1.

(a) Consider a BPSK system, where the message signal, m, is equally likely to be 1 or -1. The corresponding transmitted signal has an amplitude of A or $-A$, respectively.

(i) Determine the average energy per transmitted bit, E.

Answer: (i) $E = \frac{A^2 + A^2}{2} = A^2$

(ii) The BPSK signal is transmitted over an Additive White Gaussian Channel (AWGN) channel. The Gaussian nose has a mean of 0 and a variance of $N_0/2 = 2$. Determine the probability of bit error P_b using the formula below:

$$P_b = Q\left(\sqrt{\frac{12}{N_0}}\right)$$

Answer: (ii) $\because N_0/2 = 2$

$\therefore N_0 = 4$

$P_b = Q\left(\sqrt{\frac{12}{4}}\right) = Q(\sqrt{3}) \approx Q(1.732) \approx 4.182 \times 10^{-2}$

(iii) The BPSK signal transmitted over the AWGN channel now experiences fading. The receiver decision variable can be expressed as,

$$y = hm\sqrt{E} + w$$

where m and E are as described above, w is the AWGN described in (ii), and h is a Gaussian random variable with a mean of 2 and a variance of 1. If the decision rule is such that the estimated value of m is,

$$\hat{m} = \begin{cases} +1, if\ y > 0 \\ -1, if\ y \leq 0 \end{cases}$$

derive the expression of the average probability of bit error in terms of E and the Q-function.

Answer: (iii) $h \sim N(2, 1)$ $\quad w \sim N(0, 2)$

$m\sqrt{E}h \sim N(2m\sqrt{E}, m^2E)$ $\quad y \sim N(2m\sqrt{E}, m^2E + 2)$

$p(y \mid m=1) = \frac{1}{\sqrt{2\pi}\sqrt{E+2}} \exp\left(-\frac{y - 2\sqrt{E}}{2(E+2)}\right)$

$p(y \mid m=-1) = \frac{1}{\sqrt{2\pi}\sqrt{E+2}} \exp\left(-\frac{y + 2\sqrt{E}}{2(E+2)}\right)$

$P_b = P(y \leq 0 \mid m=1) P(m=1) + P(y > 0 \mid m=-1) P(m=-1)$

$= Q\left(\frac{2\sqrt{E}}{\sqrt{E+2}}\right) \cdot \frac{1}{2} + Q\left(\frac{2\sqrt{E}}{\sqrt{E+2}}\right) \cdot \frac{1}{2}$

$= Q\left(\sqrt{\frac{4E}{E+2}}\right)$

(iv) If the cumulative distribution function of the signal to noise ratio of a Rayleigh fading channel is expressed as,

$$1 - e^{-\frac{\gamma_0}{10}}$$

determine the outage probability when $\gamma_0 = 0.1$. Explain your answer.

Answer: (iv) $F(\gamma_0) = P[\gamma \le \gamma_0] = 1 - e^{-\frac{\gamma_0}{10}}$

$P_{out} = P[\gamma \le \gamma_{min}]$

$\gamma_{min} = \gamma_0 = 0.1$

$P_{out} = F(0.1) = 9.95 \times 10^{-3}$

(b)

(i) Consider a fading channel where the signal to noise ratio (SNR), γ, has a value of 10 with a probability of $\frac{2}{5}$ and a value of 15 otherwise. Determine the average SNR in dB of this channel.

Answer: (b) (i)

$$\gamma = \begin{cases} 10 & p = \frac{2}{5} \\ 15 & p = \frac{3}{5} \end{cases}$$

$$\bar{\gamma} = 10 \times \frac{2}{5} + 15 \times \frac{3}{5} = 2 \times 2 + 3 \times 3 = 13 \approx 11.14\,dB$$

(ii) A system employs a 2-branch Selective Diversity combining technique. Under the Rayleigh fading condition, determine the array gain of this system if the average output SNR of the combiner is,

$$\bar{\gamma} \sum_{i=1}^{2} \left(\frac{1}{i}\right)$$

where $\bar{\gamma} = 10$ is the average input SNR.

