Chapter 1

(23) Electrical Potential

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ANNOUNCEMENTS:

1.1 Electric Potential Energy

1.1.1 Electrical Potential Energy in a Uniform field

- If a positive charge moves in the direction of the field, the field does **positive** work on the charge
- The potential energy **decreases**.
- If the positive charge moves opposite the direction of the field, the field does **negative** work on the charge.
- The potential energy increases.
- If a negative charge moves in the direction of the field, the field does **negative** work on the charge.
- The potential energy **increases**.
- If the negative charge moves opposite the direction of the field, the field does **positive** work on the charge.
- The potential energy **decreases**.

Electric Potential Energy of Two Point Charges

- Doesn't depend on path taken.
- Electric potential energy only depends on the distance between the charges.
- Defined to be zero when the charges are infinitely far apart.
- Charges with the same sign have positive electric potential energy.
- Charges with opposite signs have negative electric potential energy.

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}} \tag{1.1}$$

Electric potenetial energy of two point charges

U Potential Energy

ϵ_0 Electric constant

q, q_0 Values of two charges

r Distance between two charges

Electrical Potential with Several Point Charges

- The potential energy with q_0 depends on the other charges and their distances.
- Electric potential energy is **Algebraic sum**.

$$\boxed{U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i q_{\frac{i}{r_i}}} \tag{1.2}$$

Electric potential energy q_0 due to a collection of charges

1.2 Electric Potential

Definition 1 (Electric Potential)**.** *Potential is potential energy per unit charge. The potential of a with respect to b ($V_{ab} = V_a - V_b$ equals the work done by the electric force when a unit charge moves from a to b.* •

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{1.3}$$

Electric potential due to a point charge

q value of point charge.

r distance from point charge to where potential is measured.

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}} \tag{1.4}$$

Electric potential due to a collection of point charges

1.2.1 Electric Potential and Electric Field

- Moving in the direction of the electric field, the electric potential **decreases**.
- The direction of the electric field is the direction of decreasing V.
- To move against the E-field, an external force per unit charge must be applied opposite the electric force per unit charge.
- The electric force per unit charge is the E-field.

- The **potential difference** $V_a - V_b$ equals the work done per unit charge by the external force to move from b to a:

$$V_a - V_b = -\int_a^b \overline{E} \cdot d\overline{r}$$

- Electric field can be expressed as $1\frac{N}{C} = 1\frac{V}{m}$

1.2.2 Electron Volt

- Change in potential energy U, when a charge moves from a potential of V_b to a potential of V_a is

$$U_a - U_b = q(V_a - V_b).$$

- When the potential difference is 1V for a charge q with magnitude e of the electron charge, the change in energy is defined as one electron volt (eV).

$$\boxed{1eV = 1.602 \times 10^{-19} J} \tag{1.5}$$

Electron volt

1.2.3 Electric Potential and Field of a Charged Conductor

- A solid conducting sphere of radius R has a total charge of q.
- The electric field **inside** the sphere is zero everywhere.
- The potential is the **same** at every point inside the sphere and is equal to the value at the surface.

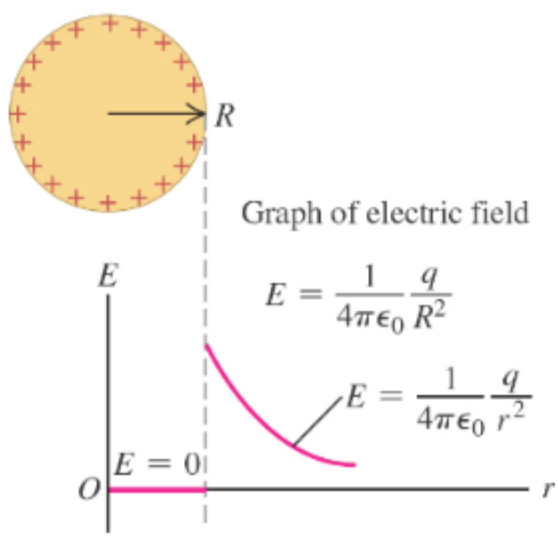

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Figure 1.1: Electric field in a conducting sphere

Ionization and Corona Discharge

- Air becomes ionized (conductor) at or above $3 \times 10^6 \frac{V}{m}$.
- For a charged sphere, $V_{\text{surface}} = E_{\text{surface}} R$
- If E_m is the E-field magnitude at which air becomes conductive **(dielectric strength)**, then V_m is the maximum potential to which it can be raised: $V_m = RE_m$.

