

Chapter 1

Systems of Linear Equations and Matrices (Cont'd)

DATE: 2020-08-19

ANNOUNCEMENTS:

20 minutes missed in lecture time will be posted as a video later today on blackboard.

Assignment: Section 1.2 Numbers 5, 6, 7, 8

1.1 Introduction

Working towards solving systems of equations in a systematic way, vs. the algebra days when you attack it at any angle with substitution and elimination. This systematic way is setting up for the theoretical

Example 1 (1). 1. take a system of equation like this

$$3x + 3y - 2z = 13$$

$$6x + 2y - 5z = 13$$

$$7x + 5y - 3z = 26$$

2. Use the algebraic operations:

(a) Multiply an equation by a nonzero constant

(b) Interchange any equation

(c) Add a constant times one equation to another equation

$$-1E2 + E3 \rightarrow E3$$

$$-6x - 2y + 5z = 13$$

$$7x + 5y - 3z = 26$$

$$x + 3y + 2z = 13$$

to get a variable in the first position with a coefficient of 1 (The equation you add to [$E3$] is always the equation you replace.)

$$6x + 2y - 5z = 13$$

$$3x + 3y - 2z = 13$$

$$x + 3y + 2z = 13$$

Interchange to get the equation with coefficient 1 on top, denoted like this:

$$E1 \leftrightarrow E3$$

$$x + 3y + 2z = 13$$

$$6x + 2y - 5z = 13$$

$$3x + 3y - 2z = 13$$

3. Use the x to eliminate the x 's from the other equations below.

$$-6E1 + E2 \rightarrow E2$$

$$-3E1 + E3 \rightarrow E3$$

$$x + 3y + 2z = 13$$

$$-16y - 17z = -65$$

$$-6y - 8z = -26$$

4. Get the next variable in column 2 with coefficient 1.

$$\frac{-1}{16}E2 \rightarrow E2$$

$$x + 3y + 2z = 13$$

$$y + \frac{17}{16}z = \frac{65}{16}$$

$$-6y - 8z = -26$$

5. Use the y to eliminate the y 's from below.

$$6E1 + E3 \rightarrow E3$$

$$x + 3y + 2z = 13$$

$$y + \frac{17}{16}z = \frac{65}{16}$$

$$-\frac{13}{8}z = -\frac{13}{8}$$

6. Get the last variable in column 3 with coefficient 1. (Continue until diagonal of coefficient 1)

$$\frac{-8}{13}E3 \rightarrow E3$$

$$x + 3y + 2z = 13$$

$$y + \frac{17}{16}z = \frac{65}{16}$$

$$z = 1$$

7. Work to eliminate variables from the bottom up.

$$-\frac{17}{16}E_3 + E_2 \rightarrow E_2$$

$$-2E_3 + E_1 \rightarrow E_1$$

$$x + 3y = 11$$

$$y = 3$$

$$z = 1$$

$$-3E_2 + E_1 \rightarrow E_1$$

$$x = 2$$

$$y = 3$$

$$z = 1$$

8. Write solution as ordered tripple: $(2, 3, 1)$

Because this system of equations could be written in this way, it has **only one** solution. There are other cases where there are problems in solving the system this way, a situation where there is no solution or infinite solutions.

Example 2 (2). Solve:

$$x + y + z = 1$$

$$-2x + y + z = -2$$

$$3x + 6y + 6z = 5$$

$$2E_1 + E_2 \rightarrow E_2$$

$$-3E_1 + E_3 \rightarrow E_3$$

$$x + y + z = 1$$

$$3y + 3z = 0$$

$$3y + 3z = 2$$

$$\frac{1}{3}E_2 \rightarrow E_2$$

$$x + y + z = 1$$

$$y + z = 0$$

$$3y + 3z = 2$$

$$-3E_2 + E_3 \rightarrow E_3$$

$$x + y + z = 1$$

$$y + z = 0$$

$$0 = 2 \leftarrow \text{false, no solution}$$

You could make the determination of no solution earlier, as soon as you catch the false statement.

Example 3 (3).

$$2x - y_z = -1$$

$$x + 3y - 2z = 2$$

$$-5x + 6y - 5z = 5$$

$$E_1 \leftrightarrow E_2$$

$$\begin{aligned}x + 3y - 2z &= 2 \\2x - y + z &= -1 \\-5x + 6y - 5z &= 5\end{aligned}$$

$$\begin{aligned}-2E_1 + E_2 &\rightarrow E_2 \\5E_1 + E_3 &\rightarrow E_3\end{aligned}$$

$$\begin{aligned}x + 3y - 2z &= 2 \\-7y + 5z &= -5 \\21y - 5z &= 15\end{aligned}$$

$$\frac{-1}{7}E_2 \rightarrow E_2$$

$$\begin{aligned}x + 3y - 2z &= 2 \\y - \frac{5}{7}z &= \frac{5}{7} \\21y - 5z &= 15\end{aligned}$$

$$-21E_2 + E_3 \rightarrow E_3$$

$$\begin{aligned}x + 3y - 2z &= 2 \\y - \frac{5}{7}z &= \frac{5}{7} \\0 &= 0\end{aligned}$$

Where the last equation is a **true** statement, but there is no coefficient of 1, is a situation where there are **infinite solutions**. You must continue to eliminate up with what you got.

$$-3E_2 + E_1 \rightarrow E_1$$

$$\begin{aligned}x + \frac{1}{7}z &= -\frac{1}{7} \\y - \frac{5}{7}z &= \frac{5}{7} \\0 &= 0\end{aligned}$$

This is now in as few variables as possible, and can give us a final solution

$$\begin{aligned}x + \frac{1}{7}z &= -\frac{1}{7} \rightarrow \boxed{x = -\frac{1}{7}z - \frac{1}{7}} \\y - \frac{5}{7}z &= \frac{5}{7} \rightarrow \boxed{y = \frac{5}{7}z + \frac{5}{7}} \\0 &= 0\end{aligned}$$

$$(x, y, z) = \left(-\frac{1}{7}z - \frac{1}{7}, \frac{5}{7}z + \frac{5}{7}, z \right)$$

Thinking about this solution in terms of vectors, $z(-\frac{1}{7}, \frac{5}{7}, 1) + (-\frac{1}{7}, \frac{5}{7}, 0)$, this describes a line.

Example 4 (4).

$$\begin{aligned} x + 3y + 4z &= 1 \\ 2x + 6y + 8z &= 2 \\ 3x + 9y + 12z &= 3 \end{aligned}$$

$$\begin{aligned} -2E_1 + E_2 &\rightarrow E_2 \\ -E_1 + E_3 &\rightarrow E_3 \end{aligned}$$

$$\begin{aligned} x + 3y + 4z &= 1 \\ 0 &= 0 \\ 0 &= 0 \end{aligned}$$

both equations with no variables are true equations, therefore **infinite solutions**, and we have one equation to solve for x .

$$x + 3y + 4z = 1 \rightarrow x = -3y - 4z + 1$$

$$(x, y, z) = (-3y - 4z + 1, y, z)$$

You can look at this as the sum of three vectors,

$$(-3y, y, 0) + (-4z, 0, z) + (1, 0, 0) \text{ or } y(-3, 1, 0) + z(-4, 0, 1) + (1, 0, 0)$$

This is just a geometric interpretation, whatever it means, but it is not a line