

Date of Birth: _____ (DD/MM/YY)

0900 h – 1100 h

Hwa Chong Institution
Mathematics Learning And Research Centre

Singapore Mathematical Olympiad for Primary Schools 2006

First Round
2 hours
(150 marks)

Instructions to Participants:

Attempt as many questions as you can.

Neither mathematical tables nor calculators may be used.

Write your answers in the answer boxes.

Working may be shown in the space below each question.

Points are awarded for correct answers only.

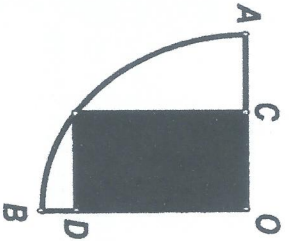
This question paper consists of 16 printed pages (including this page)

Number of correct answers for Q1 to Q10: _____ Marks (x 4) : _____

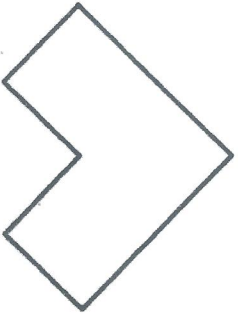
Number of correct answers for Q11 to Q20: _____ Marks (x 5): _____

Number of correct answers for Q21 to Q30: _____ Marks (x 6): _____

- 1 The diagram shows a quarter circle OAB , centre O with radius 6 cm. The shaded region is a rectangle. Find the length CD .



- 2 Draw straight lines to divide the figure into 4 identical shapes with equal areas.



- 3 A wooden plank rests on four identical rollers on horizontal ground. The circumference of each roller is 1 m. Find the distance moved by the plank after the rollers have each made one complete revolution, assuming there is no slipping.



- 4 Both Jane and Mary had a sum of money each. They wanted to buy a Mathematics book. They were each short of \$52 and \$2 respectively and after pooling their money together they still could not afford the book. Given that the price of the book is a whole number, how much did the book cost?

- 5 In a social gathering, each of the 38 boys talked to 3 girls while each girl talked to 2 boys. Find the number of girls in the social gathering.

- 6 What is the missing number in the following number sequence?

4, 6, 10, 14, 22, 26, , 38, 46, 58

- 7 In a party, there is a group of children. The boys always lie and the girls never lie. 10 children make the following statements

- A : Only one of us is a boy.
B : Only two of us are boys.
C : Only three of us are boys.
D : Only four of us are boys.
E : Only five of us are boys.
F : Only six of us are boys.
G : Only seven of us are boys.
H : Only eight of us are boys.
I : Only nine of us are boys.
J : All of us are boys.

Which of them is /are girl(s)?

8

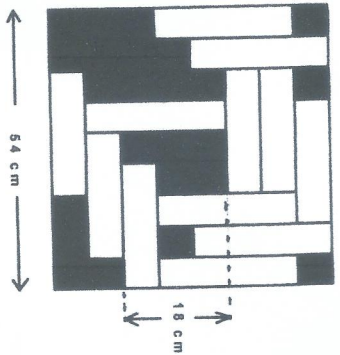
Every morning Kevin has to reach school by 8 am.

If he travels at 70 metres per minute, he would arrive 10 minutes earlier.

If he travels at 60 metres per minute, he would arrive 8 minutes earlier.

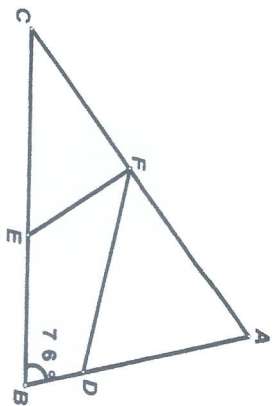
Given that he leaves home for school after 7 am, find the exact time he leaves for school.

- 9 The diagram shows 12 identical rectangles within a larger rectangle with one side 54 cm long. Find the total area of the shaded regions.

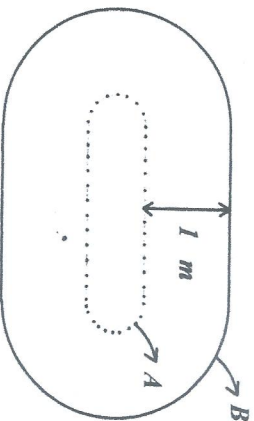


- 10 The prices of 3 types of snacks, a chocolate bar, a potato pie and a piece of cake were \$1.50, \$2.10 and \$3.30 respectively. Ben had \$14 to spend on these snacks and he bought the snacks in such a way that he maximized his spending. What was the amount he had left?