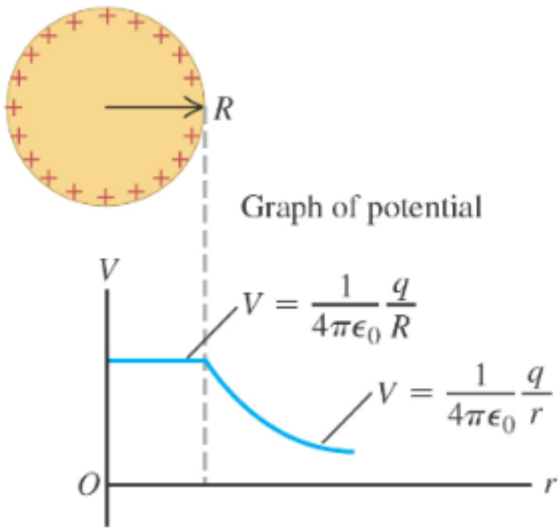

Figure 1.2: Electric potential in a conducting sphere

Oppositely Charged Parallel Plates

$$\boxed{V = Ey} \tag{1.6}$$

Potential at any height between two large oppositely charged parallel plates

V Potential (units $V = \frac{J}{C}$)

E Electric field magnitude (units $\frac{N}{C}$)

y height (m)

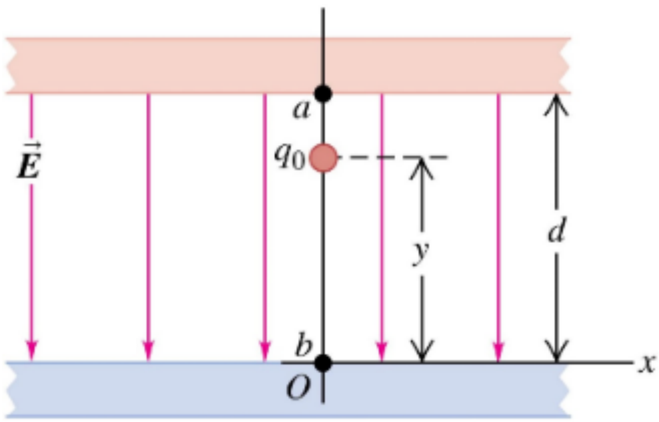

Figure 1.3: Potential between parallel plates

1.3 Equipotential Surfaces

1.3.1 Equipotential Surfaces and Field lines

- **Equipotential surfaces** have constant electric potential.
- Field lines and equipotential surfaces are always mutually perpendicular.

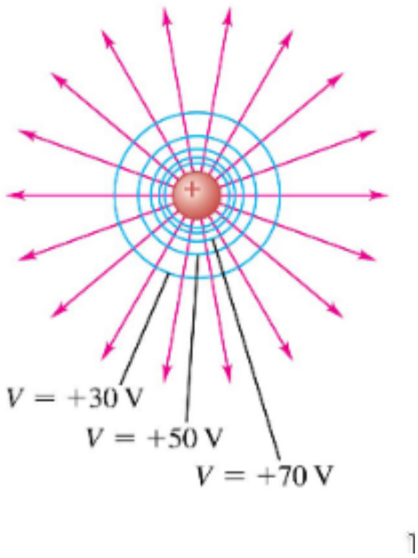

Figure 1.4: Equipotential surfaces for monopole

Equipotentials and Conductors

- When all charges are at rest
  - The surface of a conductor is always an equipotential surface
  - The electric field just outside a conductor is always perpendicular to the surface.
- If the electric field had a tangential component at the surface of a conductor, a net amount of work would be done on a test charge by moving it around a loop, which is impossible because the E-force is conservative.

