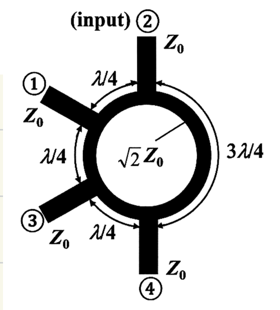
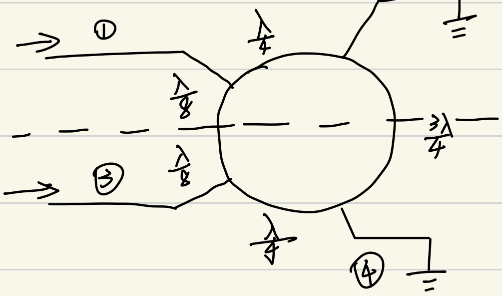
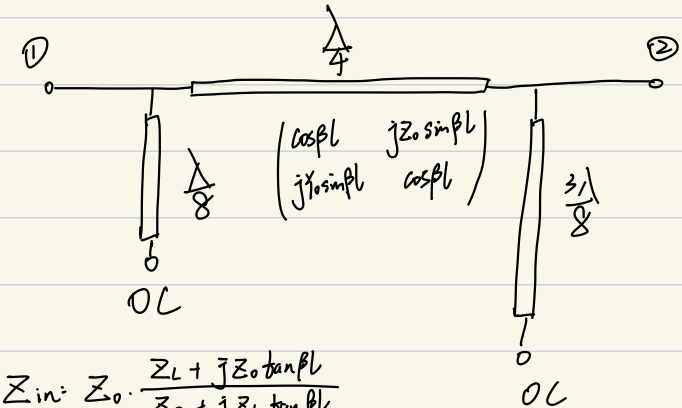
1. (a) Figure 1 on page 2 shows a four-port hybrid ring coupler circuit with the input at port 2. With the aid of even- and odd-mode bisections of the circuit, derive the even- and odd-mode ABCD and S parameters ($S_{12,e}, S_{22,e}, S_{12,o}, S_{22,o}$).



Answer : Even mode:



There is no current in the middle point. So the middle point can be seen as open circuit.



$$\begin{pmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{pmatrix}$$

$$[ABCD]_{even} = \begin{bmatrix} 1 & 0 \\ j\frac{Y_0}{\sqrt{2}} & 1 \end{bmatrix}\begin{bmatrix} 0 & j\sqrt{2}Z_0 \\ j\frac{Y_0}{\sqrt{2}} & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -j\frac{Y_0}{\sqrt{2}} & 1 \end{bmatrix}$$

$$= \begin{pmatrix} 0 & j\sqrt{2}Z_0 \\ j\frac{Y_0}{\sqrt{2}} & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -j\frac{Y_0}{\sqrt{2}} & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & j\sqrt{2}Z_0 \\ j\sqrt{2}Y_0 & -1 \end{pmatrix}$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}$$

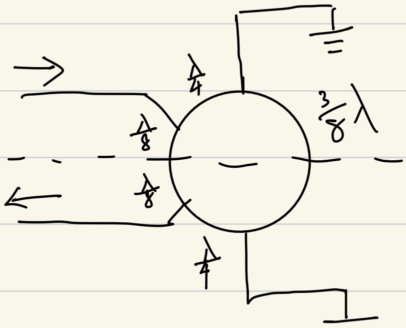
开路: $Z_L = \infty$

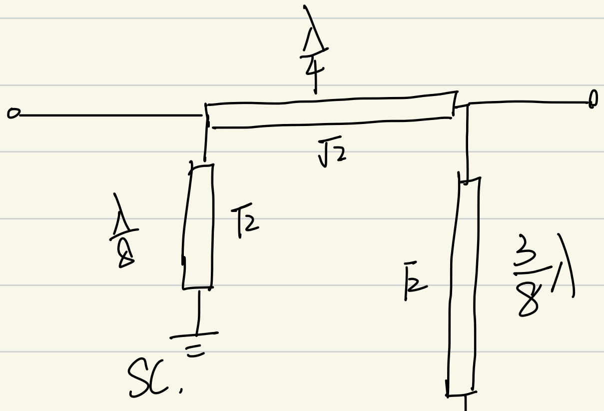
$$Z_{in} = Z_0 \cdot \frac{1}{j\tan\beta l}$$

