The Chinese High School
Mathematics Learning And Research Centre

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Asia Pacific Mathematical Olympiad for Primary Schools 2002

Invitation Round
2 hours
(60 marks)

Instructions to Participants

Attempt as many questions as you can.
Neither mathematical tables nor calculators may be used.
Working ***must*** be clearly shown in the space below each question.
Marks are awarded for both method and answer.
Each question carries 10 marks.

This question paper consists of 7 printed pages (including this page)

Question	1	2	3	4	5	6
Marks						

Total Marks for **Invitation Round** : _____

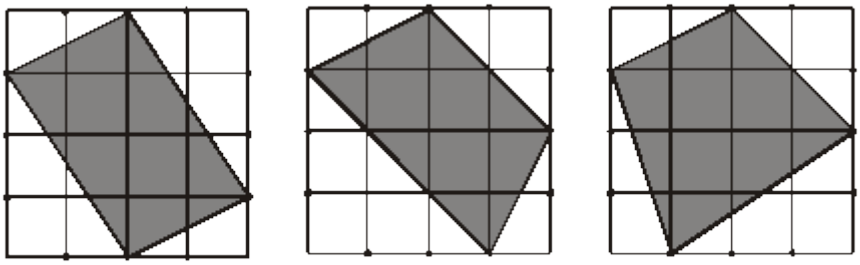
1. The following is an incomplete 9 by 9 multiplication table.

×	1	2	3	4	5	6	7	8	9
1				:			:		
2				:			:		
3				:			:		
4	. .	. .	. .	16			:		
5	. .	. .	. .	. .	. .	. .	35		
6									
7									
8									
9									

(a) Find out how many of the 81 products are odd numbers .

(b) If the multiplication table is extended up to 99 by 99, how many of the products are odd numbers ?

2. Find the area of each of the following shaded regions.



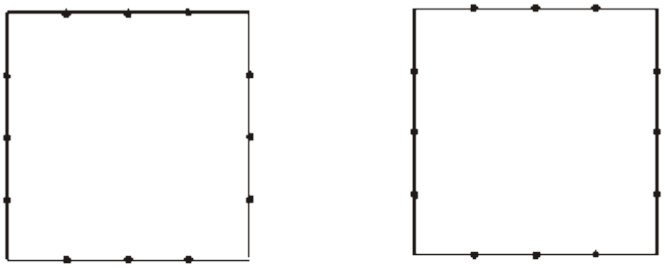
(A) (B) (C)

The shaded 4-sided figures above have been drawn with the four vertices at the dots, on each side of the square.

In the same manner,

(i) draw a 4-sided figure with the greatest possible area in (**D**),

(ii) draw a 4-sided figure with the smallest possible area in (**E**).



(D) (E)

3. Consider the following number sequence :

$$\frac{1}{2}, \frac{3}{5}, \frac{8}{13}, \frac{21}{34}, \ldots\ldots, \frac{2584}{4181}.$$

(i) Find the 5^{th} and 6^{th} numbers in the sequence.

(ii) How many numbers are there in the sequence ?

(iii) If this sequence continues, what is the number immediately after $\frac{2584}{4181}$?

4. There are two identical bottles A and B.

A contains $\frac{1}{2}$ bottle of pure honey.

B contains a full bottle of water.

First pour the water from B to fill up A and mix the content completely ;

then pour the mixture from A to fill up B and mix the content completely.

(i) What is the ratio of honey to water in B after the two pourings ?

(ii) If this process of pouring from A to B , and then from B to A, is repeated for another time, what will be the ratio of honey to water in B ?

(iii) If this process of pouring is repeated indefinitely, what will be the ratio of honey to water in B ?

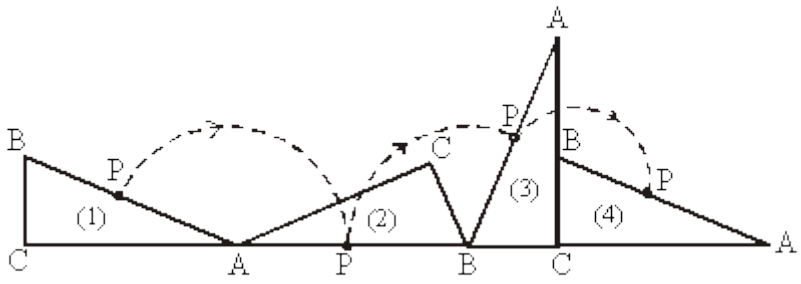
5. A right-angled triangle (1) is placed with one side lying along a straight line.

