1. (a) A shunt LC resonator comprising L_n and C_n is to be compared with a quarter-wavelength short-circuited stub of characteristic impedance Z_{0n}.

(i) Determine the admittance of the shunt LC resonator at angular frequencies near the resonance frequency, i.e., $\omega \approx \omega_0$, where $\omega_0^2 = 1/(L_n C_n)$.

(ii) Determine the admittance of the short-circuited stub at angular frequencies $\omega \approx \omega_0$, where ω_0 corresponds to 90° phase shift.

(iii) Using suitable approximations of your answers above, find the expression of Z_{0n} in terms of L_n and C_n.

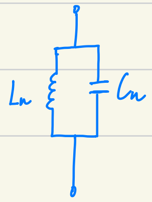
1.(a) 一个由L_n和C_n并联组成的LC谐振器，与一个特性阻抗为Z_{0n}的1/4波长短路支节相比较

(i) 计算该LC并联谐振器在接近谐振频率时的导纳 ($\omega \approx \omega_0$ $\omega_0 = \frac{1}{\sqrt{L_n C_n}}$)

(ii) 计算该短路支节在接近共振频率时的导纳 ($\omega \approx \omega_0$ ω_0对应90°相移)

(iii) 在上述计算的基础上，找出Z_{0n}用C_n和L_n如何表示。

解题思路：(i)



$$Y_L = \frac{1}{j\omega L_n} \qquad Y_c = j\omega C_n$$

$$Y = Y_L + Y_C = \frac{1}{j\omega L_n} + j\omega C_n$$

当工作在谐振频率上时 $\omega \approx \omega_0 = \frac{1}{\sqrt{L_n C_n}} + \Delta\omega$

$$Y = \frac{1}{j\omega_0 L_n} + j\omega_0 C_n = \frac{1}{j\left(\frac{1}{\sqrt{L_n C_n}} + \Delta\omega\right) L_n} + j\left(\frac{1}{\sqrt{L_n C_n}} + \Delta\omega\right)\cdot C_n$$

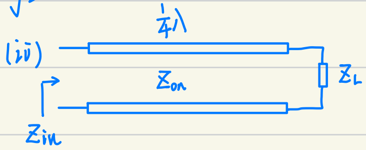
$$= \frac{1}{j\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 L_n} + j\left(1+\frac{\Delta\omega}{\omega_0}\right)\omega_0 C_n = j\omega_0 C_n\left[1+\frac{\Delta\omega}{\omega_0} - \frac{1}{1+\frac{\Delta\omega}{\omega_0}}\right] = j\omega_0 C_n \frac{\left(\frac{\Delta\omega}{\omega_0}\right)^2 + 2\frac{\Delta\omega}{\omega_0}}{1+\frac{\Delta\omega}{\omega_0}}$$

$$= \frac{\sqrt{L_n \cdot C_n}}{j\left(1+\frac{\Delta\omega}{\omega_0}\right)L_n} + j\frac{\sqrt{C_n}}{\sqrt{L_n}} + j\Delta\omega\cdot C_n = -j\sqrt{\frac{C_n}{L_n}}\cdot\frac{1}{1+\frac{\Delta\omega}{\omega_0}} + j\sqrt{\frac{C_n}{L_n}} + j\frac{\Delta\omega}{\omega_0}\cdot\sqrt{\frac{C_n}{L_n}}$$

$$= j\sqrt{\frac{C_n}{L_n}}\left[1+\frac{\Delta\omega}{\omega_0} - \frac{1}{1+\frac{\Delta\omega}{\omega_0}}\right] = j\sqrt{\frac{C_n}{L_n}}\,\frac{\left(1+\frac{\Delta\omega}{\omega_0}\right)^2 - 1}{1+\frac{\Delta\omega}{\omega_0}} = j\sqrt{\frac{C_n}{L_n}}\,\frac{\left(\frac{\Delta\omega}{\omega_0}\right)^2 + 2\frac{\Delta\omega}{\omega_0}}{1+\frac{\Delta\omega}{\omega_0}}$$

$$= j\sqrt{\frac{C_n}{L_n}}\,\frac{\Delta\omega^2 + 2\omega_0\cdot\Delta\omega}{\omega_0^2 + \omega_0\Delta\omega} \quad ☆$$

(ii)



$$Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0 \tan\beta l}{Z_0 + jZ_L \tan\beta l} \qquad \beta = \frac{2\pi}{\lambda'} \qquad l = \frac{\lambda}{4} \qquad \omega = \omega_0 + \Delta\omega$$

