# Asia Pacific Mathematical Olympiad for Primary Schools 2017

**First Round**
**2 hours**
**(150 marks)**

1. What is the smallest 4-digit number in the following sequence?

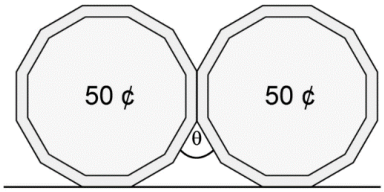
   5, 14, 29, 50, 77, …

   (SMOPS 2017 Q. 1)

2. 60% of the employees in Team *A* are males while 60% of the employees in Team *B* are females. There are 360 more employees in Team *A* than in Team *B*. If the number of female employees in Team *A* and Team *B* are equal, what is the total number of employees in the two teams?

   (SMOPS 2017 Q. 2)

3. Two fifty-cent coins are placed next to each other on a table top, forming an angle $\theta$ as shown in the figure below. Each coin takes the shape of a dodecagon with 12 sides of equal length. Find the value of $\theta$ in degrees.




   (SMOPS 2017 Q. 3)

4. What is the remainder when $6 \times 2017^{2018}$ is divided by 11?

   (SMOPS 2017 Q. 4)

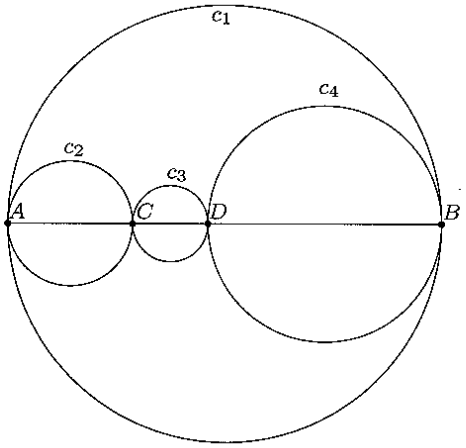
5. There were 4 questions in a mathematics test and the total number of marks was not 100. The number of marks allocated to question 1 was the highest, followed by questions 2, 3 and 4 (lowest). The number of marks allocated to question 1 was 22 and question 3 was 7 marks higher than question 4. Student A answered questions 1, 2 and 3 correctly and obtained 54 marks. Student B answered questions 2 and 4 correctly and obtained *x* marks. Find the value of *x*.

   (SMOPS 2017 Q. 5)

6. Alvin has many apples. He gives $\frac{1}{3}$ of his apples plus $\frac{2}{3}$ of an apple to Bob. He then gives $\frac{1}{4}$ of his remaining apples plus half an apple to Chris. Afterwards, he gives half of his remaining apples to David. Lastly, he gives half of his remaining apples plus half an apple to Ed. In the end, Alvin has 5 apples left. How many apples does Alvin have initially?

(SMOPS 2017 Q. 6)

7. The diagram below shows four circles $c_1$, $c_2$, $c_3$ and $c_4$ with diameters *AB*, *AC*, *CD* and *DB* respectively. If the circumference of $c_1$ is 2017, find the sum of the circumferences of all four circles.

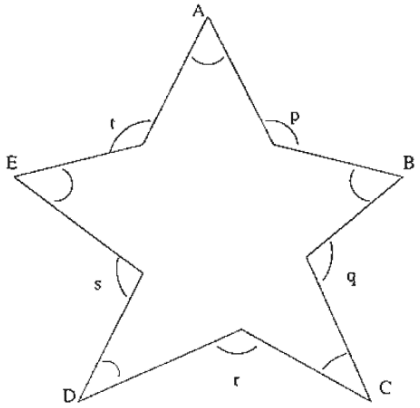



(SMOPS 2017 Q. 7)

8. Find the value of 217 × 216216217 – 216 × 217217216.

(SMOPS 2017 Q. 8)

9. In the 'star' figure below, $\angle p + \angle q + \angle r + \angle s + \angle t = 600°$ . Find the sum of $\angle A + \angle B + \angle C + \angle D + \angle E$ in degrees.




