Lecture Notes

Physics 2

Fall 2020

Jasper Runco

Instructed by:
Emily Marshman

Contents

Chapter 1

Electric Charge and Electric Field

DATE: 2020-08-17

ANNOUNCEMENTS:

Instructor - Dr. Emily Marshman
Office Hours - M 12:00pm - 1:00pm, T 10:00am - 1:00pm, W 12:00pm - 1:00am
Email - emarshman@ccac.edu
Book - University Physics with Modern Physics by Young and Freedman, 15th edition

Assignment (Sept 20 11:59): Chapter 21 Homework

Assignment (Aug 31): Lab 1

1.1 Introduction

1.1.1 Learning outcomes

- How objects become charged, and how we know it's conserved.
- How to use Coulombs law.
- Distinction between electric force and field.
- How to use idea of electric field lines.
- Calculate properties of electric charge distributions.

1.2 Electric Charge and Electric Fields

1.2.1 Electric charge

When we rub glass rods with silk, the rods become charged and repel each other.

A charged plastic rod **attracts** a charged glass rod, and both attract the cloth.

This shows there are two kinds of charge.

Electric charge and the structure of matter

The particles of the atoms are:

- the negative electrons
- the positive protons
- the uncharged neutrons

Atoms and Ions

A neutral atom has the same number of protons as electrons
A **positive ion** has one or more electrons removed.
A **negative ion** has an excess of electrons.

1.2.2 Conservation of Charge

- The proton and electron have the same magnitude of charge.
- This magnitude is **quantized** unit of charge.
- **principle of charge conservation** states the sum of all charges in a closed system is constant.

$$\boxed{\text{1 Coulomb} = \text{charge on } 6.241 \times 10^{18} protons} \tag{1.1}$$

Coulomb

$$\boxed{\text{Charge on 1 proton, +e} = 1.6 \times 10^{-19} C} \tag{1.2}$$

Proton charge

$$\boxed{\text{Charge on 1 electron, -e} = -1.6 \times 10^{-19} C} \tag{1.3}$$

Electron charge

Example 1. *Common static electricity involves charges ranging from nanocoulombs to mircrocoulombs.*
(a) How many electrons are needed to form a charge of $-2.00nC$?
(b) How many electrons must be removed from a neutral object to leave a net charge of $0.500\mu C$

Solution 1 (a).

$$\begin{aligned} \text{Charge of } e &= -1.6 \times 10^{-19} C \\ -2.00\mu C &= -2.00 \times 10^{-9} C \implies \\ \frac{-2.00 \times 10^{-9} C}{-1.6 \times 10^{-19} C} &= 1.25 \times 10^{10} \text{ electrons} \end{aligned}$$

Solution 2 (b).

$$\begin{aligned} 0.500\mu C &= 0.500 \times 10^{-19} C \implies \\ \frac{0.500 \times 10^{-9} C}{-1.6 \times 10^{-19} C} &= 3.13 \times 10^{11} \text{ electrons removed} \end{aligned}$$

Definition 1 (Conductor)**.** *A material that allows charge to flow through it easily (most metals).*

Definition 2 (Insulator)**.** *a material that does not allow charge to flow through it easily (e.g. plastic, paper, nylon, wood).*

1.3 Conductors, Insulators, and Induced Charges

1.3.1 Charge by contact

Electrons are transfered by rubbing the negatively charged rod on the metal sphere.

When the rod is removed, the electron distribute themselves over the surface.

1.3.2 Without contact (Induction)

1. An uncharged metal ball stands on an insulator
2. free electrons in the metal ball are repelled by the excess in the rod, and shift away from the rod.
3. While the rod is near, connect the ball to the ground with a conducting wire.
4. Free electrons in the metal ball are repelled by the excess in the rod, and shift away from the rod.
5. disconnect the wire and a net positive charge is left on the ball. The earth acquires an equal negative charge.

1.3.3 Electric Forces on Uncharged Objects

- negative plastic come causes shifting of charges within the neutral insulator, called **polarization**
- a charged object of **either** sign exerts an **attractive** force on an uncharged insulator.

1.4 Coulomb's Law

The magnitude of the electric force between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.

$$\boxed{F_e = k\frac{|q_1 q_2|}{r^2}} \tag{1.4}$$

Coulomb's Law

The direction of the force depends on the relative sign of the charge.

$$\boxed{k = \frac{1}{4\pi\epsilon_0} = 9.0 \times 10^9 \frac{Nm^2}{C^2}} \tag{1.5}$$

Proportionality Constant

$$\boxed{\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{Nm^2}} \tag{1.6}$$

Electric constant

Example 2 (Applying Coulomb's law).

1.5 Electric Fields

A charged object modifies the properties of the space around it. E.g. in the vicinity of a positive charge, other positives repell and negatives attract

Gravitational field:

$$
\begin{aligned}
F_g &= \frac{GMm}{r^2} \\
\frac{F_g}{m} &= \frac{GM}{r^2} \\
&\text{near earths surface} \\
\frac{\overline{F}_g}{m} &= \overline{g} = 9.8\frac{m}{s^2} \\
\overline{F}_g &= m\overline{g}
\end{aligned}
$$

Electric field:

$$
\begin{aligned}
F_e &= \frac{k\,|Qq_0|}{r^2} \\
\frac{F_e}{q_0} &= \frac{kQ}{r^2} = \overline{E}
\end{aligned}
$$

force / charge : electric field of point charge

$$\boxed{\overline{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}} \tag{1.7}$$

Electric field of a point charge

ϵ_0 Electric constant

q - Value of point charge

r - Distance from point charge to where field is measured

$\hat{r}$ **-** unit vector from point charge towards where the field is measured

Direction: electric field points **radially inward** towards negative charge and **radially outward** for positive charge.

Chapter 2

Electric Fields Cont'd

DATE: 2020-08-24
ANNOUNCEMENTS:

2.1 Electric field lines

Definition 3 (Electric field line)**.** *An imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.*

- Field lines always start at a positive charge and end at a negative charge and can never cross.
- Show the direction of the electric field at each point.
- Spacing gives idea of magnitude of field at each point.

2.1.1 Electric field lines of a dipole

Definition 4 (Electric dipole)**.** *A pair of equal and opposite electric charges.*

- The field radiates outward from the positive and towards the negative.

