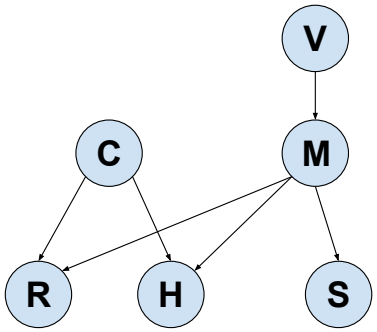
Homework 7 – Machine Learning (CS453X, Whitehill, Spring 2019)

You may complete this homework assignment either individually or in teams up to 2 people.

1. **Inference in probabilistic graphical models [15 points]**: Consider the probabilistic graphical model for a (somewhat facetious/futuristic) medical diagnostic domain[1] shown below. This graph could be used to diagnose whether a patient is suffering from a mere common cold (C) and/or the more dangerous Martian Death Flu (M), based on the patient's symptoms – whether or not the patient has a runny nose (R), whether or not the patient has a headache (H), and whether or not the patient occasionally spontaneously bursts into flames (S) – as well as relevant background information, namely whether or not he or she has previously visited Mars (V).



The probability tables for this model are shown below:

$P(V)$	
$V=0$	$V=1$
0.9999	0.0001

$P(C)$	
$C=0$	$C=1$
0.95	0.05

$P(M \mid V)$		
V	$M=0$	$M=1$
0	1.0	0.0
1	0.001	0.999

$P(R \mid C, M)$			
C	M	$R=0$	$R=1$
0	0	0.95	0.05
0	1	0.50	0.50
1	0	0.10	0.90
1	1	0.02	0.98

$P(H \mid C, M)$			
C	M	$H=0$	$H=1$
0	0	0.93	0.07
0	1	0.02	0.98
1	0	0.40	0.60
1	1	0.01	0.99

$P(S \mid M)$		
M	$S=0$	$S=1$
0	1.0	0.0
1	0.2	0.8

The graphical model itself codes various **conditional independence** relationships. We will unfortunately not discuss how these can be derived until class on Monday. To help you get started, the conditional independence assumptions that you need to finish the homework are shown below:

$$P(h \mid r, c, m) = P(h \mid c, m)$$
$$P(m \mid c) = P(m)$$
$$P(c \mid m) = P(c)$$

Using the laws of probability (e.g., Bayes rule, conditional probability, total probability) discussed in class, please *derive* (this is crucial; do not simply give the numeric answer) and compute the value of the following probabilities. (Note that you do not need to use all of the probability tables shown above to answer the questions. Which ones you need is determined by your derivation.)

[1] Adapted from `http://www.cs.cmu.edu/afs/cs.cmu.edu/project/learn-43/lib/photoz/.g/web/glossary/bayesnet.html`.

(a) $P(C = 1 \mid R = 1, H = 1, M = 0)$, i.e., if the patient has a runny nose and a headache but does not have Martian Death Flu, then what's the probability that they have a common cold?

(b) $P(M = 1 \mid H = 1, C = 0)$, i.e., if the patient has a headache but not a common cold, then what's the probability that they have Martian Death Flu?

There is no code you need to submit. Just submit your derivations, calculations, and final answers in a PDF (`homework7_WPIUSERNAME1.pdf`
or `homework7_WPIUSERNAME1_WPIUSERNAME2.pdf` for teams).