Date: 2020-09-06
Announcements:

0.0.1 Integrals

Definition 1 (Integral)**.**

$$\int_a^b \overline{r}(t)dt = \lim_{n\to\infty}\sum_{i=1}^{n}\overline{r}(t_i^*)\Delta t$$

$$= \left\langle \int_a^b f(t)dt, \int_a^b g(t)dt, \int_a^b h(t)dt \right\rangle$$

Example 1 (Integral)**.**

$$\int_1^4 \left\langle 4t^{\frac{3}{2}}, t^2, \cos t \right\rangle dt$$

$$= \left\langle \int_1^4 4t^{\frac{3}{2}}dt, \int_1^4 t^2 dt, \int_1^4 \cos t dt \right\rangle$$

$$= \frac{8}{5}t^{\frac{5}{2}}\Big|_1^4, \frac{1}{3}t^3\Big|_1^4, \sin t\Big|_1^4$$

$$= \left\langle \frac{256}{5} - \frac{8}{5}, \frac{64}{3} - \frac{1}{3}, \sin 4 - \sin 1 \right\rangle$$

$$= \boxed{\left\langle \frac{248}{5}, 21, \sin 4 - \sin 1 \right\rangle}$$

Example 2 (Integral)**.**

$$\int \left\langle te^{t^2}, te^t, \cos(5t) \right\rangle dt$$

$$= \left\langle \int te^{t^2}dt, \int te^t dt, \int \cos(5t)dt \right\rangle$$

$$= \left\langle \frac{1}{2}e^{t^2} + C_1, te^t - \int e^t dt, \frac{1}{5}\sin(5t) + C_3 \right\rangle$$

$$= \left\langle \frac{1}{2}e^{t^2} + C_1, te^t - -e^t + C_2, \frac{1}{5}\sin(5t) + C_3 \right\rangle$$

Sometimes, you want to pull out the constants:

$$= \left\langle \frac{1}{2}e^{t^2}, te^t - e^t, \frac{1}{5}\sin(5t) \right\rangle + \langle C_1, C_2, C_3\rangle$$

$$= \left\langle \frac{1}{2}e^{t^2}, te^t - e^t, \frac{1}{5}\sin(5t) \right\rangle + \overline{C}$$

Example 3 (Integral)**.**

$$r'(t) = \left\langle t^2, e^{3t}, \sqrt{t} \right\rangle$$

$$\overline{r}(0) = \langle 4, 1, 5 \rangle$$

Find $\overline{r}(t)$

$$\begin{aligned}
\overline{r}(t) &= \int \left\langle t^2, e^{3t}, \sqrt{t} \right\rangle dt \\
&= \left\langle \int t^2 dt, \int e^{3t} dt, \int \sqrt{t} dt \right\rangle \\
&= \left\langle \frac{1}{3}t^3 + C_1, \frac{1}{3}e^{3t} + C_2, \frac{2}{3}t^{\frac{3}{2}} + C_3 \right\rangle
\end{aligned}$$

Use initial condition:

$$\begin{aligned}
&= \left\langle C_1, \frac{1}{3} + C_2, C_3 \right\rangle = \langle 4, 1, 5 \rangle \implies \\
C_1 &= 4 \\
C_2 &= \frac{2}{3} \\
C_3 &= 5 \implies \\
\overline{r}(t) &= \boxed{\left\langle \frac{1}{3}t^3 + 4, \frac{1}{3e^{3t} + \frac{2}{3}, \frac{2}{3}t^{\frac{3}{2}} + 5} \right\rangle}
\end{aligned}$$