Text as Data

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Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2017)

1) Discovery: a hypothesis or view of the world
2) Measurement according to some organization
3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Measurement

Two approaches to measurement

1) Use an existing classification scheme to categorize documents (Today)
2) Simultaneously discover categories and measure prevalence (repurpose discovery methods) (Wednesday)

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- Positions on legislation
  $\Rightarrow$ { Support, Ambiguous, Oppose }
- Positions on Court Cases
  $\Rightarrow$ { Agree with Court, Disagree with Court }
- Liberal/Conservative Blog Posts
  $\Rightarrow$ { Liberal, Middle, Conservative, No Ideology Expressed }

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Style/Tone: How is it said?

- Taunting in floor statements
  $\Rightarrow$ { Partisan Taunt, Intra party taunt, Agency taunt, ... }
- Negative campaigning
  $\Rightarrow$ { Negative ad, Positive ad}

Regression models

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Predictions will be variable

Mean Square Error

Suppose θ is some value of the true parameter

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To reduce MSE, we are willing to induce bias to decrease variance $\rightsquigarrow$ methods that shrink coefficeints toward zero

Ridge Regression

Penalty for model complexity

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$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y})$$

Ridge Regression

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$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) \;=\; \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{J} \beta_j x_{ij} \right)^2$$

Ridge Regression

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where:

- $\beta_0 \rightsquigarrow$ intercept
- $\lambda \rightsquigarrow$ penalty parameter
- Standardized $\boldsymbol{X}$ (coefficients on same scale)

Ridge Regression $\rightsquigarrow$ Optimization

$$\boldsymbol{\beta}^{\text{Ridge}} = \text{arg min}_{\boldsymbol{\beta}} \{f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y})\}$$

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Demean the data and set $\beta_0 = \bar{y} = \sum_{i=1}^{N} \frac{y_i}{N}$

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&= (\boldsymbol{I}_j + \lambda\boldsymbol{I}_j)^{-1}\widehat{\boldsymbol{\beta}} \\
\beta_j^{\text{Ridge}} &= \frac{\widehat{\beta}_j}{1+\lambda}
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Ridge Regression$\rightsquigarrow$ Intuition (2)

$$\begin{aligned} \boldsymbol{\beta}_j &\sim \text{Normal}(0, \tau^2) \\ y_i &\sim \text{Normal}(\beta_0 + \boldsymbol{x}_i^{'}\boldsymbol{\beta}, \sigma^2) \end{aligned}$$

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$$p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) \propto \prod_{j=1}^{J} p(\beta_j) \prod_{i=1}^{N} p(y_i|\boldsymbol{x}_i, \boldsymbol{\beta})$$

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$$\log p(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) \quad = \quad -\sum_{j=1}^{J} \frac{\beta_j^2}{2\tau^2} - \sum_{i=1}^{N} \frac{(y_i - \beta_0 - \boldsymbol{x}'\boldsymbol{\beta})^2}{2\sigma^2}$$

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where:

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\end{aligned}$$

where:

- $\lambda = \frac{\sigma^2}{\tau^2}$

Ridge Regression $\rightsquigarrow$ Intuition (3)

Definition

Suppose $\boldsymbol{X}$ *is an* $N \times J$ *matrix. Then* $\boldsymbol{X}$ *can be written as:*

$$\boldsymbol{X} = \underbrace{\boldsymbol{U}}_{N \times N} \underbrace{\boldsymbol{S}}_{N \times J} \underbrace{\boldsymbol{V}'}_{J \times J}$$

Where:

$$\begin{aligned} \boldsymbol{U}'\boldsymbol{U} &= \boldsymbol{I}_N \\ \boldsymbol{V}'\boldsymbol{V} &= \boldsymbol{V}\boldsymbol{V}' = \boldsymbol{I}_J \end{aligned}$$

$\boldsymbol{S}$ *contains* $\min(N, J)$ *singular values,* $\sqrt{\lambda_j} \geq 0$ *down the diagonal and then 0's for the remaining entries*

Ridge Regression $\rightsquigarrow$ Intuition (3)

Recall: PCA:

$$\frac{1}{N}\boldsymbol{X}'\boldsymbol{X} = \underbrace{\boldsymbol{W}}_{\text{eigenvectors}} \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_J \end{pmatrix} \underbrace{\boldsymbol{W}'}_{\text{eigenvectors}}$$

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Using SVD:

Ridge Regression $\rightsquigarrow$ Intuition (3)

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$$\frac{1}{N}\boldsymbol{X}'\boldsymbol{X} = \underbrace{\boldsymbol{W}}_{\text{eigenvectors}} \begin{pmatrix} \lambda_1 & 0 & \dots & 0 \\ 0 & \lambda_2 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \lambda_J \end{pmatrix} \underbrace{\boldsymbol{W}'}_{\text{eigenvectors}}$$

Using SVD:

$$\frac{1}{N}\boldsymbol{X}'\boldsymbol{X} = \boldsymbol{V}\boldsymbol{S}' \underbrace{\left(\boldsymbol{U}'\boldsymbol{U}\right)}_{\boldsymbol{I}_J} \boldsymbol{S}\boldsymbol{V}'$$

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$$\hat{Y}^{\text{ridge}} = \sum_{j=1}^{J}\boldsymbol{u}_j\frac{\lambda_j}{\lambda_j + \lambda}\boldsymbol{u}_j'\boldsymbol{Y}$$

Degrees of Freedom for Ridge

We will say that the degrees of freedom for Ridge regression with penalty λ is

$$\text{dof}(\lambda) = \sum_{j=1}^{J} \frac{\lambda_j}{\lambda_j + \lambda}$$

Lasso Regression Objective Function

Different Penalty for Model Complexity

$$f(\boldsymbol{\beta}, \boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{N} \left(y_i - \beta_0 - \sum_{j=1}^{J} \beta_j x_{ij} \right)^2 + \lambda \sum_{j=1}^{J} \underbrace{|\beta_j|}_{\text{Penalty}}$$

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Lasso Regression Optimization

Definition

Coordinate Descent Algorithms:

Consider $g : \Re^J \rightarrow \Re$. *Our goal is to find* $\boldsymbol{x}^* \in \Re^J$ *such that* $g(\boldsymbol{x}^*) \leq g(\boldsymbol{x})$ *for all* $\boldsymbol{x} \in \Re$.

To find $\boldsymbol{x}^*$*:*

Until convergence: for each iteration t *and each coordinate* j

$$x_j^{t+1} = arg\ min_{x_j \in \Re} g(x_1^{t+1}, x_2^{t+1}, \ldots, x_{j-1}^{t+1}, x_j, x_{j+1}^t, \ldots, x_J^t)$$

Lasso Regression Optimization: Coordinate Descent

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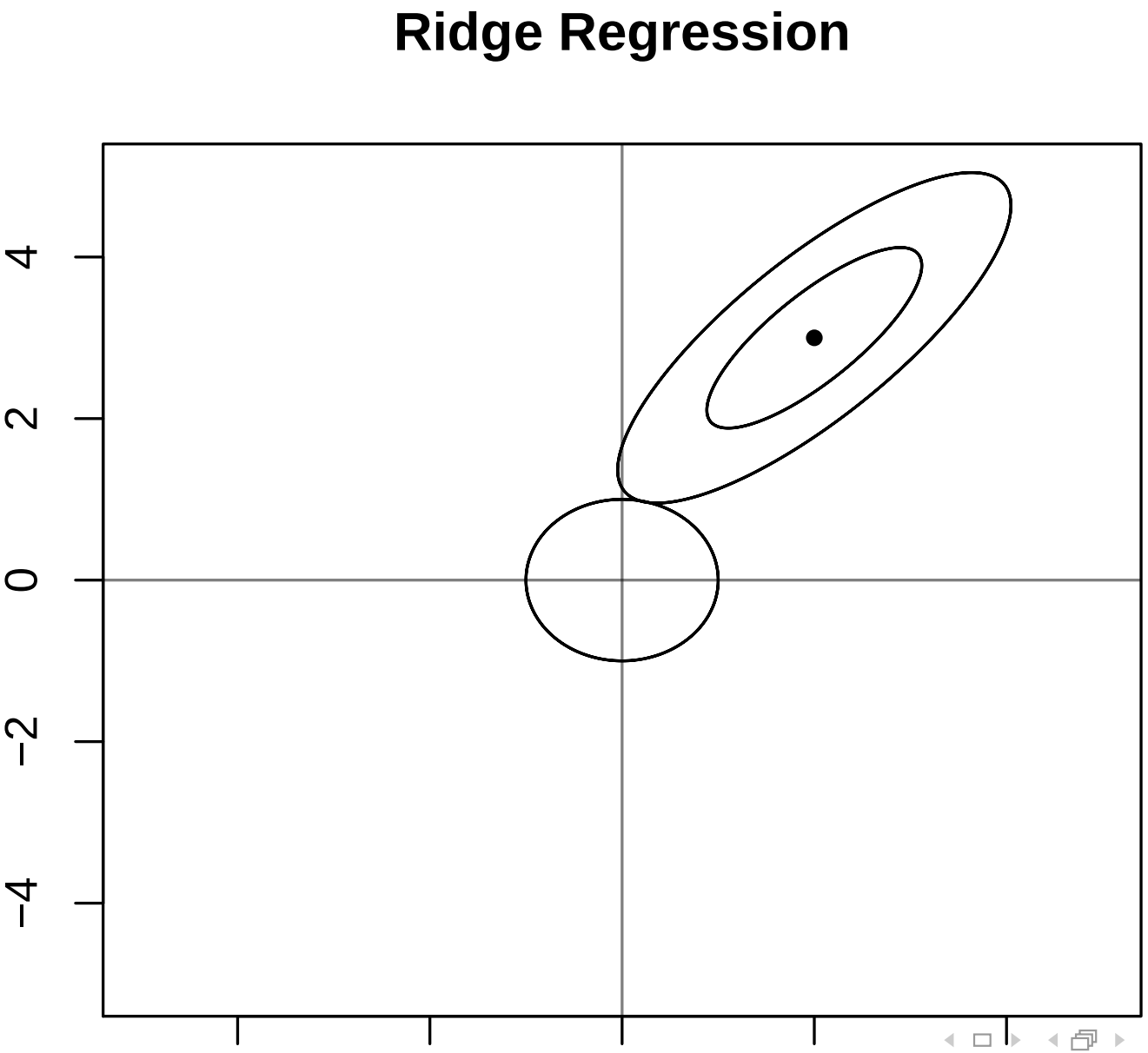
Why does LASSO induce sparsity?

