EE6111 2024-2025 Semester 2 第2题

2. (a) Consider a message as follows:

abbabaaabcaaabcdefaaaefcddefaacefdbbabaa

Determine the entropy of the above message, and compress the above message using Huffman code.

Answer: total : 40

a : 16
b : 8
c : 4
d : 4
e : 4
f : 4

$H = -\sum p(x) \log_2 p(x)$
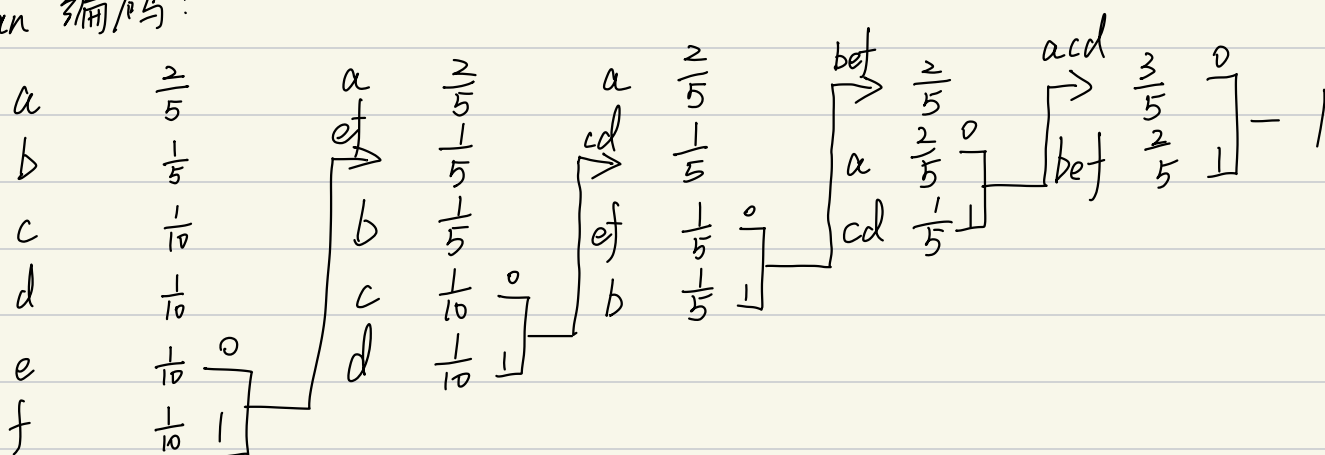
$p(a) = \frac{16}{40} = \frac{2}{5}$

$p(b) = \frac{8}{40} = \frac{1}{5}$

$p(c) = p(d) = p(e) = p(f) = \frac{4}{40} = \frac{1}{10}$

$H = -\left(\frac{2}{5} \log_2 \frac{2}{5} + \frac{1}{5} \log_2 \frac{1}{5} + \frac{4}{10} \cdot \log_2 \frac{1}{10}\right) \approx 2.32 \text{ bits}$

Huffman 编码:

a : 00
b : 11
c : 010
d : 011
e : 100
f : 101

步骤:
1. 将符号按概率从大到小排列
2. 将列表中最后两符号分别分配0和1,并将其概率融合
3. 重复,直到所有概率融合为1

若有并列融合概率 排在下方

(b)　Calculate the efficiency of the constructed codeword in (a), and propose a way to improve the efficiency.

$$\bar{L} = \sum_i p_i l_i = \frac{2}{5} \times 2 + \frac{1}{5} \times 2 + 4 \times 3 \times \frac{1}{10} = \frac{12}{5} = 2.4 \text{ bits}$$

Efficiency: $\eta = \dfrac{H}{\bar{L}} = \dfrac{2.32}{2.4} \approx 96.67\%$

平均码长计算：符号码长的数学期望

编码效率：熵除以平均码长

(c)   Compress the following message with Lempel-Ziv code:

0 1 0 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 0 0 0 0 0 1 1 1 1 0 1 0 1 0 0 0 1 0 0 0 0 1 0 0 1 1

Assuming the first two entry of the dictionary are:
Location 1   0001   Content: 0   Codeword: 00000
Location 2   0010   Content: 1   Codeword: 00001

Index	Location	Content	Codeword
1	0001	0	000000
2	0010	1	000001
3	0011	01	00011
4	0100	00	00010
5	0101	01	00111
6	0110	11	00101
7	0111	000	01000
8	1000	001	01001
9	1001	111	01101
10	1010	0000	01110
11	1011	0111	01011
12	1100	10	00100
13	1101	101	11001
14	1110	0001	01111
15	1111	00001	10101
16		0011	10001

(d)   Orthogonal frequency division multiplexing (OFDM) system is an effective multicarrier modulation with overlapping subchannels. It has the following two key elements:

- Guard Interval
- Cyclic Prefix

Please state the respective functions of the two elements, and how it helps OFDM to achieve multicarrier modulation with overlapping subchannels.

Guard Interval 用于在 **OFDM** 符号之间插入时间冗余，避免多径引起的符号间干扰（ISI）。

Cyclic Prefix 是一种特殊的 **Guard Interval**，它通过复制符号尾部到开头，使信道卷积变成循环卷积，从而保持子载波正交性，并允许子载波频谱重叠。

因此，这两者共同保证 **OFDM** 能够在多径信道下实现高效的多载波传输，同时保持频域正交，使得各子载波可以重叠而互不干扰。