| Assignment: Set 1.1 (5-10) Set 1.2 (1-22) | Assigned: 2020-08-19 | Due: n/a |

Set 1.1

Question 5-6: find the linear system in the unknowns $x_1, x_2 \ldots$.

5a $\begin{bmatrix} 2 & 0 & 0 \\ 3 & -4 & 0 \\ 0 & 1 & 1 \end{bmatrix}$

$$2x_1 = 0$$
$$3x_1 - 4x_2 = 0$$
$$x_2 = 1$$

5b $\begin{bmatrix} 3 & 0 & -2 & 5 \\ 7 & 1 & 4 & -3 \\ 0 & -2 & 1 & 7 \end{bmatrix}$

$$3x_1 - 2x_3 = 5$$
$$7x_1 + x_2 + 4x_3 = -3$$
$$-2x_2 + x_3 = 7$$

6a $\begin{bmatrix} 0 & 3 & -1 & -1 & -1 \\ 5 & 2 & 0 & -3 & -6 \end{bmatrix}$

$$3x_2 - x_3 - x_4 = -1$$
$$5x_1 + 2x_2 - 3x_4 = -6$$

6b $\begin{bmatrix} 3 & 0 & 1 & -4 & 3 \\ -4 & 0 & 4 & 1 & -3 \\ -1 & 3 & 0 & -2 & -9 \\ 0 & 0 & 0 & -1 & -2 \end{bmatrix}$

$$3x_1 + x_3 - 4x_4 = 3$$
$$-4x_1 + 4x_3 + x_4 = -3$$
$$-x_1 + 3x_2 - 2x_4 = -9$$
$$-x_4 = -2$$

Question 7-8: find the augmented matrix.

7(a)

$$-2x_1 = 6$$
$$3x_1 = 8$$
$$9x_2 = -3$$

$$\begin{bmatrix} -2 & 6 \\ 3 & 8 \\ 9 & -3 \end{bmatrix}$$

7(b)

$$6x_1 - x_2 + 3x_3 = 4$$
$$5x_2 - x_3 = 1$$

$$\begin{bmatrix} 6 & -1 & 3 & 4 \\ 0 & 5 & -1 & 1 \end{bmatrix}$$

8(a)

$$3x_1 - 2x_2 = -1$$
$$4x_1 + 5x_2 = 3$$
$$7x_1 + 3x_2 = 2$$

$$\begin{bmatrix} 3 & -2 & -1 \\ 4 & 5 & 3 \\ 7 & 3 & 2 \end{bmatrix}$$

8(b)

$$2x_1 + 2x_3 = 1$$
$$3x_1 - x_2 + 4x_3 = 7$$
$$6x_1 + x_2 - x_3 = 0$$

$$\begin{bmatrix} 2 & 0 & 2 & 1 \\ 3 & -1 & 4 & 7 \\ 6 & 1 & -1 & 0 \end{bmatrix}$$

Question 9: determine whether 3-tuple is a solution of the linear system.

$$2x_1 - 4x_2 - x_3 = 1$$
$$x_1 - 3x_2 + x_3 = 1$$
$$3x_1 - 5x_2 - 3x_3 = 1$$

9(a) $(3, 1, 1)$

$$2(3) - 4(1) - (1) = 1 [true]$$
$$(3) - 3(1) + (1) = 1 [true] \implies \boxed{\text{Is a solution}}$$
$$3(3) - 5(1) - 3(1) = 1 [true]$$

9(b) $(3, -1, 1)$

$$2(3) - 4(-1) - (1) = 1 [false]$$
$$(3) - 3(-1) + (1) = 1 [false] \implies \boxed{\text{Is not a solution}}$$
$$3(3) - 5(-1) - 3(1) = 1 [false]$$

9(c) $(13, 5, 2)$

$$2(13) - 4(5) - (2) = 1 [false]$$
$$(13) - 3(5) + (2) = 1 [false] \implies \boxed{\text{Is not a solution}}$$
$$3(13) - 5(5) - 3(2) = 1 [false]$$

9(d) $\left(\frac{13}{2}, \frac{5}{2}, 2 \right)$

$$2 \left(\frac{13}{2} \right) - 4 \left(\frac{5}{2} \right) - (2) = 1 [true]$$
$$\left(\frac{13}{2} \right) - 3 \left(\frac{5}{2} \right) + (2) = 1 [true] \implies \boxed{\text{Is not a solution}}$$
$$3 \left(\frac{13}{2} \right) - 5 \left(\frac{5}{2} \right) - 3(2) = 1 [false]$$

Question 10: determine whether 3-tuple is a solution of the linear system.

