Assignment: Homework 2	Assigned: 12SEP20	Due: 14SEP20:1600

Question 1: Determine conditions on b_1, b_2, b_3 such that system is consistent.

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$$\begin{bmatrix} 1 & 4 & 7 & b_1 \\ 2 & 5 & 8 & b_2 \\ -3 & -9 & -15 & b_3 \end{bmatrix}$$

$-2R_1 + R_2 \to R_2$
$3R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 4 & 7 & b_1 \\ 0 & -3 & -6 & b_2 - 2b_1 \\ 0 & 3 & 6 & b_3 + 3b_1 \end{bmatrix}$$

$R_2 + R_3 \to R_3$

$$\begin{bmatrix} 1 & 4 & 7 & b_1 \\ 0 & -3 & -6 & b_2 - 2b_1 \\ 0 & 0 & 0 & b_1 + b_2 + b_3 \end{bmatrix}$$

$$\implies b_1 + b_2 + b_3 = 0$$
$$\implies b_3 = -b_2 - b_3$$

$$\implies b = \boxed{\begin{bmatrix} b_1 \\ b_2 \\ -b_2 - b_3 \end{bmatrix}}$$

Question 2: Find A^{-1} if it exists or explain why it doesn't.

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$$\begin{bmatrix} 1 & -2 & 3 & 4 \\ 0 & 3 & 0 & 0 \\ 0 & 5 & 1 & 2 \\ 0 & -1 & 3 & 6 \end{bmatrix}$$

Inverse Algorithm

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 5 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$3R_4 + R_2 \to R_2$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 0 & 9 & 18 & 0 & 1 & 0 & 3 \\ 0 & 5 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & -1 & 3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$\frac{1}{9}R_2 \to R_2$
$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 5 & 1 & 2 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 2 & 0 & \frac{1}{9} & 0 & \frac{1}{3} \\ 0 & -1 & 3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$4R_4 + R_2 \to R_2$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 13 & 26 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 2 & 0 & \frac{1}{9} & 0 & \frac{1}{3} \\ 0 & -1 & 3 & 6 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_2 + R_4 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 13 & 26 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 2 & 0 & \frac{1}{9} & 0 & \frac{1}{3} \\ 0 & 0 & 16 & 32 & 0 & 0 & 1 & 5 \end{array}\right]$$

$-16R_3 + R_4 \to R_4$

$$\left[\begin{array}{cccc|cccc} 1 & -2 & 3 & 4 & 1 & 0 & 0 & 0 \\ 0 & 1 & 13 & 26 & 0 & 0 & 1 & 4 \\ 0 & 0 & 1 & 2 & 0 & \frac{1}{9} & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & -16 & -\frac{1}{3} \end{array}\right]$$

Main diagonal contains constant of zero $\implies$ non-trivial solution.
Given A is a 4×4 matrix, Equivalent Statements Theorem gives $\boxed{\text{A is not invertible}}$

$\square$

Question 3: Find A^{-1} if possible, otherwise solve another way.

$$\begin{aligned} x + 2z &= 6 \\ -x + 2y + 3z &= -5 \\ x - y &= 6 \end{aligned}$$

$$\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix}$$

If A^{-1} exists, $A^{-1}Ax = A^{-1}b$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ -1 & 2 & 3 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 + R_2 \to R_2$
$-R_1 + R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 2 & 5 & 1 & 1 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \end{array}\right]$$

$R_2 \leftrightarrow R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & -1 & -2 & -1 & 0 & 1 \\ 0 & 2 & 5 & 1 & 1 & 0 \end{array}\right]$$

$-R_2 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & 2 & 5 & 1 & 1 & 0 \end{array}\right]$$

