Chapter 1

Matrices and Matric Operations

DATE: 2020-08-26

ANNOUNCEMENTS:

Assignment: Set 1.3 (1-6, 11-16, 23, 24)

1.1 Matrices

Definition 1 (Matrix). *A rectangular array of numbers.*

$$A = \begin{bmatrix} 1 & 3 & 5 \\ 4 & 7 & 9 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$

Size: # Rows x # Columns
A : 2x3
B : 3x2

Example 1 (A General matrix). *A is $m \times n$ elements*
m-rows
n-columns

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \dots & a_1n \\ a_{21} & a_{22} & a_{23} & \dots & a_2n \\ \dots & \dots & \dots & \dots & \dots \\ a_{m_1} & a_{m_2} & a_{m_3} & \dots & a_{mn} \end{bmatrix}$$

1.1.1 Square matrix

Definition 2 (Square Matrix). *a matrix where #rows = #columns*

$$\begin{bmatrix} a_{11} & a_{22} & \dots & a1n \\ a_{21} & a_{22} & \dots & a_{2n} \\ \dots & \dots & \dots & \dots \\ a_{m_1} & a_{m_2} & \dots & a_{mn} \end{bmatrix}$$

where $a_{11} \to a_{mn}$ is the main diagonal

1.2 Matrix operations

Theorem 1 (Matrix equality). *Two matrices are defined to be equal if they have the same size and their corresponding entries are equal.*

1.2.1 addition and subtraction

Theorem 2. *The sum of matrices A and B is written $A + B$ and it is the matrix obtained by adding corresponding entries of two matrices of the same size.*

Example 2 (matrix addition).

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \end{bmatrix} + \begin{bmatrix} -3 & 4 & -7 & 8 \\ -1 & 0 & 5 & 9 \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 6 & -4 & 12 \\ 4 & 6 & 12 & 17 \end{bmatrix}$$

Notation

A - entire matrix

a_{ij} **-** individual entries

$(A + B)_{ij}$ **-** notation of entry addition $(A)_{ij} + (B)_{ij}$

$(A - B)_{ij}$ **-** $(A)_{ij-(B)_{ij}}$

Example 3 (Matrix addition).

$$\begin{bmatrix} 1 & 3 & 4 \end{bmatrix} + \begin{bmatrix} -5 & -7 & 9 \end{bmatrix} = \begin{bmatrix} -4 & -4 & 13 \end{bmatrix}$$
$$\begin{bmatrix} 1 & 3 & 4 \end{bmatrix} - \begin{bmatrix} -5 & -7 & 9 \end{bmatrix} = \begin{bmatrix} 6 & 10 & -5 \end{bmatrix}$$

1.2.2 Product of a scalar, c, and a Matrix, A

The product of a scalar and a matrix, cA, is produced by multiplying each entry of A by c.

$$(cA)_{ij} = c(A)_{ij}$$

Example 4.

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}$$
$$B = \begin{bmatrix} -1 & 5 \\ -3 & 0 \\ 9 & 7 \end{bmatrix}$$
$$3A - B$$
$$\begin{bmatrix} 3 & 6 \\ 9 & 12 \\ 15 & 18 \end{bmatrix} - \begin{bmatrix} -1 & 5 \\ -3 & 0 \\ 9 & 7 \end{bmatrix} = \begin{bmatrix} 4 & 1 \\ 12 & 12 \\ 6 & 11 \end{bmatrix}$$

1.2.3 Product of matrices

Definition 3. *The producto of two matrices A and B, written AB, is only defined when the number of columns of matrix A is equal to the number of rows of matrix B.*

$$A_{m\times r}B_{r\times n}$$

The size of the product will be the rows of A by the columns of B.

$$C_{m\times n}$$

Example 5.

$$A_{3\times 5}\ and\, B_{5\times 3}$$

5 and 5: this can be done.
3 and 3: the size of the result

Getting the entries

To find the entries in Row i and Column j of AB, single out the ith row of A and the jth column of B, multiply their corresponding entries and add the results.

Example 6.

$$\begin{bmatrix} 3 & 4 \\ 2 & 1 \\ 3 & 2 \end{bmatrix}_{3\times 2} \begin{bmatrix} -1 & 1 & 2 & 3 \\ 1 & 5 & -2 & 2 \end{bmatrix}_{2\times 4}$$

Solution 1.

$$\begin{bmatrix} 3\cdot -1+4\cdot 1 & 3\cdot 1+4\cdot 5 & 3\cdot 2+4\cdot 2 & 9+8 \\ -2+1 & 2+5 & 4+2 & 6+2 \\ -3+2 & 3+10 & 6+4 & 9+4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 23 & 14 & 17 \\ -1 & 7 & 6 & 8 \\ -1 & 13 & 10 & 13 \end{bmatrix}$$

Example 7.

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 7 \\ 2 & 1 & 9 \end{bmatrix}$$

$$X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$Ax$$

Solution 2.

$$Ax = \begin{bmatrix} 2x_1 + x_2 + 3x_3 \\ 4x_1 - x_2 + 7x_3 \\ 2x_1 + x_2 + 9x_3 \end{bmatrix}$$

Definition 4 (Transpose of a matrix)**.** *A^T is the matrix obtained when the rows and columns of A are interchanged.*

Example 8.

$$A = \begin{bmatrix} 1 & 2 & 5 \\ 9 & 3 & 1 \end{bmatrix}$$

Solution 3. $A^T = \begin{bmatrix} 1 & 9 \\ 2 & 3 \\ 5 & 1 \end{bmatrix}$