# Asia Pacific Mathematical Olympiad
for Primary Schools 2008

First Round
2 hours
(150 marks)

1. What is the difference between the sum of the first 2008 positive even numbers and the sum of the first 2008 odd numbers?

2. The sides of a triangle have lengths that are consecutive whole numbers and its perimeter is greater than 2008 cm. If the least possible perimeter of the triangle is x cm, find the value of x.

3. Find the value of $2008 \times 20072007 - 2007 \times 20082007$.

4. When a rectangular sheet of paper with length 8 cm is folded exactly into half, the ratio of its length to its width remains unchanged. If the square of the width, $(\text{width})^2$ of the original piece of paper = x cm^2, find the value of x.

5. The numbers 1 to 10 are arranged in the circles in such a way that the sum of the four numbers on each line is 21. What is the value of n?
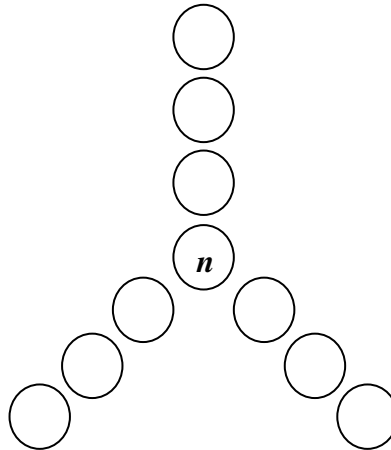

6. Find the value of $(56789 + 67895 + 78956 + 89567 + 95678) \div 5$

7. The diagram shows a circle whose circumference touches the sides and the vertices of a large and a small square respectively. If the area of the small square is $9cm^2$ and the area of the large square is x cm^2, find the value of x.
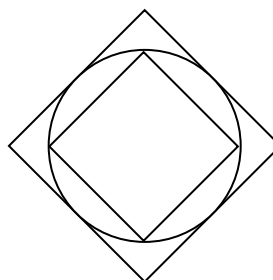
8. One hundred numbers are placed along the circumference of a circle. When any five adjacent numbers are added, the total is always 40. Find the difference between the largest and the smallest of these numbers.

9. In triangle PQR, PQ = 6cm, PR = 4cm and QR = 6cm. If sides PQ and PR are tripled while QR remains unchanged, then
    (1)   The area is tripled.
    (2)   The area increases by 9 times.
    (3)   The altitude is tripled.
    (4)   The area decreases to $0cm^2$.
    (5)   None of the above.

10. Find the last 5 digits of the sum
    $$1 + 22 + 333 + 4444 + 55555 + 666666 + 7777777 + 88888888 + 999999999.$$

11. If an arc of $80°$ on circle A has the same length as an arc of $60°$ on circle B, and that the ratio of the area of circle A to the area of circle B is a : b, find the smallest value of a + b.

12. A circle of circumference 1 m rolls around the equilateral triangle of perimeter 3 m. How many turns does the circle make as it rolls around the triangle once without slipping?
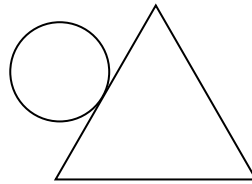

13. The diagram shows a rectangle ABCD with area 32cm². Given that area of triangle ADF = 2 cm^2, area of triangle ABE = 8 cm^2 and the area of the shaded region = x cm^2, find the value of x.
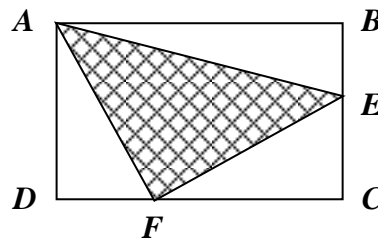

14. There are two containers A and B. Each of them contains 9 white marbles, 9 black marbles and 9 red marbles. If 10 marbles are removed from A and placed into B, how many marbles must be returned from B to A to make sure that there are at least 8 marbles of each colour in A?

15. 9^{10} is a 10-digit number. If **A** is the sum of all digits of 9^{10}, **B** is the sum of all digits of **A** and **C** is the sum of all digits of **B**, find the value of **C**.

16. A car travels from point A to B at a constant speed of V km/h. If the car increases its speed by 20%, it will reach B one hour earlier. If the car increases its speed by 25% after traveling at V km/h for 120 km, it will reach B forty eight minutes earlier. If the distance between the two towns is x km, find the value of x.

17. After all the faces of a rectangular block are painted green, the block is then cut into cubes each of volume 1 cm^3. It is found that 7 of the unit cubes have none of their faces painted green. How many of the cubes have exactly two faces painted green?

18. The diagram shows an equilateral triangle ABC with OD, OE and OF perpendicular to AB, BC and CA respectively. If OD + OE + OF = 28 cm and the height of the triangle ABC = x cm, find the value of x.
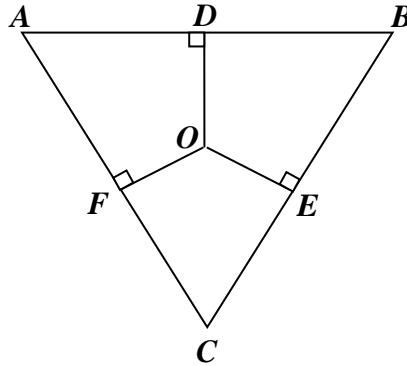

19. The diagram shows a triangle PQR. Three lines parallel to the sides of the triangle are drawn through a point O. Given that the areas of the three shaded triangles are 32 cm^2, 48 cm^2 and 96 cm^2 respectively, and the area of the triangle PQR = x cm^2, find the value of x.
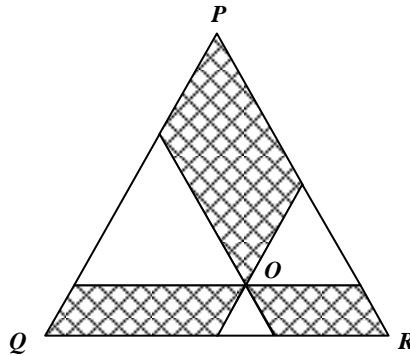