$-2R_2 + R_3 \to R_3$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

$-2R_3 + R_1 \to R_1$
$-2R_3 + R_2 \to R_2$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 3 & -2 & -4 \\ 0 & 1 & 0 & 3 & -2 & -5 \\ 0 & 0 & 1 & -1 & 1 & 2 \end{array}\right]$$

$$A^{-1} = \boxed{\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}}$$

Check:

$$\begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 6 \\ -5 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ -2 \\ 1 \end{bmatrix}$$

$$\boxed{(4, -2, 1)}$$

Question 4: Determine X.

$$\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix} X = \begin{bmatrix} 5 & -3 & 6 \\ 1 & 0 & 9 \end{bmatrix}$$

Matrix Product:
$A_{m\times r}B_{r\times n} = C_{m\times n}$
$A_{2\times 2}B_{2\times 3} = C_{2\times 3}$
$A^{-1}A = I$

$$\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix} X = \begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}^{-1} \begin{bmatrix} 5 & -3 & 6 \\ 1 & 0 & 9 \end{bmatrix}$$

$$\left[\begin{array}{cc|cc} 2 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_2$
$-2R_1 + R_2 \to R_2$
$-\frac{1}{2}R_2 \to R_2$

$$\left[\begin{array}{cc|cc} 1 & 0 & 0 & 1 \\ 0 & 1 & -\frac{1}{2} & 1 \end{array}\right]$$

$$X = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 5 & -3 & 6 \\ 1 & 0 & 9 \end{bmatrix}$$

$$\boxed{X = \begin{bmatrix} 1 & 0 & 9 \\ -\frac{3}{2} & \frac{3}{2} & 6 \end{bmatrix}}$$

Check:

$$\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 & 9 \\ -\frac{3}{2} & \frac{3}{2} & 6 \end{bmatrix} = \begin{bmatrix} 5 & -3 & 6 \\ 1 & 0 & 9 \end{bmatrix}$$

Question 5a: If symmetric, find a, b, c, and d .

$$\begin{bmatrix} a & 2a-b+c & 2a-b-4c \\ 4 & b & a-b+2c \\ -16 & -6 & c \end{bmatrix}$$

$$4 = 2a - b + c$$
$$-16 = 2a - b - 4c$$
$$-6 = a - b + 2c$$

$$\begin{bmatrix} 2 & -1 & 1 & 4 \\ 2 & -1 & -4 & -16 \\ 1 & -1 & 2 & -6 \end{bmatrix}$$

$R_3 \leftrightarrow R_1$

$$\begin{bmatrix} 1 & -1 & 2 & -6 \\ 2 & -1 & -4 & -16 \\ 2 & -1 & 1 & 4 \end{bmatrix}$$

$-2R_1 + R_2 \to R_2$
$-2R_1 + R_3 \to R_3$

$$\begin{bmatrix} 1 & -1 & 2 & -6 \\ 0 & 1 & -8 & -4 \\ 0 & 1 & -3 & 16 \end{bmatrix}$$

$-R_2 + R_3 \to R_3$
$\frac{1}{5}R_3 \to R_3$

$$\begin{bmatrix} 1 & -1 & 2 & -6 \\ 0 & 1 & -8 & -4 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$8R_3 + R_2 \to R_2$
$-2R_3 + R_1 \to R_1$

$$\begin{bmatrix} 1 & -1 & 0 & -14 \\ 0 & 1 & 0 & 28 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$R_2 + R_1 \to R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 14 \\ 0 & 1 & 0 & 28 \\ 0 & 0 & 1 & 4 \end{bmatrix}$$

$$\boxed{(a, b, c) = (14, 28, 4)}$$

Question 5b: If lower triangle, find a, b, c, and d .

$$\begin{bmatrix} a & 2a-b+c & 2a-b-4c \\ 4 & b & a-b+2c \\ -16 & -6 & c \end{bmatrix}$$

$$0 = 2a-b+c = 2a-b-4c = a-b+2c$$

$$\begin{aligned} a &= b-2c \\ 2a &= b+4c \\ 2a &= b-c \end{aligned}$$

$$b-c = b+4c$$

$$\implies \boxed{a=b=c=0}$$