Chapter 1

Vectors and the Geometry of Space

DATE: 2020-09-01

ANNOUNCEMENTS:

Review chapter 12.1 to 12.5 and complete assignments

1.1 Three-Diensional Coordinate Systems

$$\boxed{|P_1P_2| = \sqrt{(x_1 - x_1)^2 + (y_2 - y_2)^2 + (z_2 - z_1)^2}} \tag{1.1}$$

Dinstance Formula in Three Dimensions

$$\boxed{(x - h)^2 + (y - k)^2 + (z - l)^2 = r^2} \tag{1.2}$$

Equation of a sphere with a center $C(h, k, l)$ and a radius r

$$\boxed{|\overline{a}| = \sqrt{a_1^2 + a_2^2}} \tag{1.3}$$

$$\boxed{|\overline{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}} \tag{1.4}$$

Length of a Vector

$$\boxed{\overline{a} + \overline{b} = \langle a_1 + b_1, a_2 + b_2 \rangle} \tag{1.5}$$

Vector Addition

$$\boxed{\overline{a} - \overline{b} = \langle a_1 - b_1, a_2 - b_2 \rangle} \tag{1.6}$$

Vector Subtraction

$$\boxed{c\overline{a} = \langle ca_1, ca_2 \rangle} \tag{1.7}$$

Scalar Multiplication

$$\boxed{\overline{a} \cdot \overline{b} = a_1 b_1 + a_2 b_2 + a_3 b_3} \tag{1.8}$$

Dot Product

$$\begin{aligned}
\overline{a} \cdot \overline{a} &= |\overline{a}|^2 \\
\overline{a} \cdot \overline{b} &= \overline{b} \cdot \overline{a} \\
\overline{a} \cdot (\overline{b} + \overline{c}) &= \overline{a} \cdot \overline{b} + \overline{a} \cdot \overline{c} \\
(c\overline{a}) \cdot \overline{b} &= c(\overline{a} \cdot \overline{b} = \overline{a} \cdot (c\overline{b}) \\
0 \cdot \overline{a} &= 0
\end{aligned}$$

Theorem 1 (Dot product)**.** $\overline{a} \cdot \overline{b} = |a|\,|b| \cos\theta$

Corollary 1. $\cos\theta = \frac{\overline{a} \cdot \overline{b}}{|\overline{a}||\overline{b}|}$

Direction Angles

$$\begin{aligned}
\cos\alpha &= \frac{a_1}{|\overline{a}|} \\
\cos\beta &= \frac{a_2}{|\overline{a}|} \\
\cos\gamma &= \frac{a_3}{|\overline{a}|}
\end{aligned}$$

Projections

$$\boxed{comp_a \overline{b} = \frac{\overline{a} \cdot \overline{b}}{|\overline{a}|}} \tag{1.9}$$

Scalar projection of b onto a

$$\boxed{proj_a \overline{b} = \left(\frac{\overline{a} \cdot \overline{b}}{|\overline{a}|} \right) \frac{\overline{a}}{|\overline{a}|} = \frac{\overline{a} \cdot \overline{b}}{|\overline{a}|} \overline{a}} \tag{1.10}$$

Vector projection of b onto a

$$\boxed{\overline{a} \times \overline{b} = \langle a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1 \rangle} \tag{1.11}$$

Cross product

Theorem 2. $\left|\overline{a} \times \overline{b}\right| = |\overline{a}| \left|\overline{b}\right| \sin\theta$

Theorem 3 (parallel)**.** $\overline{a} \times \overline{b} = 0 \implies$ *a and b are parallel*

$$\begin{aligned} \overline{a} \times \overline{b} &= -\overline{b} \times \overline{a} \\ (c\overline{a}) \times \overline{b} &= c(\overline{a} \times \overline{b}) = \overline{a} \times (c\overline{b}) \\ \overline{a} \times (\overline{b} + \overline{c}) &= \overline{a} \times \overline{b} + \overline{a} \times \overline{c} \end{aligned}$$

1.2 Equations of Lines and Planes

1.2.1 Lines

$$\boxed{\overline{r} = \overline{r}_0 + t\overline{v}} \tag{1.12}$$

Vector equation of a line

r vector that traces the line

$\overline{r}_0$ given point on the line ($\overline{r}_0 = \langle x_0, y_0, z_0 \rangle$)

$\overline{v}$ vector in the direction of $\overline{r}$

t parameter

$$\implies \langle x, y, z \rangle = \langle x_0 + ta, y_0 + tb, z_0 + tc \rangle$$

Parametric equations for a line through a point (x_0, y_0, z_0) parallel to the vector $\overline{v} = \langle a, b, c \rangle$

$$\begin{aligned} x &= x_0 + at \\ y &= y_0 + bt \\ z &= z_0 + ct \end{aligned}$$