

DATE: 2020-08-31

## ANNOUNCEMENTS:

Test: Friday, September 11th, or moved to Monday 14th if still catching up.

Assignment: Sec 1.4 (1-4, 51-58)

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### 0.0.1 Matrix Arithmetic

**Theorem 1** (1.4.1 Properties of matrix arithmetic). *Assuming that the sizes of the matrices are such that the indicated operations can be performed, the following rules of matrix arithmetic are valid.*

- $A + B = B + A$
- $A + (B + C) = (A + B) + C$
- $A(BC) = (AB)C$

$$A_{m \times n} B_{n \times r} C_{r \times w}$$

*"Operations must be valid"*

- $A(B + C) = AB + AC$
- $(B + C)A = BA + CA$
- $A(B - C) = AB - AC$
- $(B - C)A = BA - CA$
- $a(A + B) = aA + aB$
- $a(A - B) = aA - aB$
- $(a + b)A = aA + bA$
- $(a - b)A = aA - bA$
- $a(bC) = (ab)C$
- $a(BC) = (aB)C = B(aC)$

**How to Verify**  $a(A + B) = aA + aB$ :

1. Must show that each side produces a matrix of the same size:

let A be an  $m \times n$  matrix.

let B be an  $m \times n$  matrix

$A+B$  is defined as an  $m \times n$  matrix

$a(A+B)$  is defined and is an  $m \times n$  matrix

Let A be an  $m \times n$  matrix. Let B be an  $m \times n$  matrix

$aA$  is an  $m \times n$  matrix.

$aB$  is an  $m \times n$  matrix.

$aA + aB$  is defined and is an  $m \times n$  matrix.

2. Show that the corresponding entries of each side are equal:

$$\begin{aligned} & a(A + B) \\ & (a(A + B))_{ij} = a(a_{ij} + b_{ij}) \text{ \{ALL SCALARS\} } \implies \\ & = aa_{ij} + ab_{ij} \\ & = a(A)_{ij} + a(B)_{ij} \\ & = aA + aB \end{aligned}$$

**Example 1** (Theorem 1.4.1). *Example to be done outside class. Prove 1.4.1 Theorems. 1.4.1c is the hardest.*

*Same thing with 1.4.2*

### Inverse of a Matrix

**Definition 1** (Inverse Matrix). *If  $A$  is a square matrix, and if a matrix  $B$  of the same size can be found such that  $AB = BA = I$ , then  $A$  is said to be invertible (or nonsingular) and  $B$  is called the inverse of  $A$ . If no such  $B$  can be found, then  $A$  is said to be not invertible or singular.*