1. (a) A signal g(t) bandlimited to B Hz is sampled using a triangular pulse train

$$x_p(t) = \sum_{n=-\infty}^{\infty} \Delta\left(\frac{t-nT_s}{\tau}\right)$$

where $\Delta\left(\frac{t}{\tau}\right) = \begin{cases} 1-\frac{|t|}{\tau} & ,for\ |t| \le \tau \\ 0 & ,for\ \ |t| > \tau \end{cases}$

is a triangular pulse spread over 2τ seconds (from $-\tau$ to τ). Note that $\tau = 1/(16B)$ and the sampling period $T_s = 1/(2B)$.

(i) Find the Fourier transform of the sampling pulse centered on the origin.

(i) Answer: $x_p(t) = \Delta\left(\frac{t}{\tau}\right) \circledast \sum_{n=-\infty}^{+\infty} \delta(t-nT_s)$

$\Delta\left(\frac{t}{\tau}\right) \longleftrightarrow \tau\, sinc^2(\tau f)$ $\quad \sum_{n=-\infty}^{+\infty} \delta(t-nT_s) \longleftrightarrow \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} \delta\left(f-n\frac{1}{T_s}\right)$

$$X_p(f) = \tau\, sinc^2(\tau f) \cdot \frac{1}{T_s}\sum_{n=-\infty}^{+\infty} \delta\left(f-n\frac{1}{T_s}\right)$$

$$= \frac{\tau}{T_s}\sum_{n=-\infty}^{+\infty} sinc^2\left(\frac{\tau n}{T_s}\right)\delta\left(f-\frac{n}{T_s}\right)$$

$\Delta\left(\frac{t}{\tau}\right) \Longleftrightarrow \tau\, sinc^2(\tau f)$

$\tau = \frac{1}{16B}$ $\quad T_s = \frac{1}{2B}$ $\quad \tau/T_B = \frac{1}{8}$

X(t)是 period of T_0

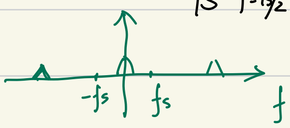
$$X_p(f) = \frac{1}{8}\sum_{n=-\infty}^{+\infty} sinc^2\left(\frac{n}{8}\right)\delta\left(f-\frac{n}{T_s}\right)$$

$X(t) = \sum_{n=-\infty}^{+\infty} C_n e^{j2\pi n f_0 t}$ $\quad C_n = \frac{1}{T}\int_{-T/2}^{T/2} X(t)\, e^{-j2\pi n f_0 t}\, dt$

(ii) Determine the Fourier series coefficients $\mathbf{C_n}$ of the periodic sampling function.

$$C_n = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} X_p(t)\, e^{-j2\pi n f_s t}\, dt = \frac{1}{T_s}\int_{-T_s/2}^{T_s/2} \Delta\left(\frac{t}{\tau}\right)\cdot e^{-j2\pi n f_s t}\, dt = \frac{1}{T_s}\cdot \tau \cdot sinc^2(\tau\, n f_s) = \frac{1}{8} sinc^2\left(\frac{n}{8}\right)$$

$C_n = \frac{1}{8} sinc^2\left(\frac{n}{8}\right)$



$$F\left[\Delta\left(\frac{t}{\tau}\right)\right] = \int_{-\infty}^{+\infty} \Delta\left(\frac{t}{\tau}\right) e^{-j2\pi f t}\, dt = \tau\, sinc^2(\tau f)$$

$$\int_{-\frac{T_s}{2}}^{+\frac{T_s}{2}} \Delta\left(\frac{t}{\tau}\right) e^{-j2\pi n f_s t}\, dt = \tau\, sinc^2(\tau n f_s)$$

(iii) Show that the sampled data signal $\bar{g}(t)$ is given by

$$\bar{g}(t) = \sum_{n=-\infty}^{\infty} \frac{1}{8}\, sinc^2\left(\frac{n}{8}\right) g(t) e^{j2\pi f_s t}$$?

where the sampling frequency $f_s = 2\,B$. Show that the signal g(t) can be totally recovered by passing the signal $\bar{g}(t)$ though an ideal low-pass filter of bandwidth B Hz that has a gain of 8.