- 11 In the triangle ABC, angle $ABC = 76^\circ$, $AD = AF$ and $CE = CF$. Find angle DFE.



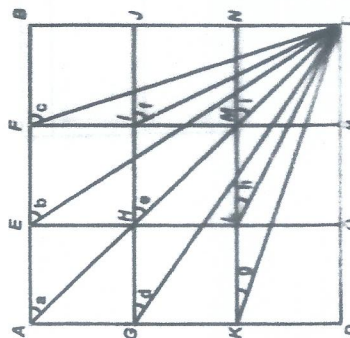
- 12 Two running tracks A (inner dotted track) and B (outer solid track) are each formed from straight parallel sides and semicircular ends. The tracks are 1 m apart. Track B is longer than track A by $\frac{22}{7}$ m. Take π as $\frac{22}{7}$.



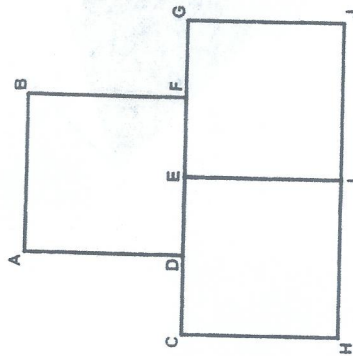
- 13 It takes 15 hours to fill up a tank when both taps A and B are turned on together. If tap A is turned on for 8 hours, then turned off, tap B will take another 50 hours to fill up the tank. How long will it take for the tank to be filled by tap B alone?

- 15 There is a row of coins, numbered from 1 to 2006, each with head facing up. In the 1st round, coins whose numbers are divisible by 1 are turned over. In the 2nd round, coins whose numbers are divisible by 2 are turned over. In the 3rd round.....
In the 2006th round, coins whose numbers are divisible by 2006 are turned over. How many coins will have heads facing up after the 2006th round of turning?

- 14 The diagram shows a square ABCD formed by 9 identical squares. Vertices A, E, F, G, H, I, K, L and M are each joined to vertex C. What is the sum of the angles a, b, c, d, e, f, g, h and i?

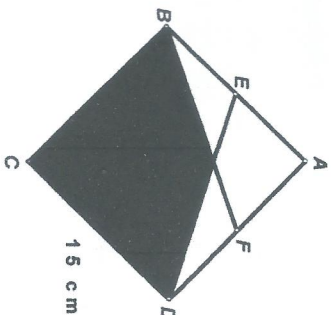


- 16 The diagram shows a figure consisting of three identical squares of side 1 m. 10 points A, B, C, D, E, F, G, H, I and J are the vertices of the squares. How many triangles, of area 1 m^2 , can be formed using any 3 of the 10 points as vertices, given that the triangles must have one of its sides either horizontal or vertical?



17 Find the value of $\left(\frac{1}{2} + \frac{345}{456} + \frac{567}{678} + 1\right) \times \left(\frac{345}{456} + \frac{567}{678} + \frac{7}{8}\right) - \left(\frac{1}{2} + \frac{345}{456} + \frac{567}{678}\right) \times \left(\frac{345}{456} + \frac{567}{678} + 1\right)$.

- 18 ABCD is a square of side 15 cm. E and F are the mid-points of AB and AD respectively. Find the area of the shaded region.



- 19 There are 3 boxes. One box contains a white ball, one a black ball and one a doll. A sentence is written on the cover lid of each of the boxes.
The sentence written on the box containing the white ball is always true.
The sentence written on the box containing the black ball is always false.
The sentence written on the box containing the doll can be true or false.

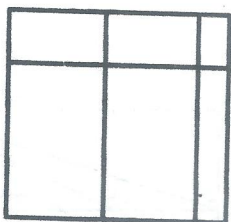
Box 1 : The sentence written on Box 2 is true.

Box 2 : Box 1 contains a black ball.

Box 3 : The doll is in Box 1.

Which box contains the doll?

- 20 The diagram shows a square formed by 6 rectangles. Given that the total perimeter of the 6 rectangles is 220 cm, find the area of the square.