Example 3 (1)**.** *A positive point charge $+Q$ is released from rest in an electric field. At any later time, the velocity of the point charge*

A. in the direction of the electric field? B. opposite the direction of the electric field? C. not enough information.

Solution 3 (1)**.** *The question does not state a uniform electric field, and remember field lines are not trajectories. C.*

Example 4 (2)**.** *What happens when I place an electric dipole in a uniform electric field pointing to the right.*

Net Force? Net Torque?

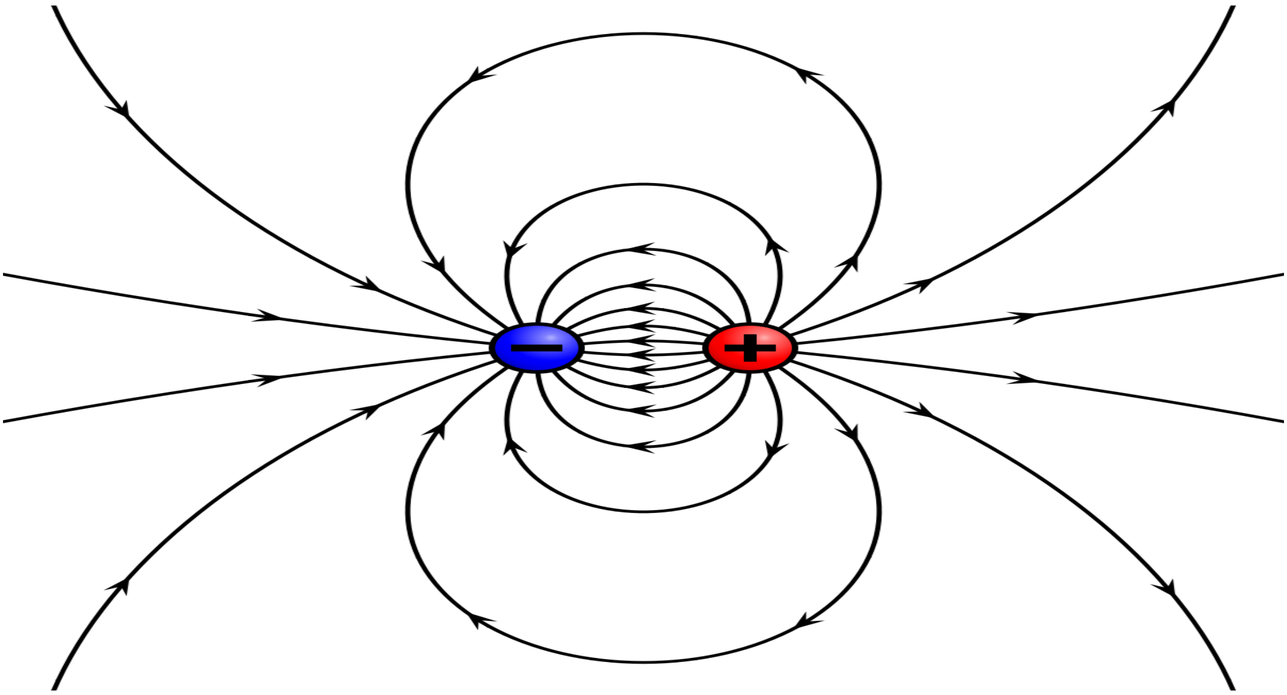


Figure 2.1: Dipole

Solution 4 (2)**.** *A. The force on the positive point charge is $F_+ = qE$ right.*
The force on the negative point charge pole is $F_- = qE$ left.
$\implies F_{net} = 0$
B. $\tau = \overline{r} \times \overline{F} = rF \sin\theta$
$\tau_+ = \left(\frac{d}{2}\right)(qE)\sin\theta$ *into page*
$\tau_- = \overline{r} \times \overline{F} = \left(\frac{d}{2}\right)(qE)\sin\theta$ *into page*
$\tau_{net} = qdE \sin\theta$ *into page*

Definition 5 (Electric dipole moment)**.**

$$\boxed{p = qd}$$

$\overline{p}$ *points from* $\boxed{-}$ *to* $\boxed{+}$
$\overline{p} \perp \overline{E}$ *, max torque*
$\overline{p} \parallel \overline{E}$ *, minimum torque*

$\tau_{net} = \overline{p} \times \overline{E}$

2.1.2 Electric field due to a charge distribution

- line of charge
- ring of charge
- disk of charge

Line of charge

Example 5 (3)**.** *What is the e-field? at point P?*

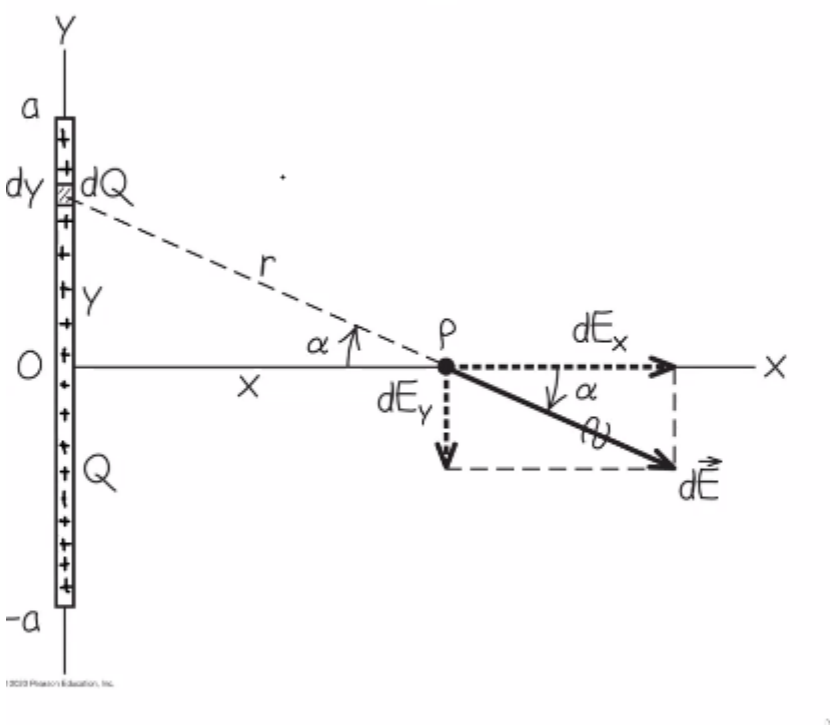


Figure 2.2: line of charge

Solution 5 (3).