(iii) Answer. The Fourier series of $x_p(t)$ is

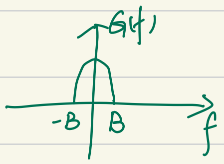
$$x_p(t) = \sum_{n=-\infty}^{+\infty} C_n\, e^{j2\pi n f_s t}$$

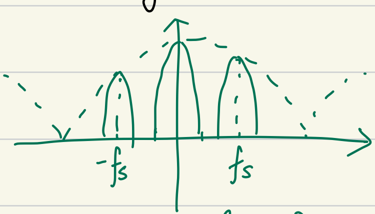
$$= \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) e^{j2\pi n f_s t}$$

When conduct the sampling, we need to multiply $x_p(t)$ with $g(t)$

$$\bar{g}(t) = g(t)\cdot x_p(t) = g(t)\sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) e^{j2\pi n f_s t} = \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) g(t)\, e^{j2\pi n f_s t}$$

$g(t) \longleftrightarrow G(f)$





$$\bar{g}(t) \longleftrightarrow \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) G(f-nf_s)$$

$f_s > 2B$

$g(t) \longleftrightarrow G(f)$

$$\bar{G}(f) = G(f)\cdot X_p(f) = G(f)\cdot\sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right)\delta(f-nf_s)$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) G(nf_s)\,\delta(f-nf_s)$$

$$\bar{g}(t) = F^{-1}\left(\bar{G}(f)\right) = \int_{-\infty}^{+\infty} \bar{G}(f)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) G(nf_s)\, e^{j2\pi f t}\,\delta(f-nf_s)\, df$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) G(nf_s)\cdot\int_{-\infty}^{+\infty} e^{j2\pi n f_s t}\delta(f-nf_s)\, df$$

$$= \sum_{n=-\infty}^{+\infty} \frac{1}{8} sinc^2\left(\frac{n}{8}\right) G(nf_s)\cdot e^{j2\pi n f_s t}$$

$$\Delta\left(\frac{t}{2}\right) \Longleftrightarrow 2\,\mathrm{sinc}^2(2f) \qquad x(t-nT_s) \longleftrightarrow X(f)\cdot e^{-j2\pi f nT_s}$$

$$= X(f)\, e^{-j2\pi nfT_s}$$

$$\Delta\left(\frac{t-nT_s}{2}\right) \longleftrightarrow 2\,\mathrm{sinc}^2(2f)\, e^{-j2\pi nfT_s}$$

$$\sum_{n=-\infty}^{+\infty} \Delta\left(\frac{t-nT_s}{2}\right) \longleftrightarrow \sum_{n=-\infty}^{+\infty} 2\,\mathrm{sinc}^2(2f)\, e^{-j2\pi nfT_s}$$

$$e^{-j2\pi nfT_s} + e^{j2\pi nfT_s}$$

$$= 2\cos 2\pi nfT_s$$

$$= 2\,\mathrm{sinc}^2(2f) \sum_{n=-\infty}^{+\infty} e^{-j2\pi nfT_s}$$

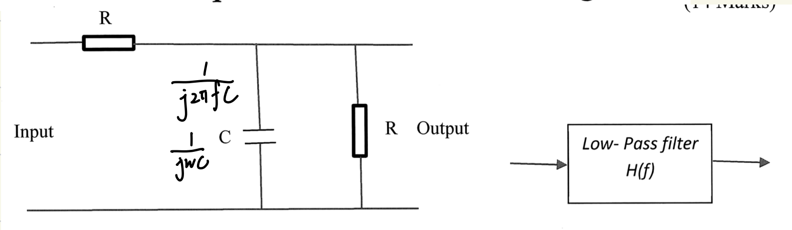
$$= 2\,\mathrm{sinc}^2(2f) \sum_{n=-\infty}^{+\infty} \cos(2\pi nfT_s)$$

$$\sum_{n=-\infty}^{+\infty} 1\cdot e^{-j2\pi nfT} \longleftrightarrow \frac{1}{T}\sum_{n=-\infty}^{+\infty} \delta\left(f-\frac{n}{T}\right)$$

$$1 \xleftrightarrow{DFT} \delta(t)$$

$$X_p(f) = \frac{1}{8}\sum_{n=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{n}{8}\right)\delta\left(f-\frac{n}{T_s}\right)$$

(b) Consider a low-pass filter as shown in Figure 1.



(i) Determine the transfer function $H(f)$, and its impulse response $h(t)$.

