Date: 2020-09-04

Announcements:

Assignment: section 1.5 (1-20)

Watch weekend video.

0.1 Elementary Matrices and Methods for Finding A^{-1}

Definition 1 (Row Equivalent). *Matrices A and B are called Row Equivalent if either is obtained from the other by a sequence of row operations.*

Definition 2 (Elementary Matrix). *A matrix E is called an elementary matrix if it can be obtained from the identity matrix by performing one row operation.*

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$E = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The product EA, the result is the same as performing the elementary row operation on a yourself.

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$2R_2 \to R_2$$

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 8 & 10 & 12 \\ 7 & 8 & 9 \end{bmatrix}$$

Is the same as:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

Theorem 1. *Every elementary matrix is invertible and its inverse is also an elementary matrix.*

Equivalent Statements Theorem

Theorem 2. *If A is an $n \times n$ matrix, then the following are equivalent, that is all are true or all are false,*

A A is invertible.

B $Ax = 0$ has only the solution c $\begin{bmatrix} 0 \\ 0 \\ \cdots \\ 0 \end{bmatrix}$ *(trivial solution).*

C The reduced row echelon form of A is I_n.

D A is expressible as a product of elementary matrices.

Proof: equivalent statements.

- Assume A true $\rightarrow$ B true $\rightarrow$ C true $\rightarrow$ D true $\rightarrow$ A true
- Assume A is invertible $\rightarrow A^{-1}$ exists and $AA^{-1} = A^{-1}A = I_n$
- $Ax = 0$
- $(A^{-1}Ax = A^{-1}0 \rightarrow Ix = \begin{bmatrix} 0 \\ 0 \\ .. \\ 0 \end{bmatrix}$
- $\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1n} & | & 0 \\ a_{21} & a_{22} & \dots & a_{2n} & | & 0 \\ a_{n1} & a_{n2} & \dots & a_{nn} & | & 0 \end{bmatrix} \begin{bmatrix} x_1 & & & | & 0 \\ & x_2 & & | & 0 \\ & & x_3 & | & 0 \end{bmatrix}$
- $[A|0]$, perform row operations and get $[I|0]$

$$E_n \dots E_2 E_1 A = I$$

$$E_1^{-1} E_2^{-1} \dots E_{n-1}^{-1} E_n^{-1} E_n \dots E_2 E_1 A = E_1^{-1} E_2^{-1} \dots E_{n-1}^{-1} E_n^{-1} I$$

- (theorem) If A and B are invertible then (AB) is invertible $\rightarrow$

$$(AB)^{-1} = B^{-1}A^{-1}$$

- $\rightarrow$ A is true.

$\square$