Asia Pacific Mathematical Olympiad
for Primary Schools 2013

First Round
2 hours
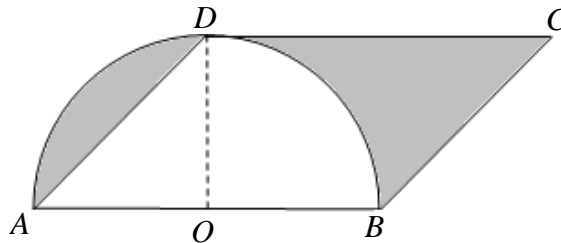**(150 marks)**

1. Find the last 5 digit of the sum.
$$127354 + 27354 + 7354 + 354 + 54 + 4.$$
(SMOPS 2013 Q.1)

2. Find the sum of all two-digit numbers whose units digit and tens digit are both even.
(SMOPS 2013 Q.2)

3. The diagram shows a semi-circle with centre O overlapped with a parallelogram $ABCD$. The diameter, AB, of the semi-circle is 12cm. Find the total area of the shaded regions in cm^2.
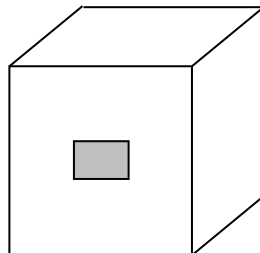

(SMOPS 2013 Q.3)

4. Abel, Ben and Charlie took part in SMOPS 2012, which comprised 30 questions. They had correctly answered 26, 23 and 18 questions respectively. What is the least possible number of questions that were answered correctly by all 3 students?
(SMOPS 2013 Q.4)

5. Find the value of $555 \times 554555 - 554 \times 555554$.
(SMOPS 2013 Q.5)

6. The diagram shows a cube with side length 5 cm. If a rectangular tunnel with dimensions 2 cm by 3 cm is made in the middle of the cube, find the amount of increase in the total surface area of the resulting solid in cm^2.

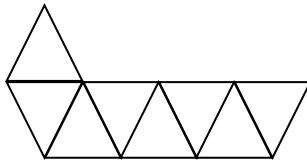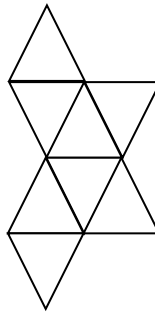(SMOPS 2013 Q.6)

7. The average value of four whole numbers $\overline{a,b5}$, $\overline{c17}$ and $\overline{d432}$, where a, b, c and d each represents the first digit of a number, is 1735. Find the value of $a + b + c + d$.

    (SMOPS 2013 Q.7)

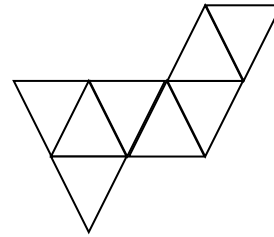8. The diagram shows a regular octahedron, which is a solid composed of eight equilateral triangles.
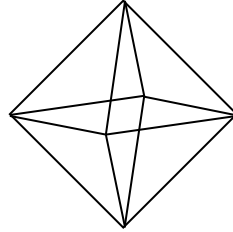
    Which of the following patterns can be folded into a regular octahedron?

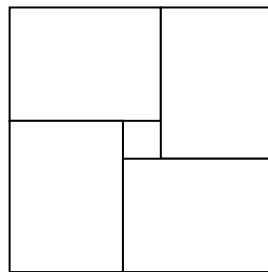 | Pattern (1) | Pattern (2) | Pattern (3) |

    (SMOPS 2013 Q.8)

9. The sum of 2 prime numbers is equal to 2013. Find the product of these two numbers.

    (SMOPS 2013 Q.9)

10. The figure shows a big square, which is divided into four identical rectangles and one small square. Given that the area of the small square is 16 cm^2, the area of each rectangle is 140 cm^2, find the width of each rectangle in cm.
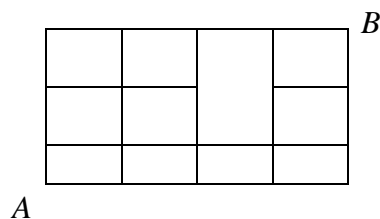

    (SMOPS 2013 Q.10)

11. A 5-digit number written in the form $\overline{24abc}$ has the last three digits unknown. If this number is divisible by 3, 4 and 5 respectively, find the greatest possible value that $\overline{abc}$ can take.
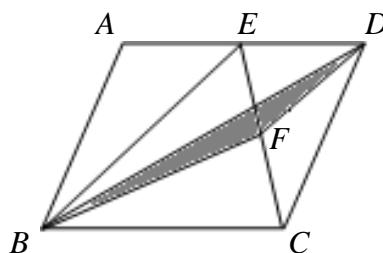
    (SMOPS 2013 Q.11)

12. In the diagram, an ant is moving from *A* to *B*. If the ant is only allowed to move to the right or upwards along the grid lines, how many different paths are there from *A* to *B*?

