Detailed Answers to NTUPC 2023

Mechanics

1. Correct answer D. This is the correct answer from Newton's third law: the action and the reaction forces are equal in magnitude but opposite in direction.

   Choice A. First half of sentence is correct, but second half is wrong.

   Choice B. The two forces mentioned act on different bodies (earth & you), so we should not add them up to obtain the net force on you.

   Choice C. First sentence is wrong – the ground <u>does</u> exert an upward force on you. Second sentence is partially right – our muscles causes us to exert a downward force on the earth.

   Choice E. $F_1 = F_2$ always.

2. Correct answer E.

   Choice A. This is only true if the two spring scales are attached in parallel.

   Choice B. Both scales feel the force exerted by the 10 kg mass, so neither of them should read zero.

   Choice C. Both scales feel the force exerted by the 10 kg mass, so neither of them should read zero.

   Choice D. The sum of the two reads 10 kg <u>only</u> if they are connected in parallel.

3. Correct answer A. Slope of the hill gets "gentler" as the ball rolls down, so acceleration decreases. Though acceleration decreases, it is always acting downwards, so the speed of the ball keeps on increasing and this is due to the conversion of the gravitational potential energy to the kinetic energy (conservation of energy).

   Choice B. The speed cannot decrease due to conservation of energy.

   Choice C. Acceleration is smaller for gentler slope as one can see from the force analysis.

   Choice D. Speed cannot be constant.

   Choice E. Again, speed cannot decrease due to potential energy converting to kinetic energy.

4. Correct answer D. The force of the rope acting on the monkey is always the same as the force of the rope acting on the bananas because the rope is tight. So all motions (acceleration etc.) of the monkey is identical to that of the bananas.

Choice A. If this is the case, the rope will change its length.

Choice B. The statement is obviously wrong.

Choice C. This directly contradicts the statement that the monkey is moving away from the ground with speed *v*.

Choice E. This directly contradicts the statement that the monkey is moving away from the ground with speed *v*.

5. Correct answer D. This question requires the student to use energy conservation. The water that leaves the bottom of the dam would leave with kinetic energy that equals the gravitational potential energy at the top of the dam. This means that the power generated is (efficiency) × (rate of flow) × (density) × g × (50 metres). Equating this to 50 MW produces a rate of flow of 340.8 m^3/s which is approximately 341 m^3/s.

Choice A. This answer results from a miscalculation where the student may have incorrectly input the value of the power needed.

Choice B. This answer may result from a miscalculation when the conversion efficiency of the dam is not taken into account.

Choice C. This is a wild guess.

Choice E. This is the wrong answer if the student used the kinetic energy but forget to take the square root.

6. Correct answer E. This is the correct answer $\omega = \sqrt{\frac{g \sin\theta}{r \cos\theta}}$, r is the radius of the circle.

Choice A. This is obtained from $\omega = \sqrt{\frac{g \sin\theta}{r}}$ by taking $mg \sin\theta$ as the centripetal force.

Choice B. This is obtained if weight is taken as centripetal force $mg = mr\omega^2$ so $\omega = \sqrt{\frac{g}{r}}$.

Choice C. This is obtained from $\omega = \sqrt{\frac{g \sin\theta}{L \cos\theta}}$ where wrong radius is used.

Choice D. This is obtained from $\omega = \sqrt{\frac{g\cos\theta}{r}}$ by taking $mg\cos\theta$ as the centripetal force.

7. Correct answer B.

Choice A. Students will choose this if they interchange the sine and cosine terms.

Choice C. Students will choose this if they forgot about the sine term in the horizontal direction.

Choice D. Students will choose this if they forgot about the cosine term in the vertical direction.

Choice E. Students will choose this if they forgot to include the sine term in the horizontal direction and cosine term in the vertical direction.

8. Correct answer A. There is always a normal force to the wheel.

Choice B. This is a front wheel drive, so the force in pushing the car forward is coming from the front wheel, so B is not the right answer. B is the answer for the back wheel, when it feels the frictional force.

Choice C. There is always a normal force to the wheel, so the student will choose C or D if they miss that.

Choice D. There is always a normal force to the wheel, so the student will choose C or D if they miss that.

Choice E. This is a wild guess.

9. Correct answer A. The force in pushing the liquid up is coming from the pressure difference between that at the top of the straw and at the bottom of the straw, times the cross sectional area A. This pressure difference is $0.1P_o$. Thus, $0.1P_oA = \rho ALg \rightarrow L = \frac{P_o}{10\rho g}$

Choice B. If the student mistook the force pushing the liquid with 0.9 P_o, they may choose B or E.

Choice C. If the student forgot to take the difference, they will choose C.

Choice D. If the student made a mistake in the algebraic calculation, they may choose D.

Choice E. If the student mistook it with 0.9 P_o, they may choose B or E.

10. Correct answer D.

Choice A. This answer will be picked if the torques of *F* and *Mg* are computed wrongly, and the student double count the weight and its torque

Choice B. This answer will be picked if the torques of *F* and *Mg* are computed wrongly.

Choices C and E. This answer will be picked if the student double counts the weight and the torque from the weight.

11. Correct answer B. This question requires an understanding that the bomb has acquired a horizontal velocity of v, so that it would reach the target X in a time of $t = x/v$. In that time *t*, it would travel a vertical distance of $h = \frac{1}{2}gt^2$ so that t is also given by $\sqrt{\frac{2h}{g}}$. Equating the two expressions gives the answer.

Choice A. The student has neglected the horizontal velocity of the bomb.

Choice C. The student is simply applying the kinematic formula $v^2 = u^2 + 2gs$ without considering how the vertical and horizontal motions are related as the bomb moves in air.

Choice D. The student is simply choosing a "convenient" answer.

