Date: 2020-08-28

Announcements:

Exam date will be updeated on Blackboard today. **Assignmtent: first homework due Wednesday**

0.0.1 Trace of a Matrix

Definition 1 (Trace)**.** *If A is a square matrix, then the trace of the matrix, dentoed by $tr(A)$ is the sum of the entries on the main diagonal.*

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$$tr(A) = 15$$

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 1 & 5 & 9 \end{bmatrix}$$

$$tr(A) \text{ not defined}$$

0.0.2 Linear Combinations

Definition 2 (Linear combination)**.** *If $A_1, A_2, \ldots, A_r$ are matrices of the same size and if $C_1, C_2, \ldots, C_r$ are scalars, then an expression of the form*

$$C_1A_1 + C_2A_2 + \ldots + C_rA_r$$

is called a linear combination of $A_1, A_2, \ldots, A_r$ with coefficients $C_1, C_2, \ldots, C_r$.

Theorem 1 (1.3.1)**.** *If A is an $m \times n$ matrix and if x is an $n \times 1$ column vector, then the product Ax can be expressed as a linear combination of the column vectors in A in which the coefficients are the entries of x.*

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \\ 5 & 9 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 32 \\ 37 & 68 \end{bmatrix}$$

We begin thinking about this in different ways.

$$= \begin{bmatrix} 16 & 32 \\ 37 & 68 \end{bmatrix} \implies \begin{bmatrix} A_{b_1} & A_{b_2} \end{bmatrix}$$

In general:

$$AB = A[b_1b_2 \ldots b_n] = [Ab_1Ab_2 \ldots Ab_n]$$

Nothing says you can't partition differently.

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\end{bmatrix}\begin{bmatrix}3 & 1\\ -1 & 2\\ 5 & 9\end{bmatrix}$$
$$\implies \begin{bmatrix}a_1\\ a_2\end{bmatrix} B$$
$$= \begin{bmatrix}a_1 B\\ a_2 B\end{bmatrix}$$

0.1 Inverse and Algebraic Properties of Matrices

The idea behind this section is we want to see these matrices like numbers in arithmetic.

Definition 3 (zero matrix)**.** *a matrix of all zero's.*

Definition 4 (identity matrix)**.** *An $n \times n$ identity matrix is a square matrix with 1's on the main diagonal and 0's everywhere else.*

Definition 5 (Additive inverse)**.**

$$A + (-A) = 0$$

Definition 6 (multiplication inverse)**.** A^{-1}