Date: 2020-09-11

Announcements:

Test on Wednesday, 8:50
Assignment: due Monday 4:00 PM

0.1 (1.7) Definitions of Matrices

Definition 1 (Diagonal Matrix)**.** *A square matrix where all the entries off of the main diagonal are zero.*

$$\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0\\0 & 3 & 0\\0 & 0 & 5\end{bmatrix}$$

$$\begin{bmatrix}1 & 0 & 0 & 0\\0 & 0 & 0 & 0\\0 & 0 & 5 & 0\\0 & 0 & 0 & 6\end{bmatrix}$$

Properties of diagonal matrices

- Multiplication is easy. Multiplying a matrix by a diagonal from the left gives the entry in the diagonal times the row:

$$\begin{bmatrix}1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}4 & 0 & 0\\0 & 5 & 0\\0 & 0 & 6\end{bmatrix} = \begin{bmatrix}4 & 0 & 0\\0 & 10 & 0\\0 & 0 & 18\end{bmatrix}$$

$$\begin{bmatrix}-1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix}\begin{bmatrix}1 & 2\\3 & 4\\5 & 6\end{bmatrix} = \begin{bmatrix}-1 & -2\\6 & 8\\15 & 18\end{bmatrix}$$

- Multiplying from the right gives the diagonal entry times the column.

$$\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\end{bmatrix}\begin{bmatrix}-1 & 0 & 0\\0 & 2 & 0\\0 & 0 & 3\end{bmatrix} = \begin{bmatrix}-1 & 4 & 9\\-4 & 10 & 18\end{bmatrix}$$

- Square a diagonal squares the entries.

$$D = \begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}$$

$$D^2 = \begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix}\begin{bmatrix}2 & 0 & 0\\0 & 3 & 0\\0 & 0 & 4\end{bmatrix} = \begin{bmatrix}4 & 0 & 0\\0 & 9 & 0\\0 & 0 & 16\end{bmatrix}$$

- Inverting the diagonal gives the reciprical of the entries

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix} D^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 & | & 1 & 0 & 0 \\ 0 & 3 & 0 & | & 0 & 1 & 0 \\ 0 & 0 & 4 & | & 0 & 0 & 1 \end{bmatrix} =$$

$$\begin{bmatrix} 1 & 0 & 0 & | & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 0 & | & 0 & \frac{1}{3} & 0 \\ 0 & 0 & 1 & | & 0 & 0 & \frac{1}{4} \end{bmatrix}$$

- If it is invertible

$$D = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

  D^{-1} (D is not invertible.)

Definition 2 (Triangular Matrices)**. Upper Triangular** *All non-zero entries are on the main diagonal or above.*

Lower Triangular *All non-zero entries are on the main diagonal or below.*

Example 1 (Upper triangular matrix)**.**

$$\begin{bmatrix} 1 & 3 & 4 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

Example 2 (Lower triangular matrix)**.**

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 3 & 0 \\ 4 & 5 & 6 \end{bmatrix}$$

Theorem 1 (1.7.1)**.** .

1. *The **transpose** of a **lower** triangular is **upper** triangular and the transpose of an upper triangular matrix is lower triangular.*

2. *The **product** of **two lower** triangular matrices is **lower** triangular and the product of two upper triangular matrices is upper triangular.*

3. *A triangular matrix is **invertible if** and only if it's **diagonal** entries are all **non-zero**.*

4. *The **inverse** of an invertible **lower** triangular matrix **is lower** triangular and the inverse of an invertible **upper** triangular matrix **is upper** triangular.*

0.1.1 Symmetric Matrix

Definition 3 (Symmetric Matrix)**.** *A square matrix where* $A = A^T$*.*

$$(A)_{ij} = (A)_{ji}$$

Example 3 (Symmetric matrix)**.**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

$$A^T = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 5 & 4 \\ 3 & 4 & 6 \end{bmatrix}$$

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Theorem 2. *If* A *and* B *are symmetric matrices with the same size, and if* k *is any scalar, then*

1. A^T *is symmetric.*
2. $A + B$ *and* $A - B$ *are symmetric.*
3. kA *is symmetric.*

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Theorem 3. *The product of tow symmetric matrices is symmetric if and only if the matrices commute.* •