Choice E. The student has chosen an answer that is dimensionally correct, but missing the factor 2 inside the square root.

12. Correct answer E. The spaceship orbits the earth with a centripetal force due to earth's gravitational force. The pendulum also orbits the earth under the same conditions. As a result, there is no net acceleration between the pendulum and spaceship with the assumption that $R_S = R_P$., and hence no net force between them so that the tension in the string is zero.

Choice A. This would be the answer if the pendulum and spaceship were to be stationary and not orbiting the earth.

Choice B. The student is aware of the gravitational forces on the spaceship and the pendulum and has thought about the reaction between the two. However, he has neglected the orbiting motions of both objects which take up the forces, leaving a net force of zero between them.

Choice C. The student is thinking of the centripetal force on the pendulum and assumes that the string experiences an opposite tension in order to keep the pendulum at rest. However, he has neglected the motion of the spaceship and the gravitational forces.

Choice D. The student is thinking of the centripetal force on the spaceship and assumes that it is pulling against the pendulum. As in choice C, there are more details in the motion that he has neglected.

13. Correct answer A. The gravitational potential energy between objects A and B is given by $-\frac{Gm_Am_B}{R}$. From this we see that doubling m_A and tripling R gives $-\frac{2}{3}\frac{Gm_Am_B}{R}$ which is a higher (less negative) than the initial value. The negative sign convention arises from the choice of zero potential energy when the objects are infinitely far apart.

Choice B. This choice is clearly not true and represents a guess.

Choice C. This choice may derive from the fact that the force is weaker as a result of the greater separation. But force is only the negative gradient of the potential energy, i.e. the change of potential energy with displacement. For an attractive force like gravity, the potential energy is higher at larger separation.

Choice D. Gravitational potential energy is a scalar and has no direction. It is always negative and does not even change sign.

Choice E. Gravitational force is conservative, so that the work done against it is independent of the path chosen. Thus, the increase of potential energy arising from a larger R by doing work on one of the objects is independent of the path chosen.

14. Correct answer D. For this question, we have to really understand that force is the rate of change of momentum. The momentum of the flatcar increases from the additional mass of the sand which is μt, as well as from the increase in velocity v. Both are supported by the force F so that the flatcar. In other words,

Change in momentum = Force × Time interval

So, $\Delta p = F\Delta t$. At time $t = 0$, the momentum p is also 0. At time $t = t$, the momentum is $p = (m + \mu t)v = Ft$. Hence, we get $v = \frac{Ft}{m+\mu t}$.

Choice A. This answer is not dimensionally correct, because μ does not have the dimension of mass m. Indeed, the flatcar has a mass $(m + \mu t)$ after time t.

Choice B. The answer is dimensionally plausible and is derived from $F = ma$, giving $(m + \mu t)\frac{dv}{dt} = F$, instead of $F = \frac{dp}{dt}$ which yields $v\frac{dm}{dt} + (m + \mu t)\frac{dv}{dt} = F$. Integration of the former by taking F, m and μ as constants leads to choice B, while integrating the latter gives the correct answer D.

Choice C. This answer replaces the mass $(m + \mu t)$ after time t with the time-averaged mass $\left(m + \frac{\mu t}{2}\right)$. This is not correct as the impulse Ft is (Final momentum – Initial momentum).

Choice E. This answer neglects the contribution of additional sand to the momentum change. The mass at time t is not m but $(m + \mu t)$.

15. Correct answer E. When the body is fully immersed, the buoyant force is always $\rho V g$, which accounts for $\rho V g h$. When the body is being lifted out of the water, the average force is $\frac{\rho V g}{2}$, which accounts for $\frac{\rho V g l}{2}$.

Choice A. The student ignored the buoyant force.

Choice B. The student ignored the length of the body.

Choice C. The student ignored the distance *h*.

Choice D. The student assumed the buoyant force is constant.

Thermodynamics and Matter

16. Correct answer B. Ideal gas molecules are approximated as point particles with mass *m*.

Choices A and C. These are true statements about ideal gas molecules.

Choices D and E. Unlike real gas molecules, ideal gas molecules do not possess any potential energy – only kinetic energy.

17. Correct answer C. This question tests the conceptual understanding of the first law of thermodynamics especially with respect to its application to the physics of ideal gas. When heat is added to a small mass of air in the atmosphere, according to the ideal gas law: $P = \rho RT$, either the pressure increases or the density decreases. Because small pressure difference in the atmosphere always equalizes first, the density of the small mass of air decreases. With the density of the small mass of air now lower than the surrounding air, buoyant forces causes the small mass of air to rise. As the small mass of air rises, it expands in volume to equilibrate with the lower pressure outside it, and thus it pushes against the atmosphere. In this way, the given heat to the small mass of air not only increases the internal energy of the small mass of air by increasing its temperature, it also provides for the work that expands the small mass of air against the atmosphere, i.e., $Q = \Delta U + W$, which is the first law of thermodynamics.

Choice A. If the heated air mass does not rise or sink, then it must have the same density as the surrounding air. The temperature increase is correct, but it must lead to a pressure increase and corresponding volume increase and reduction in density, contradicting the earlier statement.

Choice B. Both statements are correct here, but choice C is more accurate since work is also done to expand it.

Choices D and E. These are clearly wrong by common observation, since heated gases do not sink. One can also reason by the temperature increase leading to pressure and volume increases and the corresponding reduction in density.

