Chapter 1

Determinants

DATE: 2020-09-21

ANNOUNCEMENTS:

assignment: 2.1 1-5 odd, 15-25 odd

1.1 Determinants by Co-factor Expansion

Definition 1 (Determinant). *If A is an $n \times n$ matrix, the the number obtained by multiplying the entries of any row or any column of A by the corresponding cofactors and adding the results together is called the determinant of A and the sums are called the cofactor expansion of A.*

$$detA = a_{i1}c_{i1} + a_{i2}c_{i2} + \ldots + a_{in}c_{in}$$

-cofactor expansion about the i^{th} row.

$$detA = a_{1j}c_{1j} + a_{2j}c_{2j} + \ldots + a_{nj}c_{nj}$$

-cofactor expansion about the j^{th} column. •

Definition 2 (Minor). *If A is a square matrix, then minor entry of a_{ij} is denoted M_{ij} and is defined to be the determinant of the submatrix obtained when the i^{th} row and j^{th} column of A are deleted.* •

Definition 3 (Co-factor). *The number $(-1)^{i+j}M_{ij}$ is denoted C_{ij} and is called the co-factor entry of a_{ij}.* •

Example 1 (3x3).

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$M_{23} = 8 - 14 = -6$$

$$C_{23} = (-1)^5(-6) = -1 \cdot -6 = 6$$

•

Example 2 (Determinant).

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{aligned} detA &= a_{11}c_{11} + a_{12}c_{12} \\ &= a_{11}(-1)^{1+1}a_{22} + a_{12}(-1)^{1+2}a_{21} \\ &= a_{11}a_{22} - a_{12}a_{21} \end{aligned}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\begin{aligned} detA &= a_{11}c_{11} + a_{21}c_{21} \\ &= a_{11}(-1)^{1+1}a_{22} + a_{21}(-1)^{1+2}a_{12} \\ &= a_{11}a_{22} - a_{12}a_{21} \end{aligned}$$

•

Example 3 (3X3 Determinant).

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\begin{aligned} detA &= a_{11}c_{11} + a_{12}c_{12} + a_{13}c_{13} \\ &= a_{11}(-1)^2\left[a_{22}a_{33} - a_{23}a_{32}\right] \\ &+ a_{12}(-1)^3\left[a_{21}a_{33} - a_{23}a_{31}\right] \\ &+ a_{13}(-1)^4[a_{21}a_{32} - a_{22}a_{31}] \\ &= a_{11}a_{22}a_{33} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a32 - a_{13}a_{22}a_{31} \end{aligned}$$

•

Example 4 (3X3 Determinant). *"Using the minor entries and the coeficient with a sign, knowing what the cofactor would be"*

$$A = \begin{bmatrix} 1 & 3 & 11 \\ -2 & 4 & -3 \\ -1 & 5 & -7 \end{bmatrix}$$

$$\begin{aligned} detA &= 1[-28 + 15] - 3[14 - 3] + 1[-10 - (-4)] \\ &= 1[-13] - 3[11] + [-6] \\ &= -52 \end{aligned}$$

"Doing cofactor expansion about second row"

$$\begin{aligned} detA &= +2[-21 - 5] + 4[-7 - (-1)] - 3[5 - (-3)] \\ &= 2(-26) + 4(-6) + 3(8) \\ &= -52 \end{aligned}$$

"Doing cofactor expansion about third column"

$$\begin{aligned} detA &= +1[-10+4] + 3[5-(-3)] - 7[4-(-3)] \\ &= 1(-6) + 3(8) - 7(10) \\ &= -52 \end{aligned}$$

•

Example 5 (Making 3X3 Determinant Easy)**.**

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 0 & 4 \\ -1 & 5 & 9 \end{bmatrix}$$

$$detA = 0 + 0 - 5[8-3] = -25$$

•