Comparing Ridge and LASSO



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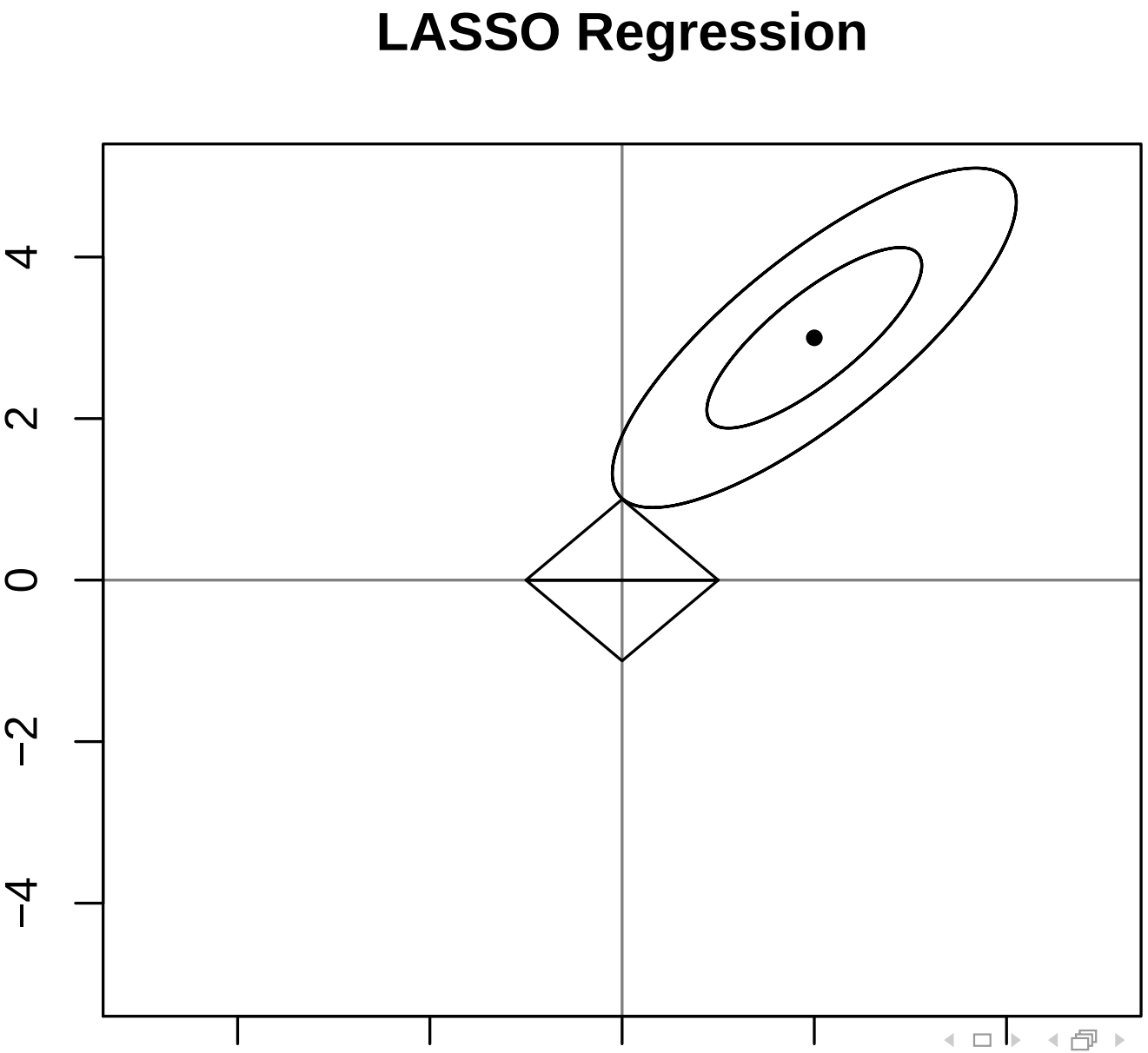

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Ridge and LASSO: The Elastic-Net

Combining the two criteria $\rightsquigarrow$ Elastic-Net

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$$\begin{aligned} L\left(Y_i, f(\hat{\boldsymbol{\beta}}, \boldsymbol{x}_i)\right) &= \left(Y_i - f(\hat{\boldsymbol{\beta}}, \boldsymbol{x}_i)\right)^2 \\ &= \left|Y_i - f(\hat{\boldsymbol{\beta}}, \boldsymbol{x}_i)\right| \\ &= I\left(Y_i = I(f(\hat{\boldsymbol{\beta}}, \boldsymbol{x}_i) > \tau)\right) \end{aligned}$$

Training and Test Sets

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Even if no division, useful to think about systematic components of data.

Loss Functions and Model Complexity

Suppose that we have:

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$$\text{Error} = \text{E}\left[\text{E}[L(\boldsymbol{Y}, f(\hat{\boldsymbol{\beta}}, \boldsymbol{X}))|\mathcal{T}]\right]$$

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&= \sigma_\epsilon^2 + \left[f(\boldsymbol{x}_0) - \text{E}[\hat{f}(\boldsymbol{x}_0)]\right]^2 + E[\left(\hat{f}(\boldsymbol{x}_0) - E[\hat{f}(\boldsymbol{x}_0)]\right)^2]
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&= \text{Irreducible error} + \text{Bias}^2 + \text{Variance}
\end{aligned}$$

Probit Regression (for motivational purposes)

Suppose:

$$\begin{aligned} Y_i &\sim \text{Bernoulli}(\pi_i) \\ \pi_i &= \Phi(\boldsymbol{\beta}' \boldsymbol{x}_i) \end{aligned}$$

where $\Phi(\cdot)$ is the cumulative normal distribution.
Implies log-likelihood

$$\log \mathrm{L}(\boldsymbol{\beta}|\boldsymbol{X}, \boldsymbol{Y}) = \sum_{i=1}^{N} \left[Y_i \log \Phi(\boldsymbol{\beta}' \boldsymbol{x}_i) + (1 - Y_i) \log(1 - \Phi(\boldsymbol{\beta}' \boldsymbol{x}_i)) \right]$$

Log-likelihood is a loss function $\rightsquigarrow$ overly optimistic: improves with more parameters

How Do We Build A Model?

There are many ways to fit models

And many choices made when performing model fit

How do we choose?

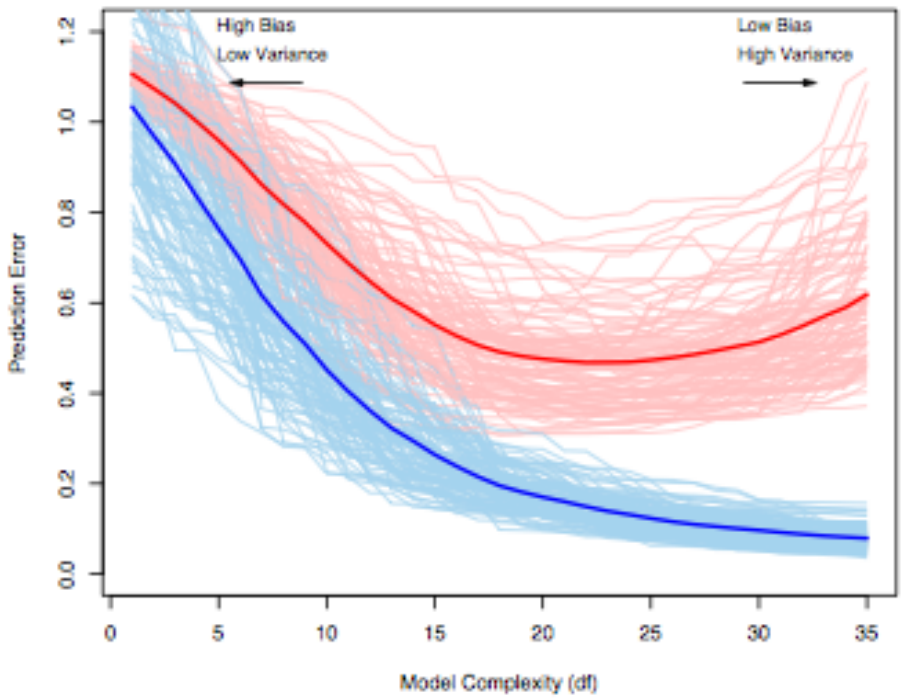

FIGURE 7.1. *Behavior of test sample and training sample error as the model complexity is varied. The light blue curves show the training error* $\overline{\mathrm{err}}$*, while the light red curves show the conditional test error* $\mathrm{Err}_{\mathcal{T}}$ *for 100 training sets of size 50 each, as the model complexity is increased. The solid curves show the expected test error* Err *and the expected training error* $\mathrm{E}[\overline{\mathrm{err}}]$.