18. Correct answer C. The estimate of $P_{cool} = 2$ GW cooling power to cool down a power plant that produces $W = 1$ GW of electricity comes from Carnot cycle with $\eta = 60\%$ efficiency, assuming a boiler temperature of $T_h = 700$ K and a river temperature of $T_c = 300$ K (might be higher in Singapore), hence $P_{cool} = \frac{1}{\eta}\left(\frac{T_h}{T_c} - 1\right) W \simeq 2.22$ GW. Since it is a given, it is not necessary to compute it. Increase of temperature ΔT is just given by $\Delta T = \frac{P_{cool}}{CM}$, where $M = 5 \times 10^5$ kg/s is the mass flow rate in the river, $C = 4187$ J/K $\cdot$ kg the specific heat capacity of water. Numerical application yields $\Delta T \simeq 0.95$ °C. A detailed calculation is actually not needed: river's temperature must increase. So, the only choice is in between C and D. Correct answer is obtained either by using orders of magnitude, or just the fact that plants with similar performances are operated worldwide without boiling water in rivers.

Choice A. A detailed calculation is actually not needed: river's temperature must increase.

Choice B. If the student uses 1 GW in the calculation above, the answer will be off by a factor of ½. This choice is meant as a distractor.

Choice D. Similar to choice B, this option is a distractor for the student who uses 1 GW in the calculation and makes a small mistake in the order of magnitude.

Choice E. This option is a distractor for the student who messes up the order of magnitude in the calculations.

19. Correct answer E. This question tests on both the concept of the first law of thermodynamics and the ideal gas law. First, the student needs to be aware that whether the system traverses from X to Y via path 1 or path 2, the change in internal energy ΔU is the same because the system starts at the same state X and ends at the same state Y. Therefore, we expect $U_1 = U_2$ when the system is at state Y. From the pressure-volume curve, we see that the system expends more work via path 1 than path 2 (observe the area under each transition curve). In order for the same change in internal energy for both paths, more heat needs to flow into the system that traverses path 1 than path 2 according to the conservation of energy. Thus, we have $Q_1 > Q_2$.

Choices A to D. The other equality and inequality signs in these options are meant to make these distractors mathematically consistent.

20. Correct answer D. The triple point of water at 0.01°C is also the melting point of ice at a pressure of 0.006 atm. From the knowledge that the melting point of ice at a pressure of 1 atm is 0 °C, we deduce that raising the surrounding pressure lowers the melting point of ice. So point (i) is correct. But in addition, a lower melting point implies that the average kinetic energy of ice molecules is lower. The deficit in energy available for melting must be made up by work done by the environment on the ice when the ice melts. This implies that ice must contract upon melting, so that a higher environmental pressure at melting is associated with more work done on the ice. So point (ii) is also implied.

Choice A. This correctly deduces point (i), but neglects point (ii).

Choice B. This neglects point (i) which is a more obvious deduction than point (ii).

Choice C. While the triple point of water is used to calibrate the absolute temperature scale in kelvins, statement (iii) is NOT a deduction from the fact of the triple point of water. The student is too preoccupied with the use of the triple point rather than understanding its meaning.

Choice E. This omits the obvious point (i) while focusing erroneously on point (iii).

21. Correct answer B. This tests the student's understanding of the first law of thermodynamics and adiabatic process. There are no specific values for the students to calculate, so they will have to imagine how the process happens and what the consequences are, which requires a deeper understanding of the relevant physics principles. Start from $Q = \Delta U + W$. Imagine moving along the adiabatic curve from S_1 to S_2: $Q = 0$, $W > 0$, so $\Delta U < 0$, which means that $U_2 < U_1$. Another way of knowing this is that when intersected, an adiabatic curve always has a steeper slope than the isothermal curve. As a result, S_2 must sit on an isothermal curve with lower T, and thus lower U. Then, if we move from S_1 to S_2 along the solid curve, compared to the adiabatic process, less work is done because the solid curve is always below the dashed curve. As a result, now $\Delta U + W < 0$, and thus $Q < 0$.

Choices A, C, D and E. The other equality and inequality signs in these options are meant to make these distractors mathematically consistent.

22. Correct answer A. The work done in a cycle is given by the enclosed area. We can divide the enclosed area into OBCO (negative net work done by gas) and ODAO (positive net work done by gas). They are equal and of opposite sign. Hence, the work done is 0.

Choice B. This is simply a wild guess.

Choice C. The student only considers the process from D to A.

Choice D. The student only considers the process from A to B or C to D.

Choice E. The student only considers the process from B to C.

23. Correct answer D. The ideal gas equation governs the behaviour of the ideal gas: $pV = nRT$, where p = pressure, V = volume, n = number of moles, R = universal molar gas constant, T = absolute temperature. Thus, for V to shrink, we need to reduce n or T, as p is always kept constant through the flexible walls. Hence this choice is definitely false: the impermeable walls lead to n being constant, while the heat insulating walls lead to T being constant. Therefore, the assumptions in Choice D are contradictory to the statement that the volume is halved and hence must be false.

Choice A. The walls of the container conduct heat and so the gas cools as heat is transferred to the cooler exterior. As temperature of the gas falls, the ideal gas equation allows the volume to be halved by keeping p and n constant. So, this statement is plausible.

Choice B. The semi-permeable walls allow gas to diffuse out and so n decreases while p and T remain constant. Thus, this statement is also plausible.

Choice C. From the kinetic theory of gases, the average kinetic energy is directly proportional to T. So this is plausible as in Choice A.

Choice E. If the walls are both semi-permeable (allowing n to decrease) and conduct heat (allowing T to fall), the ideal gas equation will certainly predict the volume to shrink where p is kept constant.

24. Correct answer D. Pressure of the water increases with depth because the pressure provides an upward force that cancels the weight of the water above so that (ii) is true. At the same time, the bubble gathers more steam as surrounding water vapourises into it so that (iii) is also true. Hence with smaller external pressure and more steam, the bubble gets larger as it rises to the surface.

Choice A. The student is thinking that hot fluids rise from convection currents. However, boiling water is well-mixed and has a temperature equal to its boiling point. Typically, heat is absorbed at the bottom of the pot and mixing occurs as the hot water rises. So the temperature at the surface cannot be higher than at the bottom.