$\because Z_0 = Z_{0n} \quad Z_L = 0$ $\qquad \lambda = \frac{c}{\omega_0/2\pi} \qquad \lambda' = \frac{c}{(\omega_0+\Delta\omega)/2\pi}$

$\therefore Z_{in} = Z_{0n}\cdot\frac{jZ_{0n}\tan\beta l}{Z_{0n}}$ $\qquad \beta = \frac{2\pi}{\frac{c\cdot 2\pi}{\omega_0+\Delta\omega}} = \frac{\omega_0+\Delta\omega}{c}$

$$= jZ_{0n}\cdot\tan\left(\frac{\pi c}{2\omega_0}\cdot\frac{\omega_0+\Delta\omega}{c}\right) = jZ_{0n}\tan\left[\frac{\pi}{2}\cdot\left(1+\frac{\Delta\omega}{\omega_0}\right)\right] \qquad l = \frac{2\pi c}{4\omega_0} = \frac{\pi\cdot c}{2\omega_0}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\cdot\tan B}$$

原式 $= jZ_{0n}\cdot\dfrac{\tan\frac{\pi}{2} + \tan\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}}{1 - \tan\frac{\pi}{2}\cdot\tan\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}} = j\cdot Z_{0n}\cdot\left(-\cot\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}\right) = -jZ_{0n}\cdot\cot\left(\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}\right)$

$$\therefore Y_{in} = 1/Z_{in} = j\frac{1}{Z_{0n}}\cdot\tan\left(\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}\right) \quad ☆$$

(iii) 若 $Y_{in} = Y$，则 $j\frac{1}{Z_{0n}}\tan\left(\frac{\pi\Delta\omega}{2\omega_0}\right) = j\sqrt{\frac{C_n}{L_n}}\,\frac{\Delta\omega^2 + 2\Delta\omega\cdot\omega_0}{\omega_0^2 + \Delta\omega\cdot\omega_0}$

$$\therefore Z_{0n} = \lim_{\Delta\omega\to 0}\sqrt{\frac{L_n}{C_n}}\cdot\frac{\tan\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}}{\frac{\Delta\omega^2 + 2\Delta\omega\cdot\omega_0}{\omega_0^2 + \Delta\omega\cdot\omega_0}} = \sqrt{\frac{L_n}{C_n}}\cdot\lim_{\Delta\omega\to 0}\frac{\frac{\pi}{2}\frac{\Delta\omega}{\omega_0}\cdot\omega_0^2}{\Delta\omega^2 + 2\Delta\omega\cdot\omega_0} = \sqrt{\frac{L_n}{C_n}}\cdot\frac{\pi}{2}\cdot\omega_0\lim_{\Delta\omega\to 0}\frac{\Delta\omega}{\Delta\omega^2 + 2\Delta\omega\,\omega_0}$$

$$= \sqrt{\frac{L_n}{C_n}}\cdot\frac{\pi}{2}\cdot\omega_0\cdot\frac{1}{2\omega_0} = \frac{\pi}{4}\sqrt{\frac{L_n}{C_n}}$$

$$Z_{0n} = \frac{\pi}{4}\sqrt{\frac{L_n}{C_n}} \quad ☆$$

$$\lim_{\Delta W \to 0} \frac{j\sqrt{\frac{L_n}{C_n}}\ \frac{\Delta W^2 + 2\Delta W \cdot W_0}{W_0^2 + \Delta W \cdot W_0}}{j2\Delta W \cdot L_n}$$

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x-x_0) + \frac{f''(x_0)}{2!}(x-x_0)^2 \ldots$$

$$\frac{1}{W_0 + \Delta W} = \frac{1}{W_0} - \frac{\Delta W}{W_0^2}$$

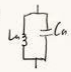
$$\left(\frac{1}{W}\right)' = -\frac{1}{W^2}$$

$$= \frac{1}{2L_n}\sqrt{\frac{L_n}{C_n}} \lim_{\Delta W \to 0} \frac{\Delta W^2 + 2\Delta W \cdot W_0}{\Delta W(\Delta W\, W_0 + W_0^2)}$$

$$W_0^2 = \frac{1}{L_n C_n} = \frac{1}{C_n}\sqrt{\frac{C_n}{L_n}} \cdot \frac{1}{\sqrt{L_n C_n}} = \frac{1}{W_0^2}$$

$$W_0 C_n = \frac{1}{W_0 L_n}$$

(i)



$$Y_{LC} = jWC_n + \frac{1}{jWL_n}$$

$W \approx W_0$

$W = W_0 + \Delta W$. $\quad W_0^2 = \frac{1}{L_n C_n}$.