(i) Answer. $Z_{shunt} = \dfrac{1}{j2\pi fC + \frac{1}{R}} = \dfrac{R}{j2\pi RCf + 1}$

$$H(f) = \frac{Z_{shunt}}{R + Z_{shunt}} = \frac{\frac{R}{j2\pi RCf+1}}{R + \frac{R}{j2\pi RCf+1}} = \frac{1}{j2\pi RCf + 1 + 1} = \frac{1}{j2\pi RCf + 2} = \frac{1}{RC}\cdot\frac{1}{\frac{2}{RC} + j2\pi f}$$

$$\exp(-\alpha t)\, u(t) \longleftrightarrow \frac{1}{\alpha + j2\pi f}$$

$$h(t) = F^{-1}[H(f)] = \frac{1}{RC}\exp\left(-\frac{2}{RC}t\right) u(t)$$

(ii) Determine the Noise Equivalent Bandwidth B_{eq} for this filter $H(f)$.

$$B_{eq} = \frac{\int_{-\infty}^{\infty}|H(f)|^2 df}{2|H(0)|^2}$$

$$\int_{-\infty}^{+\infty}|H(f)|^2 df = \int_{-\infty}^{+\infty}|h(t)|^2 dt$$

$$= \int_0^{+\infty}\frac{1}{R^2C^2}\exp\left(-\frac{4}{RC}t\right)dt$$

$$= \frac{1}{R^2C^2}\cdot\frac{RC}{4}\cdot\int_0^{+\infty}\exp\left(-\frac{4}{RC}t\right)d\left(\frac{4}{RC}t\right)$$

$$= \frac{1}{4RC}\exp(-t)\Big|_{+\infty}^{0} = \frac{1}{4RC}$$

(ii) Answer: $H(f) = \dfrac{1}{2 + j2\pi RCf}$

$$|H(f)| = \frac{1}{\sqrt{4 + 4\pi^2R^2C^2f^2}}$$

$$|H(0)|^2 = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$$

$$\int_{-\infty}^{+\infty}|H(f)|^2 df = \int_{-\infty}^{+\infty}\frac{1}{4 + 4\pi^2R^2C^2f^2}df = \frac{1}{4}\int_{-\infty}^{+\infty}\frac{df}{1 + \pi^2R^2C^2f^2} = \frac{1}{4\pi RC}\int_{-\infty}^{+\infty}\frac{d(\pi RCf)}{1 + (\pi RCf)^2}$$

$$= \frac{1}{4\pi RC}\left(\arctan(\pi RCf)\Big|_{-\infty}^{+\infty}\right) = \frac{1}{2\cdot 2\pi RC}\cdot\pi = \frac{1}{4RC}$$

$$\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$B_{eq} = \frac{\int_{-\infty}^{+\infty}|H(f)|^2 df}{2|H(0)|^2} = \frac{\frac{1}{\cancel{2}\pi RC}}{1/2} = \frac{\cancel{1}}{\pi \cancel{RC}} \quad \frac{1}{2RC}$$

2. (a) Let Z be a uniformly distributed random variable defined by the following probability density function,

$$f_Z(z) = \begin{cases} \frac{1}{2}, & -1 \le z \le 1 \\ 0, & elsewhere \end{cases}$$

Let the new two random variables be defined as $X = Z$ and $Y = Z^2$.

(i) State whether X and Y are statistically independent.

(i) Answer: X and Y are not independent

Justification: $F_Z(z) = \int_{-\infty}^{z} f_Z(z)dz = \int_{-1}^{z} \frac{1}{2}dz = \frac{1}{2}(z+1) \quad z \in [-1,1]$

$F_Z(z) = P_r\{Z \le z\} = \frac{1}{2}(z+1) \quad z \in [-1,1]$

$F_{XY}(x,y) = P_r\{X \le x, Y \le y\} = P_r\{Z \le x, Z^2 \le y\} \quad x \in [-1,1] \quad y \in [-1,1]$

$$= \begin{cases} P_r\{Z \le x, -\sqrt{y} \le Z \le \sqrt{y}\} = P_r\{-\sqrt{y} \le Z \le \min(\sqrt{y}, x)\} & -1 \le x \le 1 \quad 0 \le y \le 1 \\ 0 & else \end{cases}$$

$$= \begin{cases} P_r\{-\sqrt{y} \le Z \le \sqrt{y}\} = \frac{1}{2}(\sqrt{y}+1) - \frac{1-\sqrt{y}}{2} = \sqrt{y} & -1 \le x < 0 \quad 0 \le y \le 1 \\ P_r\{-\sqrt{y} \le Z \le x\} = \frac{1}{2}(x+1) - \frac{1-\sqrt{y}}{2} = \frac{x+\sqrt{y}}{2} & 0 \le x \le 1 \quad 0 \le y \le 1 \end{cases}$$

写不出来，反正不独立就对了

(ii) Determine the means, μ_X and μ_Y of X and Y, respectively.