Choice B. This choice has neglected point (iii), i.e. that water continually vaporizes into the bubble.

Choice C. This choice is wrong because of point (i) and also neglects point (iii).

Choice E. This choice correctly considers points (ii) and (iii) but incorrectly includes point (i).

Waves and Oscillations

25. Correct answer B. Using Archimedes' principle: the force on the cylinder is equal to the weight of the water displaced, which is $F = -(\rho_1 \cdot \pi R^2 \cdot d)g$, where d is the vertical

displacement. This acts as a spring force, $F = -kd$. The spring constant k of a harmonic oscillator of mass m is related to the angular frequency ω by $k = m\omega^2$. In this case, the mass is $m = \rho_0 \cdot \pi R^2 \cdot h$. Putting everything together,

$$k = \rho_1 \pi R^2 g = \rho_0 \pi R^2 h \omega^2$$
$$\omega^2 = (\rho_1/\rho_0)(g/h).$$

Choice A. Between Choices A and B, it is intuitive that a less dense object should oscillate with a higher frequency, because it is lighter; hence B is the answer.

Choices C, D and E. Note that this cannot be correct as they are dimensionally imbalanced: the left hand side (ω^2) has units of $[1/t^2]$, whereas the right-hand side has units of $[h/g] = [t^2]$.

26. Correct answer C. This is a paradigmatic example of the Doppler effect: the change in frequency of a wave induced by the motion of the source relative to the observer.

Choice A. This refers to observed *brightness* which is unrelated to the Doppler effect; note that the redshift or blueshift of stars *is* based on the Doppler effect but this is not what is stated.

Choice B. This is unrelated to wave phenomena.

Choice D. This refers to frequency dispersion caused by the variation of a medium's refractive index with frequency, which is a wave phenomenon but not the Doppler effect.

Choice E. This is a wild guess.

27. Correct answer E. See below for further explanation.

Choices A and B. If the input wave is monochromatic, then at steady state the wave frequency f is the same in both media. The frequency is related to the wavelength λ and wave speed v by $\lambda f = v$. Since the two media have different densities, the wave speeds are different. Hence, Choices A and B are ruled out.

Choice C. On the other hand, $v/\lambda = f$ is the same in both media, in agreement with Choice C.

Choice D. Moreover, since the media are stated to be non-dissipative, the energy fluxes must be the same throughout (i.e., energy does not appear or disappear anywhere), in agreement with Choice D.

28. Correct answer A. This is correct because X-rays are a form of electromagnetic radiation, and photons do not interact strongly with matter compared to alpha radiation (helium nuclei), beta radiation (electrons), and neutrons.

Choice B. This is incorrect: X-ray wavelengths are on the order of nanometres, much *shorter* than the relevant length scales of biological tissues (which are microns to centimetres).

Choice C. This is factually correct but irrelevant: X-ray images used in medical imaging generally do not occur at such fine resolutions.

Choice D. This is incorrect: X-rays *are* a form of ionising radiation, as X-ray photons carry enough energy to kick out the electrons in the inner-shell orbitals of atoms.

Choice E. This is a wild guess.

29. Correct answer C. From the question, we deduce that the spring force is $F = 6\text{N} = k\,(0.15 \text{ m})$, so the spring constant is $k = (6/0.15)$ N/m. For simple harmonic motion, $k = m\omega^2$, where in this case $m = 2$ kg (we are told the rod itself is "very light", so it does not contribute to the mass), and ω is the angular frequency.

Hence, the period of oscillation is

$$T = 2\pi/\omega = 2\pi\,[m/k]^{1/2} = 2\pi\,[2 \text{ kg} \times 0.15 \text{ m} / 6 \text{ N}]^{1/2} = 1.405 \text{ s}$$

Note that this can be estimated without a calculator. The quantity inside the square root is 0.15/3 = 0.05, which means the square root is a bit more than 0.2 (since the square root of 5 is a bit more than 2). Therefore, $(2\pi) \times 0.2 \approx 1.2$++, which is closest to Choice C.

Choice A.
Choice B.
Choice D.
Choice E.

30. Correct answer B. Let us sketch the rope's current shape (solid curve), along with its shape after a very brief interval of time (dashed curve). We are told that the wave is left-moving, so the dashed curve is shifted slightly to the left.

As the wave is transverse, each point on the rope moves vertically. Evidently, the point P on the solid curve is displaced downward to point P' on the dashed curve. Hence, the instantaneous velocity of P points downward.

Choice A. The student has probably mistaken that the point P moves along the rope in the direction indicated by the wave velocity. This is common where students do not understand that it is the wave that moves but each point on the rope stays at the same horizontal position.

Choice C. The student has confused the wave's motion with the motion of point P which is restricted to up-down motion. As the wave moves leftwards, point P slips into the next portion of the wave which is downwards.

Choice D. The student might have thought that point P moves in an opposite manner to the wave.

Choice E. The student has mistaken the wave as stationary. In such a case, point P being a node would remain stationary.

Electricity and Magnetism

31. Correct answer C. In electrostatic equilibrium, charges reside on the outer surface of a conducting shell. Hence the electric field is strongest on this surface.

Choice A. Students can make this choice if they do not understand that there are no charges, and hence no electric field inside a conductor in electrostatic equilibrium.

Choice B. Student can make this choice if they are confused by how we frequently replace charges on the surface of the conducting shell by a point charge at the center of the shell, which is in the hollow region.

Choice D. Student can make this choice if they are confused by the frequent choice of a point at infinity as where the electric potential is set to zero.

Choice E. This is a wild guess.

