CS453X Homework 7

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1 Inference in probabilistic graphical models

(a) $P(C=1|R=1,H=1,M=0)$.

$$= \frac{P(H=1|C=1,R=1,M=0)P(C=1|R=1,M=0)}{P(H=1|R=1,M=0)}$$
$$= \frac{P(H=1|C=1,M=0)P(C=1|R=1,M=0)}{P(H=1|R=1,M=0)}$$

Let $P(C=1|R=1,M=0)$ be $(Eq.1)$

$$\begin{aligned}(Eq.1) &= \frac{P(R=1|C=1,M=0)P(C=1|M=0)}{P(R=1|M=0)} \\ &= \frac{P(R=1|C=1,M=0)P(C=1)}{P(R=1|M=0,C=1)\times P(C=1)+P(R=1|M=0,C=0)\times P(C=0)} \\ &= \frac{0.90\times 0.05}{0.90\times 0.05+0.05\times 0.95} \\ &= 0.4865\end{aligned}$$

Let $P(H=1|R=1,M=0)$ be $(Eq.2)$

$$\begin{aligned}(Eq.2) &= \sum_{c=0}^{1} P(H=1,C=c|R=1,M=0) \\ &= \sum_{c=0}^{1} P(H=1|C=c,R=1,M=0)P(C=c|R=1,M=0) \\ &= \sum_{c=0}^{1} P(H=1|C=c,M=0)P(C=c|R=1,M=0) \\ &= 0.07\times(1-(Eq.1))+0.60\times(Eq.1) \\ &= 0.0359+0.2919 \\ &= 0.3278\end{aligned}$$

Then substitute the number back in to the original equation:

$$P(C=1|R=1,H=1,M=0) = \frac{0.60\times 0.4865}{0.3278} = 0.8904$$

(b) $P(M=1|H=1,C=0)$

$$= \frac{P(H=1|M=1,C=0)P(M=1|C=0)}{P(H=1|C=0)}$$
$$= \frac{P(H=1|M=1,C=0)P(M=1)}{P(H=1|C=0,M=0)\times P(M=0)+P(H=1|C=0,M=1)\times P(M=1)}$$

Where $P(M=m)$ can be deduced as:

$P(M=0) = P(M=0|V=0)\times P(V=0)+P(M=0|V=1)\times P(V=1) = 0.9999\times 1+0.0001\times 0.001$
$P(M=1) = P(M=1|V=0)\times P(V=0)+P(M=1|V=1)\times P(V=1) = 0+0.0001\times 0.999$

Then substitute back to the original equation, we have:

$$\begin{aligned}P(M=1|H=1,C=0) &= \frac{0.98\times(0.0001\times 0.999)}{0.07\times(0.9999\times 1+0.0001\times 0.001)+0.98\times(0.0001\times 0.999)} \\ &= 0.0014\end{aligned}$$