Chapter 1

Unit 1: Differentiation

DATE: 2020-06-01
ANNOUNCEMENTS:

1.1 A. What is a derivative?

- Geometric interpretation
- Physical interpretation
- Importance of derivatives to all measurements
  - Science
  - Enineering
  - Econ
  - PolSci

1.2 B. How to differentiate anything

Example: $e^{x \arctan(x)}$

1.2.1 Geometric interpretation:

Find the tangent line to $y = f(x)$ at $P = (x_0, y_0)$

The tangent line is defined by the equation $y - y_0 = m(x - x_0)$

Point: $y_0 = f(x_0)$

Slope: $m = f'(x_0)$

Definition 1 (Derivative). *$f'(x_0)$, the derivative of f at x_0, is the slope of the tangent line to $y = f(x)$ at the point P*

Taken for granted the geometric interpretation, we know the line through a point, but we want to annalytically describe the tangent in a way that a machine could reproduce.

First grasp this with language.

$y - y_0 = m(x - x_0$

point

$$y_0 = f(x_0).$$

slope:

$$m = f'(x_0).$$

Definition 2 (Derivativ). *$f'(x_0)$, the derivative of f at x_0, si the slope of the tangent line to $y = f(x)$ at P.*

Definition 3 (Tangent Line). *Limit of the secant lines PQ as $Q \to P$ (P fixed)*

1.2.2 Finding the Slope

To find the slope between P and Q, we denote the horizontal distance as Δx, and the height as Δf

The slope of the secand line is $\frac{\Delta f}{\Delta x}$ The slope of the tangent line is

$$\boxed{m = \lim_{\Delta x \to 0} \frac{\Delta f}{\Delta x}} \tag{1.1}$$

Slope of the tangent line

So we redefine the points P and Q to make this formula more usable,

$$P = (x_0, f(x_0).$$
$$Q = (x_0 + \Delta x, f(x_0 + \Delta x).$$

$$\boxed{f'(x_0) = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}} \tag{1.2}$$

Limit definition of derivative

Example 1 (Example 1). $f(x) = \frac{1}{x}$

Solution 1 (Example 1).

$$\begin{aligned}
\frac{\Delta f}{\Delta x} &= \frac{\frac{1}{x_0+\Delta x} - \frac{1}{x_0}}{\Delta x} \\
&= \frac{1}{\Delta x}\left(\frac{x_0 - (x_0 + \Delta x}{(x_0 + \Delta x)x_0}\right) \\
&= \frac{-1}{(x_0 + \Delta x)x_0} \to (\Delta x \to 0) \to \\
&= \frac{-1}{x_0^2}
\end{aligned}$$

Note: The calculus part of calculus is easy, but it is made hard by the putting it in the context of everything learned up to that point.

Example 2 (Geometry problem). *Find areas of triangles enclosed by the axes and tangent to $y = \frac{1}{x}$*

Solution 2 (Geometry problem). $y = \frac{1}{x}$

Pick a point $(x_0, y_0$ *on this curve, and we must find the base and the height. Find the formula for the tangent line:*

$$y - y_0 = \frac{-1}{x_0^2}(x - x_0).$$

Find the x*-intercept:*

$$\begin{aligned}
&(y = 0) \\
0 - \frac{1}{x_0} &= \frac{-1}{x_0^2}(x - x_0) \\
&= \frac{-x}{x_0^2} + \frac{1}{x_0} \\
\frac{x}{x_0^2} &= \frac{2}{x_0} \\
&= 2x_0
\end{aligned}$$