Answer: (ii) $\bar{\gamma}_\Sigma = \bar{\gamma} \sum_{i=1}^{2} \left(\frac{1}{i}\right) = 10\left[1 + \frac{1}{2}\right] = 15$

$$g = \frac{\bar{\gamma}_\Sigma}{\bar{\gamma}} = \frac{15}{10} = 1.5$$

$\therefore$ The array gain is 1.5

(iii) Consider a 1 x 2 SIMO system. The transmitted signal has an average energy of 2 V^2. The channel noise, v, is AWGN with zero mean and a variance of 0.5. The channel gains are $h_1 = 1 + i$ and $h_2 = 1 - i$, respectively where $i = \sqrt{-1}$. The received signals, y_1 and y_2, are processed to produce the output signal which is the estimated transmitted signal given by the following formula,

$$\hat{s} = h_1^* y_1 + h_2^* y_2$$

where $y_i = h_i s + v, i = 1, 2$, h_i^* is the complex conjugate of h_i. Determine the *SNR* of the output signal of this system.

Answer: (iii) $\hat{s} = h_1^* h_1 s + h_1^* v + h_2^* h_2 s + h_2^* v$

$= 4s + (h_1^* + h_2^*) v$

$= 4s + 2v$

$$SNR = \frac{E[16 s^2]}{E[4 v^2]} = \frac{4 \cdot 2}{0.5} = 16$$

The SNR of the output signal of the system is 16

(iv) Can the SIMO system described in (iii) achieve diversity if h_1 and h_2 are statistical independent? Explain your answer.

Answer: (iv) $y_1 = h_1 s + v$

$y_2 = h_2 s + v$

if h_1 and h_2 are statistical independent,

which means the symbol travel through 2 independent channels,

the fading can be mitigated.

So the SIMO system can achieve diversity.

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3. (a) FDD is a common duplexing scheme used in cellular systems. Describe what is a FDD scheme. For a FDMA FDD cellular system, what is the typical relationship between the number of FDMA channels and the number of trunk channels?

Answer 3 (a) FDD is the frequency division duplex. Uplink and downlink transmission happens in different frequency band.

For FDMA FDD, the number of trunk channels is half of the number of the FDMA channels.

(b) A FDMA FDD cellular system has a system bandwidth of 6 MHz and each of its FDMA channels has a bandwidth of 50 kHz. If the system has a cluster size of 4, what is the number of trunk channels in each cell?

Answer: (b) $B_w = 6\text{MHz}$ $B_s = 50\text{kHz}$

Number of the total FDMA channels. $n = \frac{B_w}{B_s} = \frac{6\times10^6}{50\times10^3} = 120$

Number of FDMA channels in a cell: $n_{cell} = \frac{n}{N_c} = \frac{120}{4} = 30$

Number of trunk channels in a cell: $n_t = \frac{n_{cell}}{2} = 15$

(c) For a blocking probability of 1% and considering the "blocked calls cleared model", what is the number of subscribers that can be supported in a cell by the same system described above if the average traffic flow needed by each subscriber is 0.1 Er? If the system covers an area of two million residents with 1000 cells, what is the market penetration rate? Assuming that every cell is of the same size of 3 km^2 approximately, what is the average spectral efficiency of the system in Er/MHz/km^2?

Answer: (c) GoS: $P_b = 1\%$ Blocked call cleared model, so look up the Erlang B Table

The supported offered traffic in a cell is $A = 8.108$ Erlang

The number of subscribers in a cell should be $n_s = \frac{A}{A_u} \approx 81$

If the system has 1000 cells, then the total number of subscribers of 81000

The penetration rate is $R = \frac{81000}{2\times10^6} \approx 4.05\%$

The modulation spectrum efficiency is $\eta_m = \frac{8.108 \times 4}{6 \times 3 \times 4} \approx 0.45$ Erlang/MHz/km^2

The multiple access spectrum efficiency is $\eta_a = \frac{30\times2\times50}{6000} = 1$

The system spectrum efficiency is $\eta = \eta_m \cdot \eta_a = 0.45\times1 = 0.45$ Erlangs/MHz/km^2

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3. (d) It is found that the above cellular system experience too much co-channel interference. As a result, 120° sectorized antennas were incorporated into all base stations of the system. In addition, the system operator plans to double the penetration rate determined in part (c) and expand the system capacity by adopting the cell splitting technique. Approximately, how much smaller should the 'radius" of the micro cell be compared to that of the original cell, assuming that each cell has a hexagonal geometry and the 'radius' is the distance between the center and the vertex of the cell? Explain your answer.