$$
\begin{aligned}
dE &= \frac{kdQ}{r^2} \\
\int dE_y &= 0 \textit{ (symetry)} \\
dE_x &= \frac{kdQ}{r^2}\cos\alpha \\
E_x &= \int \frac{kdQ}{r^2}\cos\alpha \\
r^2 &= x^2 + y^2 \\
\cos\alpha &= \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}} \\
\textit{define linear charge density } \lambda &= \textit{ charge/length} \\
\lambda &= \frac{Q}{2a} \\
dQ &= \lambda dy \\
E_x &= \int \frac{k\lambda dyx}{(x^2+y^2)\sqrt{x^2+y^2}} \\
&= \int_{-a}^{a} \frac{k\lambda xdy}{(x^2+y^2)^{\frac{3}{2}}} \\
E_x &= \boxed{\frac{KQ}{x\sqrt{a^2+x^2}}}
\end{aligned}
$$

Example 6 (4). *Ring of charge*

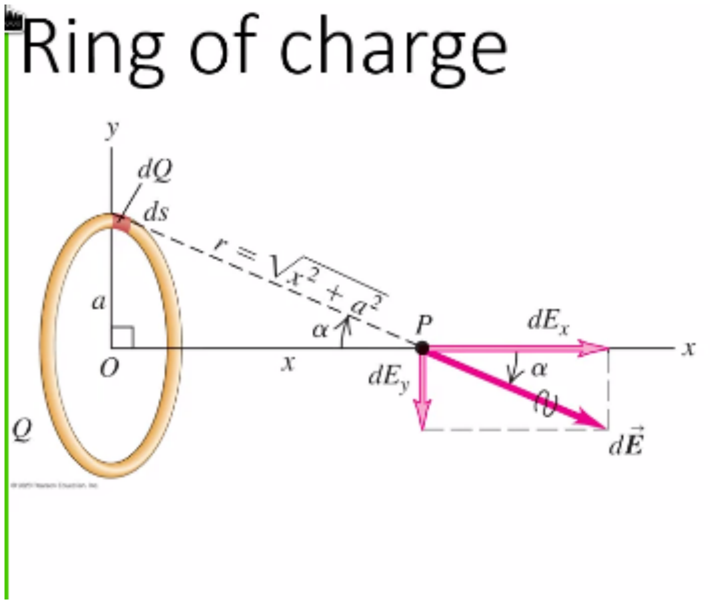


Figure 2.3: ring of charge

Solution 6 (4)**.** *Practice problem.*

Example 7 (Disk of charge)**.** *Find the electric field at point P.*

Solution 7 (Disk of charge)**.**

$$
\begin{aligned}
dE_x &= \frac{kdQ}{r^2}\cos\theta \\
r'^2 &= r^2 + x^2 \\
\cos\theta = \frac{x}{r'} &= \frac{x}{\sqrt{r^2+x^2}} \\
\textit{surface charge density}\,\sigma &= \frac{\textit{charge}}{\textit{area}} \\
dQ &= \sigma dA = \sigma 2\pi r dr \\
dA &= 2\pi r dr \\
\int dE_x &= \int_0^R \frac{k\sigma 2\pi r dr}{r^2+x^2}\frac{x}{\sqrt{r^2+x^2}} \\
E_x &= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{\frac{R^2}{x^2}+1}}\right]
\end{aligned}
$$

further: think about R»x

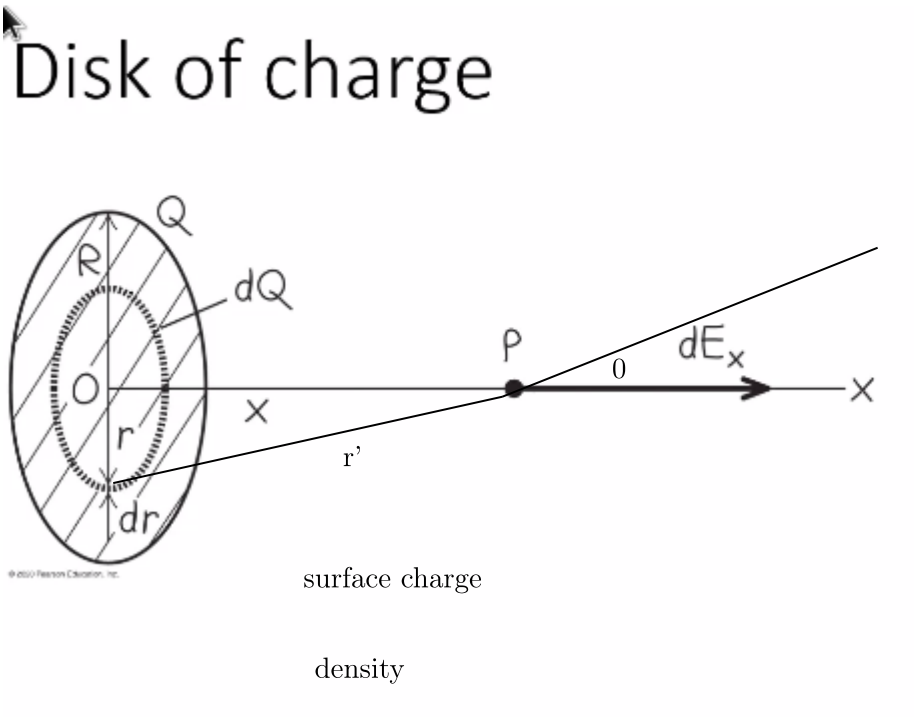


Figure 2.4: disk of charge

2.2 Chapter 22: Gauss's Law

2.2.1 Objectives

- Define electric flux and calculate through surfaces
- Define gauss's law
  - used to determine charge distribution given known e-field
  - to determine field given charge distribution
- know when Gauss's law can be used
- Do calculations with Gauss's law

Electric field due to a charged conducting sphere

We could use the electric field equations, but things get messy. Gauss's Law gives simplicity through symetry.

2.2.2 Intro to Gauss's law

- Given any general charge distribution, we surround it with an imaginary surface
- We look at the field at various points on imaginary surface
- GL is a relationship between the field at all points and total charge enclosed. it helps find the field for symmetric charge distributions.

