Chapter 1

(24) Capacitors and Dielectrics

Date: 2020-09-14

Announcements:

Exam: Can start on Tuesday, (4 hour time limit) Due Sunday 11:59 PM (Chapter 21-23) 23 questions - 14 conceptual 9 problem solving E field for point charges and charge distributions, relationship between force charge and electric field, use gausses law for e field for symmetric charge distribution and find charge distribution, find potential due to point charges, relationship between electric potential and electric potential energy, relationship between potential and electric field

Lab: due Sep 21

1.1 Capacitors and Capacitance

Definition 1 (Capacitor)**.** .

- *Two conductors separated y an insulator (or vacume)*
- *When **charged**, two conductors have equal magnitude and opposite sign*
- *Zero net charge*

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$$\boxed{C = \frac{Q}{V_{ab}}} \tag{1.1}$$

Capacitance of a capacitor

Q Magnitude of charge on each conductor

V_{ab} Potential difference between $+Q$ and $-Q$

$$\boxed{F = \frac{C}{V}} \tag{1.2}$$

Farad - Unit of Capacitance

Definition 2 (Parallel plate capacitors)**.** .

- *Two parallel conducting plates separated by a small distance compared to dimensions.*
- *Uniform field and charge distribution over opposing surfaces.*

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$$\boxed{C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}} \tag{1.3}$$

Capacitance of Parallel Plates

Q Magnitude of charge on each plate

A Area of each plate

d Distance between plates

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The Electric field of parallel plates:

$$\sigma = \text{ charge / area}$$
$$E = \frac{\sigma}{\epsilon_0}$$

1.2 Capacitors in Series and Parallel

1.2.1 Capacitors in Series

- The capacitors have the same charge Q.
- Their potential differences add:

$$V_{ac} + V_{cb} = V_{ab}$$

Finding Equivalent Capacitance:

The potential drop across both capacitors must be the sum of the potential drop across each. The charge on C_1 and C_2 must be the same for the individual capacitors.

$$C = \frac{Q}{V}$$
$$Q_1 = Q_2$$
$$V_{ab} = V_1 + V_2$$
$$V = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$$
$$V = Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right)$$
$$\frac{V}{Q} = \frac{1}{C_1} + \frac{1}{C_2}$$
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \tag{1.4}$$

Series Equivalent Capacitance

1.2.2 Capacitors in Parallel

- Have same potential drop V.
- Charge on each depends on its capacitance:

$$Q_1 = C_1 V, Q_2 = C_2 V$$

Finding Equivalent Capacitance:
The potential drop across both capacitors must be the same. The charge on C_1 and C_2 must sum to the total charge.

$$\begin{aligned} C &= \frac{Q}{V} \\ Q &= Q_1 + Q_2 \\ V_1 &= V_2 \\ Q &= C_1 V_1 + C_2 V_2 \\ C_{eq} V &= C_1 V + C_2 V \\ C_{eq} &= C_1 + C_2 \end{aligned}$$

$$\boxed{C_{eq} = C_1 + C_2} \tag{1.5}$$

Parallel Equivalent Capacitance

1.3 Energy Storage in Capacitors and Electric-Field Energy

1.3.1 Energy Stored in a Capacitor

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V} \tag{1.6}$$

Capacitor Potential Energy

U Potential energy stored in a capacitor

Q Magnitude of charge on each plate

C Capacitance

V Potential difference between plates

Derivation of Capacitor Potential Energy

Energy stored in capacitor? $\rightarrow$ Calculate work done to charge capacitor.
$V = \frac{Q}{C}$ (Final charge is Q and final potential is V after charging)
$v = \frac{q}{C}$ (v and q at some intermediate time)

$$\begin{aligned}
W &= \Delta u = Q\Delta V \\
dW &= vdq \\
\int dW &= \int_0^Q \frac{q}{C}dq \\
W &= \frac{q^2}{2C}\bigg|_0^Q \\
W &= \frac{Q^2}{2C}
\end{aligned}$$

1.3.2 Electric-Field Energy

energy density $= \frac{\text{energy}}{\text{volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}}{Ad} =$

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2} \tag{1.7}$$

Electrical Density in Vacuum

u Energy density for a vacuum

E Magnitude of electric field

1.4 Dielectrics

Most capacitors have noncuducting material between plates, such as Mylar.

Increasing Capacitance

- Connect electrometer across a charged capacitor, with magnitude Q on each and potential difference V_0
- An uncharged sheet of dielectric between the plates, the potential difference decreases to smaller V value.
- Since Q is unchanged, capacitance $C = \frac{Q}{V}$ **increased**.
- The field decreases due to **polarization** within the dielectric, induced surface charges.

$$E = \frac{E_0}{K}$$

E Decreased E-field

K dielectric constant, unitless

$$\begin{aligned}
V &= Ed \\
\implies V &= \frac{V_0}{K} \\
\implies C &= KC_0
\end{aligned}$$

$$\boxed{\epsilon = K\epsilon_0} \tag{1.8}$$

Permitivity of a Dielectric

K dielectric constant, unitless

$$\boxed{C = KC_0 = K\epsilon_0 \frac{A}{d = \epsilon \frac{A}{d}}} \tag{1.9}$$

Capacitance of parallel-plate capacotor, dielectric between plates

$$\boxed{u = \frac{1}{2} K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2} \tag{1.10}$$

Electric energy density in a dielectric