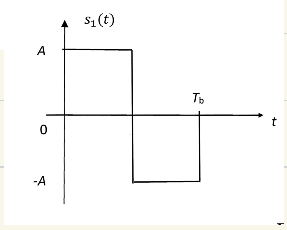
(ii) Answer: $\mu_x = E[X] = E[Z] = 0$

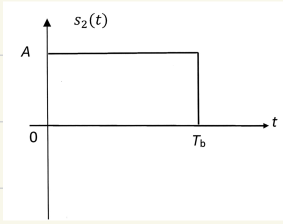
$\mu_y = E[Y] = E[Z^2] = \int_{-1}^{1} \frac{1}{2} z^2 dz = \frac{1}{2} \cdot \frac{1}{3} z^3 \Big|_{-1}^{1} = \frac{1}{3}$

(iii) Let the covariance function be defined as $\text{Cov}(X, Y) = E[(X- \mu_X)(Y- \mu_Y)]$. Derive the $\text{Cov}(X, Y)$, state whether the random variables are uncorrelated and explain any significance of this result.

(iii) Answer: $\text{Cov}(X,Y) = E[(X-\mu_x)(Y-\mu_y)] = E[Z(Z^2-\frac{1}{3})] = E[Z^3] - \frac{1}{3}E(Z) = 0$

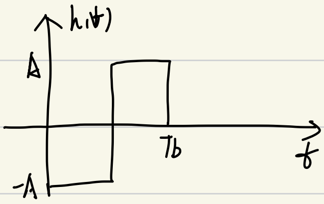
(b) The two orthogonal signals, $s_1(t)$ and $s_2(t)$, depicted in Figure 2 are used to transmit binary '1' and '0', respectively, where A is the amplitude and T_b is the bit duration. The data waveform is sent over an additive white Gaussian noise (AWGN) channel with a two-sided power spectral density of $N_0/2$.

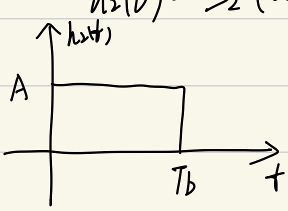




(i) Suppose the receiver is implemented by two filters matched to $s_1(t)$ and $s_2(t)$, respectively. Find and sketch the impulse responses of the matched filters.

(b) (i) Answer: $h_1(t) = S_1(T_b - t)$ $\qquad$ $h_2(t) = S_2(T_b - t)$





(ii) Determine the outputs $y_1(T_b)$ and $y_2(T_b)$ of the two matched filters at $t = T_b$ when the transmitted signal is $s_1(t)$. Note that each output contains a noise component due to AWGN. Find the mean and the variance of each noise component.

(ii) Answer:

$$y_1(T_b) = (s_1(t) + n(t)) * h_1(t) = s_1(t) * h_1(t) + n(t) * h_1(t) = \int_0^{T_b} s_1^2(t)\,dt + \int_0^{T_b} n(t)\, s_1(t)\,dt$$

$$= A^2 T_b + \int_0^{T_b} s_1(t)\, n(t)\,dt$$

$$y_2(T_b) = (s_1(t) + n(t)) * h_2(t) = s_1(t) * h_2(t) + n(t) * h_2(t) = \int_0^{T_b} s_1(t) s_2(t)\,dt + \int_0^{T_b} n(t)\, s_2(t)\,dt$$

$$= 0 + \int_0^{T_b} s_2(t)\, n(t)\,dt = \int_0^{T_b} s_2(t)\, n(t)\,dt$$

$$n_1(T_b) = \int_0^{T_b} s_1(t)\, n(t) \qquad n_2(T_b) = \int_0^{T_b} s_2(t)\, n(t)\,dt$$

$$E[n_1(T_b)] = \int_0^{T_b} s_1(t)\, E[n(t)]\,dt = 0 \qquad E[n_2(T_b)] = 0$$

$$Var[n_1(T_b)] = E[n_1^2(T_b)] - E^2[n_1(T_b)] = E[n_1^2(T_b)] = E\left[\int_0^{T_b} s_1(t_1)\, n(t_1)\,dt_1 \int_0^{T_b} s_1(t_2)\, n(t_2)\,dt_2\right]$$

$$= E\left[\int_0^{T_b}\int_0^{T_b} n(t_1)\, n(t_2)\, s_1(t_1)\, s_1(t_2)\,dt_1\,dt_2\right] = \int_0^{T_b}\int_0^{T_b} E[n(t_1)\, n(t_2)]\, s_1(t_1)\, s_2(t_2)\,dt_1\,dt_2$$

$$= \int_0^{T_b}\int_0^{T_b} R_n(t_1 - t_2)\, s_1(t_1)\, s_1(t_2)\,dt_1\,dt_2 = \int_0^{T_b}\int_0^{T_b} \frac{N_0}{2}\,\delta(t_1 - t_2)\, S(t_1)\, S(t_2)\,dt_1\,dt_2$$

$$\frac{N_0}{2}\int_0^{T_b} s_1(t_2)\, s_1(t_2)\,dt_2 = \frac{N_0}{2} T_b A^2$$

$$Var[n_2(T_b)] = \frac{N_0}{2} T_b A^2$$

(iii) Compute the average probability of bit error in terms of the Q-function, i.e., the probability $\Pr[y_1(T_b) > y_2(T_b)]$. State any necessary assumptions.

