Chapter 1

Vector Functions

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ANNOUNCEMENTS:

1.1 Vector Functions and Space Curves

Vectors

$$
\begin{aligned}
\overline{r(t)} &= \langle f(t), g(t), h(t) \rangle \\
&= f(t)\hat{\imath} + g(t)\hat{\jmath} + h(t)\hat{k} \\
\overline{r(t)} &= \left\langle t^2 + 1, \frac{1}{t-1}, \sqrt{t+3} \right\rangle \\
\overline{r(2)} &= \left\langle 5, 1, \sqrt{5} \right\rangle
\end{aligned}
$$

Domain

$$
\begin{aligned}
t^2 + 1 &\rightarrow \text{ polonomial, Domain: } (-\infty, \infty) \\
\frac{1}{t+1} &\rightarrow \text{ Rational, Domain: } (-\infty, 1) \cup (1, \infty) \\
\sqrt{t+3} &\rightarrow \text{ Radical, Domain: } [-3, \infty) \\
\text{Domain of } \overline{r(t)} &= [-3, 1) \cup (1, \infty)
\end{aligned}
$$

Example 1 (Domain)**.**

$$
\overline{r(t)} = \left\langle \cos t, \ln(t + r), \frac{t}{t^2 - 23} \right\rangle
$$

$$\begin{aligned} \cos t &\to (-\infty, \infty) \\ \ln(t+4) &\to (-4, \infty) \\ \frac{t}{t^2-25} &\to (-\infty,-5) \cup (-5,5) \cup (5,\infty) \\ \text{Domain } &= (-4,5) \cup (5,\infty) \end{aligned}$$

Graphs

2-D plane curve

3-D space curve

Plane curve

$$\overline{r(t)} = \langle t-3, t^2 \rangle$$

$$\begin{array}{l|l} \ldots & \\ t=-2 & \langle -5, 4 \rangle \\ t=-1 & \langle -4, 1 \rangle \\ t=0 & \langle -3, 0 \rangle \\ t=1 & \langle -2, 1 \rangle \\ t=2 & \langle -1, 4 \rangle \\ \ldots & \end{array}$$

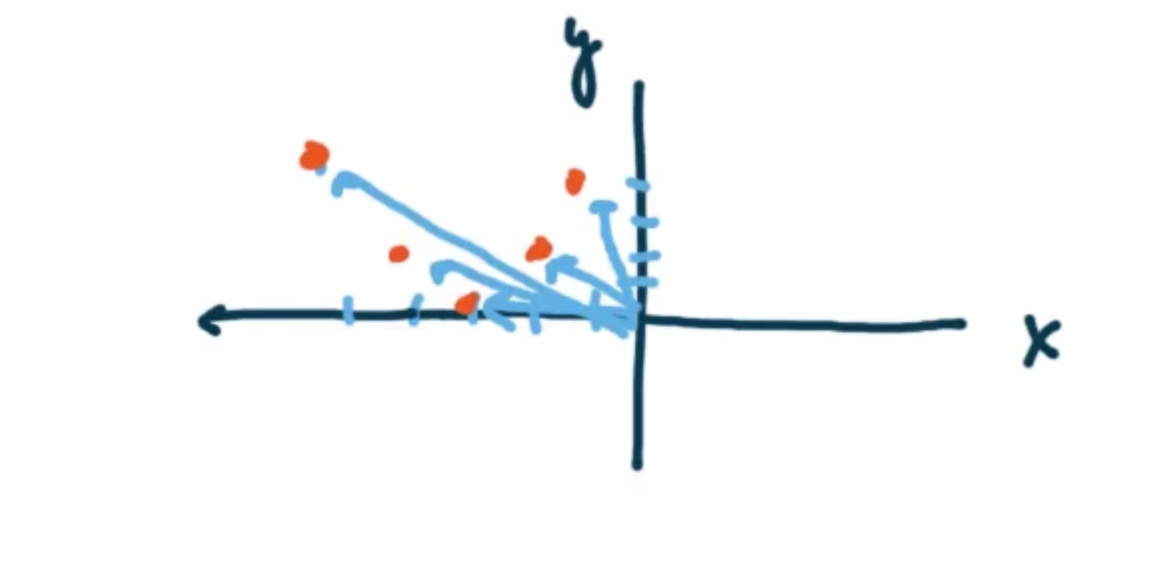

Figure 1.1: 2-D vector graph

Space Curve

$$\overline{r(t)} = \langle 3+t, 4-2t, t-5 \rangle$$

$$
\begin{aligned}
x &= 3 + t \\
y &= 4 - 2t \\
z &= t - 5
\end{aligned}
$$

(Parametric form)

$$\overline{r(t)} = \langle 3, 4t, -5 \rangle + \langle t, -2t, t \rangle$$

$$\left(\overline{r(t)} = \langle 3, 4t, -5 \rangle + t \langle 1, -2, 1 \rangle \right)$$

(Equation of line in 3-D)

$$x - 3 = \frac{y - 4}{-2} = z + 5$$

(Symetric form)

1.1.1 Limit

Definition 1 (Limit of a Vector)**.** *If* $\overline{r(t)} = \langle f(t), g(t), h(t), \rangle$*, then*

$$\lim_{t \to a} \overline{r(t)} = \left\langle \lim_{t \to a} f(t), \lim_{t \to a} g(t), \lim_{t \to a} h(t) \right\rangle$$

provided the limits of the component functions exist.

Example 2 (Limit of a Vector)**.**

$$\overline{r(t)} = \left\langle \frac{\sin^2 t}{t^2}, te^{-t}, t^2 + 1 \right\rangle$$

$$
\begin{aligned}
\lim_{t \to 0} \overline{r(t)} &= \lim_{t \to 0} \left\langle \frac{\sin^2 t}{t^2}, te^{-t}, t^2 + 1 \right\rangle \\
&= \left\langle \lim_{t \to 0} \frac{\sin^2 t}{t^2}, \lim_{t \to 0} te^{-t}, \lim_{t \to 0} t^2 + 1 \right\rangle \\
&= \langle a, b, c \rangle
\end{aligned}
$$

$$
\begin{aligned}
a &= \frac{0}{0} (\textit{ L'H Rule }) \\
a &= \lim_{t \to 0} \frac{2 \sin t \cos t}{2t} \\
a &= \lim_{t \to 0} \frac{\sin t \cos t}{t} \\
a &= \frac{0}{0} (\textit{ L'H Rule }) \\
a &= \lim_{t \to 0 \frac{-\sin^2 t + \cos^2 t}{1}} \\
a &= 1 \\
b &= 0 \\
c &= 1 \implies
\end{aligned}
$$

$$= \boxed{\langle 1, 0, 1 \rangle}$$