32. Correct answer C. In electrostatic equilibrium, the electric field inside a conductor is zero. Therefore, any excess charge resides entirely on the surface of the conductor. If there were a net charge in the interior, the electric field there would be non-zero, which contradicts the conditions of electrostatic equilibrium.

Choice A. This can be true if the object is an insulator instead of a conductor.

Choice B. Students may be aware that at an observation point far away from the conductor, the charges on the conductor behave as if they are concentrated at the center of the conductor.

Choice D. This choice might sound reasonable, if the student does not understand how charges distribute themselves in a conductor.

Choice E. Students may pick this choice, if they believe the other options are meant to trick them.

33. Correct answer C. This question is based on electric potential. The electric potential at the origin is the sum of the potentials from each individual charge. The electric potential

at the origin is proportional to the sum of the charges because each charge is the same distance from the origin. Using this observation, we can then determine that the sum of charges in the four cases are: A = 0 ; B = 0 ; C = $-3q$ and D = $+q$. Here, C has negative value for the total charge hence leading to negative electric potential → smallest potential among the 4 arrangements.

Choice A. This is probably due to sum of charges equal to 0, leading student to think the system has smallest potential.

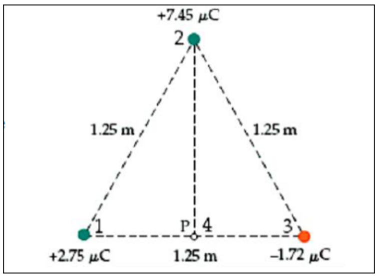
Choice B. This is probably due to sum of charges equal to 0, leading student to think the system has smallest potential.

Choice D. This is probably a wild guess.

Choice E. This is probably a wild guess.

34. Correct answer A. This question requires knowledge of electric potential and Conservation of Energy. To solve this question, we find the electric potential at Point P by summing the potentials due to the 3 charges at the vertices of the triangle. Then we multiply this potential by the value of the fourth charge, +4.82 μC, to determine its electric potential energy.

Now setting the potential energy equal to its kinetic energy to find the speed of the fourth charge when it is infinitely far away from the other three charges.



Steps to solve:

1. Distance from +7.45 μC to Point P:

$$r_2 = \sqrt{(1.25\text{ m})^2 - (\tfrac{1}{2}1.25\text{ m})^2} = 1.0825\text{ m}$$

2. Sum of potential from each of the three charges:

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} = k\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$
$$= \left(8.99\times10^9\ \tfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{2.75\times10^{-6}\text{ C}}{\frac{1}{2}(1.25\text{ m})} + \frac{7.45\times10^{-6}\text{ C}}{1.0825\text{ m}} + \frac{-1.72\times10^{-6}\text{ C}}{\frac{1}{2}(1.25\text{ m})}\right] = \boxed{76.7\text{ kV}}$$

3. Apply Conservation of Energy Principle:

$$q_4 V + 0 = 0 + \tfrac{1}{2} m v^2$$

$$v = \sqrt{\frac{2q_4}{m} V} = \sqrt{\frac{2\left(4.82\times 10^{-6}\ \text{C}\right)}{0.00233\ \text{kg}}\left(76{,}700\ \text{V}\right)} = \boxed{17.8\ \text{m/s}}$$

Choice B. Student may have used the absolute value of charge −1.72 μC in the calculation hence getting the wrong answer.

Choice C. Probably made a mistake in calculations.

Choice D. Probably a wild guess.

Choice E. Probably made a mistake in calculations.

35. Correct answer B. The electric potential at a point is the sum of the potentials from each individual charge, where $V = \Sigma \frac{kq}{R} = \frac{k}{R}\left(-\frac{Q}{\sqrt{2}} + 3Q - \frac{Q}{\sqrt{2}}\right) = +\frac{1.6kQ}{R}$.

Choice A. Student assumed that electric potential is proportional to sum of the charges and that each charge is the same distance from point P.

Choice C. This is a wild guess.

Choice D. Opposite signs used.

Choice E. Student did not take into account the sign of charges during calculations.

36. Correct answer A. A magnetic force, being perpendicular to the direction of motion, cannot do any work, and thus has zero power.

Choice B. Student can make this choice if they vaguely remembers that half of the power produced by electromagnetic induction has to be lost as heat.

Choice C. Student can make this choice if they believe that all the power consumed by the light bulb is produced by electromagnetic induction.

Choice D. Student can make this choice if they believe that electromagnetic induction produces more power than is consumed by the light bulb, but that this is not twice the amount consumed by the light bulb.

Choice E. Student can make this choice if they believe that half of the power produced by electromagnetic induction is consumed by the light bulb, and the other half lost as heat.

37. Correct answer A. The electric field lines point towards negative charges and away from positive charges. The electric potential at a distance r from a point charge q is $V =$

$\frac{kq}{r}$. In Sketch 1, A is nearest to a negative point charge; hence, $V_A < V_B$. In Sketch 2, B is nearest to a negative point charge; hence, $V_A > V_B$. In Sketch 3, A is nearest to a positive point charge; hence, $V_A > V_B$.

Choice B, C, D and E. The student may not have considered the positive and negative point charges based on direction of electric field lines.

38. Correct answer A. The three resistors are arranged in parallel. $\frac{1}{R_{eq}} = 1 + \frac{1}{2} + \frac{1}{3} \rightarrow R_{eq} = 0.5\ \Omega$.

Choice B, C, D and E. Incorrect interpretation of arrangement of resistors in series and parallel.

39. Correct answer D. The resistance of a conductor is proportional to its cross-sectional area and inversely proportional to its length. Given that $x > y > z$, the largest cross-sectional area is given by the side *xy* and the smallest cross-sectional area is given by side *yz*, the highest and lowest resistance are across terminals TU & PQ respectively. Since current is inversely proportional to the resistance, the current is lowest across terminals TU and highest across terminals PQ.