(iii) Answer: When we assume that the probability of sending 1 or 0 is equal likely.

$$P_e = P_r[y_1(T_b) > y_2(T_b)] = P_r\{y_2(T_b) > y_0\} \qquad y_0 = \frac{a_2 + a_1}{2} = \frac{A^2 T_b}{2}$$

$$y_2(T_b) \sim N\left(0, \frac{N_0 T_b A^2}{2}\right)$$

$$P_e = P_r\left\{\frac{y_2(T_b)}{\sqrt{\frac{N_0 T_b A^2}{2}}} > \frac{\frac{A^2 T_b}{2}}{\sqrt{\frac{N_0 T_b A^2}{2}}}\right\} = Q\left(\frac{A \cdot \sqrt{T_b}}{\sqrt{2}\, N_0}\right) = Q\left(\sqrt{\frac{A^2 T_b}{2 N_0}}\right)$$

(iv) Suppose $s_1(t)$ is replaced with another waveform $\widetilde{s_1}(t) = -s_1(t)$. What is its effect on the average probability of the bit error in part (iii)? Explain your answer.

(iv) Answer: It will not change the BER

If the match filter still be the same one.

$y_1(T_b) = -A^2 T_b + n_1(T_b)$

$y_2(T_b) = 0 + n_2(T_b)$

$y_0 = \frac{a_1 + a_2}{2} = -\frac{A^2 T_b}{2}$

$P_b = P_r\{y_1(T_b) > y_0\} = Q\left(\sqrt{\frac{A^2 T_b}{2N_0}}\right)$

3. A block code encoder appends a single even parity bit p_0 to each block of 3 message bits (m_0, m_1, m_2) such that the summation of all the bits in the codeword yields an even result.

(a) Determine all the possible codewords of this code.

Answer:

p_0	m_0	m_1	m_2
0	0	0	0
1	1	0	0
1	0	1	0
0	1	1	0
1	0	0	1
0	1	0	1
0	0	1	1
1	1	1	1

(b) Determine the generator matrix, **G**, and the parity-check matrix, **H**, of this code in systematic form.

Answer:

$$G = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}$$

$H = (1\ 1\ 1\ 1)$

$G = [P\ I_k]$

$H = [I_{n-k}\ P^T]$

Block code $(4, 3)$

$n=4$ $k=3$ $n-k=1$

$C = UG$

$GH^T = (0, 0, 0)^T$

(c) Show that $\mathbf{c}\mathbf{H}^T = \mathbf{0}$, where $\mathbf{c}$ is any valid codeword. Determine the syndrome for any single error received vector $\mathbf{r}$.

Answer: $C = UG$ $GH^T = (0, 0, 0) = 0$

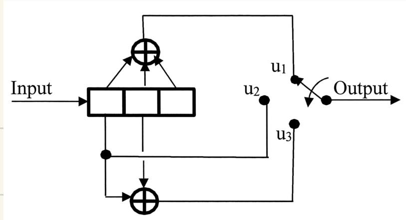
$CH^T = UGH^T = U(GH^T) = U \cdot 0 = 0$

$\therefore CH^T = 0$, when c is a valid

When there is a one bit error in r, then $r = c + e$

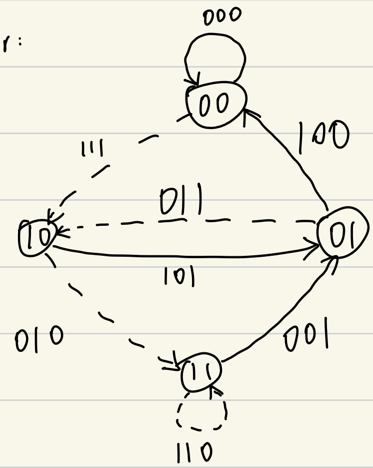
e	Syndrome eH^T
(1 0 0 0)	(1)
(0 1 0 0)	(1)
(0 0 1 0)	(1)
(0 0 0 1)	(1)

(d) The convolutional encoder diagram is shown in Figure 3.



