Date: 2020-09-09
Announcements:

0.0.1 Inversion Algorithm

To find the inverse of an invertible matrix A, find a sequence of Elementary row operations that reduce A to the Identity and then perform that same sequence of operations on I_n to obtain A^{-1}

Example 1 (Inversion Algorithm)**.**

$$A = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix}$$

$$R_1 + R_2 \to R_1$$

$$A = \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ -1 & 3 & 4 & | & 0 & 1 & 1 \\ 0 & 4 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\to \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 4 & 4 & | & 1 & 1 & 0 \\ 0 & 4 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

$$\frac{1}{4}R_2 \to R_2$$

$$\to \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 4 & 3 & | & 0 & 0 & 1 \end{bmatrix}$$

$$-4R_2 + R_3 \to R_3$$

$$\to \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & -1 & | & -1 & -1 & 1 \end{bmatrix}$$

$$-R_3 \to R_3$$

$$\to \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 1 & | & \frac{1}{4} & \frac{1}{4} & 0 \\ 0 & 0 & 1 & | & 1 & 1 & -1 \end{bmatrix}$$

If a zero results in the main diagonal of the matrix you are inverting, it is not invertible.

$$-R_3 + R_2 \to R_2$$

$$\to \begin{bmatrix} 1 & 1 & 0 & | & 1 & 0 & 0 \\ 0 & 1 & 0 & | & \frac{-3}{4} & \frac{-3}{4} & 1 \\ 0 & 0 & 1 & | & 1 & 1 & -1 \end{bmatrix}$$

$$-R_2 + R_1 \to R_1$$

$$\to \begin{bmatrix} 1 & 0 & 0 & | & \frac{7}{4} & \frac{3}{4} & 1 \\ 0 & 1 & 0 & | & \frac{-3}{4} & \frac{-3}{4} & 1 \\ 0 & 0 & 1 & | & 1 & 1 & -1 \end{bmatrix}$$

$$\boxed{A^{-1} = \begin{bmatrix} \frac{7}{4} & \frac{3}{4} & 1 \\ \frac{-3}{4} & \frac{-3}{4} & 1 \\ 1 & 1 & -1 \end{bmatrix}}$$

To prove it, multiply the origional by the inverse to get the identity:

$$AA^{-1} = I$$

$$A^{-1}A = I$$

$$AA^{-1} = \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} \begin{bmatrix} \frac{7}{4} & \frac{3}{4} & 1 \\ \frac{-3}{4} & \frac{-3}{4} & 1 \\ 1 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 01 & 0 & \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1}A = \begin{bmatrix} \frac{7}{4} & \frac{3}{4} & 1 \\ \frac{-3}{4} & \frac{-3}{4} & 1 \\ 1 & 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 0 \\ -1 & 3 & 4 \\ 0 & 4 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 01 & 0 & \\ 0 & 0 & 1 \end{bmatrix}$$