<div align="right">(SMOPS 2013 Q.12)</div>

13. The diagram shows a parallelogram *ABCD*. *E* is the midpoint of *AD*. *F* is the midpoint of *EC*. If the area of the triangle *BFD* is 9 cm^2, find the area of the parallelogram *ABCD* in cm^2.

<div align="right">(SMOPS 2013 Q.13)</div>

14. If we write $\dfrac{2013}{1990}$ in the form

$$a + \cfrac{1}{b + \cfrac{1}{c + \cfrac{1}{d + \cfrac{1}{e}}}}$$

where *a*, *b*, *c, d* and *e* are positive integers, what is the value of $a + b + c + d + e$?

<div align="right">(SMOPS 2013 Q.14)</div>

15. In the given diagram, find the value (in degrees) of
$$\angle A + \angle B + \angle C + \angle D + \angle E + \angle F$$
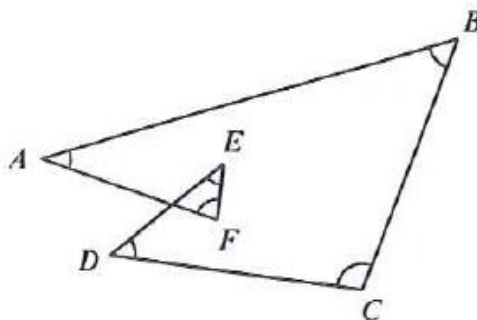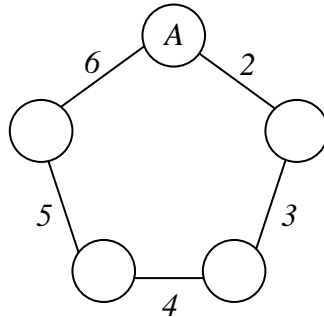

<div align="right">(SMOPS 2013 Q.15)</div>

16. In how many different ways can four children share 8 identical chocolates so that each child get at least one?

(SMOPS 2013 Q.16)

17. In the given diagram, each circle contains a natural number and the diagram satisfies the following conditions:


i. The number labelled along each edge represents the difference between the numbers in the two circles joined by the edge.
ii. The sum of the numbers in the 5 circles is equal to 1979.

Find the number in circle *A*.

(SMOPS 2013 Q.17)

18. A certain type of water bottle is sold at $10 in both Store A and Store B. Mrs. Lim would like to buy a few water bottles for a Children's Home. Store A sells the water bottle with an offer of "Buy 5 Get 1 Free" (no free bottles for buying fewer than 5 water bottles); store B gives a 15% discount for customers who buy 4 or more water bottles. What is the least amount of money (in $) that Mrs. Lim needs to spend in order to get 14 water bottles?

(SMOPS 2013 Q.18)

19. A square of side length 18 cm is inscribed in a circle. Semi-circle are constructed on its sides, as shown in the diagram. Find the total area of the four shaded lunes in cm^2.
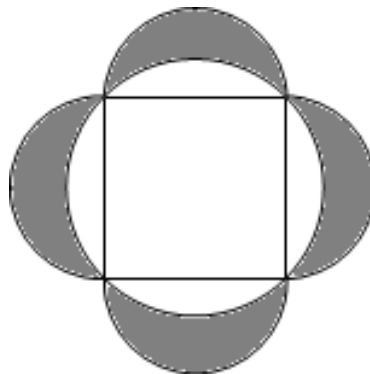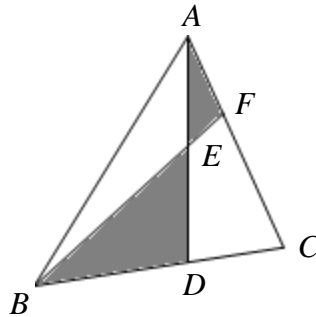

(SMOPS 2013 Q.19)

20. Four teams participated in a soccer tournament. Each team played against all other teams once each. 3 points were awarded for a win, 1 point for a draw and 0 points for a loss. At the end of the tournament, the four teams have obtained 5, 1, x, 6 points respectively. Find the value of x.