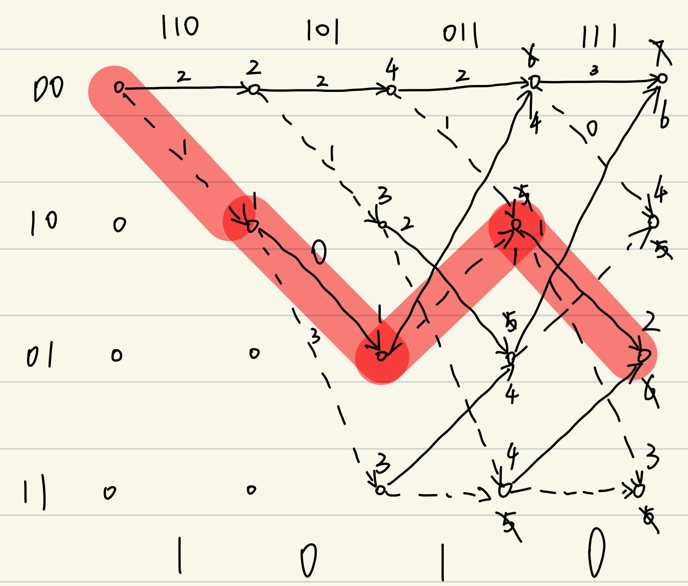
(i) Draw the state diagram.

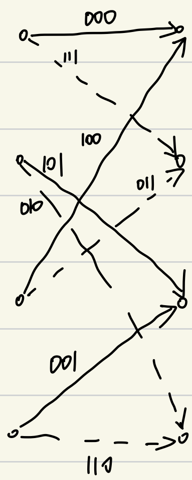
Answer:



(ii) The received sequence (starting from left to right) corresponding to an output sequence of the encoder is 110 101 011 111. Using the Viterbi algorithm, find the encoded sequence (from the path with smallest path metric). If a tie occurs, take the upper branch.

Answer



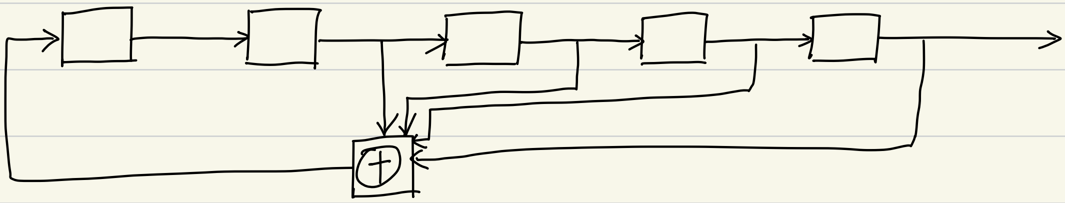


The encoded sequence is 111 101 011 101.

(1 0 1 0)

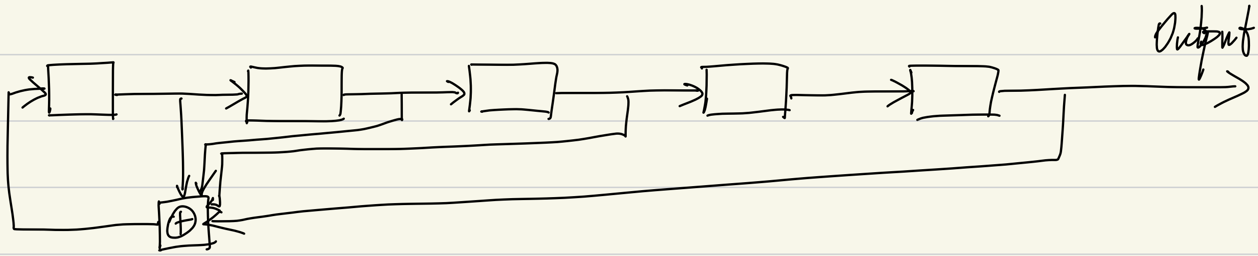
4. (a) A preferred pair of m-sequences $\mathbf{m}_1$ can be generated using primitive polynomial 101111 and initial shift-register content 10001, and $\mathbf{m}_2$ can be generated using primitive polynomial 111101 and initial shift-register content 00110. Determine and write down the first 8 chips of the Gold code generated from $\mathbf{m}_1$ and $\mathbf{m}_2$. (Note: No need to compute the entire code.)