(SMOPS 2017 Q. 9)

10. Given that $\dfrac{A}{B} = 2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{2 - \cfrac{1}{\ldots \quad \ldots \quad \ldots}}}}$ (2017 layers)

$$\ldots 2 - \frac{1}{2 - \dfrac{2016}{2017}}$$

where $A$ and $B$ are positive integers, find the value of $A + B$.

Note that $2 - \dfrac{1}{2 - \dfrac{2016}{2017}}$ is the 1st layer.

(SMOPS 2017 Q. 10)

11. A number $P$ has 9 factors whereas another number $Q$ has 10 factors. If the lowest common multiple of $P$ and $Q$ is 2800, find the number $Q$.

(SMOPS 2017 Q. 11)

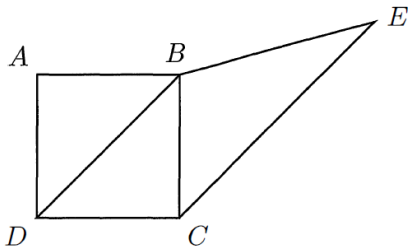
12. Mrs Lee bought $\dfrac{2}{3}$ as many pens as pencils. Each pen cost \$1.20 more than each pencil. The total cost of the pens was \$12.60 more than the total cost of the pencils. Given that Mrs Lee spent a total of \$66.60 on all the pens and pencils, find the number of pencils she bought.

(SMOPS 2017 Q. 12)

13. Given that $\dfrac{a}{b} = 1 - \dfrac{1}{2} + \dfrac{1}{3} - \dfrac{1}{4} + \dfrac{1}{5} - \dfrac{1}{6} + \dfrac{1}{7} - \dfrac{1}{8} + \dfrac{1}{9} - \dfrac{1}{10}$, such that $a$ and $b$ have no common divisor greater than 1, find the value of $a$.

(SMOPS 2017 Q. 13)

14. In the following figure, $ABCD$ is a square, $BD$ is parallel to $EC$ and $BD = BE$. Find $\angle BEC$ in degrees.




(SMOPS 2017 Q. 14)

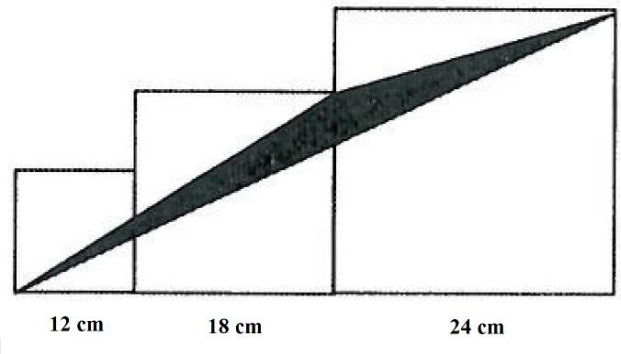
15. Motorcyclist *A* travels from Town *P* to Town *Q* whereas motorcyclist *B* travels from Town *Q* to Town *P*. Both motorcyclists started their journey at the same time. They met at a location which was 80 km from Town *Q*. Both motorcyclists continued their journey in their respective directions after they met. When both motorcyclists reached their respective destinations, they immediately travelled back to their respective starting points. If both motorcyclists met again, without overtaking, at a location which was 60 km from Town *P*, find the distance between Town *P* and Town *Q*.

(SMOPS 2017 Q. 15)

16. 2017 students arrange themselves in a single file.
In the first round of counting, they number themselves 1, 2, 3, 1, 2, 3, 1, 2, … and so on from left to right.
In the second round of counting, they number themselves 1, 2, 3, 4, 5, 1, 2, 3, 4, 5, 1, 2, … and so on from right to left.
Find the number of students whose difference between their numbers in the first and second rounds of counting is 1.

(SMOPS 2017 Q. 16)

17. The figure as shown below is made up of three squares of sides 12 cm, 18 cm and 24 cm respectively. Find the area of the shaded triangle.




(SMOPS 2017 Q. 17)

18. A pump is filling a tank with water at a constant rate while water in the tank is also drained by 3 identical pipes. Starting from a full tank of water with water being pumped into it, the tank can be completely drained by the 3 identical pipes in 35 minutes. If an additional identical pipe is installed to drain the tank, a full tank of water can be completely drained in 20 minutes when water is pumped into it. How many minutes will it take for the pump to fill an empty tank with water to its brim without any pipe to drain water from the tank?