$$x + 2y - 2z = 3$$
$$3x - y + z = 1$$
$$-x + 5y - 5z = 5$$

10(a) $\left(\frac{5}{7}, \frac{8}{7}, 1 \right)$

$$\left(\frac{5}{7} \right) + 2 \left(\frac{8}{7} \right) - 2(1) = 3 [false] \implies \boxed{\text{Is not a solution}}$$
$$3 \left(\frac{5}{7} \right) - \left(\frac{8}{7} \right) + (1) = 1$$
$$- \left(\frac{5}{7} \right) + 5 \left(\frac{8}{7} \right) - 5(1) = 5$$

10(b) $\left(\frac{5}{7}, \frac{8}{7}, 0 \right)$

$$\left(\frac{5}{7}\right) + 2\left(\frac{8}{7}\right) - 2(0) = 3 [true]$$

$$3\left(\frac{5}{7}\right) - \left(\frac{8}{7}\right) + (0) = 1 [true]$$

$$-\left(\frac{5}{7}\right) + 5\left(\frac{8}{7}\right) - 5(0) = 5 [true] \implies \boxed{\text{Is a solution}}$$

10(c) $(5, 8, 1)$

$$(5) + 2(8) - 2(1) = 3 [false] \implies \boxed{\text{Is not a solution}}$$
$$3(5) - (8) + (1) = 1$$
$$-(5) + 5(8) - 5(1) = 5$$

10(d) $\left(\frac{5}{7}, \frac{10}{7}, \frac{2}{7}\right)$

$$\left(\frac{5}{7}\right) + 2\left(\frac{10}{7}\right) - 2\left(\frac{2}{7}\right) = 3 [true]$$

$$3\left(\frac{5}{7}\right) - \left(\frac{10}{7}\right) + \left(\frac{2}{7}\right) = 1 [true]$$

$$-\left(\frac{5}{7}\right) + 5\left(\frac{10}{7}\right) - 5\left(\frac{2}{7}\right) = 5 [true] \implies \boxed{\text{Is a solution}}$$

10(e) $\left(\frac{5}{7}, \frac{22}{7}, 2\right)$

$$\left(\frac{5}{7}\right) + 2\left(\frac{22}{7}\right) - 2(2) = 3 [true]$$

$$3\left(\frac{5}{7}\right) - \left(\frac{22}{7}\right) + (2) = 1 [true]$$

$$-\left(\frac{5}{7}\right) + 5\left(\frac{22}{7}\right) - 5(2) = 5 [true] \implies \boxed{\text{Is a solution}}$$

Set 1.2

Question 1-2: Determine if in row eschelon, reduced row, both, or neither.

1(a)   Both

1(b)   Both

1(c)   Both

1(d)   Both

1(e)   Row eschelon form

1(f)   Both

1(g)   Row eschelon form

2(a)   Row eschelon form

2(b)   Neither

2(c)   Both

2(d)   Row Eschelon

2(e)   Neither

2(f)   Neither

2(g)   Both

Question 3-4: Solve the System.

3(a) $\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 5 \end{bmatrix}$

$$-2E_3 + E_2 \rightarrow E_2$$

$$\begin{bmatrix} 1 & -3 & 4 & 7 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$-4E_3 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & -3 & 0 & -13 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$3E_2 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -37 \\ 0 & 1 & 0 & -8 \\ 0 & 0 & 1 & 5 \end{bmatrix}$$

$$\boxed{(-37, -8, 5)}$$

3(b) $\begin{bmatrix} 1 & 0 & 8 & -5 & -6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$

$$-4E_3 + E_2 \rightarrow E_2$$

$$\begin{bmatrix} 1 & 0 & 8 & -5 & -6 \\ 0 & 1 & 0 & -13 & -5 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$$-8E_3 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & 0 & 0 & -13 & -22 \\ 0 & 1 & 0 & -13 & -5 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$$

$$\boxed{(-22 + 13x_4, -5 + 13x_4, 2 - x_4, x_4)}$$

3(c) $\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 1 & 6 & 5 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$-E_3 + E_2 \rightarrow E_2$$

$$\begin{bmatrix} 1 & 7 & -2 & 0 & -8 & -3 \\ 0 & 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$2E_2 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & 7 & 0 & 0 & -2 & -11 \\ 0 & 0 & 1 & 0 & 3 & -4 \\ 0 & 0 & 0 & 1 & 3 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\boxed{(-11 + 2x_4 - 7x_2, x_2, -4 - 3x_4, 9 - 3x_4, x_5)}$$