Answer: (d) Double the penetration rate $R_p = 8.1\%$

The user density is $d = \frac{2\times10^6}{1000\times3} \approx 666.67$ user/km²

Assume the Radius of a cell is R

The size of a cell $S = R\cdot\frac{\sqrt{3}}{2}R\cdot\frac{1}{2}\times6 = \frac{3\sqrt{3}}{2}R^2$

The size of a sector in the cell is $S_s = \frac{\sqrt{3}}{2}R^2$

The number of subscribers in a sector is $n_s = S_s\cdot d\cdot R_p = \frac{\sqrt{3}}{2}R^2\times666.67\times8.1\%$

The number of channels in a sector: $\frac{15}{3} = 5$; GoS: $P_d = 1\%$

Look up the Erlang B table, and find the offered traffic is 1.361 Erlang.

$\frac{A}{A_u} = n_s \qquad \frac{1.361}{0.1} = \frac{\sqrt{3}}{2}R^2\times666.67\times8.1\%$

$R \approx 0.54$ km

The radius should be 0.54 km.

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4.

(a) A newly built city near a reclamation land requires a new wireless telecommunication service to be provided for the next 5 years. The mobile operator starts by designing a hexagonal cellular mobile network to provide these telecommunication services.

The distance between the centers of two adjacent hexagonal cells is 1.3 km. Determine the cell radius R, the number of cells N_c in a cluster, and the distance D between the centers of the reference cell and its first-tier co-channel cell. You many assume from any particular reference cell, the first-tier co-channel cell is at coordinates (0, 2) using the hexagonal lattice coordinate system.

Answer 4 (a) $\sqrt{3}R = 1.3$ km $\therefore R \approx 0.75$ km

The cell Radius is approximately 0.75 km

$D = 2 \times 1.3 = 2.6$ km

The re-use distance D is 2.6 km

$N_c = i^2 + j^2 + ij = 4$

The number of cells in a cluster is 4

(b) Suppose 3-sectored antennas are used at each base station. The total number of channels $C_v = 540$. The parameters used for this design are:

Coverage area 400 km^2;
Projected population of 5 million.
Market penetration rate = 20%
Holding time of a call H = 65 seconds
Busy hour call arrival rate per user λ=1.5 calls/hour

State and justify any necessary assumptions. Find the blocking probability P_B and the trunking gain T_g.

Answer: (b) The number of channels for each cell : $\frac{540}{4} = 135$

The numbers of channels for each sector : $\frac{135}{3} = 45$

The traffic for each user : $A_u = \lambda \cdot h = 1.5 \times 65/3600 \approx 0.027$ Erlang

The Size of a cell $S = \frac{3\sqrt{3}}{2} R^2 \approx 1.46$ km^2

The number of users for a cell is $\frac{1.46}{400} \times 5 \times 10^6 = 18250$

The number of users for a sector is $\frac{18250}{3} \approx 6083$

The traffic in a sector is around : $A_s = n \cdot A_u \cdot R_p \approx 32.85$ Erlang

Assume the queuing model is 'blocked call clear'

Looking up the Erlang B Table, $P_B = 1\%$

$T_g = \frac{6083 \times 20\%}{45} \approx 27.04$

The trunking gain of the system is $T_g = 27.04$

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4 (c) All routing protocols in Ad Hoc Networks need to perform two basic functions. List and explain these two basic functions. In addition, describe how these routing protocols uses the knowledge of the instantaneous connectivity of the network. List two types of topology-based protocols approaches for the Ad Hoc Network and explain on their functionality.

Answer: (c) The function of routing protocols are routing and forwarding.

Routing is find an optimum route from source to destination.

Forwarding is to let the nodes know which port should they forward the packet to.

The two basic functions of routing protocols in Ad Hoc Networks are:

1. Route Discovery : Finding an available path between source and destination.
2. Route Maintenance: Keeping the path active and recovering from failure.

These protocols are knowledge of instantenous connectivity to dynamically adapt to changes in network topology. They rely on neighbor discovery, link status monitoring, and timely updates to ensure routes remain valid.

Two types of topology-based protocols:

Proactive protocols (DSDV): Maintain routes to all nodes at all times; low delay but high overhead.

Reactive protocols (AODV, DSR): Create routes only when needed; lower overhead but higher initial latency