(SMOPS 2013 Q.20)

21. In the diagram, the area of triangle ABC is 40. Given that $2BD = 3CD$ and $AE = DE$, find the area of the shaded region.
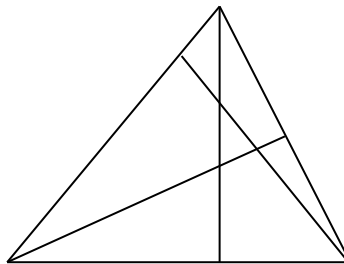

(SMOPS 2013 Q.21)

22. If integers are selected randomly from 1 to35, what is the minimum numbers of integers that need to be selected such that among the chosen numbers we can always find two integers whose difference is divisible by 7?

(SMOPS 2013 Q.22)

23. A team of workers are sent to two construction sites A and B respectively. The amount of work to be done at construction site A is 50% more than that at construction site B. In the morning, the number of workers sent to construction site A is 3 times the number of workers sent to site B. In the afternoon, the ratio of workers sent to construction site A and B is 7: 5. By the end of the day, the work at construction site A is fully completed, while construction site B still requires 8 workers to work for another full day. Assuming the workers work that the same rate, find the total numbers of workers in this team.
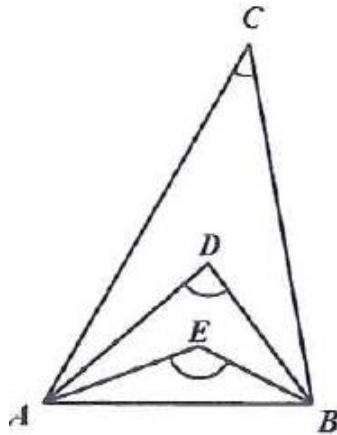
(SMOPS 2013 Q.23)

24. Find the total number of triangles in the figure below.

(SMOPS 2013 Q.24)

25. A car and a motorcycle started travelling towards each other at the same instant, from cities A and B respectively. 72 minutes later, they met along the road and continued to travel towards their destinations. Given that the speed of the car is $1\frac{1}{3}$ times that of the motorcycle, how many minutes after the car reached city B would the motorcycle reach city A?

(SMOPS 2013 Q.25)

26. Given four prime numbers a, b, c, and d, if the product of $a \times b \times c \times d$ is the sum if 55 consecutive positive integers, find the smallest possible value of $a + b + c + d$.

(SMOPS 2013 Q.26)

27. A particular brand of car tyre lasts 300 km on a front wheel or 450 km on a rear wheel. By interchanging the front and rear tyres, what is the greatest distance, in km, that can be travelled using a set of four tyres of this brand?

(SMOPS 2013 Q.27)

28. In triangle ABC, AD and AE trisect angle CAB, BD and BE trisect angle CBA. If the ratio of angle C to angle D is $1:2$, find the value of angle E in degrees.

(SMOPS 2013 Q.28)

29. The sum of 10 positive integers, not necessarily distinct, is 1001. If d is the greatest common divisor of the 10 numbers, find the maximum possible value of d.

(SMOPS 2013 Q.29)

30. How many different ways are there to select 2 distinct integers from $\{2000,2001,2002,\cdots,2014,2015\}$ such that the product of the 2 numbers is divisible by 6?

(Note: order is not important, choosing 2001 and 2002 is the same as choosing 2002 and 2001.)

(SMOPS 2013 Q.30)

Number of correct answers for Q1 to Q10 :   _____   Marks (×4) : _____
Number of correct answers for Q11 to Q20 :   _____   Marks (×5) : _____
Number of correct answers for Q21 to Q30 :   _____   Marks (×6) : _____

Answers:

SMOPS 2013					
1	62474	11	960	21	15
2	1080	12	23	22	8
3	36	13	72	23	72
4	7	14	100	24	17
5	1109	15	360	25	42
6	38	16	35	26	32
7	24	17	393	27	360
8	2	18	118	28	135
9	4022	19	324	29	91
10	10	20	4	30	47