Chapter 1

Cylinders and Quatric Surfaces

DATE: 2020-09-03

ANNOUNCEMENTS:

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Definition 1 (Cylinder)**.** *A cylinder is a surface that consists of al lines (called rulings) that are parallel to a given line and pass through a given plane curve.*

Definition 2 (plane curve)**.** *A plane curve is any graph in a 2-d space.*

1.1 Quadric Surfaces

Definition 3 (Quadric surface)**.** *The most general form is* $Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fxz + Gx + Hy + Iz + J = 0$ *where A-J are constants.*

Using rotation and translation will be brought into one of the two standard forms:

Definition 4 (Standard quadric forms)**.**

$$Ax^2 + By^2 + Cz^2 + J = 0$$

or

$$Ax^2 + By^2 + Iz = 0$$

Example 1 (What we do with this)**.** $z = \frac{x^2}{9} + \frac{y^2}{4}$

Solution 1. *You should know this is an Elliptic Paraboloid and what that looks like.*

What if you dont recognize the quadric surface form?

Looking at the equation, figure out **TRACES**, either **horizontal** (z = constant) or **vertical**

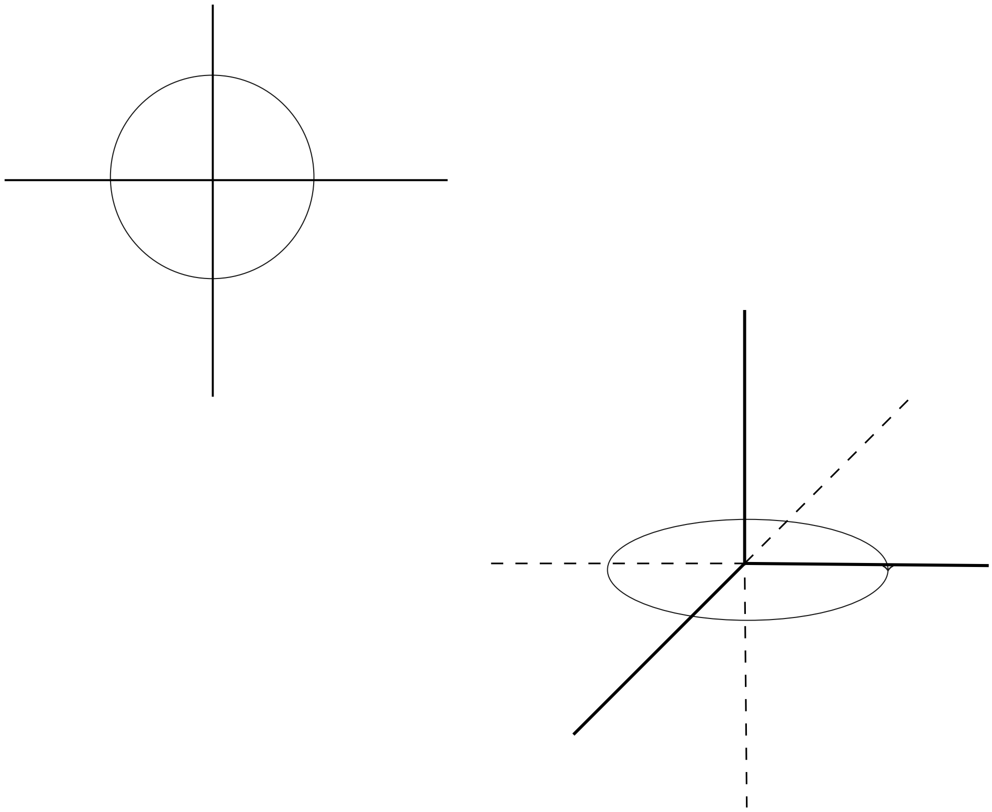

Figure 1.1: How to draw a circle in Calc 3

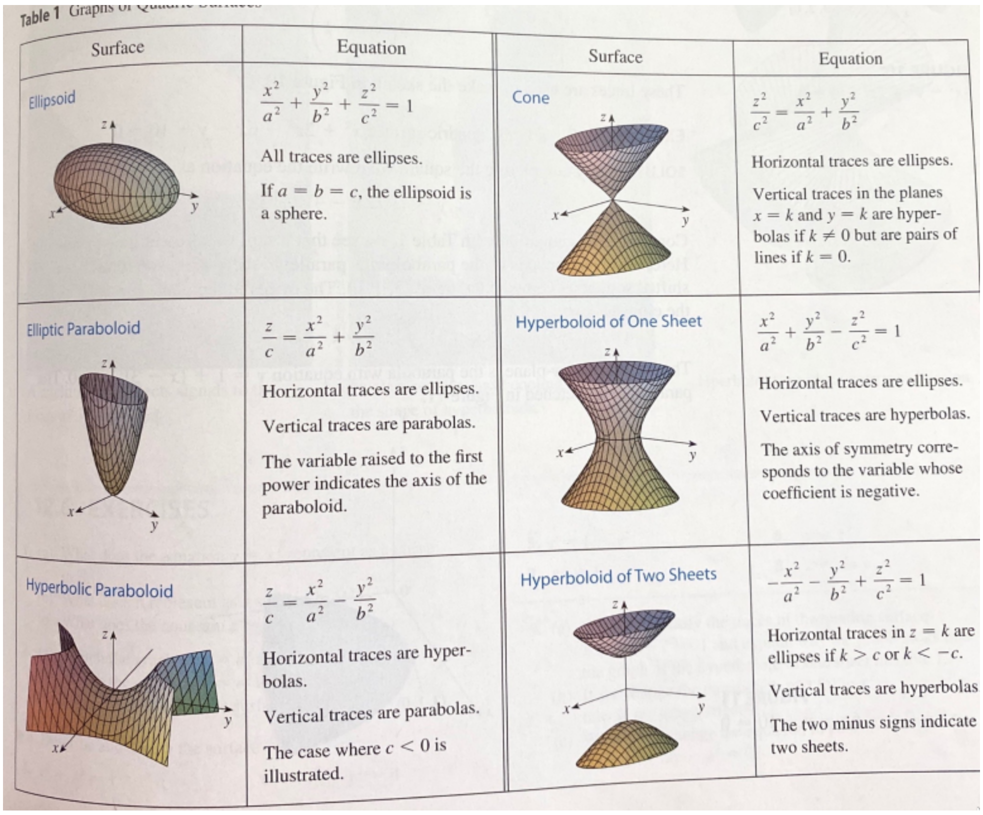

Table 1 Graphs of Quadric Surfaces

Surface	Equation	Surface	Equation
Ellipsoid	$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ All traces are ellipses. If $a = b = c$, the ellipsoid is a sphere.	Cone	$\frac{z^2}{c^2} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces in the planes $x = k$ and $y = k$ are hyperbolas if $k \neq 0$ but are pairs of lines if $k = 0$.
Elliptic Paraboloid	$\frac{z}{c} = \frac{x^2}{a^2} + \frac{y^2}{b^2}$ Horizontal traces are ellipses. Vertical traces are parabolas. The variable raised to the first power indicates the axis of the paraboloid.	Hyperboloid of One Sheet	$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 1$ Horizontal traces are ellipses. Vertical traces are hyperbolas. The axis of symmetry corresponds to the variable whose coefficient is negative.
Hyperbolic Paraboloid	$\frac{z}{c} = \frac{x^2}{a^2} - \frac{y^2}{b^2}$ Horizontal traces are hyperbolas. Vertical traces are parabolas. The case where $c < 0$ is illustrated.	Hyperboloid of Two Sheets	$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ Horizontal traces in $z = k$ are ellipses if $k > c$ or $k < -c$. Vertical traces are hyperbolas. The two minus signs indicate two sheets.

Figure 1.2: Graphs of Quadric Surfaces