$$Y_{in} = j(W_0 + \Delta W)C_n + \frac{1}{j(W_0 + \Delta W)L_n}$$

$$= j\left[(W_0 + \Delta W)C_n - \frac{1}{W_0 L_n} + \frac{\Delta W}{W_0^2 L_n}\right)$$

$$= j2\Delta W C_n.$$

$$j\left[(W_0 + \Delta W)C_n - W_0 C_n + W_0 C_n \cdot \frac{\Delta W}{W_0}\right]$$

$$= j2\Delta W C_n$$

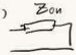
(ii)

$$\tan\left(\frac{\pi}{2} + \Delta\beta l\right) = \frac{\sin\left(\frac{\pi}{2} + \Delta\beta l\right)}{\cos\left(\frac{\pi}{2} + \Delta\beta l\right)}$$

$$= \frac{\sin\frac{\pi}{2}\cos\Delta\beta l + \cos\frac{\pi}{2}\sin\Delta\beta l}{\cos\frac{\pi}{2}\cos\Delta\beta l - \sin\frac{\pi}{2}\sin\Delta\beta l}$$

$$= \frac{\cos\Delta\beta l}{-\sin\Delta\beta l} = -\frac{1}{\tan\Delta\beta l} \approx -\frac{1}{\Delta\beta l}$$

(ii) Z_{0n}



$$Z_{in} = Z_{0n}\frac{Z_L + jZ_{0n}\tan(\beta l)}{Z_{0n} + jZ_L\tan(\beta l)}$$

$Z_L = 0$.

$$Z_{in} = jZ_{0n}\tan(\beta l)$$

$$Y_{in} = \frac{1}{Z_{in}} = \frac{1}{jZ_{0n}\tan(\beta l)} = \frac{1}{jZ_{0n}\cdot\left(-\frac{1}{\Delta\beta l}\right)} = \frac{j\Delta\beta l}{Z_{0n}}$$

$$\Delta\beta = \frac{\Delta W}{V_p} \qquad V_p = \frac{2\pi}{W_0}\lambda \qquad \Delta\beta l = \frac{\Delta W\, W_0}{2\pi\lambda}\cdot\frac{\lambda}{4} = \frac{\Delta W\, W_0}{8\pi}$$

$$Y_{in} = \frac{j\Delta W \cdot W_0}{Z_{0n}\cdot 8\pi}$$

(iii)

$$(8\pi Z_{0n})\cancel{j}2\cancel{\Delta W}C_n = \cancel{j}\cancel{\Delta W}\cdot W_0$$

$$\frac{1}{\sqrt{L_n C_n}} \cdot \frac{1}{16\pi C_n}$$

$$Z_{0n} = \frac{\frac{1}{\sqrt{L_n C_n}}}{16\pi C_n} = \frac{1}{16 C_n L_n \pi}\sqrt{\frac{L_n}{C_n}}$$

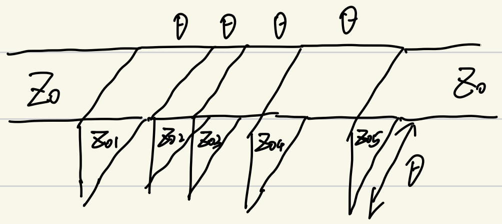
(b) Based on the quarter-wavelength short-circuited stubs and transmission line sections, design a fifth-order bandpass filter with 0.5 dB equal ripple response. The center frequency is 2 GHz, the fractional bandwidth is 0.2 and the impedance is 50 Ω. Using a table, list down the low-pass prototype values and the stub characteristic impedances. Sketch the schematic of the designed filter and highlight any potential issue with its microstrip implementation.

Answer: $N=5$ $\quad$ $X_{max}=0.5\,dB$ $\quad$ Chebyshev bandpass filter $\quad$ $f_0=2GHz$ $\quad$ $\Delta=0.2$

$Z_0=50\,\Omega$

5	1.7058	1.2296	2.5408	1.2296	1.7058	1.0000

n	g_n	Z_{0n}
1	1.7058	4.6043
2	1.2296	6.3874
3	2.5408	3.0911
4	1.2296	6.3874
5	1.7058	4.6043
6	1.0000	



$$Z_{0n}=\frac{\pi Z_0 \Delta}{4 g_n}$$

2. A microwave transistor has the following S parameters at 1 GHz in a 50-Ω system:

$$S_{11} = 0.5 \angle 180^\circ \qquad S_{12} = 0.08 \angle 30^\circ$$
$$S_{21} = 2.5 \angle 70^\circ \qquad S_{22} = 0.8 \angle -100^\circ.$$

(a) Check if the transistor is unconditionally stable using single-parameter stability criterion. Without using stability circles, discuss if it is stable to operate with the source and load reflection coefficients given by $\Gamma_s = 0.5 \angle 180^\circ$ and $\Gamma_L = 0.1 \angle 90^\circ$ respectively.

