Chapter 1

Electric Fields Cont'd

DATE: 2020-08-24
ANNOUNCEMENTS:

1.1 Electric field lines

Definition 1 (Electric field line)**.** *An imaginary line or curve whose tangent at any point is the direction of the electric field vector at that point.*

- Field lines always start at a positive charge and end at a negative charge and can never cross.
- Show the direction of the electric field at each point.
- Spacing gives idea of magnitude of field at each point.

1.1.1 Electric field lines of a dipole

Definition 2 (Electric dipole)**.** *A pair of equal and opposite electric charges.*

- The field radiates outward from the positive and towards the negative.

Example 1 (1)**.** *A positive point charge $+Q$ is released from rest in an electric field. At any later time, the velocity of the point charge*

A. in the direction of the electric field? B. opposite the direction of the electric field? C. not enough information.

Solution 1 (1)**.** *The question does not state a uniform electric field, and remember field lines are not trajectories. C.*

Example 2 (2)**.** *What happens when I place an electric dipole in a uniform electric field pointing to the right.*

Net Force? Net Torque?

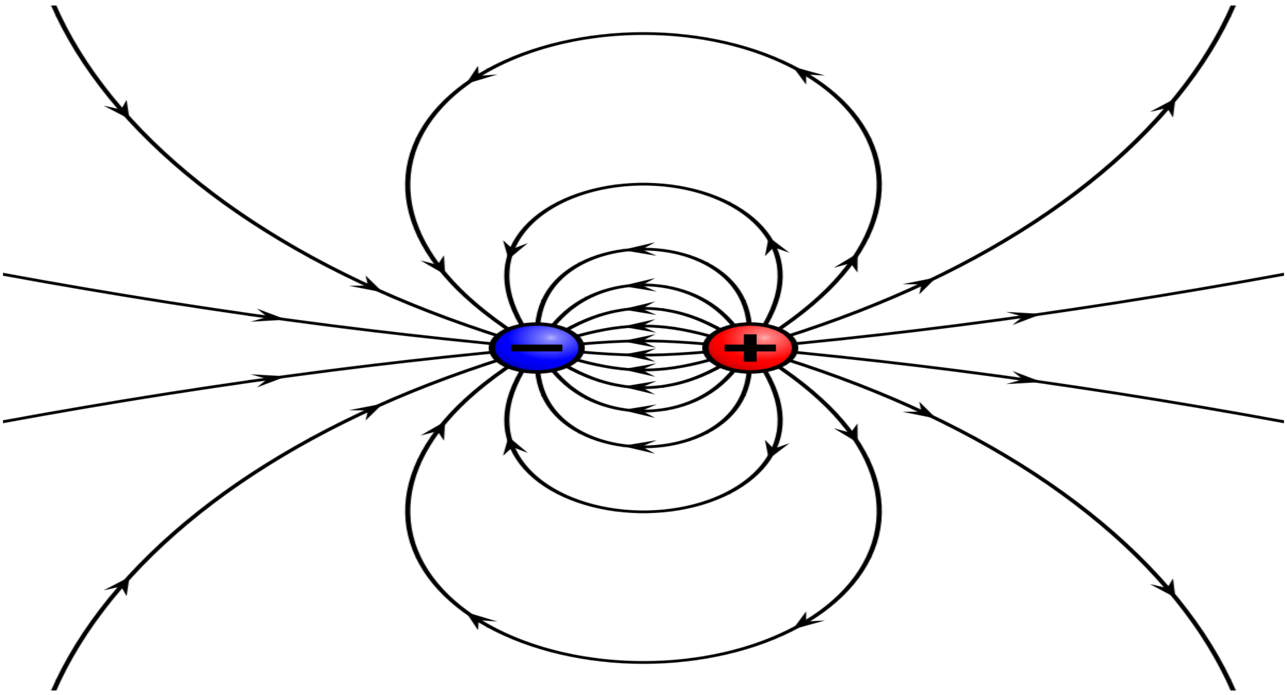

Figure 1.1: Dipole

Solution 2 (2). *A. The force on the positive point charge is $F_+ = qE$ right.*
The force on the negative point charge pole is $F_- = qE$ left.
$\implies F_{net} = 0$
B. $\tau = \overline{r} \times \overline{F} = rF\sin\theta$
$\tau_+ = \left(\frac{d}{2}\right)(qE)\sin\theta$ *into page*
$\tau_- = \overline{r} \times \overline{F} = \left(\frac{d}{2}\right)(qE)\sin\theta$ *into page*
$\tau_{net} = qdE\sin\theta$ *into page*