21. There are 2006 oranges.

The 1st group of people consumed $\frac{1}{2}$ of the oranges,

the 2nd group of people consumed $\frac{1}{3}$ of the remaining oranges,

the 3rd group,

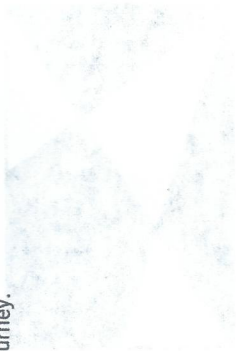
and the 2005th group of people consumed $\frac{1}{2006}$ of the remaining oranges.

Find the number of oranges left.

23. Ling Ling started from town A and planned to reach town B by noon.

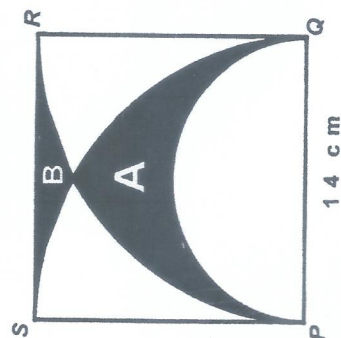
For the first half of the journey her speed was $\frac{21}{23}$ of the planned speed.

If she arrived in town B at noon, she must have travelled at _____ of the planned speed in the second half of the journey.



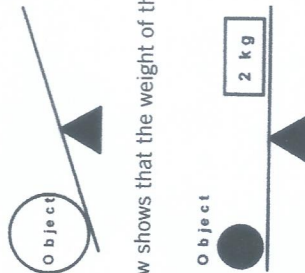
22. The diagram shows a semicircle with diameter 14 cm. PQS and QPR are two quarter circles with P and Q as centres respectively. Find the difference between the areas of shaded region A and region B.

Take π as $\frac{22}{7}$.



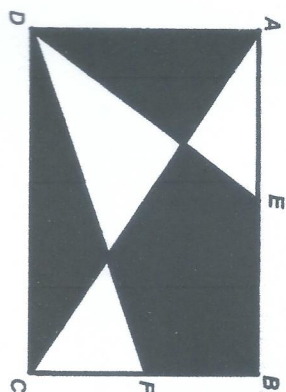
24.

The diagram shows a beam balance. By placing standard weights and the object to be measured at the ends of the beam, it can measure the weight of an object. What is the minimum number of standard weights required to weigh any object from 1 to 40 kg, given that the standard weights and the object can have only whole number weights?



[Note : The diagram below shows that the weight of the object is 2 kg. 2 kg is a standard weight.]

- 25 ABCD is a rectangle with area 150 cm^2 . E and F are the mid-points of AB and BC respectively. Find the total area of the shaded parts.

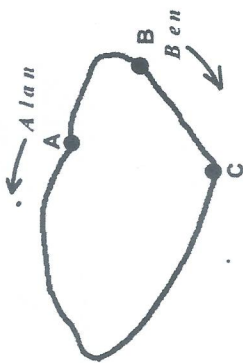


- 26 Find the value of $1 + \frac{1}{3} + \frac{1}{6} + \frac{1}{10} + \frac{1}{15} + \frac{1}{21} + \dots + \frac{1}{300}$.

- 27 How many 3-digit numbers have a remainder of 7, 2 and 3 when divided by 9, 5 and 4 respectively?

- 28 Samuel wants to distribute 174 identical marbles into a number of boxes. Given that each box can have 12 to 22 marbles and that each box contains different number of marbles, find the number of different ways to do the task.


- 29 Alan and Ben start running, at the same instant, along the running track from A and B respectively. They are running away from each other until they meet each other at C. After which, Ben turns to run in the same direction as Alan. When Ben passes B, Alan passes A. Given that Alan has run 800 m in total when he meets Ben the second time and that $AB = BC$, find the total length of the running track. Assume that their speeds have been constant throughout the journey.



- 30 Take a number between 100 and 999. The digits of the number are then rearranged so that no digit occupies its original position. The difference between these two numbers is less than 100 and is a perfect cube. List all such possible numbers.

[Note : 8, 27 and 64 are examples of perfect cubes]

Answer Key (APMOPS 2006 Round 1)

1	6	16	17
2		17	$\frac{3}{8}$
3	2	18	150
4	53	19	1
5	57	20	484
6	34	21	1
7	I	22	35
8	7.38 a.m.	23	$\frac{21}{19}$
9	864	24	4
10	\$0.20	25	100
11	52	26	$1\frac{23}{25}$
12	$6\frac{2}{7}$	27	5
13	90	28	1
14	405	29	320
15	1962	30	192, 219, 780, 807, 891, 918