Choice A. This choice incorrectly assumes the resistance across terminals RS has the higher resistance.

Choice B. This choice incorrectly assumes the resistance across terminals RS has the lower resistance.

Choice C. Student probably assumed $R \propto \frac{L}{A}$ or $I \propto R$.

Choice E. This choice incorrectly assumes the resistance across terminals RS has the higher resistance.

40. Correct answer B. This question looks at the power dissipated in a dc and ac circuit.

For DC current, $P = I^2R$

For AC current, $\frac{1}{4}P = \frac{1}{2}I_o^2R$

$$I_o = \frac{1}{\sqrt{2}}I$$

Choice A. This assumes that the peak current is directly proportional to the power.

Choice C. This is a wild guess.

Choices D and E. This is probably due to mistakes in the relationships between the peak / rms currents and average power.

41. Correct answer C. The lower limit of the potential difference between terminals P and Q is 0 V, when the slider is moved towards Q. The upper limit of the potential difference between terminals P and Q occurs when the slider is moved away from Q → $V = 25\left(\frac{4}{1+4}\right) = 20 \text{ V}$

Choices A and B. Mistake in exponent value.

Choice D. This is a convenient answer.

Choice E. Misconception of the potential difference due to position of the terminals P and Q.

42. Correct answer C. The question requires an understanding that the electric potential in the whole volume of the mercury drop is a constant value. Therefore we need only to calculate the potential at the surface of the drop as though the charge is located at the centre of the drop, even though all charges are distributed on the conducting droplet surface. The potential is given by $V = \frac{Q}{4\pi\varepsilon_o r}$, where Q is the charge on the drop and r is the radius of the sphere. If eight identical drops fuse into one large drop, the total charge has increased by 8 while the radius has increased by 2. Thus, the potential has increased by 4 times.

Choice A. The student is thinking the potential will not change as all the droplets had the same initial potential. What the student must appreciate is that the charges from the eight drops are now are closer and denser when they appear in one large drop, giving rise to a higher potential.

Choice B. The student is making a guess.

Choice D. The student appreciates that the potential is proportional to the charge but has omitted the increase in radius.

Choice E. The student is making a guess.

43. Correct answer B. (i) is not correct because the electric potential does not have to be zero when the electric field is zero. (iii) is not correct because the electric field can become zero in between opposite charges.

Choices A and D. Student can make this choice if they are under the erroneous impression that the electric potential is zero when the electric field is zero.

Choices C and E. Student can make this choice if they are under the erroneous impression that the electric field can only become zero far from a charge.

44. Correct answer D. This question is a common test in understanding circuits. We see that the current in the circuit is given by $I = \frac{\varepsilon}{r+R}$ and that the power dissipated by the

load is $P = I^2R$, which can be rewritten as $P = \frac{\varepsilon}{R+\frac{r^2}{R}+2r}$. By maximizing P wrt R, we obtain the condition $R = r$. The obvious condition of having R as large as possible does not work out here, because this reduces I.

Choices A. B and C. The student has not understood circuits and power dissipation and has probably used am incorrect maximization procedure.

Choice E. The student is mistaken that the largest R will give the highest power dissipated by the load. This is not the case because I in the circuit will be small.

45. Correct answer C. The induced emf is given by $\varepsilon = -\frac{d\phi}{dt}$. After addition of the shorting wire, two circuits are possible: (i) shorting wire + half loop with Bulb A, or (ii) shorting wire + half loop with Bulb B. Circuit (i) includes the region of changing magnetic field but circuit (ii) does not include the magnetic field. Hence Bulb B will go out, and as a result, Bulb A will becomes brighter as the resistance of the circuit is now reduced due to the absence of Bulb B.

Choice A. The student has not understood electromagnetic induction. There will be emf induced in the circuit and the current must pass through one of the bulbs. So, both bulbs <u>cannot</u> go out.

Choice B. The student has not understood that the induced emf is produced when the circuit encompasses the flux change, i.e. there must flux linkage between the magnetic and the circuit. He may be thinking that the loop with a smaller area would be more likely to experience the induced emf.

Choice D. The student has not understood the effect of the shorting wire which provides an easier alternative path for the current to flow, so one of the must go out.

Choice E. While it is true that the direction of emf stems from the change in magnetic field (increasing or decreasing), this does not affect the path of the current through the half-loop (A or B). Hence the lighting of the bulbs is determined by the geometry of the shorting wire wrt the half-loops.

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46. Correct answer C. At first, A has half the activity so $\lambda_A N_A = \frac{1}{2}\lambda_B N_B$,
Then after time t, they have the same activity

$$\lambda_A N'_A = \lambda_B N'_B$$
$$\lambda_A N_A e^{-\lambda_A t} = \lambda_B N_B e^{-\lambda_B t}$$

but $\lambda_A N_A = \frac{1}{2}\lambda_B N_B$, so

$$\frac{1}{2}\lambda_B N_B e^{-\lambda_A t} = \lambda_B N_B e^{-\lambda_B t}$$
$$e^{-\lambda_A t} = 2e^{-\lambda_B t}$$
$$-\lambda_A t = \ln 2 - \lambda_B t$$

then use $\tau\lambda = \ln 2$,

$$-\ln 2 \frac{t}{\tau_A} = \ln 2 - \ln 2 \frac{t}{\tau_B}$$
$$\frac{t}{\tau_A} = \frac{t}{\tau_B} - 1$$

Choice A. Choosing this option may be due to a misconception that "same activity" means "same half-life".

Choice B. Choosing this option may be due to a misconception that "half the activity" means "half the decay constant" and the decay constant is inversely proportional to the half-life.

Choice C. Half-life does not depend on the sample size.