2.2.3 Charge and electric flux

In boxes, there are positive charges within producing outward electric flux.

When boxes have negative charge inside, there is an inward electric flux.

When $\overline{E} = 0$ there is no electric flux in or out of the box.

What happens if there is no net charge inside the box?

- There is an electric field, but it flows in and out on either half
- Thus no net electric flux into or out of box.

What happens if there is charge near the box, but not inside?

- The net electric flux through the box is zero.

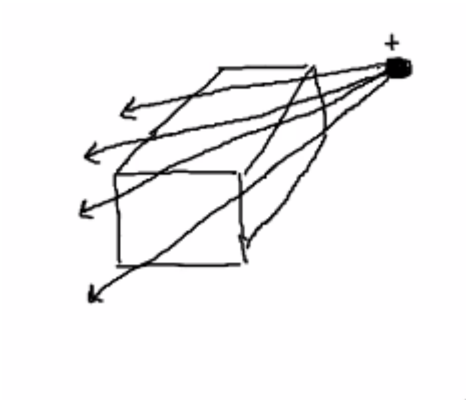


Figure 2.5: zero net flux

Quantifying electric flux

Net electric flux is **directly proportional** to net amount of charge within surface.

Net electric flux is **independent** of the size of the closed surface.

Example 8. *Suppose a Gaussian surface with rectangular sides and positive point charge $+q$ at it's center, and the surface doubles, but charge remains $+q$, what happens to the flux?*

Solution 8. *Remains the same*

Example 9. *Spherical Gaussian surface 1 has $+q$ at it's center. Spherical Gaussian surface 2, same size encloses the charge but is not centered on it. Compare flux through surface 1 and 2.*

Solution 9. *The same*

Calculating electric flux

- Consider a surface
- What affects amount of flux passing through surface?
- "net" analogy

Definition 6 (Electric flux)**.**

$$\begin{aligned}\Phi_E &= \Sigma \overline{E}_i \cdot d\overline{A}_i \\ &= \int E \cos \phi dA \\ &= \int E_\perp dA \\ &= \int \overline{E} \cdot d\overline{A} \\ &\rightarrow \int \overline{E} \cdot d\overline{A} \left(\frac{Nm^2}{C} \right)\end{aligned}$$

Φ_E **-** *Electric flux through a surface*

E **-** *Magnitude of Electric Field*

ϕ **-** *Angle between $\overline{E}$ and normal to surface*

dA **-** *Element of surface area*

$E_\perp$ **-** *Component of $\overline{E}$ perpendicular to surface*

$d\overline{A}$ **-** *Vector element of surface area*

Chapter 3

Gauss's Law

Date: 2020-08-31

Announcements:

Work on homework, reading quiz, and Lab 2

$$\boxed{\Phi_E = \oint \overline{E} \cdot d\overline{A} = Q_{encl} \frac{Q_{encl}}{\epsilon_0}} \tag{3.1}$$

Flux Equation

Φ_E **-** Electric flux through a closed surface of area A = surface integral of $\overline{E}$

Q_{encl} **-** Total charge enclosed by surface

ϵ_0 **-** Electric constant

Example 10 (Simplest case: Spherical symmetry)**.** *Use Gauss's Law to calculate electric field due to a point charge*

$$\overline{E} = \frac{kq}{r^2}\hat{r}$$

Solution 10. $|E|$ *same at all points on Gaussian surface*

$$\oint \overline{E} \cdot d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$

$$\oint E \cos\phi d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$

$$d\overline{A}$$

points normal and out from Gaussian surface
Because $\overline{E}$ and $d\overline{A}$ are both normal to the round surface, their angle is 0

$$\oint E dA = \frac{q}{\epsilon_0}$$

E constant on Gaussian surface

$$E \oint dA = \frac{q}{\epsilon_0}$$

$$E4\pi r^2 = \frac{q}{\epsilon_0}$$
$$E = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} = \frac{kq}{r^2}$$

radially outward.

3.0.1 Applications

Conductor

- Suppose we construct a Gaussian surface inside a conductor.
- Because $\overline{E} = 0$ everywhere on the surface, Gauss's law requires the net charge inside the surface be zero.
- Under **electrostatic** conditions (charges not in motion), any excess charge on a solid conductor resides entirely on the conductor's surface.
- **Therefore, the electric field inside a conductor is always zero**

Conductor with cavity

- Cavity inside a conductor
- If a charge of $1nC$ resides inside the cavity, and the field inside the conductor is always zero, then there must be a $-1nC$ charge distributed around the surface of the cavity.

Review

- Electric field due to a point charge: $\overline{E} = \frac{kq}{r^2}\hat{r}$
- Electric field due to an infinite line of charge: $\overline{E} = \frac{\lambda}{2\pi\epsilon_0 r}\hat{r}$ (λ linear charge density)
- Electric field due to an infinite sheet of charge: $\overline{E} = \frac{\sigma}{2\epsilon_0}$ uniform field (σ surface charge density)

Field of a charged conducting sphere

- Positive charge Q is distributed on spherical conductor with radius R. Find magnitude of electric field at point P and distance r from the center of the sphere.
- for $r < R$, we have symetry so we chose a Gaussian surface that is spherical with radus r.

$$E = 0$$

- for r>R, we construct a Gaussian surface enclosing the charged sphere.

$$\oint \overline{E} \cdot d\overline{A} = \frac{q_{enc}}{\epsilon_0}$$
$$\oint EdA\cos\phi = q_{enc}/\epsilon_0$$
$$\oint EdA\cos 0 = q_{enc}/\epsilon_0$$
$$\oint EdA = Q/\epsilon_0$$
$$E\oint dA = Q/\epsilon_0$$
$$E4\pi r^2 = Q/\epsilon_0$$
$$E = Q/4\pi r^2\epsilon_0$$
$$E = kQ/r^2$$

- The result is the same as for a point charge.