Definition 3 (Electric dipole moment).

$$\boxed{p = qd}$$

$\overline{p}$ *points from* $\boxed{-}$ *to* $\boxed{+}$
$\overline{p} \perp \overline{E}$ *, max torque*
$\overline{p} \parallel \overline{E}$ *, minimum torque*

$\tau_{net} = \overline{p} \times \overline{E}$

1.1.2 Electric field due to a charge distribution

- line of charge
- ring of charge
- disk of charge

Line of charge

Example 3 (3). *What is the e-field? at point P?*

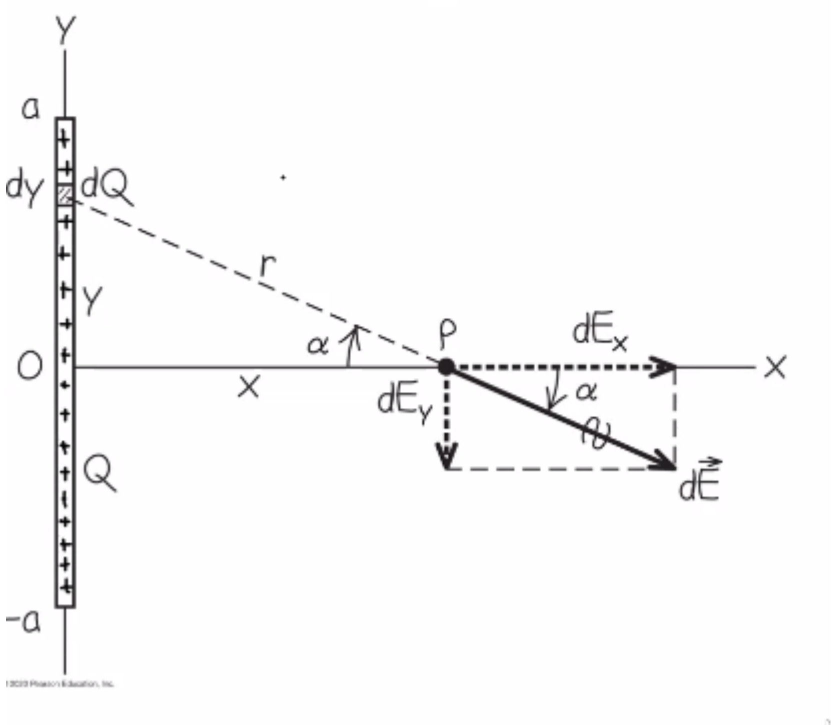

Figure 1.2: line of charge

Solution 3 (3)**.**

$$
\begin{aligned}
dE &= \frac{kdQ}{r^2} \\
\int dE_y &= 0 \textit{ (symetry)} \\
dE_x &= \frac{kdQ}{r^2}\cos\alpha \\
E_x &= \int \frac{kdQ}{r^2}\cos\alpha \\
r^2 &= x^2 + y^2 \\
\cos\alpha &= \frac{x}{r} = \frac{x}{\sqrt{x^2+y^2}} \\
\textit{define linear charge density } \lambda &= \textit{ charge/length} \\
\lambda &= \frac{Q}{2a} \\
dQ &= \lambda dy \\
E_x &= \int \frac{k\lambda dyx}{(x^2+y^2)\sqrt{x^2+y^2}} \\
&= \int_{-a}^{a} \frac{k\lambda xdy}{(x^2+y^2)^{\frac{3}{2}}} \\
E_x &= \boxed{\frac{KQ}{x\sqrt{a^2+x^2}}}
\end{aligned}
$$

