Lecture Notes

Physics

Spring 2020

Jasper Runco

Instructed by
Sal Khan
Assisted by
n/a

Contents

List of Figures

Chapter 1

One-dimensional motion

Date: 2020-05-17
Announcements:

1.1 Acceleration

$$\boxed{a = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t}} \tag{1.1}$$

D: acceleration

Example 1. *A neurotic tiger shark starts from rest and speeds up uniformly to 12 meters per second in a time of 3 seconds. What was the magnitude of the average acceleration?*

Solution 1. $a = 4\frac{m}{s^2}$

Example 2. *A bald eagle is flying to the left at 34 meters per second, a gust of wind blows back causing it to slow down with a constant acceleration of a magnitude 8 meters per second. What will be its speed after 3 seconds?*

Solution 2. *final speed* $= +10\frac{m}{s}$

Chapter 2

Kinematic formulas

Date: 2020-05-21
Announcements:

2.1 What are the kinematic formulas?

The kinematic formulas are how we relate the five kinematic (constant acceleration) variables below.

Table 2.1: Kinematic variables

Δx	Displacement
t	Time interval
v_0	Initial velocity
v	Final velocity
a	Constant acceleration

If acceleration is constant, we can use a kinematic formula to solve for the unknown. The kinematic formulas are written as the following four equations:

$$v = v_0 + at$$
$$\Delta x = \left(\frac{v + v_0}{2}t\right)$$
$$\Delta x = v_0 t + \frac{1}{2}at^2$$
$$v^2 = v_0^2 + 2a\Delta x.$$

Definition 1 (Free flying object). *All free flying objects – also called projectiles – on Earth, regardless of mass, have a constant downward acceleration due to gravity*

Constant 1 (Acceleration due to gravity). $g = 9.81\frac{m}{s^2}$

2.2 Selecting kinematic formula

Each formula is missing one variable, so we choose the one that has three variables we already know and the one we are looking for.

2.2.1 Problem solving tip:

Note that each of the formulas is missing one of the variables:

Table 2.2:

1.	$v = v_0 + at$	(This formula is missing Δx.)
2.	$\Delta x = \left(\frac{v+v_0}{2}\right) t$	(This formula is missing a.)
3.	$\Delta x = v_0 t + \frac{1}{2}at^2$	(This formula is missing v.)
4.	$v^2 = v_0^2 + 2a\Delta x$	(This formula is missing t.)

2.3 Deriving the first kinematic formula, $v = v_0 + at$

This one is just a rearranged version of the definition of acceleration,

$$a = \frac{\Delta v}{\Delta t}.$$

Now we replace Δv with the definition of the change in velocity.

$$a = \frac{v - v_0}{\Delta t}.$$

Finally solve for v to get

$$v = v_0 + a\Delta t.$$

and Δt just becomes t to arrive at the **first kinematic formula**

$$\boxed{v = v_0 + at} \tag{2.1}$$

first kinematic formula

2.4 Deriving the second kinematic formula, $\Delta x = \left(\frac{v+v_0}{2}\right)t$

Visualize this kinematic formula by looking at a velocity graph for an object with constant acceleration (constant slope) with some initial velocity. See figure **??**

The area under the graph gives displacement Δx. We can break this down into a rectangle and a triangle. The height of the blue rectangle is v_0, so its area is $v_0 t$. The base of the red triangle is t and its height is $v - v_0$, so the are of the triangle is $\frac{1}{2}t\,(v - v_0)$. The total area will be

$$\Delta x = v_0 t + \frac{1}{2}vt - \frac{1}{2}v_0 t.$$

We can simplify to get

$$\Delta x = \frac{1}{2}vt + \frac{1}{2}v_0 t.$$

And finally rewrite the right side to get the **second kinematic formula**

$$\boxed{\Delta x = \left(\frac{v + v_0}{2}\right)t} \tag{2.2}$$

Second kinematic formula

2.5 Deriving the third kinematic formula, $\Delta x = v_0 t + \frac{1}{2}at^2$

Reconsider the figure **??**, since the area under the graph is the displacement, we can again use geometry to derive the following:

