

Assignment: $\det(A)$ of general 4×4 matrix	Assigned: 2020-09-18	Due: 2020-09-22
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$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \end{bmatrix}$$

$$\det A = \sum_{all-perm} \delta(j_1, j_2, j_3, j_4) a_{1j_1} a_{2j_2} a_{3j_3} a_{4j_4}$$

Example Set:

1, 2, 3, 4

Permutations:

{ 1, 2, 3, 4 }
 { 1, 2, 4, 3 }
 { 1, 3, 2, 4 }
 { 1, 3, 4, 2 }
 { 1, 4, 2, 3 }
 { 1, 4, 3, 2 }
 { 2, 1, 3, 4 }
 { 2, 1, 4, 3 }
 { 2, 3, 1, 4 }
 { 2, 3, 4, 1 }
 { 2, 4, 1, 3 }
 { 2, 4, 3, 1 }
 { 3, 1, 2, 4 }
 { 3, 1, 4, 2 }
 { 3, 2, 1, 4 }
 { 3, 2, 4, 1 }
 { 3, 4, 1, 2 }
 { 3, 4, 2, 1 }
 { 4, 1, 2, 3 }
 { 4, 1, 3, 2 }
 { 4, 2, 1, 3 }
 { 4, 2, 3, 1 }
 { 4, 3, 1, 2 }
 { 4, 3, 2, 1 }

Parity:

$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 0$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 0, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 1$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 1$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 0, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 2$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 0, \alpha_2 = 2, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 2$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 0, \alpha_2 = 2, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 1$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 1, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 2$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 2$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 1, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 1, \alpha_2 = 2, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 4$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 2, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 2$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 2, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 2, \alpha_2 = 1, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 2, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 4$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 2, \alpha_2 = 2, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 4$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 2, \alpha_2 = 2, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 5$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 3, \alpha_2 = 0, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 3$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 3, \alpha_2 = 0, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 4$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 3, \alpha_2 = 1, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 4$	$\delta(j_1, j_2, j_3, j_4) = 1$
$\alpha_1 = 3, \alpha_2 = 1, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 5$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 3, \alpha_2 = 2, \alpha_3 = 0, \alpha_4 = 0$	$\sum \alpha_n = 5$	$\delta(j_1, j_2, j_3, j_4) = -1$
$\alpha_1 = 3, \alpha_2 = 2, \alpha_3 = 1, \alpha_4 = 0$	$\sum \alpha_n = 6$	$\delta(j_1, j_2, j_3, j_4) = 1$

Expand sum over all permutations

$$\det A = \sum_{all-perm} \delta(j_1, j_2, j_3, j_4) a_{1j_1} a_{2j_2} a_{3j_3} a_{4j_4}$$

$$\begin{aligned}
& \begin{aligned}
& (+)a_{11} \quad a_{22} \quad a_{33} \quad a_{44} (-)a_{11} \quad a_{22} \quad a_{34} \quad a_{43} (-)a_{11} \quad a_{23} \quad a_{32} \quad a_{44} \\
& (-)a_{12} \quad a_{21} \quad a_{33} \quad a_{44} (+)a_{12} \quad a_{21} \quad a_{34} \quad a_{43} (+)a_{12} \quad a_{23} \quad a_{31} \quad a_{44} \\
& (+)a_{13} \quad a_{21} \quad a_{32} \quad a_{44} (-)a_{13} \quad a_{21} \quad a_{34} \quad a_{42} (-)a_{13} \quad a_{22} \quad a_{31} \quad a_{44} \\
& (-)a_{14} \quad a_{21} \quad a_{32} \quad a_{43} (+)a_{14} \quad a_{21} \quad a_{33} \quad a_{42} (+)a_{14} \quad a_{22} \quad a_{31} \quad a_{43} \\
& (+)a_{11} \quad a_{23} \quad a_{34} \quad a_{42} (+)a_{11} \quad a_{24} \quad a_{32} \quad a_{43} (-)a_{11} \quad a_{24} \quad a_{33} \quad a_{42} \\
& (-)a_{12} \quad a_{23} \quad a_{34} \quad a_{41} (-)a_{12} \quad a_{24} \quad a_{31} \quad a_{43} (+)a_{12} \quad a_{24} \quad a_{33} \quad a_{42} \\
& (+)a_{13} \quad a_{22} \quad a_{34} \quad a_{41} (+)a_{13} \quad a_{24} \quad a_{31} \quad a_{42} (-)a_{13} \quad a_{24} \quad a_{32} \quad a_{41} \\
& (-)a_{14} \quad a_{22} \quad a_{33} \quad a_{41} (-)a_{14} \quad a_{23} \quad a_{31} \quad a_{42} (+)a_{14} \quad a_{23} \quad a_{32} \quad a_{41}
\end{aligned}
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