DATE: 2020-09-20
ANNOUNCEMENTS:

0.1 Partial Derivatives

Definition 1 (Partial Derivative).

$$z = f(x, y)$$

The partial derivative of f with respect to x

$$f_x = \frac{\partial f}{\partial x} = \frac{\partial z}{\partial x} = z_x$$

How the output z changes with respect to x - the y stays constant.

$$\frac{\partial z}{\partial x} = \lim_{h \to 0} \frac{f(x+h, y) - f(x, y)}{h}$$

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Example 1 (Partial Derivative).

$$\begin{aligned} z = f(x, y) &= x^2 y^3 \\ \frac{\partial z}{\partial x} &= \lim_{h \to 0} \frac{(x+h)^2 y^3 - x^2 y^3}{h} \\ &= \lim_{h \to 0} \frac{y^3 \left((x+h)^2 - x^2\right)}{h} \\ &= y^3 \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h} \\ &= y^3 \lim_{h \to 0} \frac{h(2x+h)}{h} \\ &= y^3 \lim_{h \to 0} 2x + h = \boxed{2xy^3} \end{aligned}$$

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0.1.1 Higher Order Derivatives

Theorem 1 (Clairaut's Theorem). *Suppose that f is defined on a disk D that contains the point (a,b). If the function f_{xy} and f_{yx} are both continuous on D, then the*

$$f_{xy}(a, b) = f_{yx}(a, b)$$

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Corollary 1 (Clairaut's Corollary).

$$f_{xxy} = f_{xyx} = f_{yxx}$$
$$f_{yyx} = f_{yxy} = f_{xyy}$$

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