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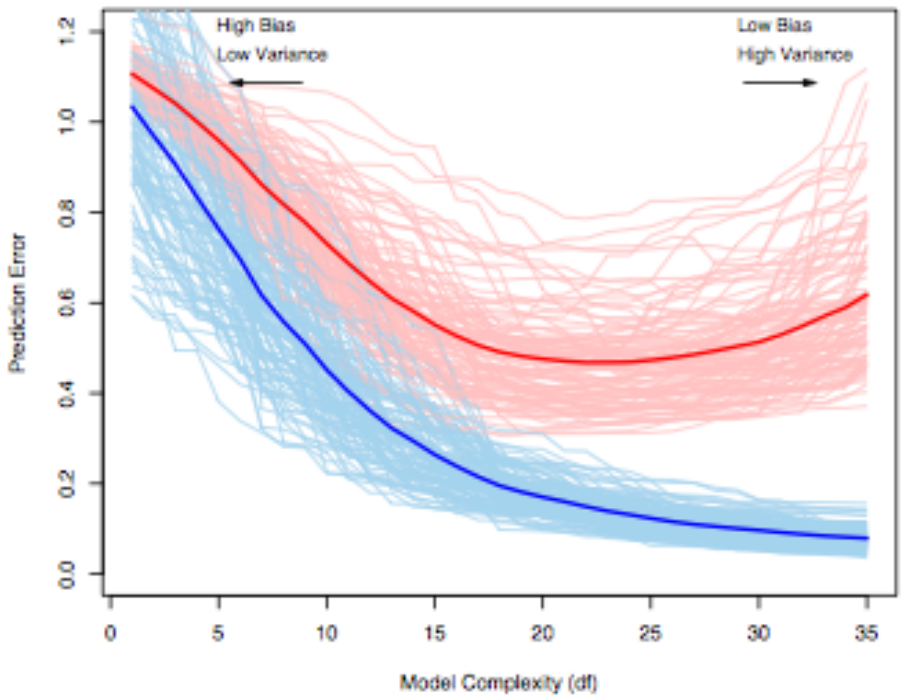

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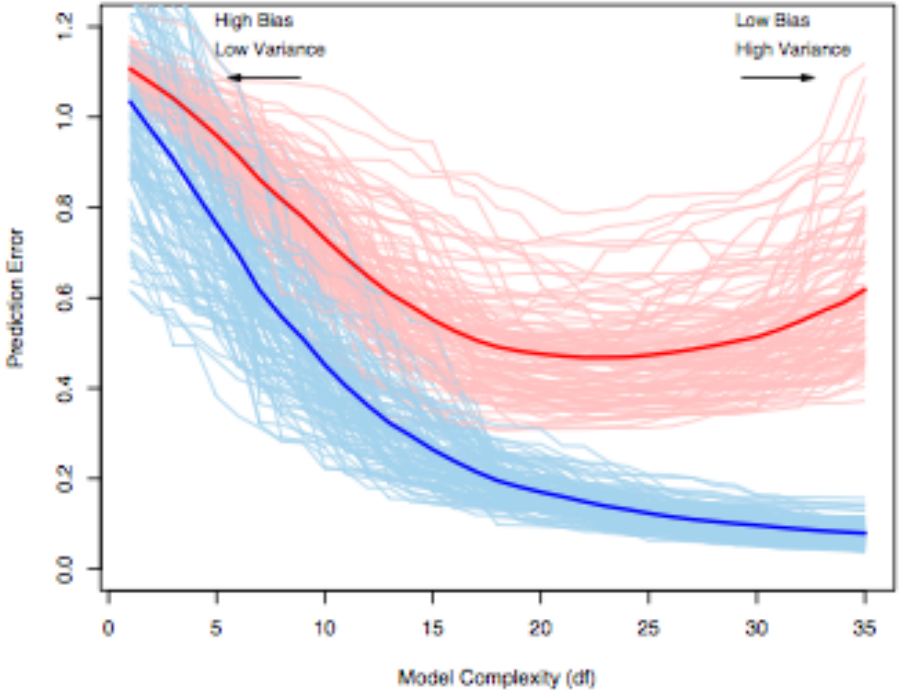
Bad way to choose: within sample model fit (HTF Figure 7.1)



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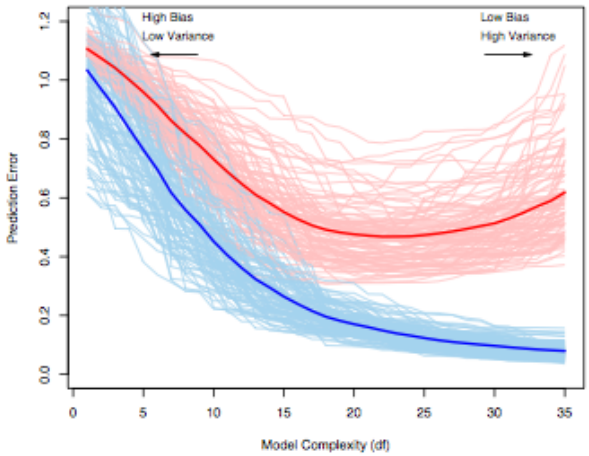

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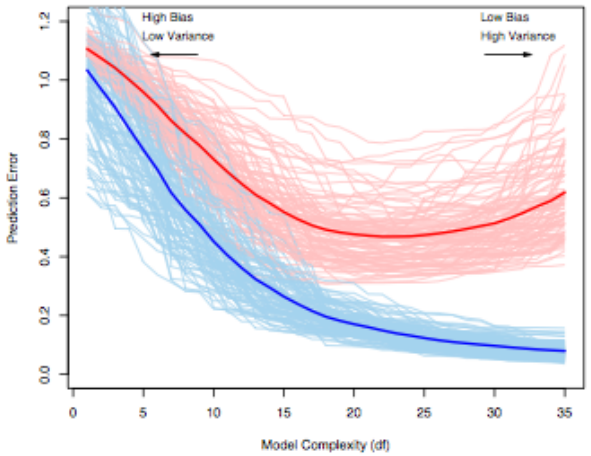

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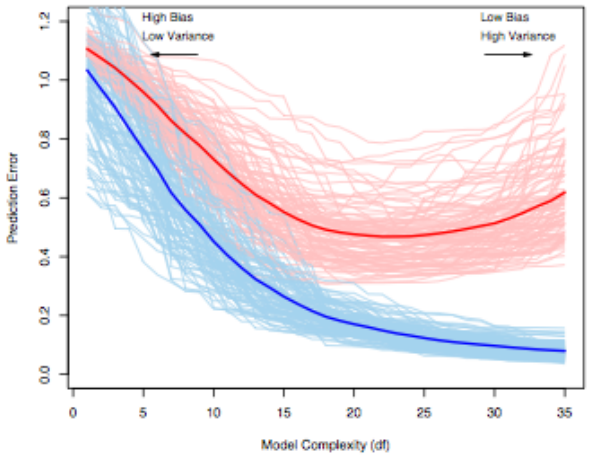

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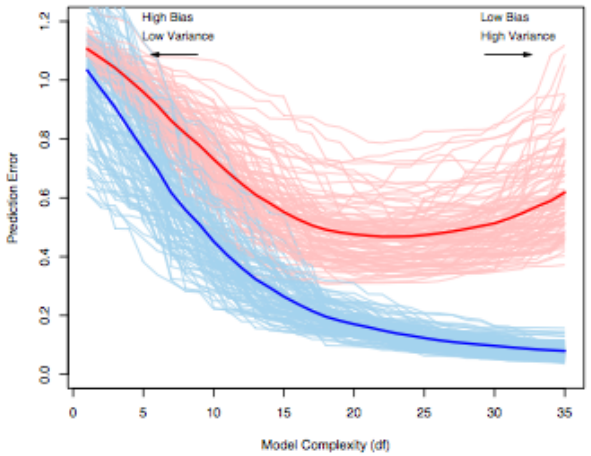

