DATE: 2020-09-22
ANNOUNCEMENTS:

0.1 Tangent Planes and Linear Approximations

Example 1 (Tangent Plane).

$$f(x,y) = x^2\sqrt{y}$$

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Finding $f(2,9)$ would be easy.
Finding $f(1.98,9.03)$ by hand would be hard.
Approximate $f(2,9)$. Do better than (12).
At the point on the surface at $(2,9)$, there is a tangent plane that touches at $(2,9,12)$.
The "old" z value is 12 and there will be a change in the x and y to afect z.

$$\text{old z-value } + \frac{\text{change in z}}{\text{change in x}}(\text{ change in x }) + \frac{\text{change in z}}{\text{change in y}}(\text{ change in y })$$

$$12 + \frac{\partial z}{\partial x}(x-2) + \frac{\partial y}{\partial x}(y-9)$$

Theorem 1. *Suppose f has continuous partial derivatives. An equation of the tangetn plane to the surface $z = f(x,y)$ at the point (x_0, y_0, z_0) is*

$$\boxed{z = z_0 + f_x(x_0,y_0)(x-x_0) + f_y(x_0,y_0)(y-y_0)} \tag{1}$$

Tangent Plane

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Example 2 (Tangent Plane Cont'd). *Approximate $f(1.98,9.03)$ using the equation of the tangent plane at $(2,9,12)$.*

$$f(x,y) = x^2\sqrt{y}$$

1. $f(2,9) = 12$
2. $f_x(x,y) = 2x\sqrt{y} \to f_x(2,9) = 2(2)\sqrt{9} = 12$
3. $f_y(x,y) = \frac{1}{2}x^2y^{-\frac{1}{2}} \to f_y(2,9) = \frac{2^2}{2\sqrt{9}} = \frac{2}{3}$
4. *Tangent Plane*

$$z = 12 + 12(x-2) + \frac{4}{9}(y-9)$$

5. *Solve for approximation*

$$\begin{aligned}
f(1.98, 9.03) &\approx 12 + 12(1.98 - 2) + \frac{2}{3}(9.03 - 9) \\
&\approx 12 + 12(\frac{-2}{100}) + \frac{2}{3}(\frac{3}{100}) \\
&\approx 12 - \frac{24}{100} + \frac{2}{100} \\
&\approx 12 - 0.22 \\
&\approx \boxed{11.78}
\end{aligned}$$

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