3(d) $\begin{bmatrix} 1 & -3 & 7 & 1 \\ 0 & 1 & 4 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\boxed{\text{No solution}}$$

4(a) $\begin{bmatrix} 1 & 0 & 0 & -3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 7 \end{bmatrix}$

$$\boxed{(-3, 0, 7)}$$

4(b) $\begin{bmatrix} 1 & 0 & 0 & -7 & 8 \\ 0 & 1 & 0 & 3 & 2 \\ 0 & 0 & 1 & 1 & -5 \end{bmatrix}$

$$\boxed{(8 + 7x_4, 2 - 3x_4, -5 - x_4, x_4)}$$

4(c) $\begin{bmatrix} 1 & -6 & 0 & 0 & 3 & -2 \\ 0 & 0 & 1 & 0 & 4 & 7 \\ 0 & 0 & 0 & 1 & 5 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$

$$\boxed{-2 + 6x_2 - 3x_5, x_2, 7 - 4x_5, 8 - 5x_5, x_5)}$$

4(d) $\begin{bmatrix} 1 & -3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

$$\boxed{\text{No solution}}$$

Question 5-8: solve by Gaussian elimination.

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$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$$

$$E_1 + E_2 \rightarrow E_2$$

$$-3E_1 + E_3 \rightarrow E_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$$-E_2 \rightarrow E_2$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & -10 & -2 & -14 \end{bmatrix}$$

$$10E_2 + E_3 \rightarrow E_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{bmatrix}$$

$$-\frac{1}{52}E_3 \rightarrow E_3$$

$$\begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$5E_3 + E_2 \rightarrow E_2$$

$$-2E_3 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$-E_2 + E_1 \rightarrow E_1$$

$$\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

$$\boxed{(3, 1, 2, x_4)}$$

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8   Question 5: Solve equations

$$x_1 + x_2 + 2x_3 = 8$$
$$-x_1 - 2x_2 + 3x_3 = 1$$
$$3x_1 - 7x_2 + 4x_3 = 10$$

$E1 + E2 \rightarrow E2$

$$x_1 + x_2 + 2x_3 = 8$$
$$+[-x_1 - 2x_2 + 3x_3 = 1]$$
$$\overline{}$$
$$-x_2 + 5x_3 = 9$$

$-1(E2) \rightarrow E2$

$$x_2 - 5x_3 = -9$$

$E3 - 3(E1) \rightarrow E3$

$$3x_1 - 7x_2 + 4x_3 = 10$$
$$-3[x_1 + x_2 + 2x_3 = 8]$$
$$\overline{}$$
$$-10x_2 - 2x_3 = -12$$

$-\frac{1}{10}E3 \rightarrow E3$

$$x_1 + x_2 + 2x_3 = 8$$
$$x_2 - 5x_3 = -9$$
$$x_2 + \frac{1}{5}x_3 = \frac{6}{5}$$

$E3 - E2 \rightarrow E3$

$$x_2 + \frac{1}{5}x_3 = \frac{6}{5}$$
$$-[x_2 - 5x_3 = -9]$$
$$\overline{}$$
$$\frac{26}{5}x_3 = \frac{51}{5}$$

$(\frac{5}{26})E3 \rightarrow E3$

$$x_1 + x_2 + 2x_3 = 8$$
$$x_2 - 5x_3 = -9$$
$$x_3 = \frac{51}{26}$$

$E2 + 5(E3) \rightarrow E2$

$$x_2 - 5x_3 = -9$$
$$+5[x_3 = \frac{51}{26}]$$
$$\overline{\qquad\qquad x_2 = \frac{21}{26}}$$

$$x_1 + x_2 + 2x_3 = 8$$
$$x_2 = \frac{21}{26}$$
$$x_3 = \frac{51}{26}$$

$E1 - E2 - 2(E3) \rightarrow E1$

$$x_1 + x_2 + 2x_3 = 8$$
$$-[x_2 = \frac{21}{26}]$$
$$-2[x_3 = \frac{51}{26}]$$
$$\overline{\qquad\qquad x_1 = 8 - \frac{21}{26} - \frac{102}{26}}$$
$$x_1 = \frac{85}{26}$$

$$x_1 = \frac{85}{26}$$
$$x_2 = \frac{21}{26}$$
$$x_3 = \frac{51}{26}$$

$$\boxed{\left(\frac{85}{26}, \frac{21}{26}, \frac{51}{26}\right)}$$

Question 2: solve equations.

$$2x_1 + 2x_2 + 2x_3 = 0$$
$$-2x_1 + 5x_2 + 2x_3 = 1$$
$$8x_1 + x_2 + 4x_3 = -1$$