$$S_{12e} = \frac{2(AD-BC)}{A+B/Z_0+CZ_0+D} = \frac{2(-1-2)}{1+j\sqrt{2}+j\sqrt{2}-1} = \frac{-6}{j2\sqrt{2}} = j\frac{3\sqrt{2}}{2}$$

$$S_{22e} = \frac{-A+B/Z_0-CZ_0+D}{A+B/Z_0+CZ_0+D} = \frac{-1+j\sqrt{2}-j\sqrt{2}-1}{1+j\sqrt{2}+j\sqrt{2}-1} = \frac{-2}{j2\sqrt{2}} = j\frac{\sqrt{2}}{2}$$

Odd mode:





$$\begin{pmatrix} \cos\beta l & jZ_0\sin\beta l \\ jY_0\sin\beta l & \cos\beta l \end{pmatrix}$$

$$Z_{in} = Z_0\frac{Z_L+jZ_0\tan\beta l}{Z_0+jZ_L\tan\beta l} = jZ_0\tan\beta l$$

SC.

$$[ABCD]_{odd} = \begin{pmatrix} 1 & 0 \\ -j\frac{Y_0}{\sqrt{2}} & 1 \end{pmatrix}\begin{pmatrix} 0 & j\sqrt{2}Z_0 \\ j\frac{Y_0}{\sqrt{2}} & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ j\frac{Y_0}{\sqrt{2}} & 1 \end{pmatrix} = \begin{pmatrix} 0 & j\sqrt{2}Z_0 \\ j\frac{Y_0}{\sqrt{2}} & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ j\frac{Y_0}{\sqrt{2}} & 1 \end{pmatrix} = \begin{pmatrix} -1 & j\sqrt{2}Z_0 \\ j\sqrt{2}Y_0 & 1 \end{pmatrix}$$

$$S_{120} = \frac{2(AD-BC)}{A+B/Z_0+Z_0C+D} = \frac{2(-1-2)}{-1+j\sqrt{2}+j\sqrt{2}+1} = \frac{-6}{j2\sqrt{2}} = j\frac{3\sqrt{2}}{2}$$

$$S_{220} = \frac{-A+B/Z_0-Z_0C+D}{A+B/Z_0+Z_0C+D} = \frac{2}{j2\sqrt{2}} = -j\frac{\sqrt{2}}{2}$$

(b) Design a coupled line filter with fifth-order 3 dB equal ripple band-pass response, center frequency 2 GHz, bandwidth 200 MHz and impedance 50 Ω. Using a table, list down the low-pass prototype values, normalized inverter parameters, even and odd-mode characteristic impedances of the coupled line sections. Sketch the schematic diagram of the designed filter.

Answer: $N=5$ $\alpha_{max}=3dB$ Chebyshev bandpass filter $f_0=2GHz$ $BW=200MHz$ $Z_0=50\Omega$

5	3.4817	0.7618	4.5381	0.7618	3.4817	1.0000

$\Delta=\frac{0.2}{2}=0.1$

n	g_n	Z_0J_n	J_n	Z_{0e}	Z_{0o}
1	3.4817	0.2124	4.248×10^{-3}	62.88	41.64
2	0.7618	9.645×10^{-2}	1.929×10^{-3}	55.29	45.64
3	4.5381	8.448×10^{-2}	1.6896×10^{-3}	54.58	46.13
4	0.7618	8.448×10^{-2}	1.6896×10^{-3}	54.58	46.13
5	3.4817	9.645×10^{-2}	1.929×10^{-3}	55.29	45.64
6	1.0000	0.2124	4.248×10^{-3}	62.88	41.64

$$Z_0J_1=\sqrt{\frac{\pi\Delta}{2g_1g_0}}$$
$$Z_0J_n=\frac{\pi\Delta}{2\sqrt{g_ng_{n-1}}}$$
$$Z_0J_{N+1}=\sqrt{\frac{\pi\Delta}{2g_Ng_{N+1}}}$$

$$Z_{0e}=Z_0\left[1+Z_0J_n+(Z_0J_n)^2\right]$$
$$Z_{0o}=Z_0\left[1-Z_0J_n+(Z_0J_n)^2\right]$$