Asia Pacific Mathematical Olympiad for Primary Schools 2009

First Round
2 hours
(150 marks)

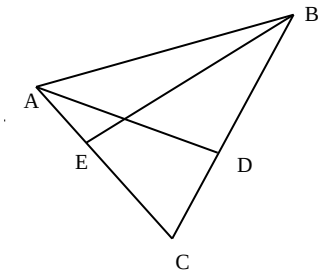
1. Given that the product of two whole numbers $m \times n$ is a prime number, and the value of m is smaller than n, find the value of m.

2. Given that $(2009 \times n - 2009) \div (2008 \times 2009 - 2006 \times 2007) = 0$, find the value of n.

3. Find the missing number x in the following number sequence.
   2, 9, -18, -11, x, 29, -58, -51,…

4. Jane has 9 boxes with 9 accompanying keys. Each box can only be opened by its accompanying key. If the 9 keys have been mixed up, find the maximum number of attempts Jane must make before she can open all the boxes.

5. The diagram shows a triangle ABC with $AC = 18cm$ and $BC = 24cm$. D lies on BC such that AD is perpendicular to BC and E lies on AC such that BE is perpendicular to AC. Given that $BE = 20cm$ and $AD = x\ cm$, find the value of x.



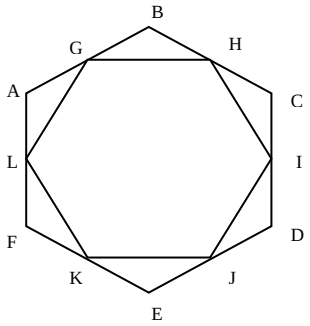
6. A language school has 100 pupils, 69% of the pupils study French, 79% study German, 89% study Japanese and 99% study English. Given that at least x% of the students study all four languages, find the value of x.

7. Find the value of x.

89	35	9	1
35	x	7	1
9	7	5	1
1	1	1	3

8. Given that $9 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$ where $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ and n_9 are consecutive numbers, find the value of the product $n_1 \times n_2 \times n_3 \times n_4 \times n_5 \times n_6 \times n_7 \times n_8 \times n_9$.

9. The diagram shows a regular 6-sided figure ***ABCDEF***. *G, H,I, J, K* and *L* are mid-points of *AB, BC, CD, DE, EF* and *FA* respectively. Given that the area of ***ABCDEF*** is 100cm^2 and the area of ***GHIJKL*** is x cm^2, find the value of x.

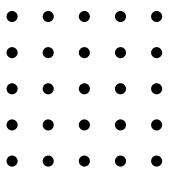


10. Three pupils A, B and C are asked to write down the height of a child, the circumference of a circle, the volume of a cup and the weight of a ball. Their responses are tabulated below:

Pupil	Height of the child (cm)	Circumference of the circle (cm)	Volume of the cup (cm^3)	Weight of the ball (g)
A	90	22	250	510
B	70	21	245	510
C	80	22	250	520

If each pupil has only two correct responses, and the height of the child is x cm, find the value of x.

11. The diagram shows a square grid comprising 25 dots. A circle is attached to the grid. Find the largest possible number of dots the circle can pass through.



12. Jane and Peter competed in a 100m race. When Peter crossed the finishing line, Jane just crossed the 90m mark. If Peter were to start 10m behind the starting line, the distance between them when one of them crosses the finishing line is x m. Find the value of x.

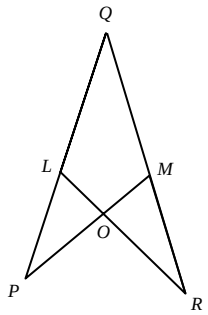
13. Given that (1+2+3+4+5+4+3+2+1) × (123454321) = x^2, find the value of x.

14. A $5 \times 5 \times 5$ cube is to be assembled using only $1 \times 1 \times 1$ cube(s) and $1 \times 1 \times 2$ cuboid(s). Find the maximum number of $1 \times 1 \times 2$ cuboid(s) required to build this $5 \times 5 \times 5$ cube.

15. Given that $(n_1)^2 + (2n_2)^2 + (3n_3)^2 + (4n_4)^2 + (5n_5)^2 + (6n_6)^2 + (7n_7)^2 + (8n_8)^2 + (9n_9)^2 = 285$, find the value of $n_1 + n_2 + n_3 + n_4 + n_5 + n_6 + n_7 + n_8 + n_9$ if $n_1, n_2, n_3, n_4, n_5, n_6, n_7, n_8$ and n_9 are non-zero positive whole numbers.

16. A circle and a square have the same perimeter. Which of the following statement is true?
(1) Their areas are the same.
(2) The area of the circle is four times the area of the square.
(3) The area of the circle is greater than that of the square.
(4) The area of the circle is smaller than that of the square.
(5) None of the above.

17. As shown in the diagram, the points *L* and *M* lie on *PQ* and *QR* respectively. *O* is the point of intersection of the lines *LR* and *PM*. Given that *MP* = *MQ*, *LQ* = *LR*, *PL* = *PO* and $\angle POR = x^o$, find the value of *x*.