Answer:



State	Output
10001	
11000	1
11100	0
01110	0
10111	0
11011	1
11101	1
11110	1
11111	0

$m_1 = 10001110$



State	Output
00110	
10011	0
01001	1
00100	1
10010	0
11001	0
11100	1
11110	0
11111	0

$m_2 = 01100100$

Goldcode = 11101010

(b) In a single-cell direct-sequence CDMA downlink, scrambled Walsh Hadamard (WH) codes are used as CDMA spreading codes. The scrambling code is the Gold code generated in part (a). There are 2 users in the cell. The cell base-station (BS) wants to transmit bit 1 to user A and assigns a WH code [0110 0110]. At the same time the BS wants to transmit bit 0 to user B and assigns another WH code [1111 0000]. Determine the CDMA signal vector to be transmitted by the BS. Express your answer using the signal mapping 0 → +1 and 1 → −1.

Answer: Scramble code: 1 1 1 0 1 0 1 0

WH_A = 0 1 1 0 0 1 1 0
Gold = 1 1 1 0 1 0 1 0
$Scrambled_A$ = 1 0 0 0 1 1 0 0
-1 +1 +1 +1 -1 -1 +1 +1
1 : +1 -1 -1 -1 +1 +1 -1 -1

WH_B = 1 1 1 1 0 0 0 0
Gold = 1 1 1 0 1 0 1 0
$Scrambled_B$ = 0 0 0 1 1 0 1 0
+1 +1 +1 -1 -1 +1 -1 +1
0 : +1 +1 +1 -1 -1 +1 -1 +1

+1 -1 -1 -1 +1 +1 -1 -1
+1 +1 +1 -1 -1 +1 -1 +1
+2 0 0 -2 0 +2 -2 0

The signal vector transmitted BS is [+2 0 0 -2 0 +2 -2 0]

(c) A noisy multipath signal vector **r** of the transmitted signal by the BS described in part (b) received by the two users is (where the left most digit is the earliest received):

$$\mathbf{r} = [\,+1\ \ +0.5\ \ +2\ \ -1\ \ -0.5\ \ -3\ \ -1\ \ -2\ \ -0.5\ \ -0.5\,]$$

The channel contains 2 paths separated by a 2-chip duration. Demodulate **r** using an Equal-Gain Combining (EGC) RAKE receiver for user A. Assume the signal mapping $0 \rightarrow +1$ and $1 \rightarrow -1$.

Answer:

$r = p[n] + p[n-2]$

$r = [+1\ \ +0.5\ \ +2\ \ -1\ \ -0.5\ \ -3\ \ -1\ \ -2\ \ -0.5\ \ -0.5]$

$p[n] = [+1\ \ +0.5\ \ +1\ \ -1.5\ \ -1.5\ \ -1.5\ \ +0.5\ \ -0.5\ \ -1\ \ 0\ \ 0\ \ 0]$

$p[n-2] = [0\ \ 0\ \ +1\ \ +0.5\ \ +1\ \ -1.5\ \ -1.5\ \ -1.5\ \ +0.5\ \ -0.5\ \ -1\ \ 0]$

Use EGC then after combination the signal.

$S_{EGC} = [+2\ \ +1\ \ +2\ \ -3\ \ -3\ \ -3\ \ +1\ \ -1]$

$A = [-1\ \ +1\ \ +1\ \ +1\ \ -1\ \ -1\ \ +1\ \ +1]$

$\Sigma S_{EGC} \cdot A = -2 + 2 + 2 - 3 + 3 + 3 + 1 - 1 = 5 > 0$

$\therefore$ A sent a bit of '0'

$$\mathbf{r} = [\,+1\ \ +0.5\ \ +2\ \ -1\ \ -0.5\ \ -3\ \ -1\ \ -2\ \ -0.5\ \ -0.5\,]$$

