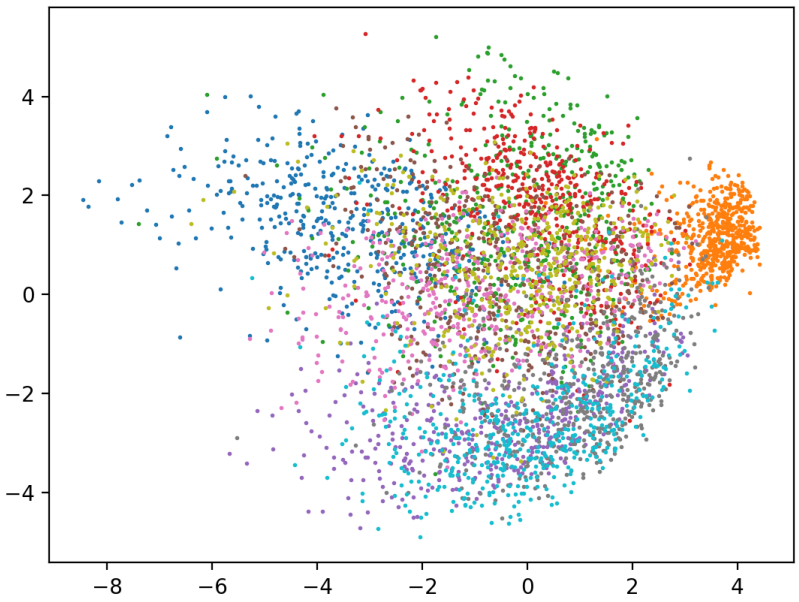
Homework 5 – Machine Learning (CS453X, Whitehill, Spring 2019)

You may complete this homework assignment either individually or in teams up to 2 people. You should use the same MNIST dataset as you did for Homework #3.

1. **Principal Component Analysis (PCA) [30 points]**: In this problem you will implement Principal Component Analysis (PCA). Note that you are required to implement PCA from scratch; you may not use any off-the-shelf software (e.g., `sklearn`). Apply the PCA algorithm you implemented to the 5,000 examples in the `small_mnist_test_*.py` files. In particular, for each image $\mathbf{x}$ in the MNIST test set, project it onto the *first* and *second* principal components, i.e., the two eigenvectors of $\tilde{\mathbf{X}}\tilde{\mathbf{X}}^\top$ with the highest and second-highest associated eigenvalues. Call these two projections p_1 and p_2; then plot (p_1, p_2) of each MNIST image on a 2-D plane, where the *color* of each point is determined by the class label of that image $\mathbf{x}$. You should obtain an image similar to the following. Note, however, that the colors in your image could differ (since they are arbitrary); also, the *orientation* of the image could also be different (since the eigenvectors of a given matrix are not unique).



You should attach the figure that you rendered as either a PNG or a PDF.

Submit your Python code in a file called `homework5_WPIUSERNAME1.py` (or `homework5_WPIUSERNAME1_WPIUSERNAME2.py` for teams). Submit your figure (either a PDF or PNG) using an analogous naming convention.