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2. (a) Consider a received LZ (Lempel–Ziv) message as follows:

00001 00000 00010 00111 01000 00101 00011 00110
01011 01110 01011 01110 10010 10111 10001 11011

01010 01111

Given the first two entry of the dictionary are:

Location 1	0001	Content: 1	Codeword: 00001
Location 2	0010	Content: 0	Codeword: 00000

Please decode the received LZ message as shown above.

Answer: LZ Decoding

1: Build the dictionary

Location	Binary	Content	Codeword
1	0001	1	00001
2	0010	0	00000
3	0011	10	00010
4	0100	101	00111
5	0101	1010	01000
6	0110	01	00101
7	0111	11	00011
8	1000	100	00110
9	1001	10101	01011
10	1010	110	01110
11	1011	10100	01010
12	1100	111	01111
13	1101	101010	10010
14	1110	101001	10111
15	1111	1001	10001
16		1010101	11011

字典中也暗含原序列的顺序信息

解码后得到:

1 0 10 101 1010 01 11 100 10101 110 10100 1
11 101010 101001 1001 1010101

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2. (b) A source is generating two messages: S_1 with probability p, and S_2 with probability $1\text{-}p$.

(i) What is the value of p that requires the highest channel capacity for the source to transmit the information? What is the required channel capacity in this case, assuming that S_1 and S_2 are both single bit, and the source is generating one message per second?

Answer: 要进行无差错传输，信道容量不能大于信源熵

$H(p) = -p\log_2 p - (1-p)\log_2(1-p)$

$p=0.5$ 时，熵最大

证明: $\frac{dH}{dp} = -\log_2 p - \frac{\ln 2}{p}\cdot p + \log_2(1-p) + \frac{\ln 2}{1-p}(1-p)$

$= \log_2 \frac{1-p}{p}$

令 $\frac{dH}{dp} = 0$

$\frac{1-p}{p} = 1 \qquad p = 0.5$

$H(0.5) = +\frac{1}{2} + \frac{1}{2} = 1$ bit

$C = 1$ symbol/s $\times H = 1$ bps

(ii) What is the value of p that requires the lowest channel capacity for the source to transmit the information? What is the required channel capacity in this case, assuming that S_1 and S_2 are both single bit, and the source is generating one message per second?

(8 Marks)

Answer: $p=0$ 或 $p=1$ 时 熵最小

$H(0) = H(1) = 0$ bit

$C = 1$ symbol/s $\times H = 0$ bps

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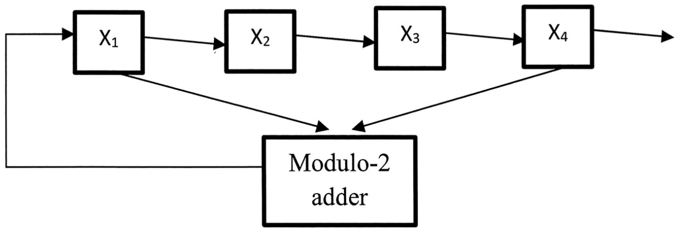
(c) Consider the generator in the circuit in Figure 2, assuming initial state is 1000, what is the 15-bits output sequence? Using the 15-bits output sequence, show the values of the autocorrelation with zero shift, and the values of the autocorrelation with two cyclic shift to the right.



(7 Marks)

Figure 2

Answer:

	State	Output
Initial	1 0 0 0	
1	1 1 0 0	0
2	1 1 1 0	0
3	1 1 1 1	0
4	0 1 1 1	1
5	1 0 1 1	1
6	0 1 0 1	1
7	1 0 1 0	1
8	1 1 0 1	0
9	0 1 1 0	1
10	0 0 1 1	0
11	1 0 0 1	1
12	0 1 0 0	1
13	0 0 1 0	0
14	0 0 0 1	0
15	1 0 0 0	1

自相关函数定义式

$$R(k) = \sum_{n=0}^{N-1} S[n] \cdot S[(n+k) \bmod N]$$

使用 -1 表示 0，1 表示 1

$$R(0) = \sum_{n=0}^{14} S^2[n] = 15$$

$$R(2) = \sum_{n=0}^{14} S[n]\, S[(n+2) \bmod 15]$$

0 0 0 1 1 1 1 0 1 0 1 1 0 0 1

0 1 0 0 0 1 1 1 1 0 1 0 1 1 0

1 -1 1 -1 -1 1 1 -1 1 1 1 -1 -1 -1 -1

$R(2) = -1$

Output sequence:

$S[n] = [0\ 0\ 0\ 1\ 1\ 1\ 1\ 0\ 1\ 0\ 1\ 1\ 0\ 0\ 1]$