Chapter 1

Determinants

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ANNOUNCEMENTS:

Consider the set $\{j_1, j_2, \ldots jn\}$ where each j_i is a positive integer.
Each possible ordering of the set (standard order) is called a permutation set
A set with n elements has $n!$ permutation sets.
In each permutation set, define a number α_i for each number.
$\alpha_i =$ the entries following j_i that are less than j_i.

Example 1 (set $\{1, 2, 3\}$)**.** *permutations:*

$$\begin{array}{ccc} 1, & 2, & 3 \\ 1, & 3, & 2 \\ 2, & 1, & 3 \\ 2, & 3, & 1 \\ 3, & 1, & 2 \\ 3, & 2, & 1 \end{array}$$

$$\begin{array}{ccc} \alpha_1 = 0, & \alpha_2 = 0, & \alpha_3 = 0 \\ \alpha_1 = 0, & \alpha_2 = 1, & \alpha_3 = 0 \\ \alpha_1 = 1, & \alpha_2 = 0, & \alpha_3 = 0 \\ \alpha_1 = 1, & \alpha_2 = 1, & \alpha_3 = 0 \\ \alpha_1 = 2, & \alpha_2 = 0, & \alpha_3 = 0 \\ \alpha_1 = 2, & \alpha_2 = 1, & \alpha_3 = 0 \end{array}$$

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$\alpha_1 + \alpha_2 + \ldots + \alpha_n =$ total number of inversions in permutation set = parity
Parity is either even or odd.
Define a function

$$\delta(j_1, j_2, \ldots j_n) = \{1, -1\}$$

Form all possible products of the form $a_{1j_1} a_{2j_2} a_{2j_3} \ldots a_{nj_n}$.
Determinant of $A = det(A)$

$$= \sum_{\text{all possible permutation sets}} \delta(j_1, j_2, \ldots, jn) a_{1j_1} a_{2j_2} a_{2j_3} \ldots a_{nj_n}$$