Chapter 1

(14) Partial Derivatives

DATE: 2020-09-20
ANNOUNCEMENTS:

1.1 Functions of Several Variables

A function of two variables is a rule that assigns to each ordered pair (x, y) in a set D a unique real number denoted by $f(x, y)$. The set D is the Domain of f and its Range is the set of values that f takes on, that is $\{f(x, y)|(x, y) \in D\}$.

Single Variables

$$y = f(x)$$
$$(x, y)$$

$$f(x) = x^2 - 2x + 3$$
$$f(-3) = (-3)^2 - 2(-3) + 3 = 18$$
$$(-3, 18)$$

Two Variables

$$z = f(x, y)$$
$$(x, y, z)$$

$$f(x, y) = x^2y + xy$$
$$f(-3, 2) = (-3)^2(2) + (-3)(2) = 12$$

$$(-3, 2, 12)$$

These are both polynomial functions.

A. Domain (1 variable): $(-\infty, \infty)$, All real numbers.
B. Domain (2 variables): $\mathbb{R}^2$

$$f(x) = \frac{\sqrt{x-3}}{x-7}$$

need $x \geq 3$, $x \neq 7$

$$[3, 7) \cup (7, \infty)$$

$$f(x, y) = \frac{\sqrt{x+y-1}}{x-y}$$

need $y \geq -x + 1$, $x \neq y$
You could graph this domain in the x,y-plane

$$f(x, y, z) = \ln(1 - x^2 - y^2 - z^2)$$

need $x^2 + y^2 + z^2 < 1$
Recognize special quadric surface, an elipsoid that is a sphere, and graph it with dotted surface to represent the domain.

1.1.1 Level Curve

The level curves of a function of two variables are the curves with equations $f(x, y) = k$ where k is a constant in the Range of f.

Definition 1 (Contour Map)**.** *A 2-d representation of a surface drawn with level curves.* •

Example 1 (Unfamiliar surface)**.**

$$f(x, y) = e^{x-y}$$

This will never output a value of 0 or negative.

$$1 = e^{x-y}$$
$$\ln 1 = x - y$$
$$y = x$$

$$2 = e^{x-y}$$
$$\ln 2 = x - y$$
$$y = x - \ln 2$$

The larger the value k gets, the more negative the y-intercept gets.
This resembles a slide that slopes towards the x-y plane. •

Example 2.

$$f(x, y, z) = \sqrt{x^2 + y^2 - z}$$

Level surfaces

$$0 = \sqrt{x^2 + y^2 - z}$$
$$z = x^2 + y^2$$

(paraboloid that maps to zero in 4-D) $(x, y, z, 0)$

$$1 = \sqrt{x^2 + y^2 - z}$$
$$z = x^2 + y^2 - 1$$

(paraboloid that maps to one in 4-D) $(x, y, z, 1)$ *This paraboloid would shift down from input 0, and this could continue...*

$$(x, y, z, k)$$

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