Choice D. Same calculation as option E but the incorrect expression $Ne^{\lambda t}$ is used instead.

47. Correct answer C. This is photoelectric effect.

Choice A. This is the effect of Incandescence.

Choice B. This is refraction.

Choice D. This is refraction.

Choice E. This is interference.

48. Correct answer A. Power is energy per second. In this case it is

$$Power = Number\ of\ photons\ per\ second \times energy\ per\ photon$$
$$= Number\ of\ photons\ per\ second \times \frac{\hbar c}{\lambda}$$

For red,

$$Power = Number\ of\ photons\ per\ second \times \frac{\hbar c}{\lambda}$$
$$= 4.0 \times 10^8 \times \frac{\hbar c}{632.8 \times 10^{-9}} = 6.32 \times 10^{14} \hbar c$$

For blue,

$$Power = Number\ of\ photons\ per\ second \times \frac{\hbar c}{\lambda}$$
$$= 1.5 \times 10^8 \times \frac{\hbar c}{480 \times 10^{-9}} = 3.13 \times 10^{14} \hbar c$$

Choice B. Choosing this option could mean that the student misunderstood power with energy of the photon. The blue photon has higher energy.

Choice C. Choosing this option could mean a miscalculation by the student.

Choice D. Choosing this option could mean a misunderstanding between power and intensity by the student.

Choice E. Choosing this option could mean a misunderstanding between power and total energy.

49. Correct answer A. Use $E = \frac{p^2}{2m}$ and $\lambda = \frac{h}{p}$ to get $\lambda = \frac{h}{\sqrt{2mE}}$ or $E = \frac{1}{2m}\left(\frac{h}{\lambda}\right)^2$. Then use conservation of energy

$$E_{1i} + E_{2i} = E_{1f} + E_{2f}$$
$$\frac{1}{m}\left(\frac{h}{\lambda_i}\right)^2 = \frac{1}{2m}\left(\frac{h}{\lambda_{1f}}\right)^2 + \frac{1}{2m}\left(\frac{h}{\lambda_{2f}}\right)^2$$
$$\frac{1}{\lambda_{2f}^2} = \frac{2}{\lambda_i^2} - \frac{1}{\lambda_{1f}^2}$$
$$\lambda_{2f} = 860.37 \text{ nm}$$

Choice B. Choosing this option could be a miscalculation using this formula:

$$\frac{1}{\lambda_{2f}^2} = \frac{2}{\lambda_i^2} + \frac{1}{\lambda_{1f}^2}$$
$$\lambda_{2f} = 488.80 \text{ nm}$$

Choice C. Choosing this option could be a miscalculation using this formula:

$$\frac{1}{\lambda_{2f}^2} = \frac{1}{\lambda_{1f}^2} - \frac{1}{\lambda_i^2}$$
$$\lambda_{2f} = 5492 \text{ nm}$$

Choice D. Choosing this option could be a miscalculation using this formula:

$$\frac{1}{\lambda_{2f}} = \frac{2}{\lambda_i} - \frac{1}{\lambda_{1f}}$$
$$\lambda_{2f} = 860.24 \text{ nm}$$

Choice E. Choosing this option could be a miscalculation using this formula:

$$\frac{1}{\lambda_{2f}} = \frac{1}{\lambda_{1f}} - \frac{1}{\lambda_i}$$
$$\lambda_{2f} = 71400 \text{ nm}$$

50. Correct answer A. The activity is λN so tripled mass means tripled N but the decay constant is a constant.

Choices B, C, D and E. This option is merely a combination of the correct answer.

51. Correct answer B. We use the photoelectric effect formula:

$$hf = \phi + eV_{stop}$$
$$V_{stop} = \frac{h}{e}f - \frac{\phi}{e}$$

So the gradient is $\frac{h}{e}$ and has nothing to do with the work function ϕ.

Choices A, D and E. This option tries to include the 2ϕ as part of the answer as confusion.

Choice C. This option is a miscalculation of the inverse of the right answer.

52. Correct answer D. We use the photoelectric effect formula:

$$\frac{hc}{\lambda} = \phi + eV_{stop}$$

$$\frac{hc}{450 \times 10^{-9}} = \phi + e(0.75)$$

$$\phi = \frac{hc}{(450 \times 10^{-9})} - e(0.75) = 2\ eV$$

Then use wavelength 300 nm,

$$\frac{hc}{300 \times 10^{-9}} = \phi + eV_{stop}$$

$$\frac{hc}{300 \times 10^{-9}} = \frac{hc}{(450 \times 10^{-9})} - e(0.75) + eV_{stop}$$

$V_{stop} = 2.13$ V

Choice A. This option is a miscalculation where 450 nm → 300nm so 0.75 V → 0.5 V proportionally.

Choice B. This option is a miscalculation where the inverse relationship of 450 nm → 300nm is used, so $V_{stop} = 1.13\ V$.

Choice C. This option is misunderstanding that the stopping potential is deduced from the work function:

$$\phi = \frac{hc}{(450 \times 10^{-9})} - e(0.75) = 2\ eV$$

Choice E. This option is a miscalculation where the wavelength difference is used:

$$\frac{hc}{(450 - 300) \times 10^{-9}} = 2\ eV + eV_{stop}$$

$$V_{stop} = 6.28\ V$$

53. Correct answer C. We use the photoelectric effect formula:

$$hf = \phi + eV_{stop}$$

From V=5 V and $KE_{max} = 1$ eV, this means stopping potential = 6V. So

$$hf = \phi + 6\ eV$$

Now $f' = 2f$,

$$hf' + 5\ eV = \phi + eV_{stop}$$

with $V_{stop} = 20$ V, then

$$h2f + 5\ eV = \phi + 20\ eV$$

then

$$2\phi + 12\ eV + 5eV = \phi + 20\ eV$$
$$\phi = 3\ eV$$

Now $f' = 2f$,

$$hf' + 5\ eV = \phi + eV_{stop}$$

with $V_{stop} = 20$ V, then

$$h2f + 5\ eV = \phi + 20\ eV$$

then

$$2\phi + 12\ eV + 5eV = \phi + 20\ eV$$
$$\phi = 3\ eV$$

Choice A. This option is a misunderstanding that $KE_{max} = 1\ eV =$ work function.