Example 4 (4)**.** *Ring of charge*

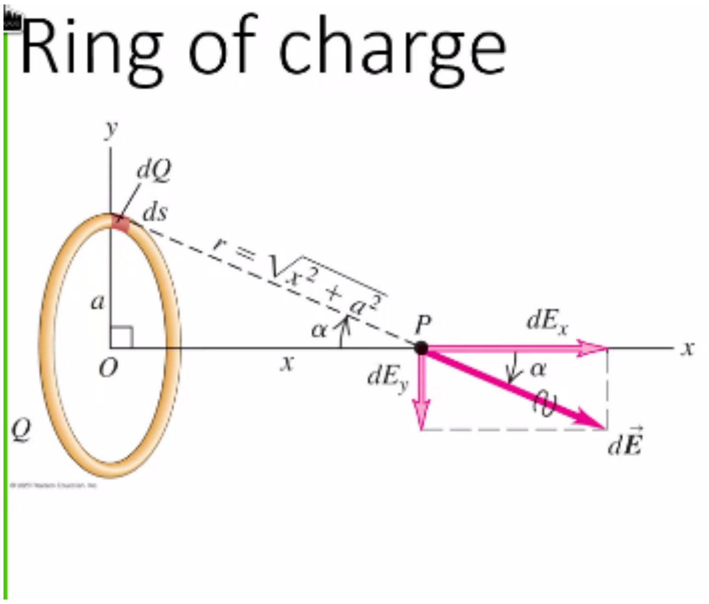

Figure 1.3: ring of charge

Solution 4 (4)**.** *Practice problem.*

Example 5 (Disk of charge)**.** *Find the electric field at point P.*

Solution 5 (Disk of charge)**.**

$$
\begin{aligned}
dE_x &= \frac{kdQ}{r^2}\cos\theta \\
r'^2 &= r^2 + x^2 \\
\cos\theta = \frac{x}{r'} &= \frac{x}{\sqrt{r^2+x^2}} \\
\textit{surface charge density}\,\sigma &= \frac{\textit{charge}}{\textit{area}} \\
dQ &= \sigma dA = \sigma 2\pi r dr \\
dA &= 2\pi r dr \\
\int dE_x &= \int_0^R \frac{k\sigma 2\pi r dr}{r^2+x^2}\frac{x}{\sqrt{r^2+x^2}} \\
E_x &= \frac{\sigma}{2\epsilon_0}\left[1 - \frac{1}{\sqrt{\frac{R^2}{x^2}+1}}\right]
\end{aligned}
$$