$$\boxed{\Delta x = v_0 t + at^2} \tag{2.3}$$

Third kinematic formula

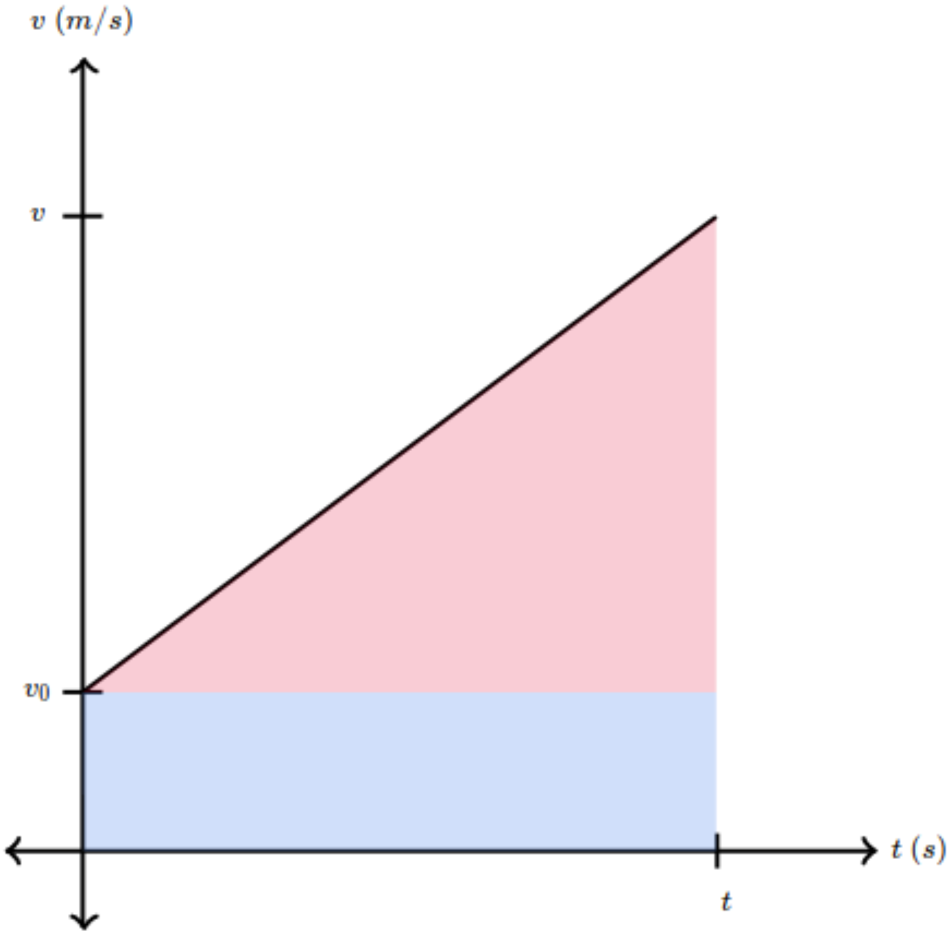

Figure 2.1: Velocity graph

2.6 Deriving the fourth kinematic formula, $v^2 = v_0^2 + 2a\Delta x$

To derive this formula, we start with the second kinematic formula:

$$\Delta x = \left(\frac{v + v_0}{2}\right) t.$$

Then we eliminate t from this formula by solving the first kinematic equation for t to get $t = \frac{v - v_0}{a}$, then substituting this into the second formula:

$$\Delta x = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right).$$

Multiplying the fractions gives

$$\Delta x = \left(\frac{v^2 - v_0^2}{2a}\right).$$

And now solve for v^2 to get the fourth kinematic equation.

$$\boxed{v^2 = v_0^2 + 2a\Delta x} \tag{2.4}$$

Fourth kinematic formula

2.7 Common confusion

Don't forget

- These equations are only true when acceleration is constant.
- Sometimes the known variables will not be explicitly given, but rather implied by **code-words** like "starts from rest" or "comes to a stop"
- All the variables but t can be negative.
- The third kinematic formula, equation **??**, might require the use of the **quadratic formula**.

2.8 Examples

Example 3 (First kinematic formula, $v = v_0 + at$)**.** *A water balloon filled with Kool-Aid is dropped from the top of a building. What is the velocity of the watter balloon after falling for* $t = 2.35$ *s?*

Solution 3. *Assuming upward is the positive direction, our known variables are*

Table 2.3:

$v_0 = 0$	(Since the water balloon was dropped from rest.)
$t = 2.35s$	(This is the time interval after which we want the velocity.)
$a_g = -9.81\frac{m}{s^2}$	(This is implied by freely falling.)

