Date: 2020-08-19

Announcements:

20 minutes missed in lecture time will be posted as a video later today on blackboard.
Assignment: Section 1.2 Numbers 5, 6, 7, 8

0.0.1 1.1 Continued

Working towards solving systems of equations in a systematic way, vs. the algebra days when you attack it at any angle with substitution and elimination. This systematic way is setting up for the theoretical

Example 1 (1). *1. take a system of equation like this*

$$3x + 3y - 2z = 13$$

$$6x + 2y - 5z = 13$$

$$7x + 5y - 3z = 26$$

2. *Use the algebraic operations:*

   (a) *Multiply an equation by a nonzero constant*

   (b) *Interchange any equation*

   (c) *Add a constant times one equation to another equation*

   $-1E2 + E3 \to E3$

$$\begin{array}{r} -6x - 2y + 5z = 13 \\ 7x + 5y - 3z = 26 \\ \hline \\ x + 3y + 2z = 13 \end{array}$$

to get a variable in the first position with a coeficient of 1 (The equation you add to [E3] is always the equation you replace.)

$$6x + 2y - 5z = 13$$

$$3x + 3y - 2z = 13$$

$$x + 3y + 2z = 13$$

Interchange to get the equation with coeficient 1 on top, denoted like this:

$E1 \leftrightarrow E3$

$$x + 3y + 2z = 13$$

$$6x + 2y - 5z = 13$$

$$3x + 3y - 2z = 13$$

3. *Use the x to eliminate the x's from the other equations below.*
   $-6E1 + E2 \rightarrow E3$
   $-3E1 + E3 \rightarrow E3$

$$\begin{aligned} x + 3y + 2z &= 13 \\ -16y - 17z &= -65 \\ -6y - 8z &= -26 \end{aligned}$$

4. *Get the next variable in column 2 with coeficient 1.*
   $\frac{-1}{16}E2 \rightarrow E2$

$$\begin{aligned} x + 3y + 2z &= 13 \\ y + \frac{17}{16}z &= \frac{65}{16} \\ -6y - 8z &= -26 \end{aligned}$$

5. *Use the y to elimintate the y's from below.*

$$6E1 + E3 \rightarrow E3$$

$$\begin{aligned} x + 3y + 2z &= 13 \\ y + \frac{17}{16}z &= \frac{65}{16} \\ -\frac{13}{8}z &= -\frac{13}{8} \end{aligned}$$

6. *Get the last variable in column 3 with coeficient 1. (Continue until diagonal of coeficient 1)*
   $\frac{-8}{13}E3 \rightarrow E3$

$$\begin{aligned} x + 3y + 2z &= 13 \\ y + \frac{17}{16}z &= \frac{65}{16} \\ z &= 1 \end{aligned}$$

7. *Work to eliminate variables from the bottom up.*
   $-\frac{17}{16}E3 + E2 \rightarrow E2$
   $-2E3 + E1 \rightarrow E1$

$$\begin{aligned} x + 3y &= 11 \\ y &= 3 \\ z &= 1 \end{aligned}$$

   $-3E2 + E1 \rightarrow E1$

$$\begin{aligned} x &= 2 \\ y &= 3 \\ z &= 1 \end{aligned}$$

8. *Write solution as ordered tripple:* $(2, 3, 1)$

Because this system of equations could be written in this way, it has **only one** solution. There are other cases where there are problems in solving the system this way, a situation where there is no soution or infinite solutions.

Example 2 (2)**.** *Solve:*

$$\begin{aligned} x + y + z &= 1 \\ -2x + y + z &= -2 \\ 3x + 6y + 6z &= 5 \end{aligned}$$

$2E_1 + E_2 \to E_2$
$-3E_1 + E_3 \to E_3$

$$\begin{aligned} x + y + z &= 1 \\ 3y + 3z &= 0 \\ 3y + 3z &= 2 \end{aligned}$$

$\frac{1}{3}E_2 \to E_2$

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 0 \\ 3y + 3z &= 2 \end{aligned}$$

$-3E_2 + E_3 \to E_3$

$$\begin{aligned} x + y + z &= 1 \\ y + z &= 0 \\ 0 &= 2 \leftarrow \textit{false, no solution} \end{aligned}$$

You could make the determination of no solution eariler, as soon as you catch the false statement.

Example 3 (3)**.**

$$\begin{aligned} 2x - y_z &= -1 \\ x + 3y - 2z &= 2 \\ -5x + 6y - 5z &= 5 \end{aligned}$$

$E_1 \leftrightarrow E_2$

$$\begin{aligned} x + 3y - 2z &= 2 \\ 2x - y + z &= -1 \\ -5x + 6y - 5z &= 5 \end{aligned}$$

$-2E_1 + E_2 \to E_2$
$5E_1 + E_3 \to E_3$

$$\begin{aligned} x + 3y - 2z &= 2 \\ -7y + 5z &= -5 \\ 21y - 5z &= 15 \end{aligned}$$

$\frac{-1}{7}E_2 \rightarrow E_2$

$$\begin{aligned} x + 3y - 2z &= 2 \\ y - \frac{5}{7}z &= \frac{5}{7} \\ 21y - 5z &= 15 \end{aligned}$$

$-21E_2 + E_3 \rightarrow E_3$

$$\begin{aligned} x + 3y - 2z &= 2 \\ y - \frac{5}{7}z &= \frac{5}{7} \\ 0 &= 0 \end{aligned}$$

*Where the last equation is a **true** statement, but there is no coeficient of 1, is a situation where there are **infinite solutions**. You must continue to elimintate up with what you got.*
$-3E_2 + E_1 \rightarrow E_1$

$$\begin{aligned} x + \frac{1}{7}z &= -\frac{1}{7} \\ y - \frac{5}{7}z &= \frac{5}{7} \\ 0 &= 0 \end{aligned}$$

This is now in as few variables as possible, and can give us a final solution

$$\begin{aligned} x + \frac{1}{7}z &= -\frac{1}{7} \rightarrow \boxed{x = -\frac{1}{7}z - \frac{1}{7}} \\ y - \frac{5}{7}z &= \frac{5}{7} \rightarrow \boxed{y = \frac{5}{7}z + \frac{5}{7}} \\ 0 &= 0 \end{aligned}$$

$$\boxed{(x, y, z) = \left(-\frac{1}{7}z - \frac{1}{7}, \frac{5}{7}z + \frac{5}{7}, z\right)}$$

Thinking about this solution in terms of vectors, $z(-\frac{1}{7}, \frac{5}{7}, 1) + (-\frac{1}{7}, \frac{5}{5}, 0)$, this describes a line.

Example 4 (4)**.**

$$\begin{aligned} x + 3y + 4z &= 1 \\ 2x + 6y + 8z &= 2 \\ 3x + 9y + 12z &= 3 \end{aligned}$$

$-2E_1 + E_2 \to E_2$
$-E_1 + E_3 \to E_3$

$$\begin{aligned} x + 3y + 4z &= 1 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

both equations with no variables are ture equations, therefore ***infinite solutions****, and we have one equations to solve for x.*

$x + 3y + 4z = 1 \to x = -3y - 4z + 1$

$\boxed{(x, y, z) = (-3y + 4z + 1, y, z)}$

You can look at this as the sum of three vectors,
$(-3y, y, 0) + (-4z, 0, z) + (1, 0, 0)$ or $y(-3, 1, 0) + z(-4, 0, 1) + (1, 0, 0)$
This is just a geometric interpretation, whatever it means, but it is not a line