20. As shown in the diagram, a straight line can cut across at most 3 squares in a 2 by 2 square and at most 5 squares in a 3 by 3 square. What is the greatest number of squares than can be cut across by a straight line in a 2008 by 2008 square?
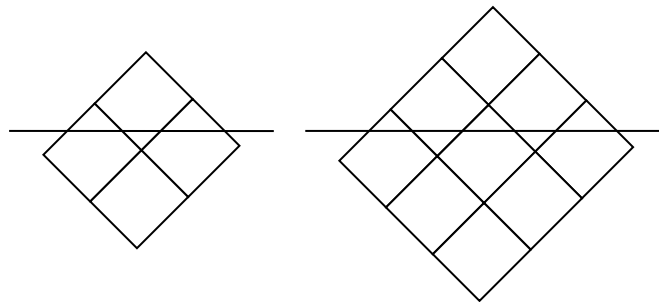
21. There are three containers. One contains red marbles, another white marbles and the third one a mixture of red and white marbles. Given that all of them are labeled wrongly and you are allowed to open only one of them to take out only one marble in order to state correctly where all the labels ought to go, which container should you open?

| red | white | mixture |
| (1) | (2) | (3) |
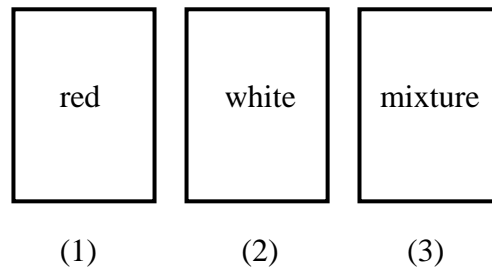
22. Given that $\underbrace{200820082008\ldots2008\ 623}_{n \text{ of } 2008}$, find the smallest value of n such that the number is divisible by 11.

23. Given that ∠ABC + ∠BCD + ∠CDE + ∠DEF + ∠EFG + ∠FGA + ∠GAB = x°, find the value of x.
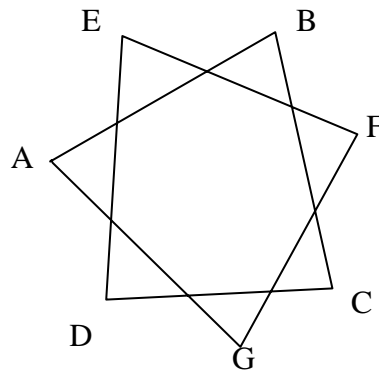
24. Peter and Jane are to take turns to subtract perfect squares from a given whole number and the one who reaches zero first is the winner, if the whole number is 29, and Peter is the first player, what perfect number must he subtract in order for him to definitely win? [Note: 4, 9 and 16 are examples of perfect squares.]

25. The inhabitants of an island are either gentlemen or liars. A gentleman always tells the truth and a liar always lies. A, B, and C are three of the inhabitants. A sailor who landed on the island asked A: "Are you a gentlemen or a liar?" A answered but he sailor could not hear clearly what he said. He then asked B, "What did A say?" B replied, "A said that he is a liar." At that instant, C immediately shouted "B is lying!"
I   It is impossible to tell whether A is a gentlemen or a liar.
II   B is a gentleman and C is a liar.
III   B is a liar and C is a gentleman.
(1)   Only I is true.
(2)   Only II is true.
(3)   Only 3 is true.
(4)   Only I and II are true.
(5)   Only I and III are true,

26. The product of n whole numbers $1 \times 2 \times 3 \times 4 \times 5 \times \ldots \times (n-1) \times n$ has twenty eight consecutive zeros. Find the largest value of n.

27. Find the largest number *n* such that there is only one whole number k that satisfies

$$\frac{8}{21} < \frac{n}{n+k} < \frac{5}{13}$$

[Note: A < C < B means that the value of C is between A and B example 4 < 9 < 16]

28. How many ways are there to distribute 28 identical marbles into 3 different boxes such that no box is empty?

29. If Peter walks up an up-going escalator at the rate of 1 step per second, he is able to reach the top in 10 steps. If he increases his rate to 2 steps per second, he can reach the top in 16 steps. Find the number of steps of escalator.

30. The diagram shows a quadrilateral ABCD. If AB = CD, angle ADB + angle CBD = 180°, angle BCD = 55° and angle BAD = x°, find the value of x.
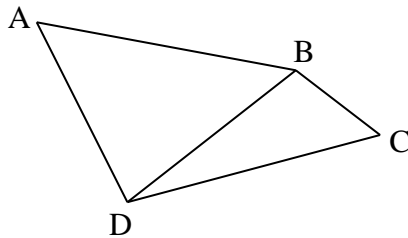
Number of correct answers for Q1 to Q10 :   _____   Marks (×4) : _____
Number of correct answers for Q11 to Q20 :   _____   Marks (×5) : _____
Number of correct answers for Q21 to Q30 :   _____   Marks (×6) : _____

Answers:

1. 2008	2. 2010	3. 2007
4. 32	5. 4	6. 77777
7. 18	8. 0	9. (4)
10. 93685	11. 25	12. 4
13. 15	14. 27	15. 9
16. 360	17. 36	18. 28
19. 288	20. 4015	21. (3)
22. 3	23. 540	24. 9
25. (5)	26. 124	27. 80
28. 351	29. 40	30. 55