Use y as the position variable because the motion is vertical. Since we don't know the displacement and weren't asked to find displacement, use the first kinematic formula, which is missing Δy.

$$v = v_0 + at$$
$$v = 0\frac{m}{s} + (-9.81\frac{m}{s^2}(2.35s)$$
$$v = -23.1\frac{m}{s}$$

Note 1. *The final velocity was negative since the water balloon was heading downward.*

Example 4 (Second kinematic formula, $\Delta x = \left(\frac{v+v_0}{2}\right)t$)**.** *A leopard is running at* $6.20\frac{m}{s}$ *and then speeds up to* $23.1\frac{m}{s}$ *in a time of* $3.3s$*. How much ground did it cover?*

Solution 4. *Assume the initial direction is positive, our known variables are*

$$v_0 = 6.20\frac{m}{s}$$
$$v = 23.1\frac{m}{s}$$
$$t = 3.30s.$$

Since we don't know and aren't asked for acceleration, we use the second kinematic formula for the horizontal direction, which is missing a.

$$\Delta x = \left(\frac{v + v_0}{2}\right) t$$
$$\Delta x = \left(\frac{23.1\frac{m}{s} + 6.20\frac{m}{s}}{2}\right)(3.30s)$$
$$\Delta x = 48.3m$$

Example 5 (third kinematic formula, $\Delta x = v_0 t + \frac{1}{2}at^2$)**.** *A student throws her pencil straight up at $18.3\frac{m}{s}$. How long does it take the pencil to reach a point $12.2m$ higher than where it was thrown?*

Solution 5. *Assuming upward is the positive direction, our known variables are*

$$v_0 = 18.3\frac{m}{s}$$
$$\Delta y = 12.2m$$
$$a = -9.81\frac{m}{s}.$$

Since we don't know and aren't asked for the final velocity, use the third kinematic formula for the vertical direction:

$$\Delta y = v_{0_y} t + \frac{1}{2}a_y t^2$$
$$\Delta y = v_{0_y} t + \frac{1}{2}a_y t^2$$

.

Note 2. *We cannot solve this algebraically if none of the terms are zero because it is a quadratic equation, so we substitute the known values and solve with the quadratic equation:*

$$0 = \frac{1}{2}(-9.81\frac{m}{s^2} + (18.3\frac{m}{s}t - 12.2m$$
$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$t = \frac{-18.3\frac{m}{s} \pm \sqrt{(18.3\frac{m}{s})^2 - 4\left[\frac{1}{2}(-9.81\frac{m}{s^2})(-12.2m)\right]}}{2\left[\frac{1}{2}(-9.81\frac{m}{s^2}\right]}$$
$$t = 0.869s \text{ and } t = 2.86s.$$

There are two answers, which tells us that the pencil reaches this height twice in it's trajectory. We will chose the smaller time $t = 0.869s$

Example 6 (fourth kinematic formula, $v^2 = v_0^2 + 2a\Delta x$)**.** *A motorcyclist starts with a speed of $23.4\frac{m}{s}$ and slows down over a length of $50.2m$ with a constant deceleration of magnitude $3.20\frac{m}{s^2}$. Assume the it is moving forward the entire trip. What is the new velocity of the motorcycle after slowing down through the $50.2m$?*

Solution 6. *Assume the initial direction of travel is the positive direction, our known variables are*

$$\begin{aligned}
v_0 &= 23.4\frac{m}{s}\\
a &= -3.20\frac{m}{s^2}\\
\Delta x &= 50.2m.
\end{aligned}$$

Since we don't know and aren't asked to find the time, we use the fourth kinematic formula for the horizontal direction

$$\begin{aligned}
v_x^2 &= v_{0x}^2 + 2a_x\Delta x\\
v_x &= \pm\sqrt{v_{0x}^2 + 2a_x\Delta x}\\
&\quad \textit{We will assume the positive answer}\\
v_x &= \sqrt{v_{0x}^2 + 2a_x\Delta x}\\
v_x &= \sqrt{(23.4\frac{m}{s})^2 + 2(-3.20\frac{m}{s^2})(50.2m)}\\
v_x &= 15.0\frac{m}{s}
\end{aligned}$$