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Announcements:

0.1 Planes on a Space Curve

Definition 1 (Normal Plane). .
Determined by Normal and Binormal vectors, $\overline{N}$ and $\overline{B}$ where $\overline{T}$ is normal vector to normal plane. The normal plane contains all lines orthogonal to $\overline{T}$ •

Definition 2 (Oscilating Plane). .
Determined by $\overline{T}$ and $\overline{N} \to \overline{B}$ is normal vector to oscilating plane. The oscilating plane lies closest to the space curve $\overline{r}(x)$ in 3-D space. •

Definition 3 (Rectifying Plane). .
Contains tangent $\overline{T}$ and binormal $\overline{B}$ vectors so that the normal vector $\overline{N}$ is normal to the rectifying plane.

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Table 1: Vectors of 3 Planes

Plane	Vectors contained	Orthogonal to
Normal	$\overline{N}$ $\overline{B}$	$\overline{T}$
Oscilating	$\overline{T}$ $\overline{N}$	$\overline{B}$
Rectifying	$\overline{T}$ $\overline{B}$	$\overline{N}$

Example 1 (Find 3 planes at a point). .
let $\overline{r}(t) = \langle \cos(3t), t, \sin(3t) \rangle$, find the equation of the Normal plane, Oscilating plane, and Rectifying plane at $t = \frac{\pi}{2}$ •

Solution 1 (Find 3 planes at a point). .
A) Normal Plane: Find Unit Tangent Vector
point: $t = \frac{\pi}{2}$

$$\overline{r}(\frac{\pi}{2}) = \left\langle \cos(\frac{3\pi}{2}), \frac{\pi}{2}, \sin(\frac{3\pi}{2}) \right\rangle$$
$$= \left\langle 0, \frac{\pi}{2}, -1 \right\rangle$$

generic vector $\left\langle x, y - \frac{\pi}{2}, z+1 \right\rangle$

$$\overline{r}'(t) = \langle -3\sin(3t), 1, 3\cos(3t) \rangle$$
$$|\overline{r}'(t)| = \sqrt{9\sin^2(3t) + 1 + 9\cos^2(3t)}$$
$$|\overline{r}'(t)| = \sqrt{9\left(\sin^2(3t) + \cos^2(3t)\right) + 1}$$
$$= \sqrt{10}$$

$$\overline{T} = \frac{1}{|r'(t)|} r'(t)$$

$$\overline{T}(t) = \frac{1}{\sqrt{10}} \langle -3\sin(3t), 1, 3\cos(3t) \rangle$$

$$\overline{T}(\frac{\pi}{2}) = \frac{1}{\sqrt{10}} \langle 0, 0, -3 \rangle$$

$$= \left\langle 0, 0, -\frac{3}{\sqrt{10}} \right\rangle$$

$$\overline{N}(\frac{3}{2}) : \left\langle 0, 0, -\frac{3}{\sqrt{10}} \right\rangle \cdot \left\langle x, y - \frac{\pi}{2}, z + 1 \right\rangle = 0$$

$$-\frac{3}{\sqrt{10}} z - \frac{3}{\sqrt{10}} = 0$$

Normal Plane:

$$\boxed{z = -1}$$

B) Rectifying Plane: Find Normal Tangent Vector

$$\overline{T}'(t) = \frac{1}{\sqrt{10}} \langle -9\cos(3t), 0, -9\sin(3t) \rangle$$

$$\left|\overline{T}'(t)\right| = \frac{1}{\sqrt{10}} \sqrt{81\cos^2(3t) + 0 + 81\sin^2(3t)}$$

$$= \frac{1}{\sqrt{10}} \sqrt{91}$$

$$= \frac{9}{\sqrt{10}}$$

$$\overline{N}(t) = \frac{1}{\frac{9}{\sqrt{10}}} \frac{1}{\sqrt{10}} \langle -9\cos(3t), 0, -9\sin(3t) \rangle$$

$$\overline{N}(t) = \frac{1}{9} \langle -9\cos(3t), 0, -9\sin(3t) \rangle$$

$$= \langle -\cos(3t), 0, -\sin(3t) \rangle$$

$$\overline{N}(\frac{\pi}{2}) = \langle 0, 0, 1 \rangle$$

$$\textit{Rectifying plane } = \langle 0, 0, -1 \rangle \cdot \left\langle x, y - \frac{\pi}{2}, z + 1 \right\rangle = 0$$

$$0x + 0(y - \frac{\pi}{2}) + 1(z + 1) = 0$$

$$z + 1 = 0$$

$$\textit{Rectifying Plane: } \boxed{z = -1}$$

C) Oscilating Plane: Find Binormal Unit Vector

$$\overline{B}(\frac{\pi}{2}) = \overline{T}(\frac{\pi}{2}) \times \overline{N}(\frac{\pi}{2})$$

$$= \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{3}{\sqrt{10}} & \frac{1}{\sqrt{10}} & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

$$= \left\langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0 \right\rangle$$

Oscilating Plane $\left\langle -\frac{1}{\sqrt{10}}, \frac{3}{\sqrt{10}}, 0 \right\rangle \cdot \left\langle x, y - \frac{\pi}{2}, z+1 \right\rangle = 0$

$$-\frac{1}{\sqrt{10}}x + \frac{3}{\sqrt{10}}y - \frac{3}{2\sqrt{10}} + 0 = 0$$

C)

$$\boxed{-2x + 6y - 3 = 0}$$

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