

1. (a) Consider the convolutional encoder as shown in Figure 1.

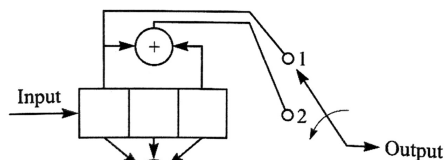
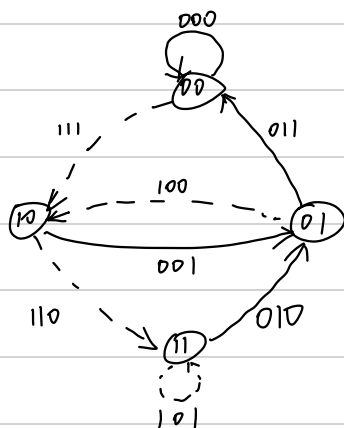


Figure 1

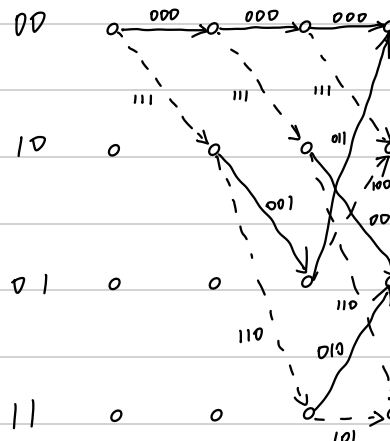
Draw the trellis diagram of the convolutional code.

状态转移图:

output  $\rightarrow$  input 1  
output  $\rightarrow$  input 0



Trellis Diagram:



解题思路:

1. 先根据编码器的结构, 画出状态转移图
2. 根据状态转移图, 画出 Trellis Diagram.

Trellis Diagram 可理解为状态转移图引入了时间序

Trellis Diagram 的初始状态默认为 00

(b) Based on the encoder in (a). What is the encoded message if you receive the followings bits:

(first to receive) 001 101 110 110 011 (last to receive)?

Assuming the registers of the encoder are of zero value at the initial stage.

received code:

001 101 110 110 011

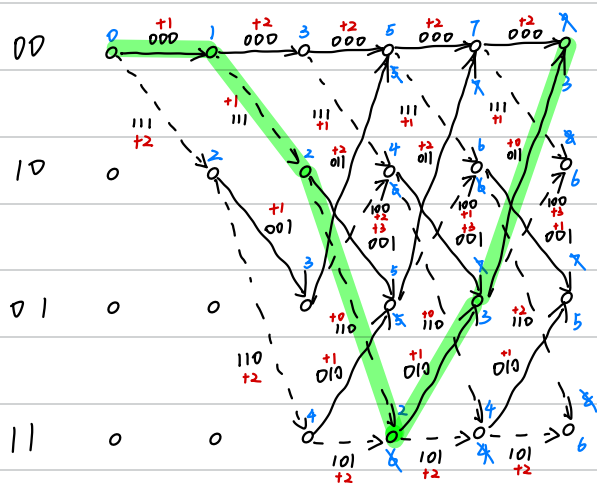
解题思路:

1. 画出对应步长的 Trellis 图, 并将接收码按时间序排在 Trellis 图上.

2. 将接收码与每一步的输出按位比较, 计算差异值

3. 将上一个节点的差异值与这个输出的差异值相加, 得到下一个点的差异值.

4. 从值最小的最后状态开始回溯, 找到最优路径, 完成解码.



0 1 1 0 0

解码结果

(c) Designing a  $k=2, n=8$  code. Just need to show the final codeword. Given the following properties:

- All-zero is one of the codeword.
- Systematic.
- Minimum distance  $d_{\min} = 5$ .
- The sum of any two codewords must yield a valid codeword in the space.

$$G = \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix}$$

$$00 : 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0$$

$$01 : 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1$$

$$10 : 1 \ 0 \ 1 \ 1 \ 1 \ 1 \ 0 \ 0$$

$$11 : 1 \ 1 \ 0 \ 1 \ 1 \ 0 \ 1 \ 1$$

解题要点: 1. 线性分组码性质:

任意、合法码字 按位模2加 的结果也是合法码字

2. 系统码生成矩阵特点:

$$G = [I_k : P_{n-k}] \quad I_k \text{ 是一个 } k \times k \text{ 单位矩阵}$$

编码过程:  $C = UG$

码字 = 被编码序列 左乘生成矩阵

举例:

$$01 \rightarrow 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1$$

$$(01) \begin{pmatrix} 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} = 0 \ 1 \ 1 \ 0 \ 0 \ 1 \ 1 \ 1$$

3. 最小码距  $d_{\min}$  等于码中最小非零码重