(SMOPS 2017 Q. 18)

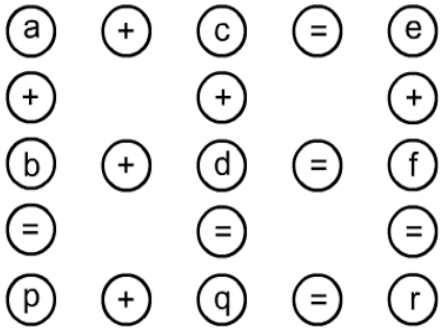
19. How many fractions of the form $\frac{a}{b}$ are there, where $a$ and $b$ have no common factors larger than 1, such that $b = a + 6$ and $\frac{a}{b} < \frac{2017}{2023}$?

(SMOPS 2017 Q. 19)

20. There are 6 red points labelled R1, R2, R3, R4, R5 and R6, and 6 green points labelled G1, G2, G3, G4, G5 and G6. Using straight lines, each red point is connected to at least one green point and each green point is connected to at least one red point. The number of green points connected to R1, R2, R3, R4 and R5 is 5, 4, 3, 2 and 2 respectively. The number of red points connected to G1, G2, G3, G4 and G5 is 4, 3, 2, 1 and 1 respectively. Find the number of red points connected to G6.

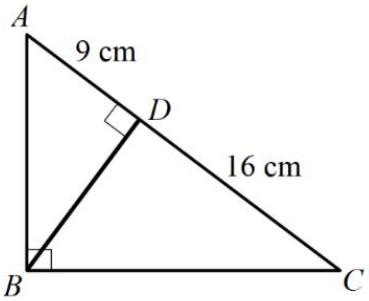
(SMOPS 2017 Q. 20)

21. 9 natural 3-digit numbers, represented by 9 different letters *a*, *b*, *c*, *d*, *e*, *f*, *p*, *q* and *r*, formed an addition sequence shown in the diagram below. Find the smallest possible value of *r*.




(SMOPS 2017 Q. 21)

22. *ABC* is a right angle triangle. *D* is a point on *AC* such that *AC* is perpendicular to *BD*. Given that *AD* = 9 cm, *CD* = 16 cm, find the area of triangle *ABC* in $cm^2$.




(SMOPS 2017 Q. 22)

23. A tank measuring 28 cm by 20 cm by 30 cm was filled with 5000 $cm^3$ of water. At 2 p.m., water was continuously added into the tank at a constant rate of 25 $cm^3$ every minute. At 3 p.m., 200 $cm^3$ of water was drawn out of the tank and subsequently, a further 200 $cm^3$ was also drawn out at the end of every hour after. At what time did the tank start to overflow for the first time? (Express your answer in HHMM format of the 24-hour clock. For example, 4 a.m. represented as 0400; 4.25 p.m. represented as 1625.)

(SMOPS 2017 Q. 23)

24. Mrs Tan and Mrs Ong were having a conversation at a park.

Mrs Tan: "Hi, how are you? How are your children? You have three if I remembered correctly. How old are they now?"

Mrs Ong: "Yes, I have three children. The product of their ages is equal to 96. The sum of their ages is equal to the number of trees in this park."

Mrs Tan counted the number of trees in the park, thought for a while and said,

"I still could not figure out the ages of your children."

Mrs Ong simultaneously replied,

"Pardon me, for I have to pick up my children from the school across the park. We can catch up again soon. Goodbye."

Finally, Mrs Tan managed to figure out all the ages of Mrs Ong's children.
How old is Mrs Ong's youngest child?