1.3.2 Potential Gradient

- The components of the electric field can be found by partial derivatives of the electric potential.
- The electric field is the negative gradient of the potential.

$$\boxed{E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}} \tag{1.7}$$

Components of electric field

$$\boxed{\overline{E} = -\Delta \overline{V}} \tag{1.8}$$

Electric field, negative gradient of potential

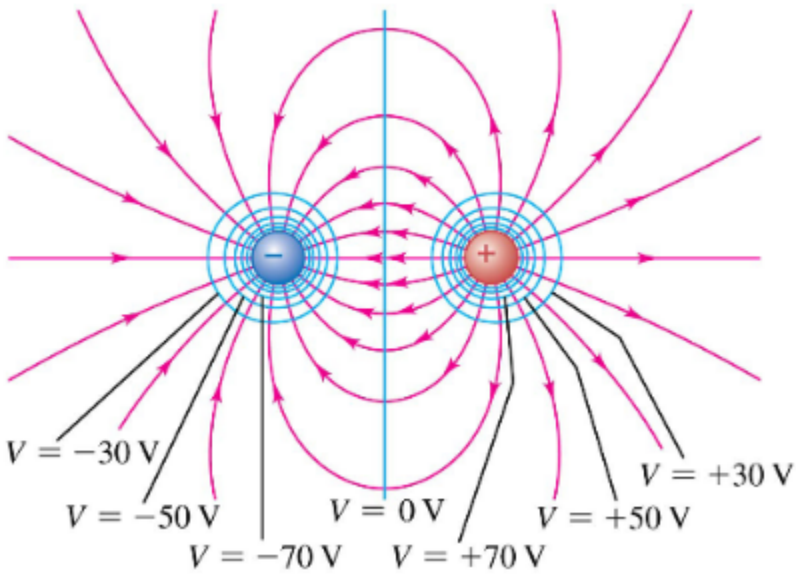

Figure 1.5: Equipotential surfaces for dipole

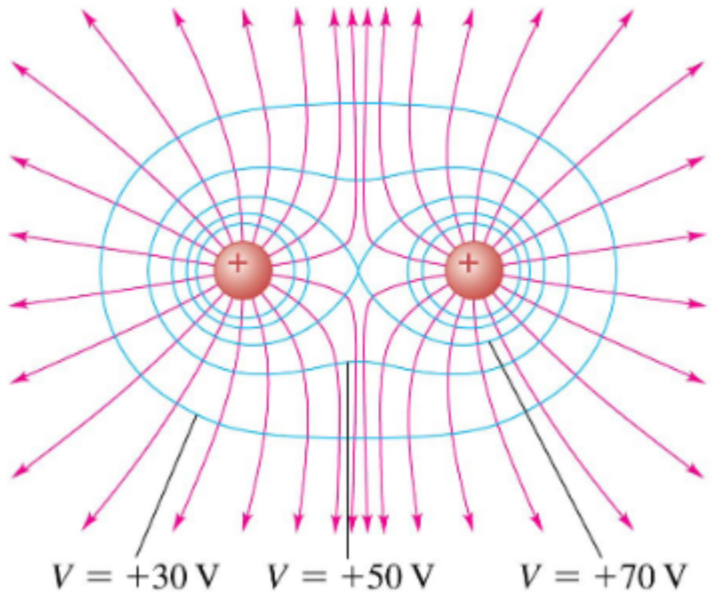

Figure 1.6: Equipotential surfaces for two equal charges

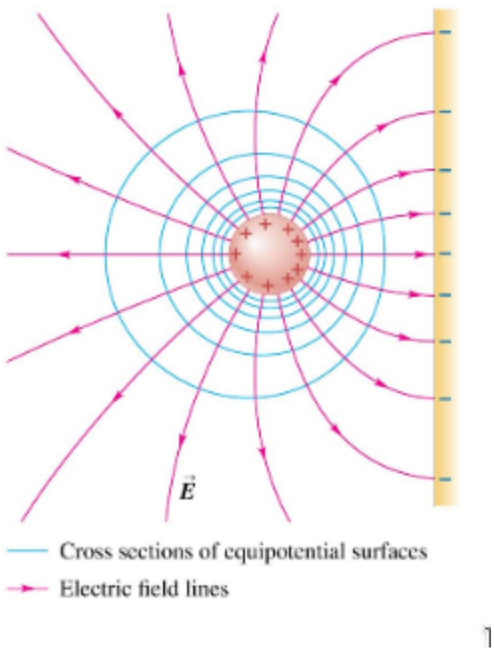

Figure 1.7: Equipotential surface and conductor