Chapter 1

Gauss's Law

DATE: 2020-08-31

ANNOUNCEMENTS:

Work on homework, reading quiz, and Lab 2

$$\boxed{\Phi_E = \oint \overline{E} \cdot d\overline{A} = Q_{encl} \frac{Q_{encl}}{\epsilon_0}} \tag{1.1}$$

Flux Equation

Φ_E **-** Electric flux through a closed surface of area A = surface integral of $\overline{E}$

Q_{encl} **-** Total charge enclosed by surface

ϵ_0 **-** Electric constant

Example 1 (Simplest case: Spherical symmetry)**.** *Use Gauss's Law to calculate electric field due to a point charge*

$$\overline{E} = \frac{kq}{r^2}\hat{r}$$

Solution 1. $|E|$ *same at all points on Gaussian surface*

$$\oint \overline{E} \cdot d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E \cos\phi d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$

$$d\overline{A}$$

points normal and out from Gaussian surface
Because $\overline{E}$ *and* $d\overline{A}$ *are both normal to the round surface, their angle is 0*

$$\oint E dA = \frac{q}{\epsilon_0}$$

E constant on Gaussian surface

$$E \oint dA = \frac{q}{\epsilon_0}$$

$$E4\pi r^2 = \frac{q}{\epsilon_0}$$
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

radially outward.

1.0.1 Applications

Conductor

- Suppose we construct a Gaussian surface inside a conductor.
- Because $\overline{E} = 0$ everywhere on the surface, Gauss's law requires the net charge inside the surface be zero.
- Under **electrostatic** conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.
- **Therefore, the electric field inside a conductor is always zero**

Conductor with cavity

- Cavity inside a conductor
- If a charge of $1nC$ resides inside the cavity, and the field inside the conductor is always zero, then there must be a $-1nC$ charge distributed around the surface of the cavity.

Review

- Electric field due to a point charge: $\overline{E} = \frac{kq}{r^2}\hat{r}$
- Electric field due to an infinite line of charge: $\overline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$ (λ linear charge density)
- Electric field due to an infinite sheet of charge: $\overline{E} = \frac{\sigma}{2\epsilon_0}$ uniform field (σ surface charge density)

Field of a charged conducting sphere

- Positive charge Q is distributed on spherical conductor with radius R. Find magnitude of electric field at point P and distance r from the center of the sphere.
- for $r < R$, we have symetry so we chose a Gaussian surface that is spherical with radus r.

$$E = 0$$

- for r>R, we construct a Gaussian surface enclosing the charged sphere.

$$\oint \overline{E} \cdot d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$
$$\oint EdA\cos\phi = q_{enc}/\epsilon_0$$
$$\oint EdA\cos 0 = q_{enc}/\epsilon_0$$
$$\oint EdA = Q/\epsilon_0$$
$$E\oint dA = Q/\epsilon_0$$
$$E4\pi r^2 = Q/\epsilon_0$$
$$E = Q/4\pi r^2\epsilon_0$$
$$E = kQ/r^2$$

- The result is the same as for a point charge.

Field of uniformly charged sphere (insulator)

- Positive charge Q uniformly throughtout volume of **insulating** sphere with radius R. Find magnitude of field at point P a distance r from center.

$$\rho_{\text{sphere}} = \text{folume charge density, charge/volume}$$

- r<R

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

- The symmetry says the E-field points out and is constant across a gaussian surface.

$$\oint EdA\cos\phi = q_{enc}/\epsilon_0$$

$$\oint EdA = q_{enc}/\epsilon_0$$

$$E\oint dA = q_{enc}/\epsilon_0$$

$$E4\pi r^2 = q_{enc}/\epsilon_0$$

$$E4\pi r^2 = q_{enc}/\epsilon_0$$

- the question: what is q_{enc}?

$$q_{enc} = \rho V_{enc} = \frac{Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3$$

$$q_{enc} = \frac{Qr^3}{R^3}$$

$$E = \frac{1}{4\pi r^2\epsilon_0}\frac{Qr}{R^3} \text{ radially out}$$

- Now, what is the field outside?

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$E4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{kQ}{r^2} \text{ radially out}$$

Field of a Uniform Line Charge

- We need to chose a Gaussian surface that is symetrical, so use a **cylinder**

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

- This has three components, the end caps and the round

$$\int_{end} \overline{E}d\overline{A} + \int_{end} \overline{E}d\overline{A} + \int_{round} \overline{E}d\overline{A}$$

- For the end caps, the field lines perpendicular to the area, so they contribute nothing to the flux

$$\int EdA\cos\phi = q_{enc}/\epsilon_0$$

$$E\int dA = q_{enc}/\epsilon_0$$

$$q_{enc} = \lambda l$$

$$E \cdot 2\pi rl = \lambda\frac{l}{\epsilon_0}$$

Infinite plane of charge

- planar symmetry

$$\sigma = \text{ charge / area}$$

- the Gaussian surface that takes advantage of this symmetry is a cylinder bisecting the plane
- cylinder: endcaps area A

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$\int_{end} \overline{E} \cdot d\overline{A} + \int_{end} \overline{E} \cdot d\overline{A} + \int_{round} \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$\int \overline{E} \cdot d\overline{A} \cos\phi + \int EdA \cos\phi = q_{enc}/\epsilon_0$$

$$2 \int EdA = q_{enc}/\epsilon_0$$

$$2EA = \sigma A/\epsilon_0$$

$$E = \sigma/2\epsilon_0 \text{ uniform outward from plane}$$

1.0.2 Conductors

$$\boxed{E_\perp = \frac{\sigma}{\epsilon_0}} \tag{1.2}$$

Electric field at surface of a conductor

Electrostatic shielding

- A conducting box immersed in uniform field
- the field of induced charges combines with the uniform field to give **zero** total field