Coulomb's Law Lab
PHY-222-AC01

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1 Theory

Coulomb's law describes a mathematical expression for the interaction of electrically charged objects. The electrical charge generates a force pair, F_E, with a magnitude that has been experimentally determined to such a degree that it is accepted as a universal law.

1.1 Definitions

Electrically charged object - Matter with more or less electrons than protons is negatively or positively charged respectively.

Force pair - The principal of Newton's third law in which every action force has a corresponding reaction force equal in magnitude and opposite in direction.

Coulomb's law -

$$F_E = k\frac{q_1 q_2}{r^2}$$

r - The distance between the charged objects measured in meters.

$q_1 q_2$ **-** The sum of the electrical charges of each object, measured in Coulombs.

k - The electric constant, the experimentally determined proportionality constant with a value of $k = 9.0 \times 10^9 \frac{Nm^2}{C^2}$

2 Objectives

First Objective
Experimentally confirm Coulomb's law.

Second Objective
Study how distance and charge affect the electric force.

Third Objective

Experimentally determine the value of the electric constant, k.

3 Experimental Data

3.1 Part One

Table 1:

$q_1 = 2\mu C$		$q_2 = 4\mu C$	
r(cm)	$r^2(m^2)$	$\frac{1}{r^2}(\frac{1}{m^2})$	$F_E(N)$
10	1.0×10^{-2}	1×10^2	7.190
9	8.1×10^{-3}	1.2×10^2	8.877
8	6.4×10^{-3}	1.6×10^2	11.234
7	4.9×10^{-3}	2.0×10^2	14.674
6	3.6×10^{-3}	2.8×10^2	19.972
5	2.5×10^{-3}	4.0×10^2	28.760
4	1.6×10^{-3}	6.3×10^2	44.938
3	9.0×10^{-4}	1.1×10^3	79.889

3.2 Part Two

Table 2:

$q_1 = 5\mu C$	$r = 6cm$
$q_2(\mu C)$	$F_E(N)$
10	124.827
9	112.344
8	99.862
7	87.379
6	74.896
5	62.414
4	49.931
3	37.448

4 Data Analysis

4.1 Part One

4.1.1 F_E with respect to r

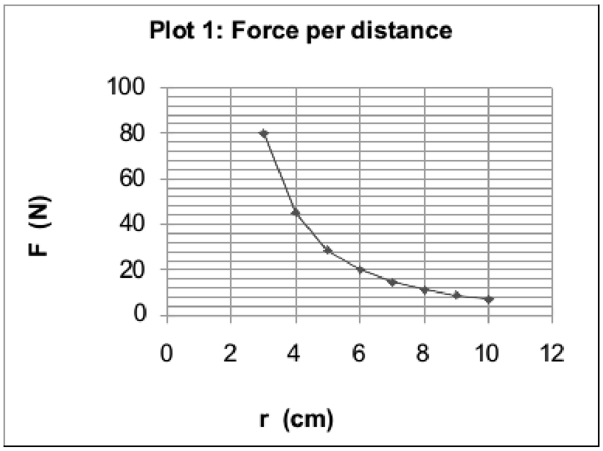

Figure 1: Force per distance

Comments on Figure 1: As the charged objects' distance (r) tends towards zero from the positive direction, the force (F) tends towards infinity. As r tends towards positive infinity, F tends towards zero.

4.1.2 F_E with respect to $\frac{1}{r^2}$ to find k

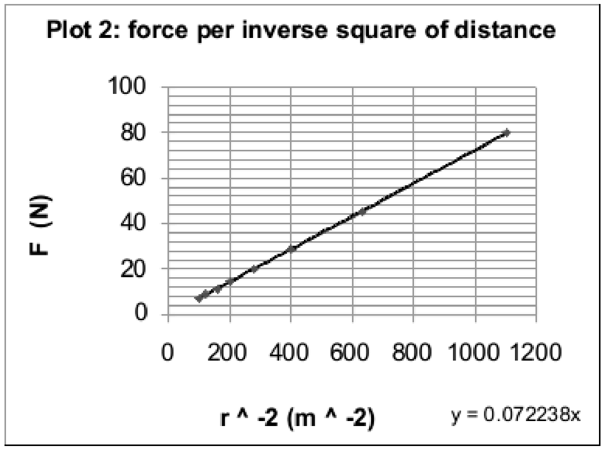

Figure 2: Force per inverse square of distance

Coulomb's Law - $F_E = k\frac{q_1 q_2}{r^2}$

Linear regression of Figure 2 data - y = 0.072238x

q_1 **-** $2\mu C$

q_2 **-** $4\mu C$

$$F_E = k\frac{(2\mu C)(4\mu C)}{r^2} \implies$$
$$F_E(N) = k\left(8\times 10^{-12}C^2\right)\left(\frac{1}{r^2}\right)\left(\frac{1}{m^2}\right)$$
$$y = 0.072238x \implies$$
$$F_E = 0.072238\left(\frac{1}{r^2}\right)$$
$$0.072238\left(\frac{1}{r^2}\right)(N) = k\left(8\times 10^{-12}C^2\right)\left(\frac{1}{r^2}\right)\left(\frac{1}{m^2}\right)$$
$$k = \frac{0.072238\left(\frac{1}{r^2}\right)(N)(m^2)}{8\times 10^{-12}C^2\left(\frac{1}{r^2}\right)}$$
$$k = \boxed{9.02975\times 10^9}\left(\frac{Nm^2}{C^2}\right)$$