Field of uniformly charged sphere (insulator)

- Positive charge Q uniformly throughtout volume of **insulating** sphere with radius R. Find magnitude of field at point P a distance r from center.

$$\rho_{\text{ sphere}} = \text{ folume charge density, charge/volume}$$

- r<R

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

- The symmetry says the E-field points out and is constant across a gaussian surface.

$$\oint EdA\cos\phi = q_{enc}/\epsilon_0$$

$$\oint EdA = q_{enc}/\epsilon_0$$

$$E\oint dA = q_{enc}/\epsilon_0$$

$$E4\pi r^2 = q_{enc}/\epsilon_0$$
$$E4\pi r^2 = q_{enc}/\epsilon_0$$

- the question: what is q_{enc}?

$$q_{enc} = \rho V_{enc} = \frac{Q}{\frac{4}{3}\pi R^3}\frac{4}{3}\pi r^3$$

$$q_{enc} = \frac{Qr^3}{R^3}$$

$$E = \frac{1}{4\pi r^2\epsilon_0}\frac{Qr}{R^3} \text{ radially out}$$

- Now, what is the field outside?

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$E4\pi r^2 = \frac{Q}{\epsilon_0}$$

$$E = \frac{kQ}{r^2} \text{ radially out}$$

Field of a Uniform Line Charge

- We need to chose a Gaussian surface that is symetrical, so use a **cylinder**

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

- This has three components, the end caps and the round

$$\int_{end} \overline{E}d\overline{A} + \int_{end} \overline{E}d\overline{A} + \int_{round} \overline{E}d\overline{A}$$

- For the end caps, the field lines perpendicular to the area, so they contribute nothing to the flux

$$\int EdA\cos\phi = q_{enc}/\epsilon_0$$

$$E\int dA = q_{enc}/\epsilon_0$$

$$q_{enc} = \lambda l$$

$$E \cdot 2\pi rl = \lambda\frac{l}{\epsilon_0}$$

Infinite plane of charge

- planar symmetry

$$\sigma = \text{ charge / area}$$

- the Gaussian surface that takes advantage of this symmetry is a cylinder bisecting the plane
- cylinder: endcaps area A

$$\oint \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$\int_{end} \overline{E} \cdot d\overline{A} + \int_{end} \overline{E} \cdot d\overline{A} + \int_{round} \overline{E} \cdot d\overline{A} = q_{enc}/\epsilon_0$$

$$\int \overline{E} \cdot d\overline{A} \cos\phi + \int E dA \cos\phi = q_{enc}/\epsilon_0$$

$$2 \int E dA = q_{enc}/\epsilon_0$$

$$2EA = \sigma A/\epsilon_0$$

$$E = \sigma/2\epsilon_0 \text{ uniform outward from plane}$$

3.0.2 Conductors

$$\boxed{E_\perp = \frac{\sigma}{\epsilon_0}} \tag{3.2}$$

Electric field at surface of a conductor

Electrostatic shielding

- A conducting box immersed in uniform field
- the field of induced charges combines with the uniform field to give **zero** total field

Chapter 4

(23) Electrical Potential

DATE: 2020-09-07

ANNOUNCEMENTS:

4.1 Electric Potential Energy

4.1.1 Electrical Potential Energy in a Uniform field

- If a positive charge moves in the direction of the field, the field does **positive** work on the charge
- The potential energy **decreases**.
- If the positive charge moves opposite the direction of the field, the field does **negative** work on the charge.
- The potential energy increases.
- If a negative charge moves in the direction of the field, the field does **negative** work on the charge.
- The potential energy **increases**.
- If the negative charge moves opposite the direction of the field, the field does **positive** work on the charge.
- The potential energy **decreases**.

Electric Potential Energy of Two Point Charges

- Doesn't depend on path taken.
- Electric potential energy only depends on the distance between the charges.
- Defined to be zero when the charges are infinitely far apart.
- Charges with the same sign have positive electric potential energy.
- Charges with opposite signs have negative electric potential energy.

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\frac{qq_0}{r}} \tag{4.1}$$

Electric potenetial energy of two point charges

U Potential Energy

ϵ_0 Electric constant

q, q_0 Values of two charges

r Distance between two charges

Electrical Potential with Several Point Charges

- The potential energy with q_0 depends on the other charges and their distances.
- Electric potential energy is **Algebraic sum**.

$$\boxed{U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \dots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i q_{\frac{i}{r_i}}} \tag{4.2}$$

Electric potential energy q_0 due to a collection of charges

4.2 Electric Potential

Definition 7 (Electric Potential)**.** *Potential is potential energy per unit charge. The potential of a with respect to b ($V_{ab} = V_a - V_b$ equals the work done by the electric force when a unit charge moves from a to b.* •

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}} \tag{4.3}$$

Electric potential due to a point charge

q value of point charge.

r distance from point charge to where potential is measured.

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}} \tag{4.4}$$

Electric potential due to a collection of point charges

4.2.1 Electric Potential and Electric Field

- Moving in the direction of the electric field, the electric potential **decreases**.
- The direction of the electric field is the direction of decreasing V.
- To move against the E-field, an external force per unit charge must be applied opposite the electric force per unit charge.
- The electric force per unit charge is the E-field.

- The **potential difference** $V_a - V_b$ equals the work done per unit charge by the external force to move from b to a:

$$V_a - V_b = -\int_a^b \overline{E} \cdot d\overline{r}$$

- Electric field can be expressed as $1\frac{N}{C} = 1\frac{V}{m}$

4.2.2 Electron Volt

- Change in potential energy U, when a charge moves from a potential of V_b to a potential of V_a is

$$U_a - U_b = q(V_a - V_b).$$

- When the potential difference is 1V for a charge q with magnitude e of the electron charge, the change in energy is defined as one electron volt (eV).

$$\boxed{1eV = 1.602 \times 10^{-19} J} \tag{4.5}$$

Electron volt

4.2.3 Electric Potential and Field of a Charged Conductor

- A solid conducting sphere of radius R has a total charge of q.
- The electric field **inside** the sphere is zero everywhere.
- The potential is the **same** at every point inside the sphere and is equal to the value at the surface.