(SMOPS 2017 Q. 24)

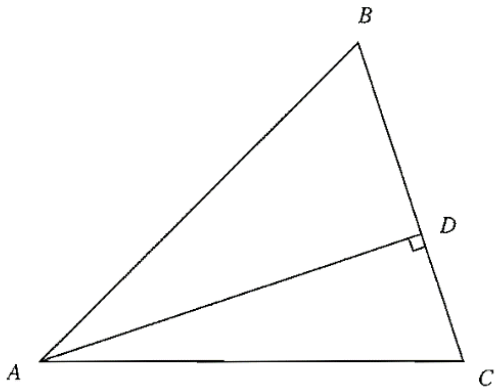
25. Given that

$$\frac{5}{1\times 2\times 3\times 4}+\frac{9}{3\times 4\times 5\times 6}+\frac{13}{5\times 6\times 7\times 8}+\ldots+\frac{57}{27\times 28\times 29\times 30}=\frac{M}{N},$$

where $M$ and $N$ have no common factor larger than 1, find the value of $M + N$.

(SMOPS 2017 Q. 25)

26. In triangle $ABC$, $D$ is a point on $BC$ such that $AD$ is perpendicular to $BC$ and $\angle BAC = 45°$. Given that $CD$ = 2 cm and $BD$ = 3 cm, find the area of triangle $ABC$ in $cm^2$.




(SMOPS 2017 Q. 26)

27. All of the faces of a rectangular block are painted red. The rectangular block is cut into unit cubes with a volume of 1 $cm^3$ each. It is known that there are 30 unit cubes with none of their faces painted red and $x$ unit cubes each with exactly two faces painted red. Find the sum of all possible values of $x$.

(SMOPS 2017 Q. 27)

28. 2017 schools participated in a big inter-school chess competition. Each school sent in 2 participants.
    The rules of the competition were:
        i) Participants do not compete with each other for more than 1 game,
        ii) Participants from the same school cannot compete with each other.

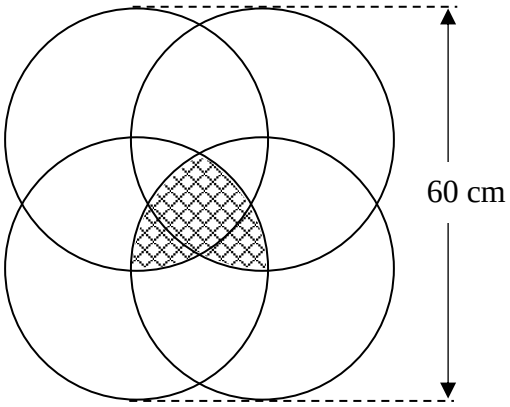
    After many rounds of the competition, it was known that every participant had completed a different number of games except participants from one of the 2017 schools. How many games had each of these 2 participants from the same school completed?

    (SMOPS 2017 Q. 28)

29. A train *X* is travelling through the countryside at a constant speed of 90km/h. Another train *Y* of the same length is travelling in the opposite direction and it takes 5 seconds for both trains to pass by each other completely. Later, train *X* enters a tunnel and it takes 50 seconds for the train to exit the tunnel completely. Find the length of the tunnel in metres.

    (SMOPS 2017 Q. 29)

30. The figure shows 4 overlapping circles with each circle passing through the centres of its adjacent 2 circles. The figure measures 60 cm vertically from top to bottom as shown below. Find the total area of the shaded parts. (Take $\pi = 3.14$)




    (SMOPS 2017 Q. 30)

*Number of correct answers for Q1 to Q10 :* _____ Marks ( ×4 ) : _____
*Number of correct answers for Q11 to Q20 :* _____ Marks ( ×5 ) : _____
*Number of correct answers for Q21 to Q30 :* _____ Marks ( ×6 ) : _____

Answers:

| SMOPS 2017 | | | | | |
|---|---|---|---|---|---|
| 1 | 1085 | 11 | 112 | 21 | 406 |
| 2 | 1800 | 12 | 27 | 22 | 150 |
| 3 | 60 | 13 | 1627 | 23 | 2256 |
| 4 | 10 | 14 | 30 | 24 | 1 |
| 5 | 25 | 15 | 180 | 25 | 1087 |
| 6 | 46 | 16 | 674 | 26 | 15 |
| 7 | 4034 | 17 | 126 | 27 | 344 |
| 8 | 433 | 18 | 28 | 28 | 2016 |
| 9 | 240 | 19 | 672 | 29 | 1125 |
| 10 | 8069 | 20 | 6 | 30 | 228 |