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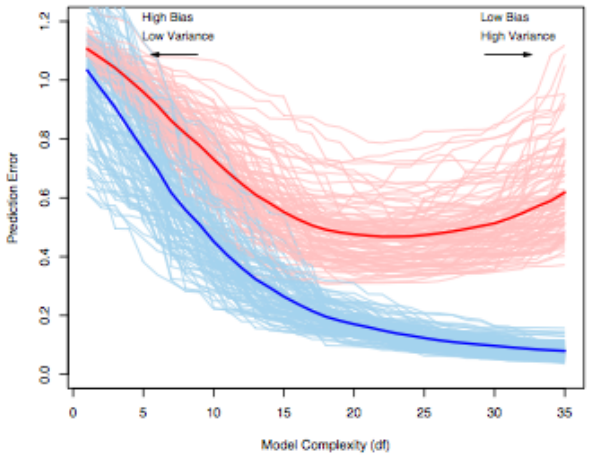

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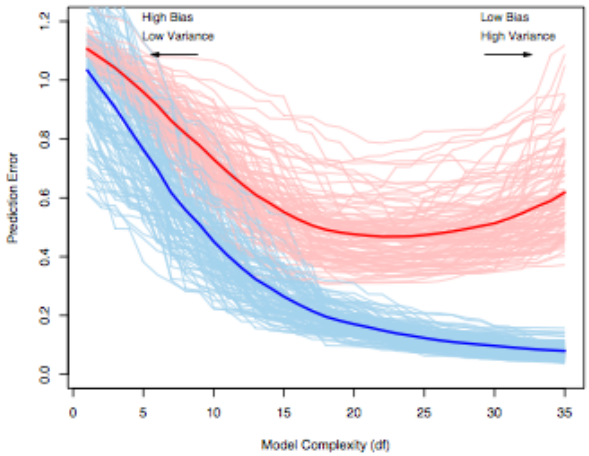

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- Reduces error in training set, increases error in test set

How Do We Choose Covariates?

Best model depends on task

- Causal inference observational study: make treatment assignment ignorable
- Prediction: improve predictive performance

Stepwise Regression

Suppose we have P covariates.
2^P potential models

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Problematic:

1) Not optimal model selection (path dependent)
2) P-value $\neq$ objective of model

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- Derived from method to estimate optimism in likelihood based models
- Derived from a method to compute similarity between estimated model and true model (under assumptions of course)
- Can be extended to general models, though requires estimate of irresolvable error

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Bayesian Information Criterion (BIC) [Schwarz Criterion]

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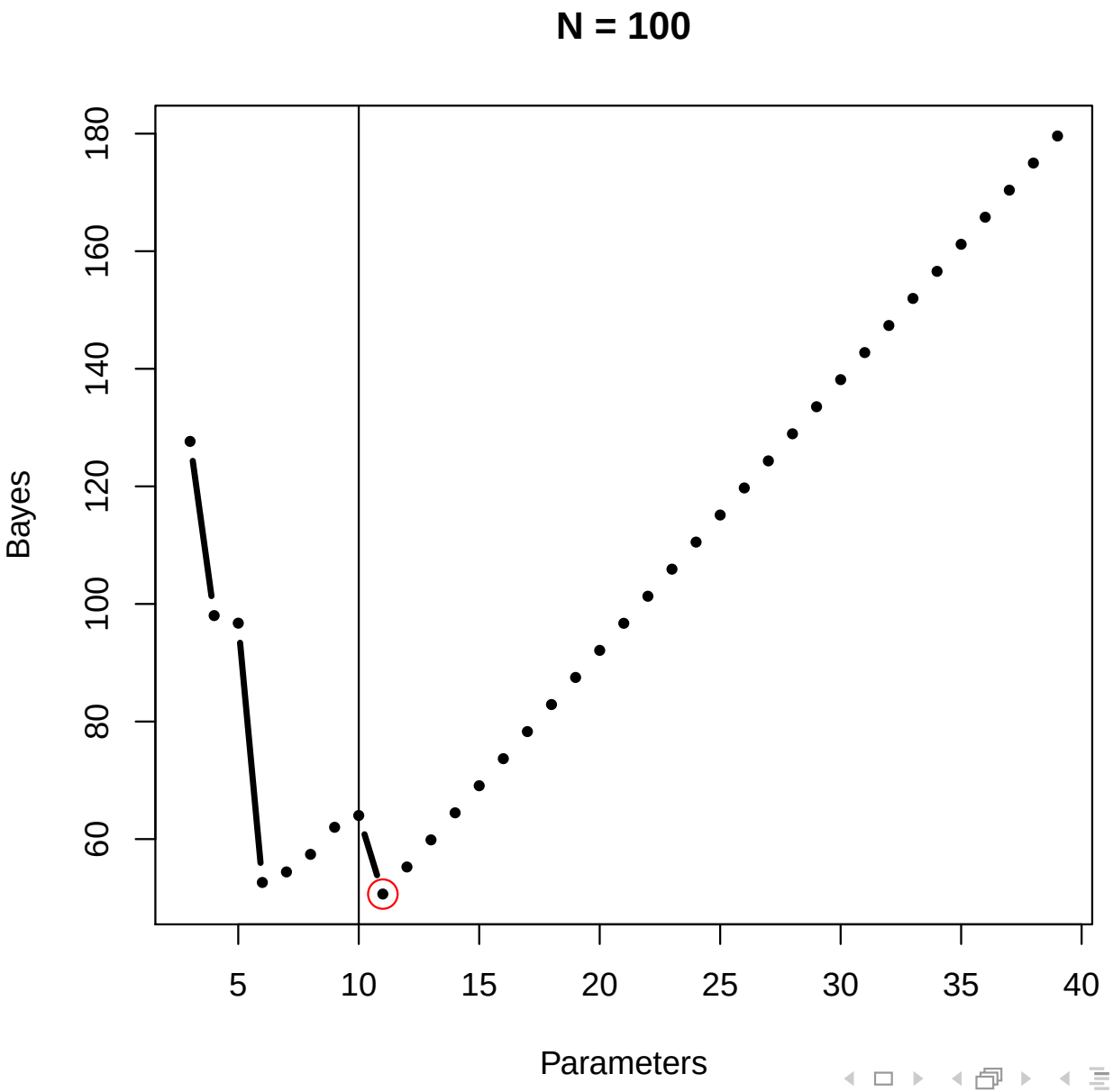
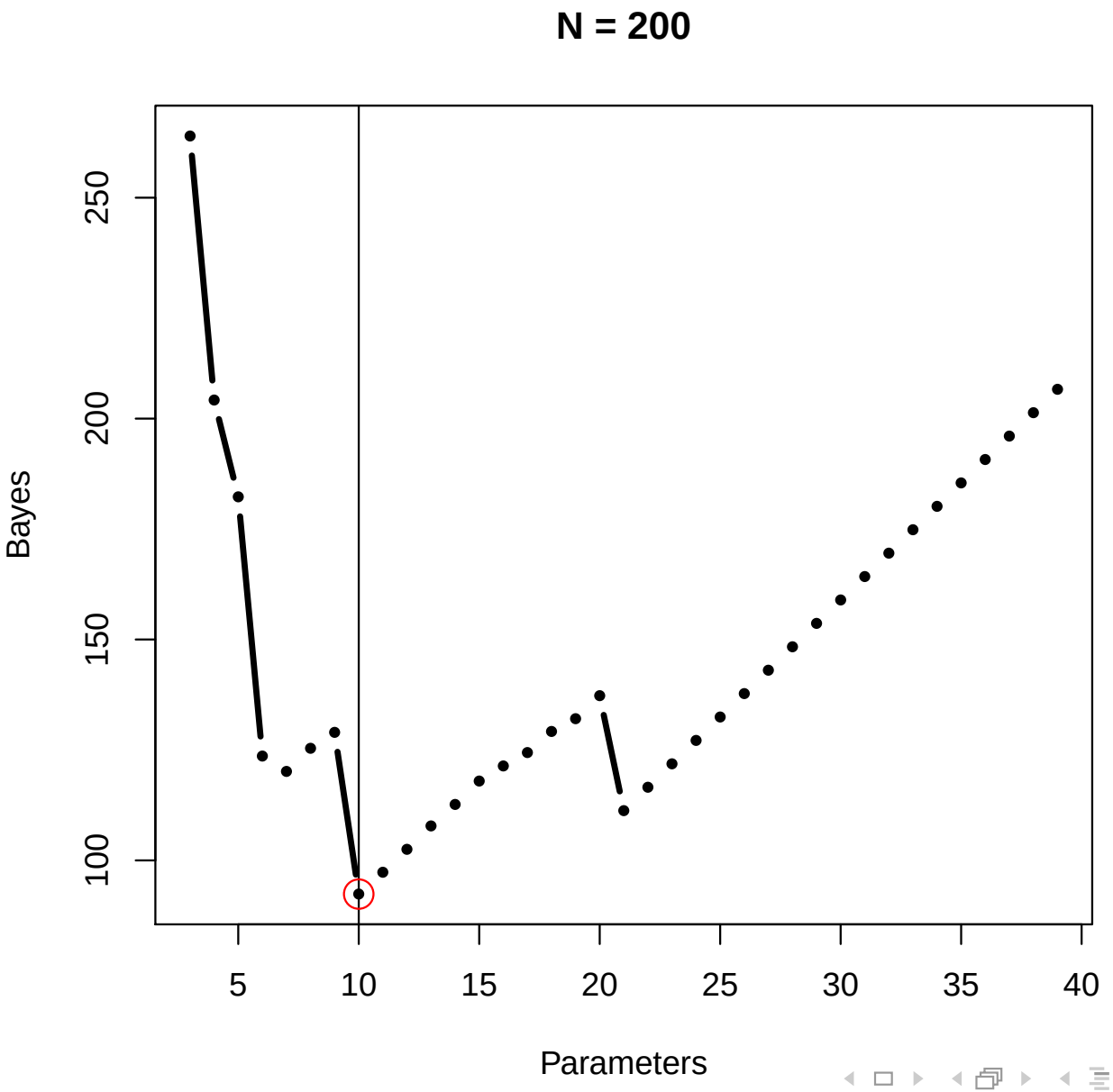
- Approximation to Bayes' factor
- Penalizes more heavily than AIC

