DATE: 2020-09-14

ANNOUNCEMENTS:

Assignment: 1.8 1,3,5,7,9,11,15,17,19

0.1 (1.8) Matrix Transformations

Definition 1 (Transformations). *If f is a function with Domain $\mathbb{R}^n$ and codomain $\mathbb{R}^m$, then we say that f is a transformation from $\mathbb{R}^n \to \mathbb{R}^m$ or that f maps from $\mathbb{R}^n$ to $\mathbb{R}^m$ which we denote by writing*

$$f : \mathbb{R}^n \to \mathbb{R}^m$$

If $m = n$, then the transformation is sometimes called an operator on $\mathbb{R}^n$. •

Example 1 (transformations).

$$\begin{aligned} 3x_1 + 4x_2 - 5x_3 + x_4 &= w_1 \\ 7x_1 - 9x_2 + 3x_3 - x_4 &= w_2 \end{aligned}$$

$$\implies \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 3 & 4 & -5 & 1 \\ 7 & -9 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

4 dimensional space would be the domain, and this transformation maps it to an output with a 2 dimensional codomain.

$$f : \mathbb{R}^2 \to \mathbb{R}^3$$

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0.1.1 Matrix Transformation Notation

When you can accomplish what you want to do with just one matrix multiplication:

$$w_{(m\times1)} = A_{(m\times n)} x_{(n\times1}$$

Then this tranformation is called Matrix Transformation, can use Matrix Transformation Notation

$$T_A : R^n \to R^m$$

$$w = T_A(x)$$

Example 2 (Matrix transformations). .

A)

$$\begin{aligned} w_1 &= x_1^2 + 3x_2 + x_3 \\ w_2 &= x_1^2 + 3x_2x_3 \end{aligned}$$

B)

$$w_1 = 4x_1 - 2x_2 + x_3$$
$$w_2 = 5x_1 - 7x_2 + 7x_3$$

A and B are both $\mathbb{R}^3 \to \mathbb{R}^2$
A) Very different situation

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} x_1 & 3 & 1 \\ x_1 & 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

This is a transformation, but we cannot use matrix multiplication so all we can say is $f : \mathbb{R}^3 \to \mathbb{R}^2$
B)

$$\begin{bmatrix} w_1 \\ w_2 \end{bmatrix} = \begin{bmatrix} 4 & -2 & 1 \\ 5 & -7 & 7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$T_A : \mathbb{R}^3 \to \mathbb{R}^2$$
$$A = \begin{bmatrix} 4 & -2 & 1 \\ 5 & -7 & 7 \end{bmatrix}$$
$$w = Ax$$
$$w = T_A(x)$$

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Example 3 (matrix transformations).

$$w_1 = 6x_1 - 5x_2 + 7$$
$$w_2 = x_2 - 3x_2 + 4$$
$$w_3 = x_1 - 7x_2 + 5$$
$$\mathbb{R}^2 \to \mathbb{R}^3$$
$$\begin{bmatrix} w_1 \\ w_2 \\ w_3 \end{bmatrix} = \begin{bmatrix} 6 & -5 \\ 1 & -3 \\ 1 & -7 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

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Theorem 1 (matrix transformation proporties). *For every matrix A, the matrix transformation* $T_A : \mathbb{R}^n \to \mathbb{R}^m$ *has the following proporties:*

1. $T_A(0) = 0$
2. $T_A(kx) = kT_A(x)$
3. $T_A(x_1 + x_2) = T_A(x_1) + T_A(x_2)$
4. $T_A(x_1 - x_2) = T_A(x_1) = T_A(x_2)$

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You should be able to identify the Domain and Codomain. Find matrix of transformation. Tell whether or not someting is a matrix transformation.