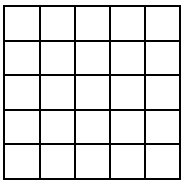


18. Given that the value of the sum $\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$ lies between $\frac{28}{29}$ and 1, find the smallest possible value of $a+b+c$ where *a*, *b* and *c* are whole numbers.

19. Jane has nine 1 cm sticks, six 2 cm sticks and three 4 cm sticks. Given that Jane has to use all the sticks to make a single rectangle, how many rectangles with different dimensions can she make?
(1) Three (2) Two (3) One (4) Zero (5) None of the above

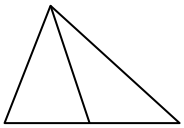
20. Peter wants to cut a 63 cm long string into smaller segments so that one or more of the segments add up to whole numbers in centimetres from 1 to 63. Find the least number of cuts he must make.

21. The diagram shows a 5 by 5 square comprising twenty five unit squares. Find the **least number** of unit squares to be shaded so that any 3 by 3 square has **exactly four** unit squares shaded.



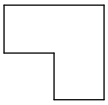
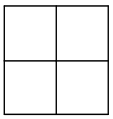
22. Peter and Jane were each given a candle. Jane's candle was 3 cm shorter than Peter's and each candle burned at a different rate. Peter and Jane lit their candles at 7 pm and 9 pm respectively. Both candles burned down to the same height at 10 pm. Jane's candle burned out after another 4 hours and Peter's candle burned out after another 6 hours. Given that the height of Peter's candle at the beginning was *x* cm, find the value of *x*.

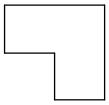
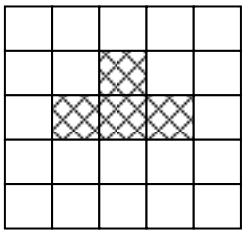
23. Three straight lines can form a maximum of one triangle. Four straight lines can form a maximum of two non-overlapping triangles as shown below. Five straight lines can form a maximum of five non-overlapping triangles. Six straight lines can form a maximum of *x* non-overlapping triangles. Find the value of *x*.



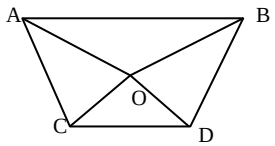
24. Given that $N = \underbrace{2 \times 2 \times 2 \times \ldots \times 2}_{\text{2009 '2's}} \times \underbrace{5 \times 5 \times 5 \times \ldots \times 5}_{\text{2000 '5's}}$, find the number of digits in *N*.

25. Jane and Peter are queuing up in a single line to buy food at the canteen. There are *x* persons behind Jane and there are *y* persons in front of Peter. Jane is *z* persons in front of Peter.
The number of people in the queue is _____ persons.
(1) $-x + y + z - 1$ (2) $x + y - z + 1$ (3) $-x + y + z$
(4) $-x + y + z + 2$ (5) $x + y - z$

26. There are 4 ways to select  from  .

Find the number of ways to select  from 

27. The diagram shows a trapezium *ABCD*. The length of *AB* is $2\frac{1}{2}$ times that of *CD* and the areas of triangles *OAB* and *OCD* are 20 cm^2 and 14 cm^2 respectively. Given that the area of the trapezium is *x* cm^2, find the value of *x*.



28. Given that

$$\frac{1}{10 + \frac{1}{9 + \frac{1}{9}}} + \frac{1}{a + \frac{1}{b + \frac{1}{b + \frac{1}{b}}}} = 1$$

where *a* and *b* are whole numbers, find the value of $a + b$.

29. A shop sells dark and white chocolates in three different types of packaging as shown in the table.

	Number of Dark Chocolate	***Number of White Chocolate***
Package *A*	9	3
Package *B*	9	6
Package *C*	6	0

Mr Tan bought a total of 36 packages which consisted of 288 pieces of dark chocolates and 105 pieces of white chocolates. How many packages of type ***A*** did he buy?

30. There are buses travelling to and fro between Station *A* and Station *B*. The buses leave the stations at regular interval and a bus will meet another bus coming in the opposite direction every 6 minutes.
Peter starts cycling from *A* towards *B* at the same time Jane starts cycling from *B* towards *A*.
Peter and Jane will meet a bus coming in the opposite direction every 7 and 8 minutes respectively.
After 56 minutes of cycling on the road, they meet each other.
Find the time taken for a bus to travel from A to B.

Number of correct answers for Q1 to Q10 : _____ Marks (×4) : _____
Number of correct answers for Q11 to Q20 : _____ Marks (×5) : _____
Number of correct answers for Q21 to Q30 : _____ Marks (×6) : _____

Answers:

1. 1	2. 1	3. 22	4. 45
5. 15	6. 36	7. 19	8. 0
9. 75	10. 80	11. 8	12. 1
13. 55555	14. 62	15. 9	16. (3)
17. 108	18. 12	19. (4)	20. 5
21. 7	22. 18	23. 7	24. 2003
25. (2)	26. 30	27. 77	28. 10
29. 13	30. 68		