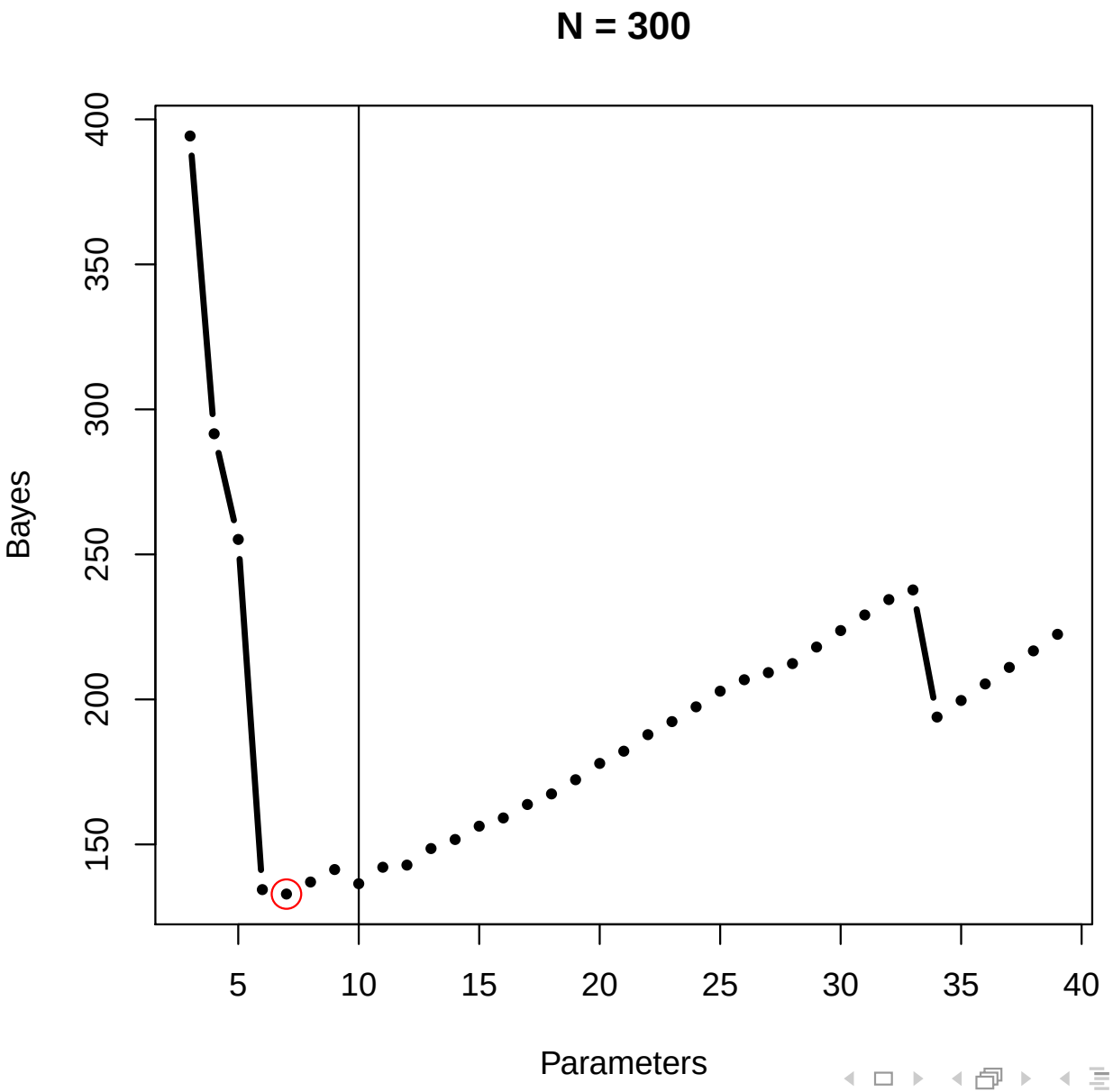
BIC or AIC?


N = 100
Bayes
60
80
100
120
140
160
180
5
10
15
20
25
30
35
40
Parameters

BIC or AIC?



BIC or AIC?


N = 300
Bayes
400
350
300
250
200
150
5
10
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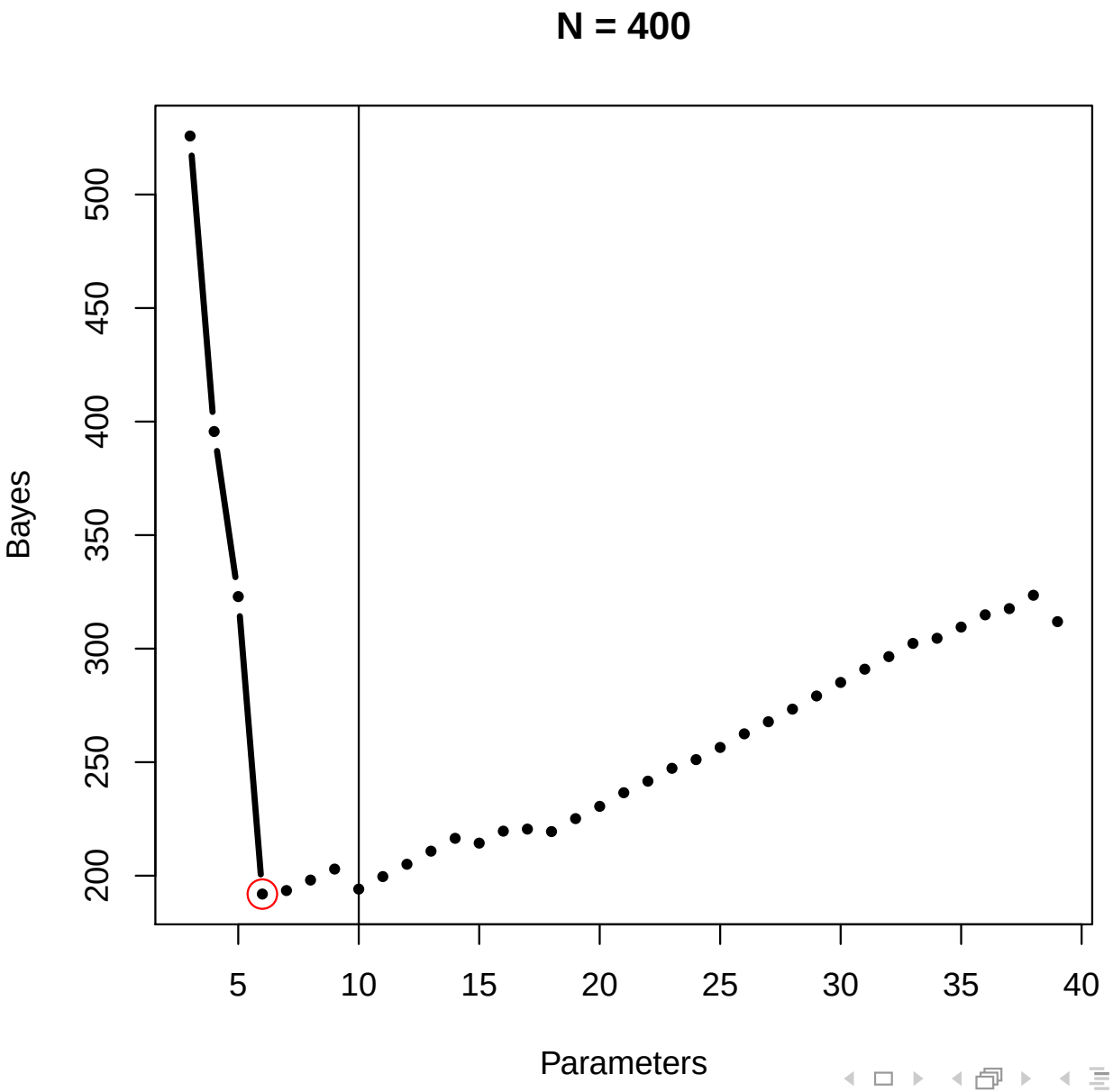
20
25
30
35
40
Parameters

BIC or AIC?


