DATE: 2020-09-22

ANNOUNCEMENTS:

0.1 The Chain Rule

$$y = f(g(x))$$

let

$$u = g(x) \rightarrow \frac{\partial u}{\partial x} = g'(x)$$
$$y = f(u) \rightarrow \frac{\partial y}{\partial u} = f'(u)$$

0.1.1 The Chain Rule (Case 1)

Suppose $z = f(x, y)$ is a differentiable function of x and y, where x and y are differentiable functions of t, Then z is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

Example 1 (Case 1 Chain Rule)**.**

$$z = x^2 y + 3x^3 y^4$$
$$x = e^{2t}$$
$$y = \cos t$$

$$\frac{\partial z}{\partial x} = 2xy + 9x^2 y^4$$
$$\frac{\partial z}{\partial y} = x^2 + 12x^3 y^3$$

$$\frac{dx}{dt} = 2e^{2t}$$
$$\frac{dy}{dt} = -\sin t$$

$$\frac{dz}{dt} = (2xy + 9x^2 y^4)2e^{2t} + (x^2 + 12x^3 y^3) \cdot -\sin t$$

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0.1.2 The Chain Rule (Case 2)

Suppose $z = f(x, y)$ is a differentiable function of x and y, where $x = g(s, t)$ and $y = h(s, t)$ are differentiable functions of t, Then z is a differentiable function of s and t, Then