4.1.3 Percent error in k

$$\% \text{ error } = \left| \frac{\text{theoretical } - \text{ experimental}}{\text{theoretical}} \right| \times 100\%$$

$$\% \text{ error } = \left| \frac{\left(9.0 \times 10^9 \frac{Nm^2}{C^2}\right) - \left(9.02975 \times 10^9 \frac{Nm^2}{C^2}\right)}{\left(9.0 \times 10^9 \frac{Nm^2}{C^2}\right)} \right| \times 100\%$$

$$\% \text{ error } = 0.330556\%$$

4.2 Part Two

4.2.1 F_E with respect to q_2

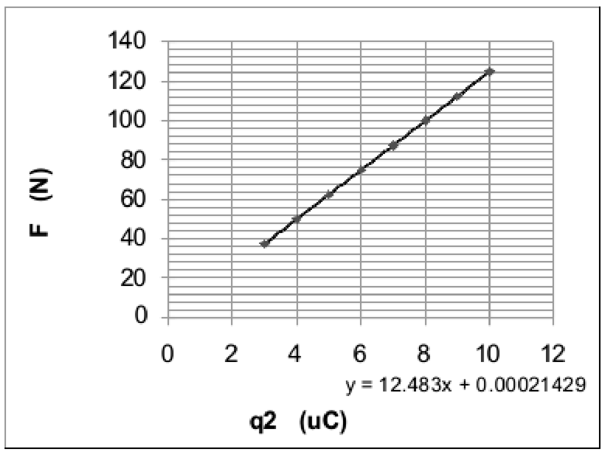

Figure 3: Force per magnitude charge on q_2

Comments on Figure 3: There is a positive linear relationship between the magnitude of charge on one object $q_2(\mu C)$ and electric force $F_E(N)$. Given the precision of F_E measurement, it appears to approach a lower bounds of $F_E = 0$ at $q_2 = 0$.

4.2.2 F_E with respect to q_2 to find k

Coulomb's Law - $F_E = k\frac{q_1 q_2}{r^2}$

Linear regression of Figure 3 data - $y = 12.483x + 0.00021429$

q_1 **-** $5\mu C$

r - $6cm$

$$
\begin{aligned}
F_E &= k\frac{(5\mu C)(q_2)}{(6cm)^2} \implies \\
F_E(N) &= kq_2(\mu C)\left(\frac{5\times10^{-6}C}{3.6\times10^{-3}m^2}\right) \\
y &= 12.483x + 0.00021429 \implies \\
F_E &= 12.483q_2 + 0.00021429 \\
(12.483q_2 + 0.00021429)(N) &= kq_2(\mu C)\left(\frac{5\times10^{-6}C}{3.6\times10^{-3}m^2}\right) \implies \\
k &= \frac{(12.483q_2 + 0.00021429)(3.6\times10^{-3}m^2)(N)}{(q_2)(5\times10^{-6}C)(10^{-6}C)} \implies \\
k &= \boxed{8.98776\times10^9\left(\frac{Nm^2}{C^2}\right)} + 1.542888\times10^5 q_2^{-1}\left(\frac{Nm^2}{C^2}\right)
\end{aligned}
$$

4.2.3 Percent error in k

$$
\begin{aligned}
\%\ \text{error} &= \left|\frac{\text{theoretical} - \text{experimental}}{\text{theoretical}}\right| \times 100\% \\
\%\ \text{error} &= \left|\frac{\left(9.0\times10^9\,\frac{Nm^2}{C^2}\right) - \left(8.98776\times10^9\,\frac{Nm^2}{C^2}\right)}{\left(9.0\times10^9\,\frac{Nm^2}{C^2}\right)}\right| \times 100\% \\
\%\ \text{error} &= 0.136\%
\end{aligned}
$$

5 Results and Conclusions

5.1 First Objective

This experiment does confirm Coulomb's law. It is observed that charged objects generate a non-contact force on each other that is proportional to the sum of their charges and inversely proportional to the square of their distance.

5.2 Second Objective

According to Figure 1, the magnitude of the force pair experienced by the charged objects increases as the distance decreases. This is an exponential relationship and the exponent that describes how the force changes depending on the distance variable is 2, meaning this is an inverse square relationship.
According to Figure 3, the magnitude of the force pair experienced by the charged objects increases as the charge on either object increases. The variable

of charge has a directly proportional effect on the force experienced by the objects. This is the case when the magnitude of the other charge is held constant, but the total effect on the force is determined by the sum of their charges.

5.3 Third Objective

Figures 2 and 3 show that that the value of $k = 9.0\times10^9\ \frac{Nm^2}{C^2}$ was experimentally confirmed with a maximum percent error of 0.330556%

6 Discussion of Experimental Uncertainty

The virtual experiment is meant to show an uncertainty generated by the precision of the force ($F_E(N)$) measurement. Tables 1 and 2 show a precision of F_E on the mN scale. This is a rounded value that makes calculation practical but leads to an error in the calculation of k and fitting error in Figure 3 where the line does not intersect the origin.