N = 400
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350
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450
500
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30
35
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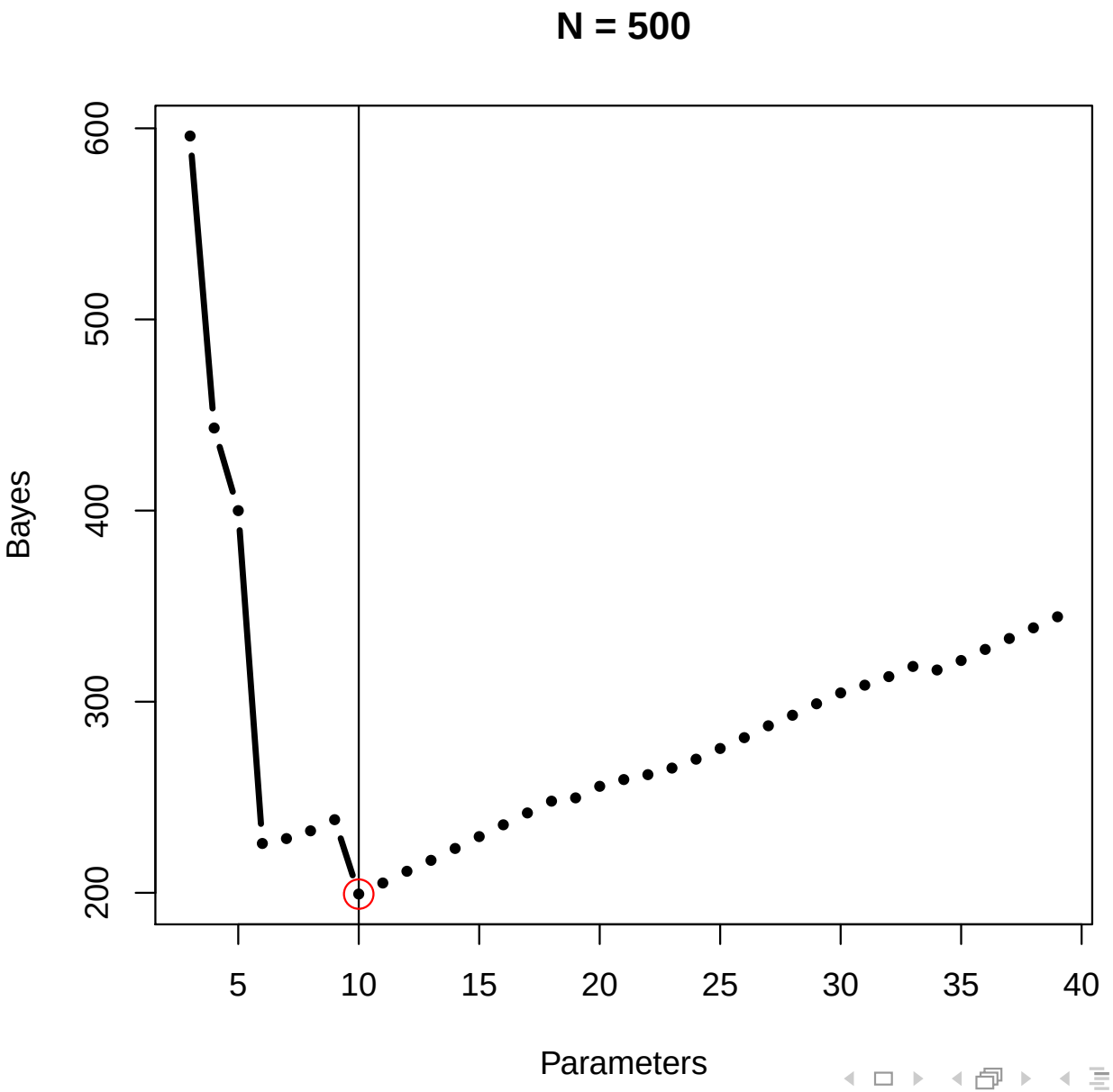
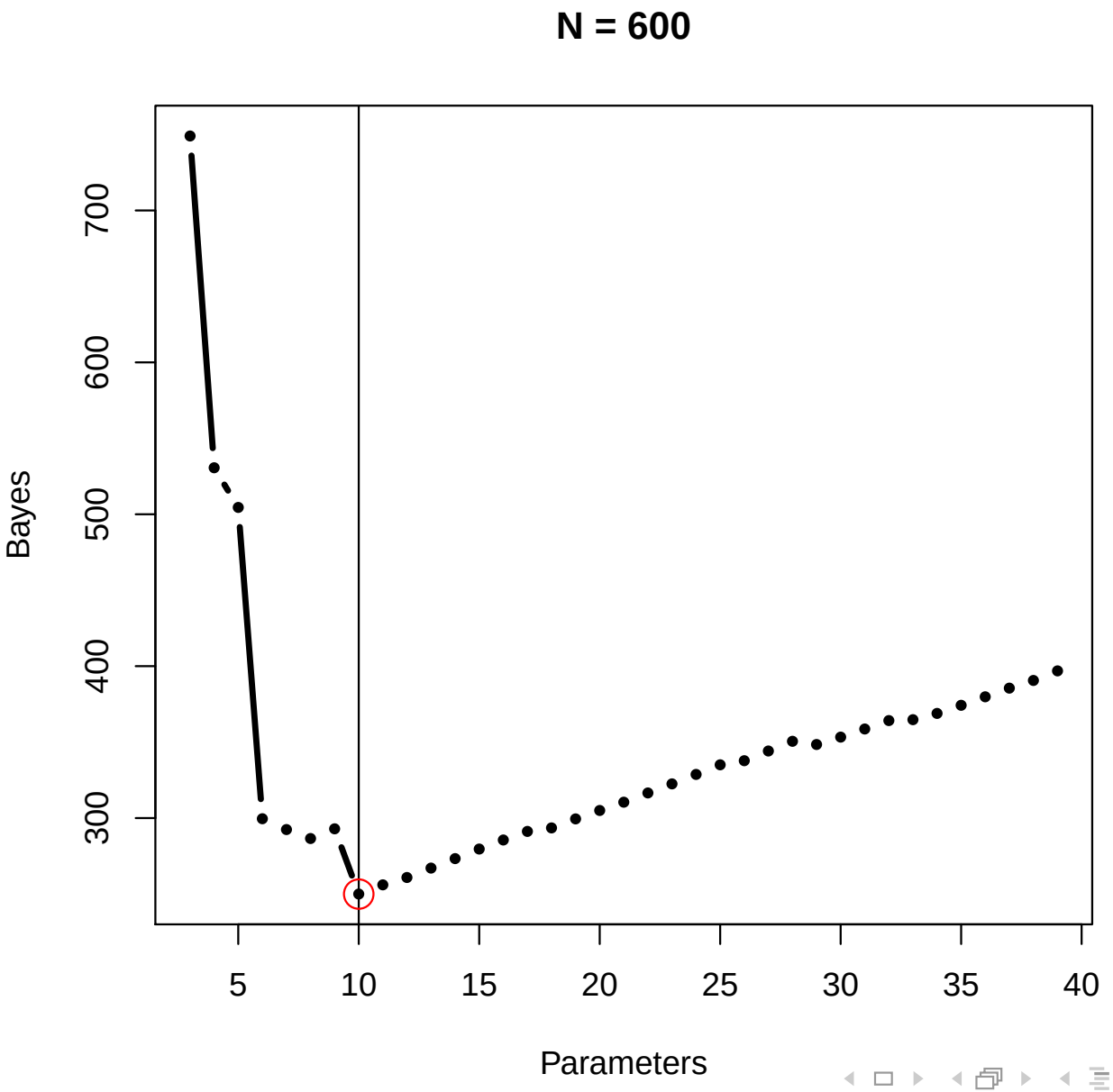
Parameters