further: think about R»x

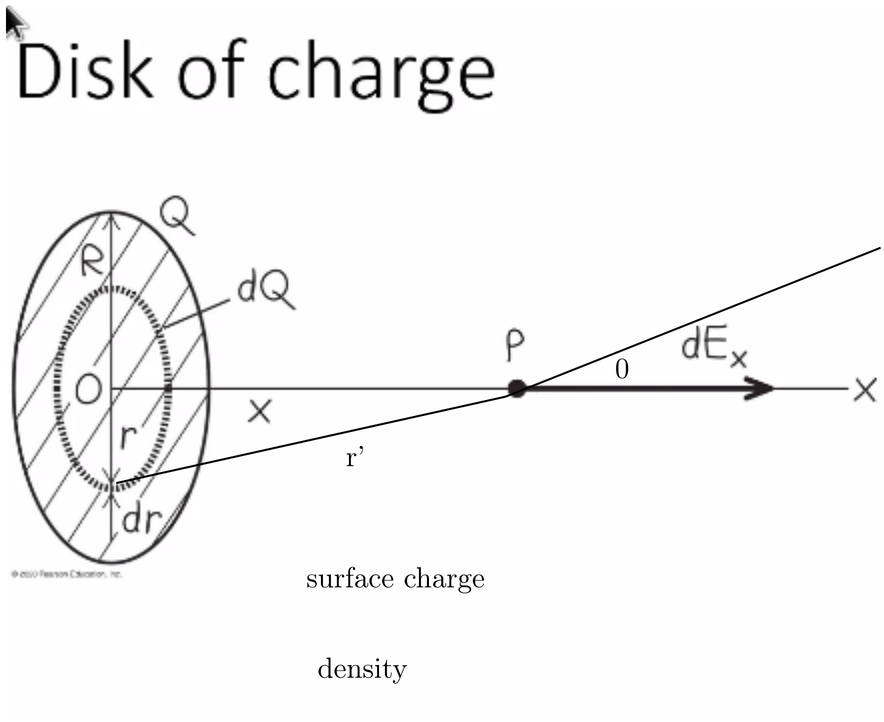

Figure 1.4: disk of charge

1.2 Chapter 22: Gauss's Law

1.2.1 Objectives

- Define electric flux and calculate through surfaces
- Define gauss's law
  - used to determine charge distribution given known e-field
  - to determine field given charge distribution
- know when Gauss's law can be used
- Do calculations with Gauss's law

Electric field due to a charged conducting sphere

We could use the electric field equations, but things get messy. Gauss's Law gives simplicity through symetry.

1.2.2 Intro to Gauss's law

- Given any general charge distribution, we surround it with an imaginary surface
- We look at the field at various points on imaginary surface
- GL is a relationship between the field at all points and total charge enclosed. it helps find the field for symmetric charge distributions.

1.2.3 Charge and electric flux

In boxes, there are positive charges within producing outward electric flux.

When boxes have negative charge inside, there is an inward electric flux.

When $\overline{E} = 0$ there is no electric flux in or out of the box.

What happens if there is no net charge inside the box?

- There is an electric field, but it flows in and out on either half
- Thus no net electric flux into or out of box.

What happens if there is charge near the box, but not inside?

- The net electric flux through the box is zero.

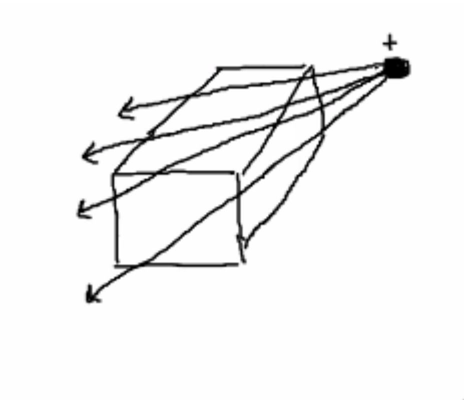

Figure 1.5: zero net flux

Quantifying electric flux

Net electric flux is **directly proportional** to net amount of charge within surface.

Net electric flux is **independent** of the size of the closed surface.

Example 6. *Suppose a Gaussian surface with rectangular sides and positive point charge $+q$ at it's center, and the surface doubles, but charge remains $+q$, what happens to the flux?*

Solution 6. *Remains the same*

Example 7. *Spherical Gaussian surface 1 has $+q$ at it's center. Spherical Gaussian surface 2, same size encloses the charge but is not centered on it. Compare flux through surface 1 and 2.*

Solution 7. *The same*

Calculating electric flux

- Consider a surface
- What affects amount of flux passing through surface?
- "net" analogy

Definition 4 (Electric flux)**.**

$$\begin{aligned}\Phi_E &= \Sigma \overline{E}_i \cdot d\overline{A}_i \\ &= \int E \cos\phi dA \\ &= \int E_\perp dA \\ &= \int \overline{E} \cdot d\overline{A} \\ &\rightarrow \int \overline{E} \cdot d\overline{A} \left(\frac{Nm^2}{C}\right)\end{aligned}$$

Φ_E **-** *Electric flux through a surface*

E **-** *Magnitude of Electric Field*

ϕ **-** *Angle between $\overline{E}$ and normal to surface*

dA **-** *Element of surface area*

$E_\perp$ **-** *Component of $\overline{E}$ perpendicular to surface*

$d\overline{A}$ **-** *Vector element of surface area*