Assignment: Homework 1	Assigned: 2020-08-28	Due: 2020-09-03

Question 1: form augmented matrix and solve using row operations.

$$\begin{aligned} x + y - z &= -1 \\ 4x - 3y + 2z &= 16 \\ 2x - 2y - 3z &= 5 \end{aligned}$$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 4 & -3 & 2 & 16 \\ 2 & -2 & -3 & 5 \end{bmatrix}$$

$-4E_1 + E_2 \to E_2$
$-2E_1 + E_3 \to E_3$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & -7 & 6 & 20 \\ 0 & -4 & -1 & 7 \end{bmatrix}$$

$-\frac{1}{4}E_3 \to E_3$
$E_2 \leftrightarrow E_3$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & \frac{1}{4} & -\frac{7}{4} \\ 0 & -7 & 6 & 20 \end{bmatrix}$$

$7E_2 + E_3 \to E_3$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & \frac{1}{4} & -\frac{7}{4} \\ 0 & 0 & \frac{24+7}{4} & \frac{80-49}{4} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & \frac{1}{4} & -\frac{7}{4} \\ 0 & 0 & \frac{31}{4} & \frac{31}{4} \end{bmatrix}$$

$\frac{4}{31}E_3 \to E_3$

$$\begin{bmatrix} 1 & 1 & -1 & -1 \\ 0 & 1 & \frac{1}{4} & -\frac{7}{4} \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$-\frac{1}{4}E_3 + E_2 \to E_2$
$E_3 + E_1 \to E_1$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$-E_2 + E_1 \to E_1$

$$\begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\boxed{(2, -2, 1)}$$

Question 2: find the solution, or breifly explain why "No Solution".

2(a) $\begin{bmatrix} 1 & 1 & 0 & 4 & 9 \\ 0 & 0 & 1 & 7 & 7 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

$$E_3 : \text{ False Statement } 0 = 4 \implies \boxed{\text{"No Solution"}}$$

2(b) $\begin{bmatrix} 1 & 0 & 3 & 2 \\ 0 & 1 & 9 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

$$x_1 + 3x_3 = 2$$
$$x_2 + 9x_3 = 5$$

$$\boxed{(2 - 3x_3, 5 - 9x_3, x_3)}$$

Question 3: scalar, matrix (size), or not defined.

A 3×3
B 4×5
C 5×5
D 4×2
E 3×4

3(a) $tr(C)$

$$C_{11} + C_{22} + C_{33}$$

$$\boxed{\text{Scalar}}$$

3(b) E^T

$$(3 \times 4)^T$$

$$\boxed{\text{Matrix } (4 \times 3)}$$

3(c) $tr(A)B$

$$(A_{11} + A_{22} + A_{33})B$$

$$\boxed{\text{Matrix } (4 \times 5)}$$

3(d) $A - E$
Matrix addition is only defined for matrices of the same size.

$$\implies \boxed{\text{Not defined}}$$

3(e) AE

$$(3 \times 3)(3 \times 4)$$

$$\boxed{\text{Matrix } (3 \times 4)}$$

3(f) $B^T D$

$$(4 \times 5)^T (4 \times 2)$$
$$(5 \times 4)(4 \times 2)$$
$$\boxed{\text{Matrix } (5 \times 2)}$$

3(g) $B - C$

$$(4 \times 5) - (5 \times 5)$$

Matrix addition is only defined for matrices of the same size.

$$\implies \boxed{\text{Not defined}}$$

3(h) $E + 3A$

$$(3 \times 4) + (3 \times 3)$$

Matrix addition is only defined for matrices of the same size.

$$\implies \boxed{\text{Not defined}}$$

Question 4: Find $p(A)$ for the polynomial $p(x) = 2x^3 - 4x + 2$ and the matrix $A = \begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix}$.

$$
\begin{aligned}
2\left[\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix}\right] - 4\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \\
2\left[\begin{pmatrix} 16 & -3 \\ -4 & 13 \end{pmatrix}\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix}\right] - 4\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \\
2\begin{pmatrix} -44 & 45 \\ 60 & 1 \end{pmatrix} - 4\begin{pmatrix} -2 & 3 \\ 4 & 1 \end{pmatrix} + 2\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} &= \\
\begin{pmatrix} -88 & 90 \\ 120 & 2 \end{pmatrix} + \begin{pmatrix} 8 & -12 \\ -16 & -4 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} &= \\
\begin{pmatrix} -88+8+2 & 90-12 \\ 120-16 & 2-4+2 \end{pmatrix} &= \\
\boxed{\begin{pmatrix} -78 & 78 \\ 104 & 0 \end{pmatrix}}
\end{aligned}
$$