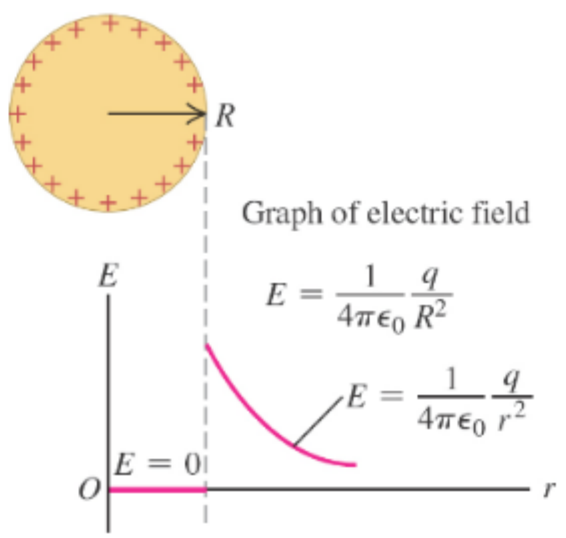
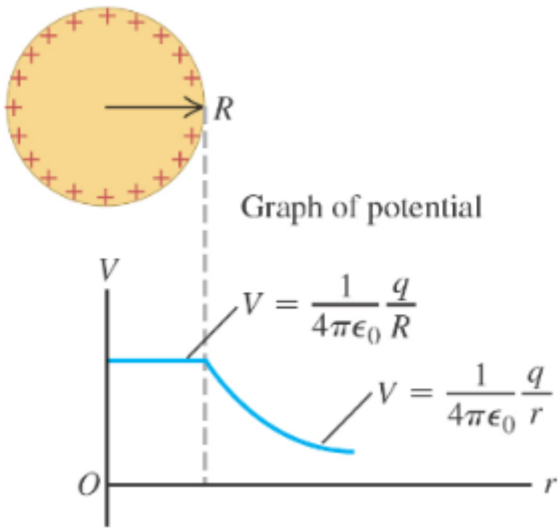


Figure 4.1: Electric field in a conducting sphere

Ionization and Corona Discharge

- Air becomes ionized (conductor) at or above $3 \times 10^6 \frac{V}{m}$.
- For a charged sphere, $V_{\text{surface}} = E_{\text{surface}} R$
- If E_m is the E-field magnitude at which air becomes conductive **(dielectric strength)**, then V_m is the maximum potential to which it can be raised: $V_m = RE_m$.



[1]

Figure 4.2: Electric potential in a conducting sphere

Oppositely Charged Parallel Plates

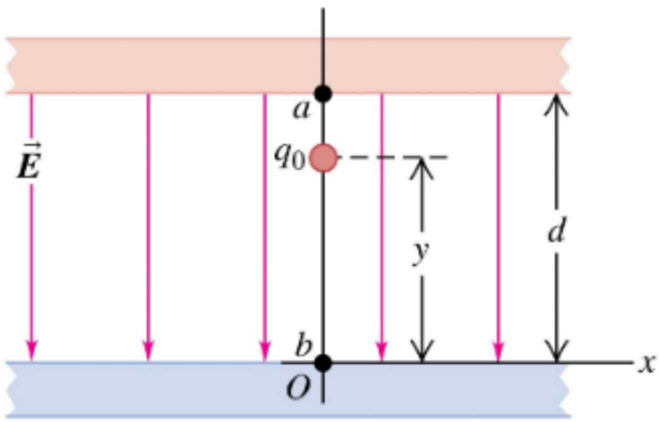
$$\boxed{V = Ey} \tag{4.6}$$

Potential at any height between two large oppositely charged parallel plates

V Potential (units $V = \frac{J}{C}$)

E Electric field magnitude (units $\frac{N}{C}$)

y height (m)



[1]

Figure 4.3: Potential between parallel plates

4.3 Equipotential Surfaces

4.3.1 Equipotential Surfaces and Field lines

- **Equipotential surfaces** have constant electric potential.
- Field lines and equipotential surfaces are always mutually perpendicular.

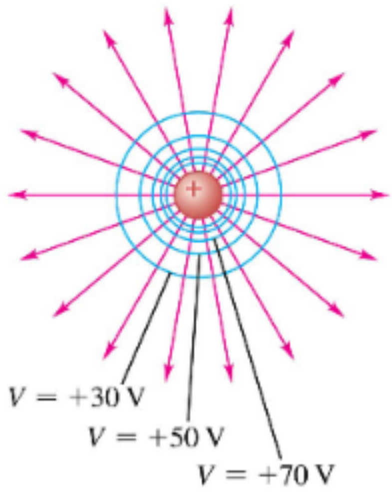


Figure 4.4: Equipotential surfaces for monopole

Equipotentials and Conductors

- When all charges are at rest
  - The surface of a conductor is always an equipotential surface
  - The electric field just outside a conductor is always perpendicular to the surface.
- If the electric field had a tangential component at the surface of a conductor, a net amount of work would be done on a test charge by moving it around a loop, which is impossible because the E-force is conservative.

4.3.2 Potential Gradient

- The components of the electric field can be found by partial derivatives of the electric potential.
- The electric field is the negative gradient of the potential.

$$\boxed{E_x = -\frac{\partial V}{\partial x}, E_y = -\frac{\partial V}{\partial y}, E_z = -\frac{\partial V}{\partial z}} \tag{4.7}$$

