Date: 2020-09-11

Announcements:

0.1 13.3 Arc Length and Curvature

0.1.1 Review

dx form

$$s = \int_a^b \sqrt{1 + (\frac{dy}{dx})^2} dx$$

dy form

$$s = \int_c^d \sqrt{1 + (\frac{dx}{dy})^2} dy$$

dt form

$$s = \int_{t_0}^{t_1} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2} dt$$

0.1.2 Calc 3 Version

Definition 1 (Arclength of a Space Curve)**.** .

Larange form

$$s = \int_{t_0}^{t_1} \sqrt{(f'(t))^2 + (g'(t))^2 + (h'(t))^2} dt$$

Leibnitz form

$$s = \int_{t_0}^{t_1} \sqrt{(\frac{dx}{dt})^2 + (\frac{dy}{dt})^2 + (\frac{dz}{dt})^2} dt$$

Vector form

$$\begin{aligned}
\overline{r}(t) &= \langle f(t), g(t), h(t) \rangle \implies \\
\overline{r}'(t) &= \langle f'(t), g'(t), h'(t) \rangle \implies \\
|\overline{r}'(t)| &= \sqrt{f'(t)^2 + g'(t)^2 + h'(t)^2} \\
s &= \int_{t_0}^{t_1} |\overline{r}'| \, dt
\end{aligned}$$

Example 1 (Find arclength$0 \leq t \leq 1$).

$$\begin{aligned}
\overline{r}'(t) &= \left\langle 2, 2t, t^2 \right\rangle \\
|\overline{r}'(t)| &= \sqrt{4 + 4t^2 + t^4} \\
&= \sqrt{(t^2+2)^2} \\
&= t^2 + 2 \\
s &= \int_0^1 t^2 + 2dt \\
&= \frac{1}{3}t^3 + 2t \Big|_0^1 \\
&= \boxed{\frac{7}{3}}
\end{aligned}$$

Definition 2 (Unit Vector).

$$\boxed{\overline{T}(t) = \frac{1}{|\overline{r}'(t)|}\overline{r}'(t)} \tag{1}$$

Unit (Tangent) Vector

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Definition 3 (Principle Unit Normal Vector).

$$\boxed{\overline{N}(t) = \frac{1}{\left|\overline{T}'(t)\right|}\overline{T}'(t)} \tag{2}$$

Normal Vector

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Definition 4 (Binormal Vector).

$$\boxed{\overline{B}(t) = \overline{T}(t) \times \overline{N}(t)} \tag{3}$$

Binormal Vector

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Example 2.

$$\overline{r}(t) = \left\langle 2t, t^2, \frac{1}{3}t^3 \right\rangle$$

$$\overline{r}'(t) = \left\langle 2, 2t, t^2 \right\rangle$$

$$|\overline{r}'(t)| = t^2 + 2$$

$$\overline{T}(t) = \frac{1}{t^2+2}\left\langle 2, 2t, t^2 \right\rangle$$

$$\overline{T}'(t) = \frac{1}{t^2+2}\left\langle 0, 2, 2t \right\rangle - (t^2+2)^{-2}(2t)\left\langle 2, 2t, t^2 \right\rangle$$

$$\overline{T}'(t) = \left\langle 0, \frac{2}{t^2+2}, \frac{2t}{t^2+2} \right\rangle + \left\langle -\frac{4t}{(t^2+2)^2}, -\frac{4t^2}{(t^2+2)^2}, -\frac{2t^3}{(t^2+2)^2} \right\rangle$$

$$\overline{T}'(t) = \left\langle -\frac{4t}{(t^2+2)^2}, -\frac{-2t^2+4}{(t^2+2)^2}, \frac{4t}{(t^2+2)^2} \right\rangle$$
$$\overline{T}'(t) = \frac{1}{(t^2+2)^2} \left\langle -4t, -2t^2+4, 4t \right\rangle$$

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