Date: 2020-08-21

Announcements:

Assignment: 1.1 (5-10) 1.2 (1-22) Solve them using Gaus-Jordan Elimination, as was shown, not gausian elimination.

0.1 Gaussian elimination

Example 1 (solve).

$$\begin{aligned} 2y + 3z &= 8 \\ 2x + 3y + z &= 5 \\ x - y - 2z &= -5 \end{aligned}$$

Definition 1 (Matrix). *A rectangular array of numbers.*

coeficient matrix

$$\begin{bmatrix} 0 & 2 & 3 \\ 2 & 3 & 1 \\ 1 & -1 & -2 \end{bmatrix}$$

Constan matrix

$$\begin{bmatrix} 8 \\ 5 \\ -5 \end{bmatrix}$$

Augmented matrix

$$\begin{bmatrix} 0 & 2 & 3 & 8 \\ 2 & 3 & 1 & 5 \\ 1 & -1 & -2 & -5 \end{bmatrix}$$

0.1.1 Reduced Row Eschelon Form

1. If a row does not consist entirely of zeros, then the first non zero entry is a 1. (Leading one).
2. If any rows that are all zero, they appear at the bottom of the matrix.
3. In any two successive rows that are not all zeros, the leading one in lower row is further to the right than the 1 in the higher row. (This qualifies as Row Eschelon Form)
4. Each column that contains a leading 1 has zeros everywhere else in that column.

Solution 1 (1).

$R_1 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 2 & 3 & 1 & 5 \\ 0 & 2 & 3 & 8 \end{bmatrix}$$

$-2R_1 + R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 5 & 5 & 15 \\ 0 & 2 & 3 & 8 \end{bmatrix}$$

$\frac{1}{5}R_2 \to R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 2 & 3 & 8 \end{bmatrix}$$

$-2R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 1 & 2 \end{bmatrix} (Row Exchelon Form)$$

$-R_3 + R_2 \to R_2$

$$\begin{bmatrix} 1 & -1 & -2 & -5 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$2R_3 + R_1 \to R_1$

$$\begin{bmatrix} 1 & -1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix} (Reduced)$$

$$x = 0$$
$$y = 1$$
$$z = 2$$

$\boxed{(0, 1, 2)}$

Example 2 (Is this Row-eschelon or Reduced?). $\begin{bmatrix} 1 & 1 & 3 & 4 & 9 \\ 0 & 0 & 1 & 7 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

Solution 2. *Answer: Just row eschelon, there must be a zero above and below every leading one.*

$-2R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 1 & 0 & -10 & -5 \\ 0 & 0 & 1 & 7 & 7 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} Reduced Row Eschelon Form$$

(R_1) $x_1 + x_2 - 10x_3 = -5$
(R_2) $x_3 + 7x_4 = 7$
(R_1) $x_1 = -5 - x_2 + 10x_3$
(R_2) $x_3 = 7 - 7x_4$

Definition 2 (n-tuple). *Ordered pair with n entries in it.*

E.g 4-tuple of solution above.

$$(x_1, x_2, x_3, x_4) = (-5 - x_2 + 10x_4, x_2, 7 - 7x_4, x_4)$$

The tuple is always going to be parameterized, replacing x_1 with s's and x_2 with t's, etc.

$$(-5 - t + 10s, t, 7 - 7s, s)$$