

DATE: 2020-09-23

ANNOUNCEMENTS:

Oct2 , Friday, Test CH2: 2.1, 2.2, 2.3

HW: 2.2 1-21 Odd

Example 1 (Determinant of 4x4).

$$\det(A) = |A|$$

$$A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & -1 & 5 \\ 3 & 0 & 1 & 4 \\ 0 & 1 & 2 & -1 \end{bmatrix}$$

$$\begin{bmatrix} + & - & & \\ & + & & \\ & & - & \\ & & & + \end{bmatrix}$$

$$\begin{aligned} \det(a) &= -2 \begin{bmatrix} 2 & -1 & 5 \\ 3 & 1 & 4 \\ 0 & 2 & -1 \end{bmatrix} + 0 = 0 + 1 \begin{bmatrix} 1 & 0 & 1 \\ 2 & -1 & 5 \\ 3 & 1 & 4 \end{bmatrix} \\ &= -2[0 - 2(8 - 15) - 1(2 + 3)] + 1[1(-4 - 5) - 0 + 1(2 + 3)] \\ &= -2(14 - 5) + (-9 + 5) = -22 \end{aligned}$$

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Theorem 1 (Determinant of triangular matrix). *If A is an $n \times n$ triangular matrix (upper, lower, diagonal), then the $\det(A)$ is the product of the entries on the main diagonal.*

$$\det A = a_{11}a_{22}a_{33} \dots a_{nn}$$

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0.1 Evaluating Determinants by Row Reduction

Theorem 2. *Let A be an $n \times n$ matrix. If A has a row of zeros, then $\det A = 0$*

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Theorem 3. *Let A be an $n \times n$ matrix. Then $\det A = \det(A^T)$*

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Theorem 4. *Let A be an $n \times n$ matrix.*

1. *If B is the matrix that results when a single row or single column of A is multiplied by a scalar k , then*

$$\det B = k \det A.$$

2. *If B is the matrix that results when two rows or two columns of A are interchanged, then*

$$\det B = -\det A.$$

3. If B is the matrix obtained when a multiple of one row or one column is added to another, then

$$\det B = \det A$$

Example 2.

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 3 & 6 & 7 & 1 \\ 4 & 8 & 9 & 2 \\ 5 & 3 & 1 & 7 \end{bmatrix}$$

find $\det A$