DATE: 2020-09-12
ANNOUNCEMENTS:

0.0.1 Oscilating Circle

Definition 1 (Oscilating Circle)**.** *The circle that lies in the oscilating plane of the curve at a point and has the same tangent, lies on the concave side of the curve and has a radius of* $\frac{1}{\kappa}$ •

To find a circle, you would need a center and the Radius. We know the radius is $\frac{1}{\kappa}$. To find the center, we must move the radius distance to the center from the edge touching the curve. This is in the direction of the unit normal vector.

Scale the unit normal vector by the length of the radius:

$$\frac{1}{\kappa}\overline{N}(t)$$

So, the vector from the origin to the center is the vector from the origin to the point on the curve + $\frac{1}{\kappa}\overline{N}$

Example 1 (oscilating circle)**.** *Given* $y = 4 - x^2$, $\overline{r}(t)0\left\langle t, 4 - t^2, 0\right\rangle$, *find the equation of the oscilating circle when t=0, at* $(0, 4)$ •

Solution 1 (oscilating circle)**.**

$$\overline{r}(t) = \left\langle t, 4 - t^2, 0\right\rangle$$

$$\kappa = \frac{|\overline{r}' \times \overline{r}''|}{|\overline{r}'|^3}$$

$$\overline{r}' = \langle 1, -2t, 0\rangle$$

$$\overline{r}'' = \langle 0, -2, 0\rangle$$

$$\overline{r}' \times \overline{r}'' = \begin{bmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -2t & 0 \\ 0 & -2 & 0 \end{bmatrix}$$

$$= \langle 0, 0, -2\rangle$$

$$|\overline{r}' \times \overline{r}''| = 2$$

$$|\overline{r}'(t)| = \sqrt{1 + 4t^2}$$

$$\kappa = \frac{2}{(\sqrt{1 + 4t^2})^3}$$

$$\kappa(0) = 2$$

$$radius = \frac{1}{2}$$

We know the normal vector is in the negative-y direction because this is a prabola centered on the axis.

$$\langle 0, -1, 0\rangle$$

$$\langle 0,4,0\rangle + \frac{1}{2}\langle 0,-1,0\rangle = \left\langle 0,\frac{7}{2},0\right\rangle$$
$$(0,\frac{7}{2})$$
$$(x-h)^2+(y-k)^2=r^2$$
$$\boxed{x^2+(y-\frac{7}{2})^2=\frac{1}{4}}$$

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Example 2 (oscilating circle)**.** *Given same as above, find oscilating circle at* $(2,0)$ •

Solution 2 (oscilating circle)**.**

$$\kappa(2)=\frac{2}{(\sqrt{1+4(2)^2})^3}$$
$$\frac{17\sqrt{17}}{2}$$

Center: $\langle 2,0,0\rangle + \frac{17\sqrt{17}}{2}\overline{N}(t)$ *We need principal normal unit vector at t=2.*

$$\overline{T}(t)=\frac{1}{\sqrt{1+4t^2}}\langle 1,-2t,0\rangle$$
$$\overline{T}'(t)=\frac{1}{\sqrt{1+4t^2}}\langle 0,-2,0\rangle-\frac{1}{2}(1+4t^2)^{-\frac{3}{2}}(8t)\langle 1,-2t,0\rangle$$
$$\overline{T}'(t)=\left\langle 0,-\frac{2}{\sqrt{1+4t^2}},0\right\rangle+\frac{-4t}{\left(\sqrt{1+4t^2}\right)^3}\langle 1,-2t,0\rangle$$
$$\overline{T}'(2)=\left\langle 0,-\frac{2}{\sqrt{1+4(2)^2}},0\right\rangle+\frac{-4(2)}{\left(\sqrt{1+4(2)^2}\right)^3}\langle 1,-2(2),0\rangle$$
$$\overline{T}'(2)=\left\langle -\frac{8}{17\sqrt{17}},\frac{-2}{17\sqrt{17}},0\right\rangle$$
$$\left|\overline{T}'(2)\right|=\sqrt{\frac{64}{17^2*17}+\frac{4}{17^2*17}}=\frac{2}{17}$$
$$\overline{N}(2)=\frac{1}{\frac{2}{17}}\left\langle -\frac{8}{17\sqrt{17}},-\frac{2}{17\sqrt{17}},0\right\rangle$$
$$\left\langle -\frac{4}{\sqrt{17}},-\frac{1}{\sqrt{17}},0\right\rangle$$

Center: $\langle 2,0,0\rangle + \frac{17\sqrt{17}}{2}\left\langle -\frac{4}{\sqrt{17}},-\frac{1}{\sqrt{17}},0\right\rangle$

$$=\langle 2,0,0\rangle+\left\langle -34,-\frac{17}{2},0\right\rangle$$
$$=\left\langle -32,-\frac{17}{2},0\right\rangle$$
$$(x-h)^2+(y-k)^2=r^2$$
$$\boxed{(x+32)^2+(y+\frac{17}{2})^2=\frac{4913}{2}}$$

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