It is rotated about point A into position (2).

It is then rotated about point B into position (3).

Finally, it is rotated about point C into position (4).

Given that AP = BP = CP = 10 cm, find the total length of the path traced out by point P. (Take $\pi = 3.14$.)



6. Figure 1 shows a street network where *A, B, ..., I* are junctions. We observe that it takes <u>at most</u> 4 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 4 steps.

$$A \xrightarrow{(1)} B \xrightarrow{(2)} E \xrightarrow{(3)} H \xrightarrow{(4)} I$$

The street network is now converted to a one-way traffic system as shown in Figure 2. In this one-way traffic system, it takes <u>at most</u> 6 steps to travel from one junction to another junction.

e.g. From *A* to *I*, we may take the following 6 steps .

$$A \xrightarrow{(1)} D \xrightarrow{(2)} E \xrightarrow{(3)} B \xrightarrow{(4)} C \xrightarrow{(5)} F \xrightarrow{(6)} I$$

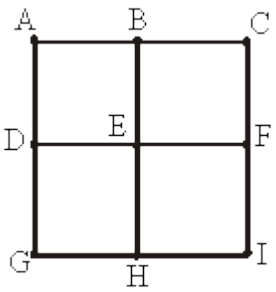


Figure 1

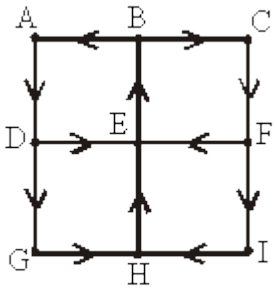


Figure 2

In Figure 3, design a one-way traffic system so that it takes **<u>at most</u>** 5 steps to travel between any two junctions.

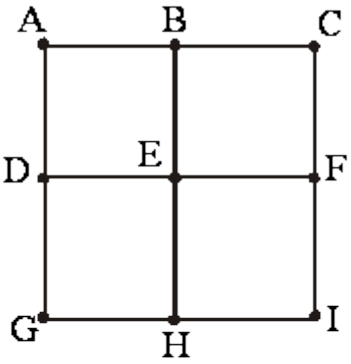


Figure 3

THE END

Singapore Mathematical Olympiad for Primary Schools 2002

Invitation Round – Answers Sheet

Question 1:

Ans: a) 25 b) 2500

Question 2:

Question 3:

Ans:

i) 5^{th} number: 55/89 6^{th} number: 144/233

ii) 2584/4181

iii) 6765/10946

Question 4:

Ans:

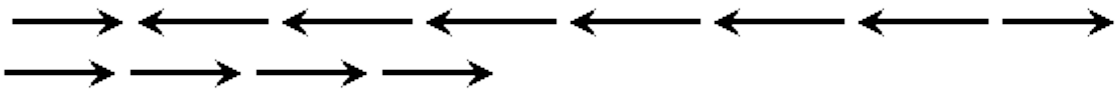
i) 1 : 3

ii) 5 : 11

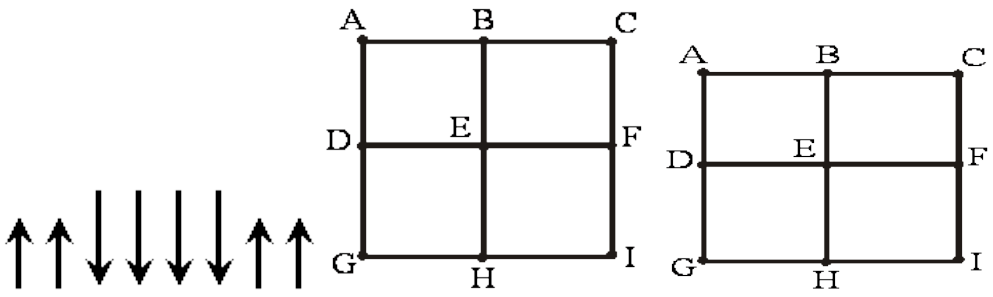
iii) 1 : 2

Question 5:

Ans: 62.8cm



Question 6


A
B
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