Answer:

$$\Delta = S_{11} \cdot S_{22} - S_{12} \cdot S_{21} = 0.2228 \angle 62.12^\circ$$

$$|\Delta| = 0.2228 < 1$$

$$K = \frac{1 + |\Delta|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12} \cdot S_{21}|} \approx 0.399 < 1$$

The transistor is not unconditionally stable

$$K_t = \frac{3 - 2|S_{11}|^2 - 2|S_{22}|^2 + |\Delta|^2 - |1 - |\Delta|^2|}{4|S_{12}S_{21}|} > 1$$

$$K_t = 0.399 < 1$$

$$C_1 = S_{11} - S_{22}^* \Delta$$

$$C_2 = S_{22} - S_{11}^* \Delta$$

$$C_1 = 0.65 \angle -170.31^\circ$$
$$C_2 = 0.69 \angle -97.18^\circ$$

$$c_s = \frac{C_1^*}{|S_{11}|^2 - |\Delta|^2} \qquad C_s = 3.24 \angle 170.31$$

$$r_s = \frac{|S_{12}S_{21}|}{||S_{11}|^2 - |\Delta|^2|} \qquad r_s = 0.9982$$

$$c_L = \frac{C_2^*}{|S_{22}|^2 - |\Delta|^2} \qquad C_L = 1.169 \angle 97.18^\circ$$

$$r_L = \frac{|S_{12}S_{21}|}{||S_{22}|^2 - |\Delta|^2|} \qquad r_L = 0.3388$$

When $\Gamma_s = 0.5 \angle 180^\circ$ / $\Gamma_L = 0.1 \angle 90^\circ$, the network is stable

(b) Using the above source and load reflection coefficients, calculate the resultant transducer power gain in dB. Design the input and output matching networks using quarter-wavelength transformers.

$$G_t = \frac{1-|\Gamma_s|^2}{|1-\Gamma_{in}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-S_{22}\Gamma_L|^2}$$

$$G_t = \frac{1-|\Gamma_s|^2}{|1-S_{11}\Gamma_s|^2}|S_{21}|^2\frac{1-|\Gamma_L|^2}{|1-\Gamma_{out}\Gamma_L|^2}$$

$$\Gamma_{in} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1-S_{22}\Gamma_L} = \frac{S_{11}-\Delta\Gamma_L}{1-S_{22}\Gamma_L}$$

$$\Gamma_{out} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1-S_{11}\Gamma_s} = \frac{S_{22}-\Delta\Gamma_s}{1-S_{11}\Gamma_s}$$

$$\Delta = S_{11}S_{22} - S_{12}S_{21}$$

Answer: $\Gamma_S = 0.5\angle 180^\circ$ $\Gamma_L = 0.1\angle 90^\circ$

$$\Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L} = 0.5214\angle -179.62^\circ$$

$$\Gamma_{out} = \frac{S_{22} - \Delta\Gamma_S}{1 - S_{11}\Gamma_S} = 0.9264\angle -97.18^\circ$$

3. (a) A four-port directional coupler is characterized by the following S parameters:

$$[S] = \begin{bmatrix} 0.02 & 0.8 & j0.5 & j0.03 \\ 0.8 & 0.02 & j0.03 & j0.5 \\ j0.5 & j0.03 & 0.02 & 0.8 \\ j0.03 & j0.5 & 0.8 & 0.02 \end{bmatrix}$$

Assuming the input is at port 1, calculate the return loss, coupling, isolation and directivity in dB. Give the relation among these quantities if any.

(10 Marks)

$C = 10\log_{10}\frac{P_1}{P_3} = -20\log_{10}|S_{13}| \approx 6.02\ dB$ $\quad I = C + D$

$I = 10\log_{10}\frac{P_1}{P_4} = -20\log_{10}|S_{14}| \approx 30.46\ dB$

$D = 10\log_{10}\frac{|P_3|}{|P_4|} = -20\log_{10}\frac{|S_{14}|}{|S_{13}|} \approx 24.44\ dB$

$RL = -20\log_{10}|S_{11}| \approx 33.98$