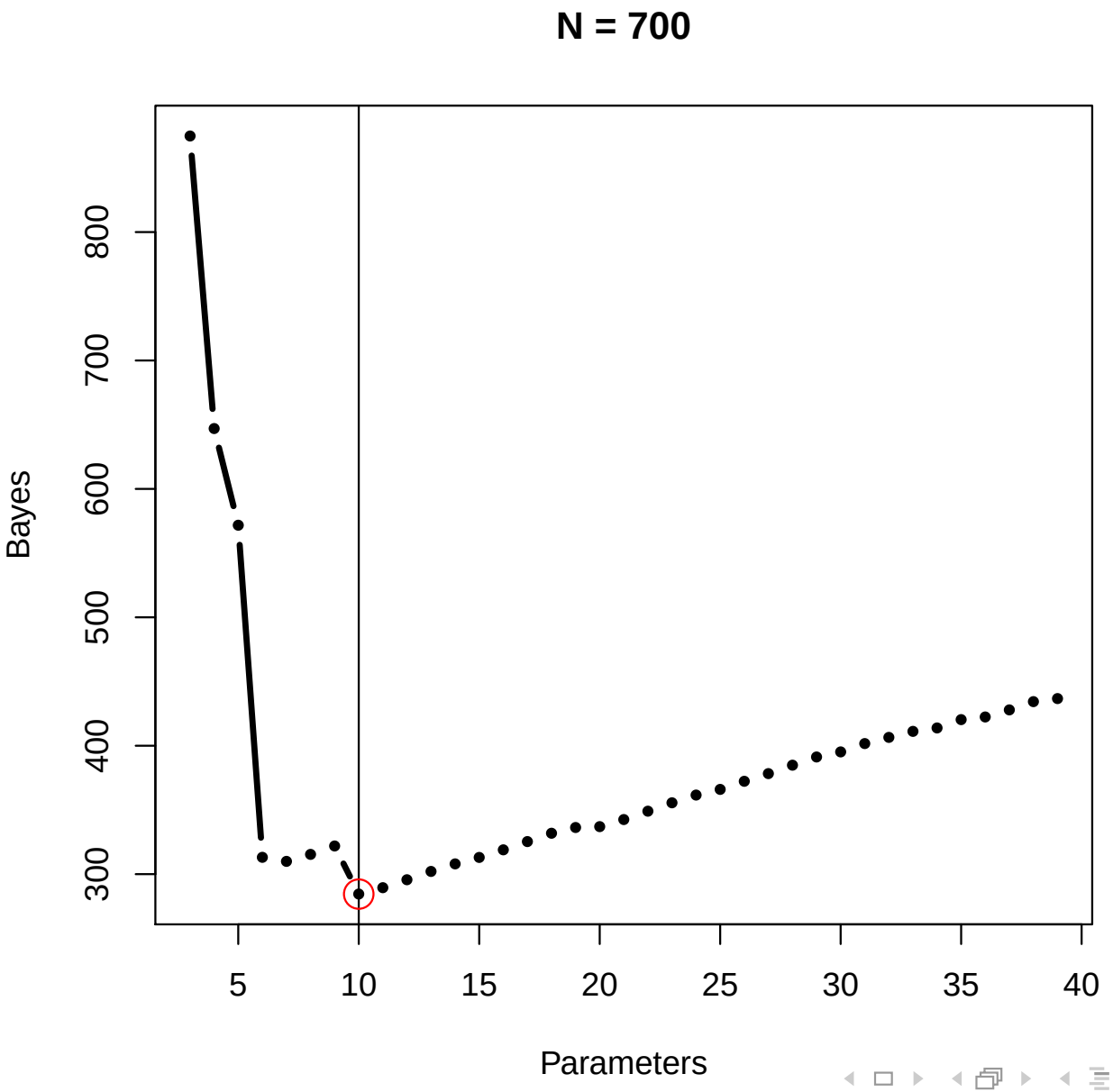
BIC or AIC?



BIC or AIC?



BIC or AIC?


N = 700
Bayes
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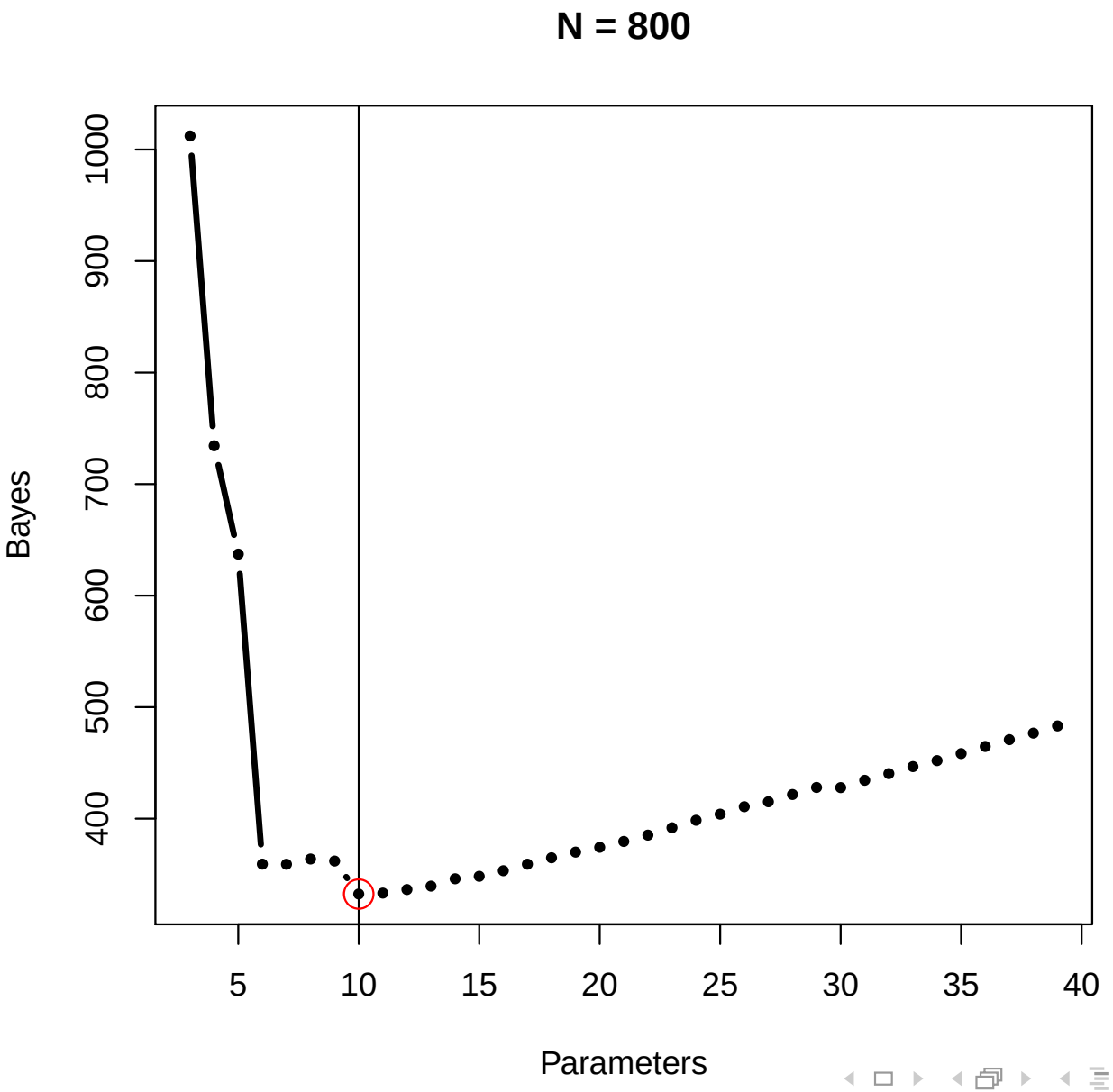
Parameters

BIC or AIC?