Components of electric field

$$\boxed{\overline{E} = -\Delta\overline{V}} \tag{4.8}$$

Electric field, negative gradient of potential

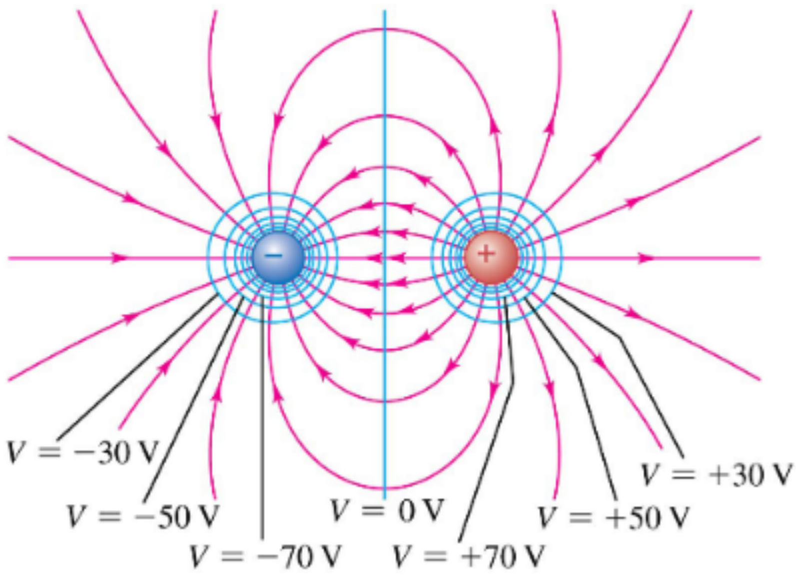


Figure 4.5: Equipotential surfaces for dipole

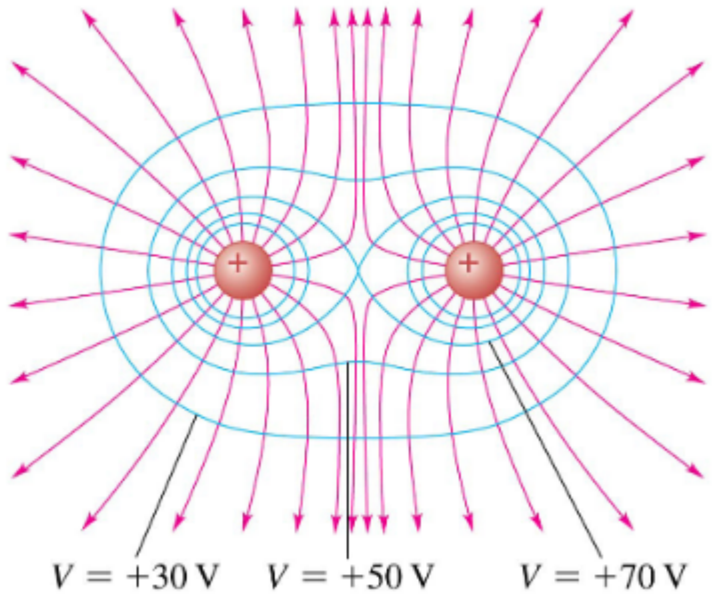


Figure 4.6: Equipotential surfaces for two equal charges

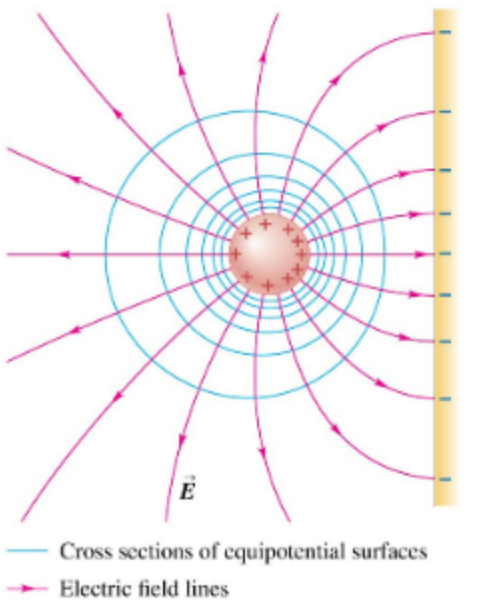


Figure 4.7: Equipotential surface and conductor

Chapter 5

(24) Capacitors and Dielectrics

Date: 2020-09-14

Announcements:

Exam: Can start on Tuesday, (4 hour time limit) Due Sunday 11:59 PM (Chapter 21-23) 23 questions - 14 conceptual 9 problem solving E field for point charges and charge distributions, relationship between force charge and electric field, use gausses law for e field for symmetric charge distribution and find charge distribution, find potential due to point charges, relationship between electric potential and electric potential energy, relationship between potential and electric field

Lab: due Sep 21

5.1 Capacitors and Capacitance

Definition 8 (Capacitor)**.** .

- *Two conductors separated y an insulator (or vacume)*
- *When **charged**, two conductors have equal magnitude and opposite sign*
- *Zero net charge*

-

$$\boxed{C = \frac{Q}{V_{ab}}} \tag{5.1}$$

Capacitance of a capacitor

Q Magnitude of charge on each conductor

V_{ab} Potential difference between $+Q$ and $-Q$

$$\boxed{F = \frac{C}{V}} \tag{5.2}$$

Farad - Unit of Capacitance

Definition 9 (Parallel plate capacitors)**.** .

- *Two parallel conducting plates separated by a small distance compared to dimensions.*
- *Uniform field and charge distribution over opposing surfaces.*

•

$$\boxed{C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}} \tag{5.3}$$

Capacitance of Parallel Plates

Q Magnitude of charge on each plate

A Area of each plate

d Distance between plates

.

The Electric field of parallel plates:

$$\begin{aligned} \sigma &= \text{ charge / area} \\ E &= \frac{\sigma}{\epsilon_0} \end{aligned}$$

5.2 Capacitors in Series and Parallel

5.2.1 Capacitors in Series

- The capacitors have the same charge Q.
- Their potential differences add:

$$V_{ac} + V_{cb} = V_{ab}$$

Finding Equivalent Capacitance:

The potential drop across both capacitors must be the sum of the potential drop across each. The charge on C_1 and C_2 must be the same for the individual capacitors.

$$\begin{aligned} C &= \frac{Q}{V} \\ Q_1 &= Q_2 \\ V_{ab} &= V_1 + V_2 \\ V &= \frac{Q_1}{C_1} + \frac{Q_2}{C_2} \\ V &= Q\left(\frac{1}{C_1} + \frac{1}{C_2}\right) \\ \frac{V}{Q} &= \frac{1}{C_1} + \frac{1}{C_2} \\ \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \end{aligned}$$

$$\boxed{\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}} \tag{5.4}$$

Series Equivalent Capacitance

5.2.2 Capacitors in Parallel

- Have same potential drop V.
- Charge on each depends on its capacitance:

$$Q_1 = C_1 V, Q_2 = C_2 V$$

Finding Equivalent Capacitance:

The potential drop across both capacitors must be the same. The charge on C_1 and C_2 must sum to the total charge.

$$\begin{aligned} C &= \frac{Q}{V} \\ Q &= Q_1 + Q_2 \\ V_1 &= V_2 \\ Q &= C_1 V_1 + C_2 V_2 \\ C_{eq} V &= C_1 V + C_2 V \\ C_{eq} &= C_1 + C_2 \end{aligned}$$

$$\boxed{C_{eq} = C_1 + C_2} \tag{5.5}$$

Parallel Equivalent Capacitance

5.3 Energy Storage in Capacitors and Electric-Field Energy

5.3.1 Energy Stored in a Capacitor

$$\boxed{U = \frac{Q^2}{2C} = \frac{1}{2} C V^2 = \frac{1}{2} Q V} \tag{5.6}$$

Capacitor Potential Energy

U Potential energy stored in a capacitor

Q Magnitude of charge on each plate

C Capacitance

V Potential difference between plates

Derivation of Capacitor Potential Energy

Energy stored in capacitor? → Calculate work done to charge capacitor.

