Date: 2020-09-12

Announcements:

0.0.1 Curvature

Definition 1 (Curvature)**.** *.*

- *A measure of how quickly the curve changes direction at that point*
- *The magnitude of the rate of change of the unit tangent vector with respect to arc length.*

$$\kappa = \left| \frac{d\overline{T}}{ds} \right|$$

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Problem: don't want to use the arclength.

$$\overline{T}(t) \to \frac{d\overline{T}}{dt}$$

$$\frac{d\overline{T}}{dt} \cdot \frac{dt}{ds} \to$$

$$\left| \frac{d\overline{T}}{dt} \right|$$

$$\frac{\frac{d\overline{T}}{ds}}{\frac{ds}{dt}}$$

$$\implies \kappa = \frac{\frac{d\overline{T}}{dt}}{\frac{ds}{dt}}$$

Arclength so far function:

$$s = \int_a^t |\overline{r}'(u)| \, du$$

$$\frac{ds}{st} = |\overline{r}'(t)|$$

$$\left| \frac{ds}{st} \right| = |\overline{r}'(t)|$$

Definition 2 (Curvature)**.**

$$\kappa = \frac{\left| \overline{T}'(t) \right|}{|\overline{r}'(t)|}$$

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Example 1 (Curvature).

$$\overline{r}(t) = \langle 3t+2, t, t-7 \rangle$$

, find κ •

Solution 1 (Curvature).

$$r'(t) = \langle 3, 1, 1 \rangle$$
$$|r'(t)| = \sqrt{9+1+1} = \sqrt{11}$$
$$\overline{T} = \frac{1}{\sqrt{11}} \langle 3, 1, 1 \rangle$$
$$\overline{T}' = \frac{1}{\sqrt{11}} \langle 0, 0, 0 \rangle$$
$$\overline{T}' = \langle 0, 0, 0 \rangle$$
$$\left| \overline{T}' \right| = 0$$
$$\kappa = \frac{0}{\sqrt{11}} = 0$$

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Example 2 (Curvature). *let* $\overline{r}(t) = \left\langle \sqrt{2}t, e^t, e^{-t} \right\rangle$*, Find* κ •

Solution 2 (Curvature).

$$r'(t) = \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$$
$$|r'(t)| = \sqrt{2 + e^{2t} + e^{-2t}}$$
$$|r'(t)| = \sqrt{e^{2t} + 2 + e^{-2t}}$$
$$|r'(t)| = \sqrt{(e^t + e^{-t})^2}$$
$$|r'(t)| = e^t + e^{-t}$$
$$\overline{T}(t) = \frac{1}{e^t + e^{-t}} \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$$
$$\overline{T}(t) = \frac{e^t}{e^{2t} + 1} \left\langle \sqrt{2}, e^t, -e^{-t} \right\rangle$$
$$\overline{T}(t) = \frac{1}{e^{2t} + 1} \left\langle \sqrt{2}e^t, e^{2t}, -1 \right\rangle$$
$$\overline{T}'(t) = \frac{1}{e^{2t} + 1} \left\langle \sqrt{2}e^t, 2e^{2t}, 0 \right\rangle + -1(e^{2t}+1)^{-2} 2e^{2t} \left\langle \sqrt{2}e^t, e^{2t}, -1 \right\rangle$$
$$\overline{T}'(t) = \left\langle \frac{\sqrt{2}e^t}{e^{2t}+1}, \frac{2e^{2t}}{e^{2t+1}}, 0 \right\rangle + \left\langle \frac{-2\sqrt{2}e^{3t}}{(e^{2t}+1)^2}, \frac{-2e^{4t}}{(e^{2t}+1)^2}, \frac{2e^{2t}}{(t^{2t+1})^2} \right\rangle$$
$$\overline{T}'(t) = \left\langle \frac{\sqrt{2}e^{3t} + \sqrt{2}e^t - 2\sqrt{2}e^{3t}}{(e^{2t}+1)^2}, \dots \right\rangle$$
$$\overline{T}'(t) = \left\langle \frac{-\sqrt{2}e^{3t} + \sqrt{2}e^t}{(e^{2t}+1)^2}, \dots \right\rangle$$
$$\overline{T}'(t) = \left\langle \frac{-\sqrt{2}e^{3t} + \sqrt{2}e^t}{(e^{2t}+1)^2}, \frac{2e^{2t}}{(e^{2t}+1)^2}, \frac{2e^{2t}}{(e^{2t}+1)^2} \right\rangle$$
$$\overline{T}'(t) = \frac{\sqrt{2}e^t}{(e^{2t}+1)^2} \left\langle 1 - e^{2t}, \sqrt{2}e^t, \sqrt{2}e^t \right\rangle$$

$$\left|c\overline{V}\right| = |c|\,|\overline{v}|$$

$$\left|\overline{T}'(t)\right| = \frac{\sqrt{2}e^t}{(e^{2t}+1)^2}\sqrt{1-2e^{2t}+e^{4t}+2e^{2t}+2e^{2t}}$$

$$\left|\overline{T}'(t)\right| = \frac{\sqrt{2}e^t}{(e^{2t}+1)^2}\sqrt{e^{4t}+2e^{2t}+1}$$

$$\left|\overline{T}'(t)\right| = \frac{\sqrt{2}e^t}{(e^{2t}+1)^2}\sqrt{(e^{2t}+1)^2}$$

$$\left|\overline{T}'(t)\right| = \frac{\sqrt{2}e^t}{e^{2t}+1}$$

$$\kappa = \frac{\frac{\sqrt{2}e^t}{e^{2t}+1}}{e^t+e^{-t}}$$

$$\kappa = \frac{\frac{\sqrt{2}e^{2t}}{e^{2t}+1}}{e^{2t}+1}$$

$$\kappa = \frac{\sqrt{2}e^{2t}}{(e^{2t}+1)^2}$$

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This example worked, but is a contrived textbook version of the problem. A better formula is still needed..