform density ρ, radius a and thickness t.