$V = \frac{Q}{C}$ (Final charge is Q and final potential is V after charging)

$v = \frac{q}{C}$ (v and q at some intermediate time)

$$\begin{aligned}
W &= \Delta u = Q\Delta V \\
dW &= vdq \\
\int dW &= \int_0^Q \frac{q}{C} dq \\
W &= \frac{q^2}{2C}\Big|_0^Q \\
W &= \frac{Q^2}{2C}
\end{aligned}$$

5.3.2 Electric-Field Energy

energy density $= \frac{\text{energy}}{\text{volume}} = \frac{\frac{1}{2}CV^2}{Ad} = \frac{\frac{1}{2}\frac{\epsilon_0 A}{d}}{Ad} =$

$$\boxed{u = \frac{1}{2}\epsilon_0 E^2} \tag{5.7}$$

Electrical Density in Vacuum

u Energy density for a vacuum

E Magnitude of electric field

5.4 Dielectrics

Most capacitors have noncuducting material between plates, such as Mylar.

Increasing Capacitance

- Connect electrometer across a charged capacitor, with magnitude Q on each and potential difference V_0
- An uncharged sheet of dielectric between the plates, the potential difference decreases to smaller V value.
- Since Q is unchanged, capacitance $C = \frac{Q}{V}$ **increased**.
- The field decreases due to **polarization** within the dielectric, induced surface charges.

$$E = \frac{E_0}{K}$$

  E Decreased E-field

  K dielectric constant, unitless

$$\begin{aligned}
V &= Ed \\
\implies V &= \frac{V_0}{K} \\
\implies C &= KC_0
\end{aligned}$$

$$\boxed{\epsilon = K\epsilon_0} \tag{5.8}$$

Permitivity of a Dielectric

K dielectric constant, unitless

$$\boxed{C = KC_0 = K\epsilon_0 \frac{A}{d = \epsilon \frac{A}{d}}} \tag{5.9}$$

Capacitance of parallel-plate capacotor, dielectric between plates

$$\boxed{u = \frac{1}{2}K\epsilon_0 E^2 = \frac{1}{2}\epsilon E^2} \tag{5.10}$$

Electric energy density in a dielectric

Chapter 6

Currents, Resistance, and Electromotive Force

DATE: 2020-09-21
ANNOUNCEMENTS:

6.1 Current

Definition 10 (Current)**.** *A current is any motion of charge form one region to another.* •

Quantifying Current

- Model a wire as a cylinder:
- $n = \frac{\text{\# of moving charges}}{\text{volume}}$
- Volume $= dxA$
- Volume $= (\overline{v}_d dt)A$
- Amount of charge that folows through cylinder:

$$dQ = q \cdot \text{ number of charges } = q \cdot n \cdot V$$

$$dQ = qn\overline{v}_d dtA$$

$$\frac{dQ}{dt} = I = qn\overline{v}_d A$$

$$\boxed{I = n|q|v_d A} \tag{6.1}$$

Current

I Current

q charge per particle

n # of moving Charges

v_d Drift velocity

A Cross sectional area of conductor

units Ampers, Amps, A

Scalar quantity, cw or ccw in a curcuit

$$\boxed{\overline{J} = \frac{I}{A} = |q|\, n\overline{v}_d} \tag{6.2}$$

Vector Current Density

I Current

A Cross sectional area

units $\frac{A}{m^2}$

6.1.1 Direction of Current Flow

- **Conventional Current** is treated as a flow of positive charges.
- I a metallic conductor, the charges moving are electrons, but the conventional current points in the opposite direction.
- The vector **Current Density** is always in the same direction as the electric field.

6.2 Resistivity

Definition 11 (Resistivity)**.** *The resistivity of a material is the ratio of the electric field in the material to the current density it causes. A measure of a materia's opposition to flow. Depends on material and temperature.* ***Conductivity*** *is the recipricol of resistivity* •

$$\boxed{\rho = \frac{E}{J}} \tag{6.3}$$

Resistivity

rho Resistivity

E Electric field

J Current densityCurrent density

$$\boxed{\rho(T)v = \rho_0[1 + \alpha(T - T_0)]} \tag{6.4}$$

Temperature dependence of resistivity

6.3 Resistance

6.3.1 Resistance and Ohm's Law

The **resistance** of a conductor is $R = \rho\frac{L}{A}$.

The potential across a conductor is given by Ohm's Law:

$$\boxed{V = IR} \tag{6.5}$$

Ohm's Law

$$\boxed{R = \frac{\rho L}{A}} \tag{6.6}$$

Resistance Equation

L Length of conductor

A Cross sectional area

6.4 Electromotive Froce and Circuits

Electromotive force (emf) makes current flow from low to high potential. A circuit that provides emf is called a **source of emf**. SI units of $1V = 1\frac{J}{C}$. A flashlight battery has an emf of 1.5V; it does 1.5 J of work on every coulomb of charge that passes through it. Use symbol $\mathcal{E}$ (cursive E) for emf.

When a battery is connected, electrons must reach the positive charge. When they do:

$$W_{net} = W_b - W_c = \Delta KE = 0(\text{ steady state })$$

$$W_b = W_e$$

$$q\mathcal{E} = qV_{ab}$$

$$\mathcal{E} = V_{ab}$$

$U_i \implies$ powering circuit elements. Rise in PE = loss in potential energy.

$$\mathcal{E} = V_{ab} = IR$$

This is energy balance, what ever emf is put in is used by the resistors.

6.4.1 Internal Resistance

Real sources contain some **internal resistance, r**.

$$\boxed{V_{ab} = \mathcal{E} - Ir} \tag{6.7}$$

Potential of source with internal resistance

V Terminal voltage, source with internal resistance.

$\mathcal{E}$ emf of source.

I Current through source.

r Internal resistance of source.

-Ir Loss of potential in battery.

6.5 Energy and Power in Electric Circuits

- A circuit element causes a net transfer of energy into or out of the circuit.
- the time rate of energy transfer is power, denoted by P

$$\boxed{P = V_{ab} I = I^2 R = \frac{V_{ab}^2}{R}} \tag{6.8}$$

Power delivered to or extracted from a circuit element

V Voltage across circuit element.

I Current in circuit element.

Unit Watts, W