Choice B. This option is a misunderstanding that double the incident frequency leads to doubling of $KE_{max} = 2\ eV =$ work function.

Choice D. This option is simply sequential to the previous options.

Choice E. This option is simply sequential to the previous options.

54. Correct answer D. This option is correct because decaying is an independent random event.

Choice A. Statement (i) is correct but it is not the only correct statement.

Choice B. Statement (ii) is correct but it is not the only correct statement.

Choice C. This option is incorrect because decaying is an independent random event.

Choice E. Decaying is a random event, nuclei do not definitively decay.

55. Correct answer D. Activity is λN then half life is related as $\tau\lambda = \ln 2$, so Activity $= \frac{N \ln 2}{\tau}$

Initially they have the same activity,

$$\frac{N_s \ln 2}{\tau_s} = \frac{N_c \ln 2}{\tau_c}$$
$$\frac{N_s}{N_c} = \frac{\tau_s}{\tau_c} = \frac{28}{5.3}$$

Choice A. This option is a variation of the correct option.

Choice B. This option is an incorrect statement.

Choice C. This option is a variation of the correct option. Students may also misunderstood activity to be the number of nuclei.

Choice E. This option is an incorrect statement.

56. Correct answer A. Use the decay equation

$$N = N_0 e^{-\lambda t}$$

And $\tau\lambda = \ln 2$, so

$$N = N_0 e^{-\frac{t \ln 2}{\tau}}$$

$$t = \frac{\tau}{\ln 2} \ln \frac{N_0}{N} = \frac{4}{\ln 2} \ln 10 = 13.28 \text{ days}$$

Choice B. This option is a multiple of the correct answer.

Choice C. This option is a multiple of the correct answer.

Choice D. This option is a multiple of the correct answer.

Choice E. This option is a multiple of the correct answer.

57. Correct answer C. At maximum, the entire energy E of an electron can be converted to an X-ray photon. Then the formula for a photon $E = hf$ gives the frequency of the X-ray photon to be E/h.

Choice A. Statement (i) is incorrect as not all the energy is converted to X-rays.

Choice B. Statement (i) is incorrect as not all the energy is converted to X-rays.

Choice D. Statement (iii) is also correct.

Choice E. Statement (ii) is also correct

58. Correct answer B. Given that,

$$\frac{N_{A0}}{N_{B0}} = \frac{6}{4}$$

Then $\frac{N_A}{N_B} = \frac{3}{7} = \frac{N_{A0} e^{-\lambda_A t}}{N_{B0} e^{-\lambda_B t}} = \frac{6e^{-\lambda_A t}}{4e^{-\lambda_B t}}$ so $\frac{e^{-\lambda_A t}}{e^{-\lambda_B t}} = \frac{12}{42}$ so $e^{-(\lambda_A - \lambda_B)t} = \frac{2}{7}$ so $\lambda_A - \lambda_B = -\frac{1}{t} \ln \frac{2}{7}$

Then $\frac{N_A'}{N_B'} = \frac{1}{9} = \frac{N_A e^{-\lambda_A t'}}{N_B e^{-\lambda_B t'}} = \frac{3e^{-\lambda_A t'}}{7e^{-\lambda_B t'}}$ so $e^{-(\lambda_A - \lambda_B)t'} = \frac{7}{27}$ then $t' = -\frac{1}{\lambda_A - \lambda_B} \ln \frac{7}{27} = \frac{t}{\ln \frac{2}{7}} \ln \frac{7}{27} = 3.23$ days.

Choice A. A miscalculation assuming proportion:

$$3\ days\ \times \frac{3}{7} = 1.29\ days$$

Choice C. A miscalculation assuming proportion:

$$3\ days\ \times \frac{6}{4} = 4.5\ days$$

Choice D. A miscalculation assuming proportion:

$$3\ days\ \times \frac{7}{3} = 7\ days$$

Choice E. A miscalculation assuming proportion:

$$3\ days\ \times \frac{9}{1} = 27\ days$$

59. Correct answer B. The student needs to understand that the energy in a light beam is the sum total from its photons, each carrying an energy of hf. As blue photons possess higher f and are more energetic, there will be less of them present in the light beam, as compared to the red ones which possess lower f.

Choice A. The student has not understood that a light beam is made up of photons, each carrying an energy of hf so the 2 beams having the same energy implies that they have different numbers of photons.

Choice C. The student is thinking of hf, which is larger for blue photons. However, higher photon energy does not lead to more photons in the beam but the contrary is true.

Choice D. Having higher energy (= hf) does not enable the blue photons to move faster. The student has mistaken photon energy for kinetic energy of particles (= ½ mv^2) which determine the speed of the particles.

Choice E. The red photons having lower energy are more abundant than the blue photons but they do not move any faster. The student has mistaken more for faster.

60. Correct answer A. Here we wish to demonstrate interference but the condition to achieve a reasonable value of λ is the same: small momentum, i.e. small mass and small velocity.

Choice B. Large velocity will lead to very small λ which makes the interference fringes very close to each other and thus very difficult to observe.

Choice C. Planck's constant h is a universal constant, and we have no ability to choose its value.

Choice D. Small mass and large velocity will compensate each other, so that λ remains small.

Choice E. The student has understood the necessary condition for interference but alas, the value of h cannot be varied.