$-1\ \ +1\ \ +1\ \ +1\ \ -1\ \ -1\ \ +1\ \ +1$

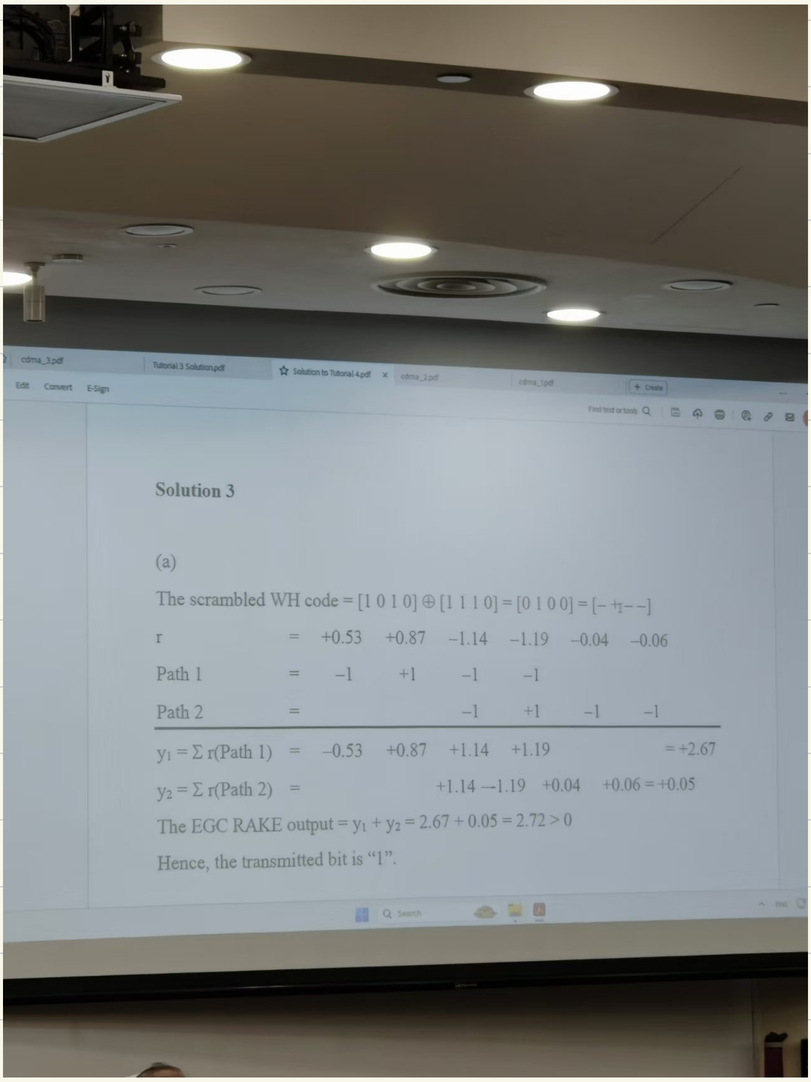
$\qquad\quad -1\ \ +1\ \ +1\ \ +1\ \ -1\ \ -1\ \ +1\ \ +1$

$P_1 = \cancel{1} + 0.5 + \cancel{2}\ \cancel{1} + 0.5 + \cancel{3} - \cancel{1} - \cancel{2} = 1$

$P_2 = -2 - 1 - 0.5 - \cancel{3} + \cancel{1} + \cancel{2} - 0.5 - 0.5 = -4.5$

$EGC = P_1 + P_2 = -3.5 < 0$

$\therefore$ transmitted is '1'



(d) Now consider a single-cell direct-sequence CDMA uplink system with frequency-selective fading channel with L equal-power slow Rayleigh paths. The CDMA system has a data bandwidth of 10 kHz and a spread bandwidth of 10 MHz. There are 76 equal-power users in the cell sharing the band. With only one signal being transmitted by a user in the cell (uplink transmission), the received E_b/N_0 at the cell BS receiver is 16 dB. Given that the bit-error rate (BER) performance of the Maximum Ratio Combining (MRC)-RAKE receiver with L fingers is

$$\text{BER} \approx \binom{2L-1}{L} \prod_{i=1}^{L} \left[\left(\frac{4E_b}{N_0 + I_0} \right)_i \right]^{-1}$$

where E_b, N_0 and I_0 are the average energy per bit, channel noise power spectral density and average interference power spectral density, respectively, find the BER of the base-station receiver if the base-station uses:

(i) An MRC-RAKE combiner with 3 RAKE fingers,

Answer: (i) $f_b = 10\text{kHz}$ $\quad f_c = 10\text{MHz}$ $\quad PG = \frac{f_c}{f_b} = \frac{10\times10^6}{10\times10^3} = 1000$

$M = 76 \quad L = 3$

$$\frac{S}{MAI} = \frac{E_b \cdot R_b}{I_0 \cdot R_c} = \frac{E_b}{I_0} \cdot \frac{T_c}{T_b} = \frac{E_b}{I_0} \cdot \frac{1}{PG} = \frac{P}{(M-1)P} = \frac{1}{75}$$

$$\frac{E_b}{I_0} = \frac{1000}{75}$$

$$\frac{E_b}{N_0} = 16\text{dB} = 40$$

$$\frac{N_0 + I_0}{E_b} = \frac{1}{40} + \frac{75}{1000} = \frac{25+75}{1000} = 0.1$$

$$\frac{N_0 + I_0}{4E_b} = 0.025$$

$$BER = \binom{5}{3} \cdot (0.025)^3 \approx 1.5625\times10^{-4}$$

(ii) A conventional receiver without RAKE combining.

$$BER = Q\left(\sqrt{\frac{E_b}{N_0 + I_0}}\right) = Q(\sqrt{10}) \approx Q(3.16) \approx 7.888\times10^{-4}$$