N = 800
Bayes
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500
600
700
800
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1000
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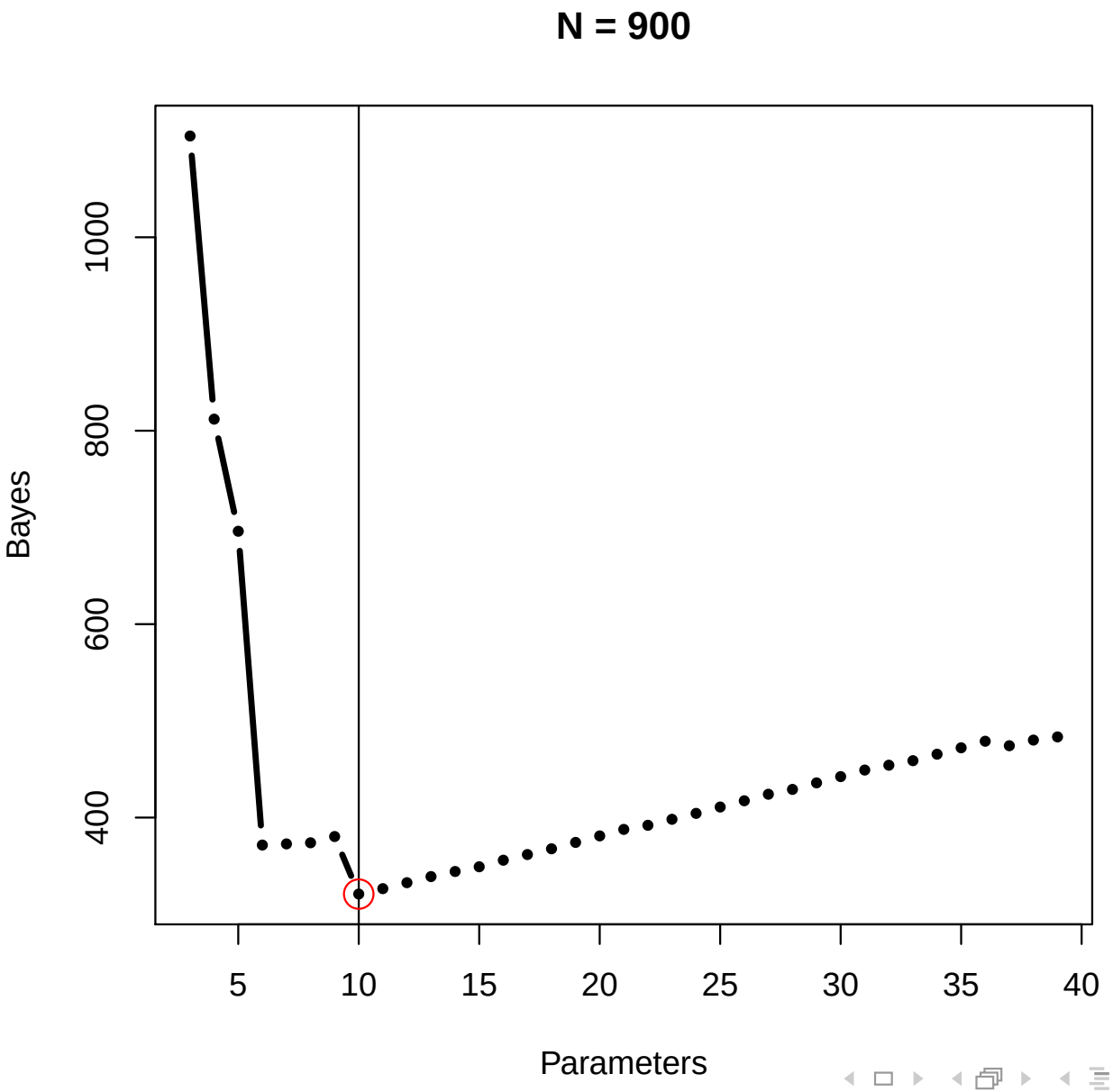
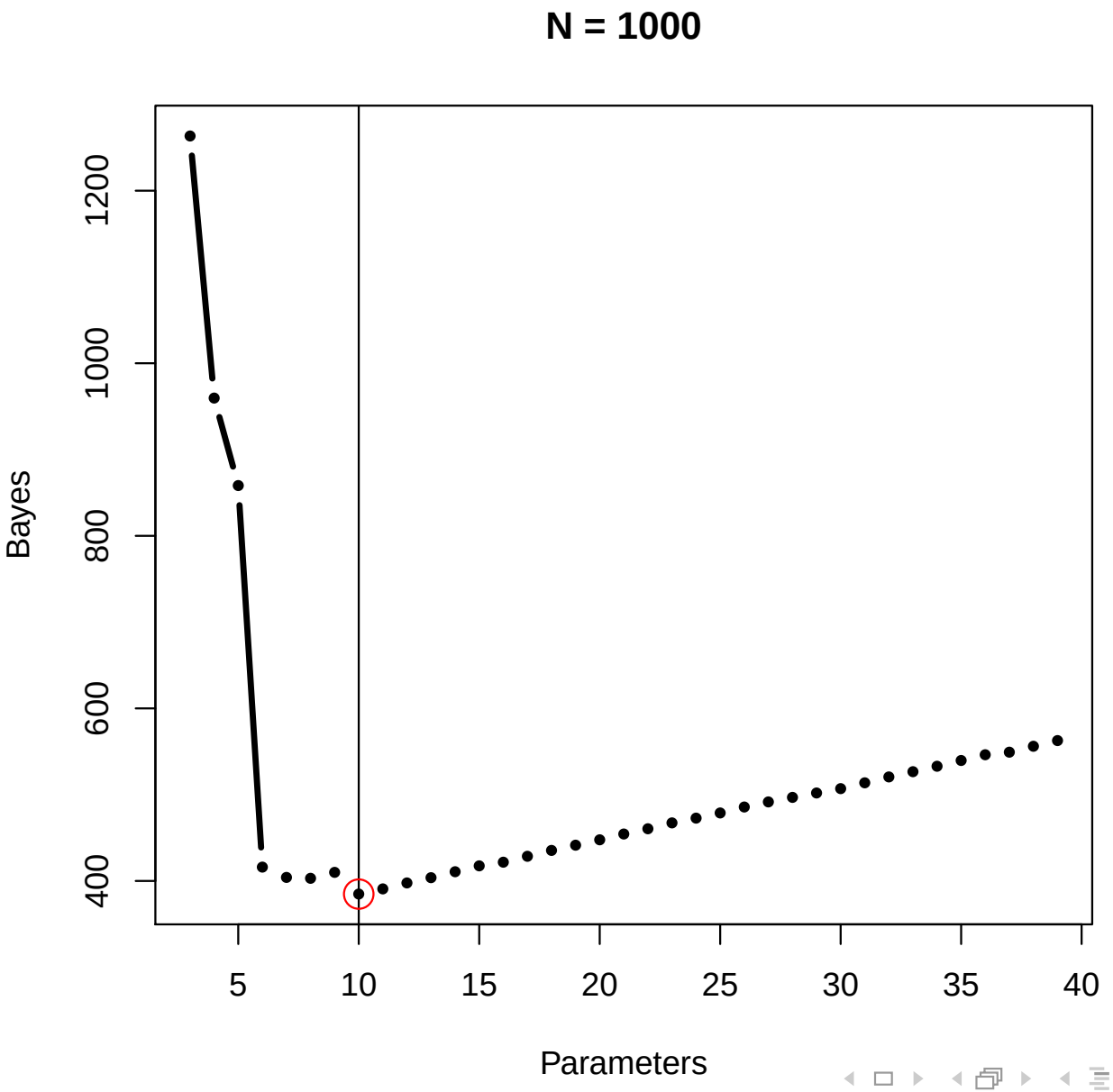
Parameters

BIC or AIC?



BIC or AIC?


N = 1000
Bayes
400
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1000
1200
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35
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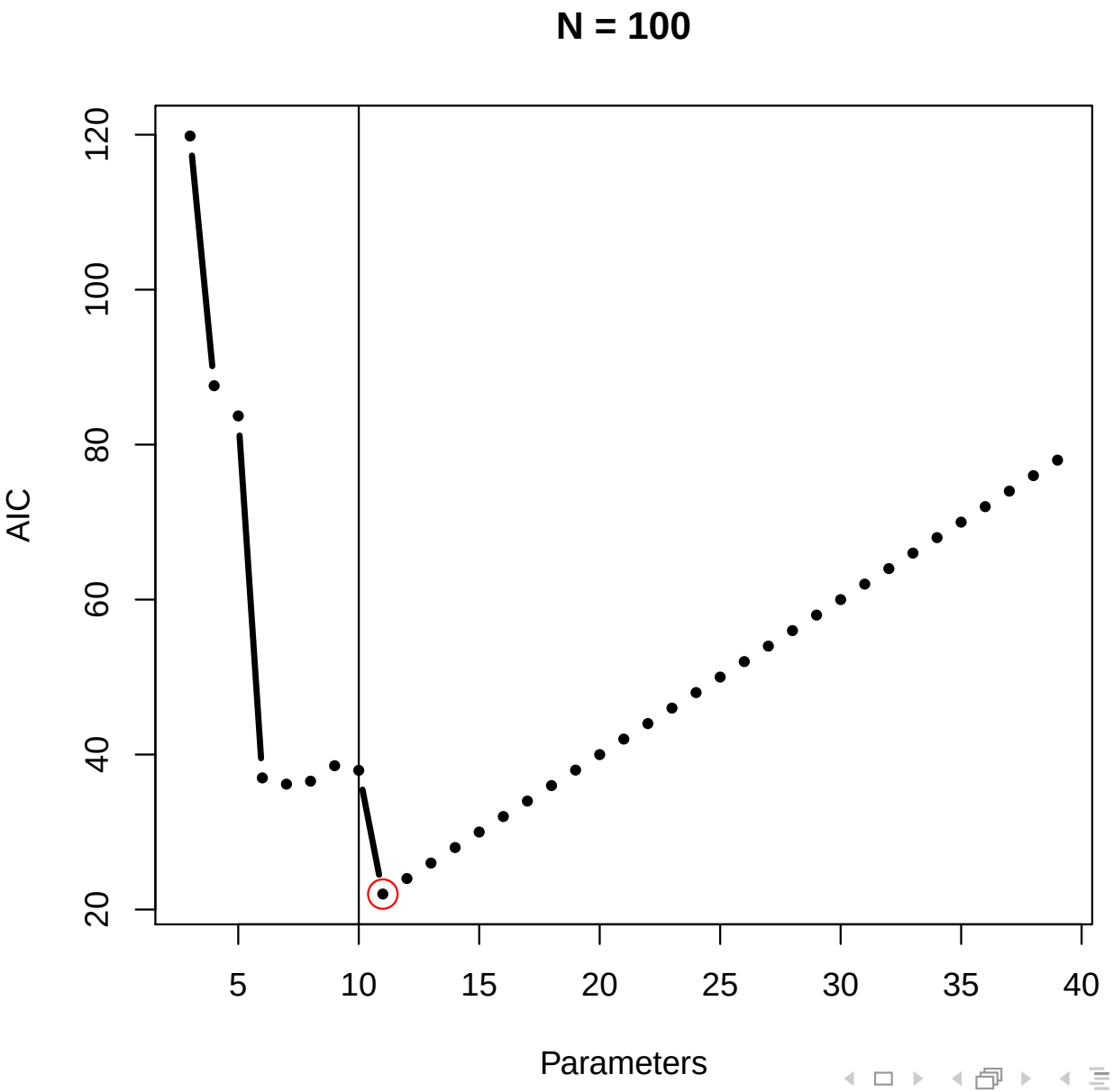
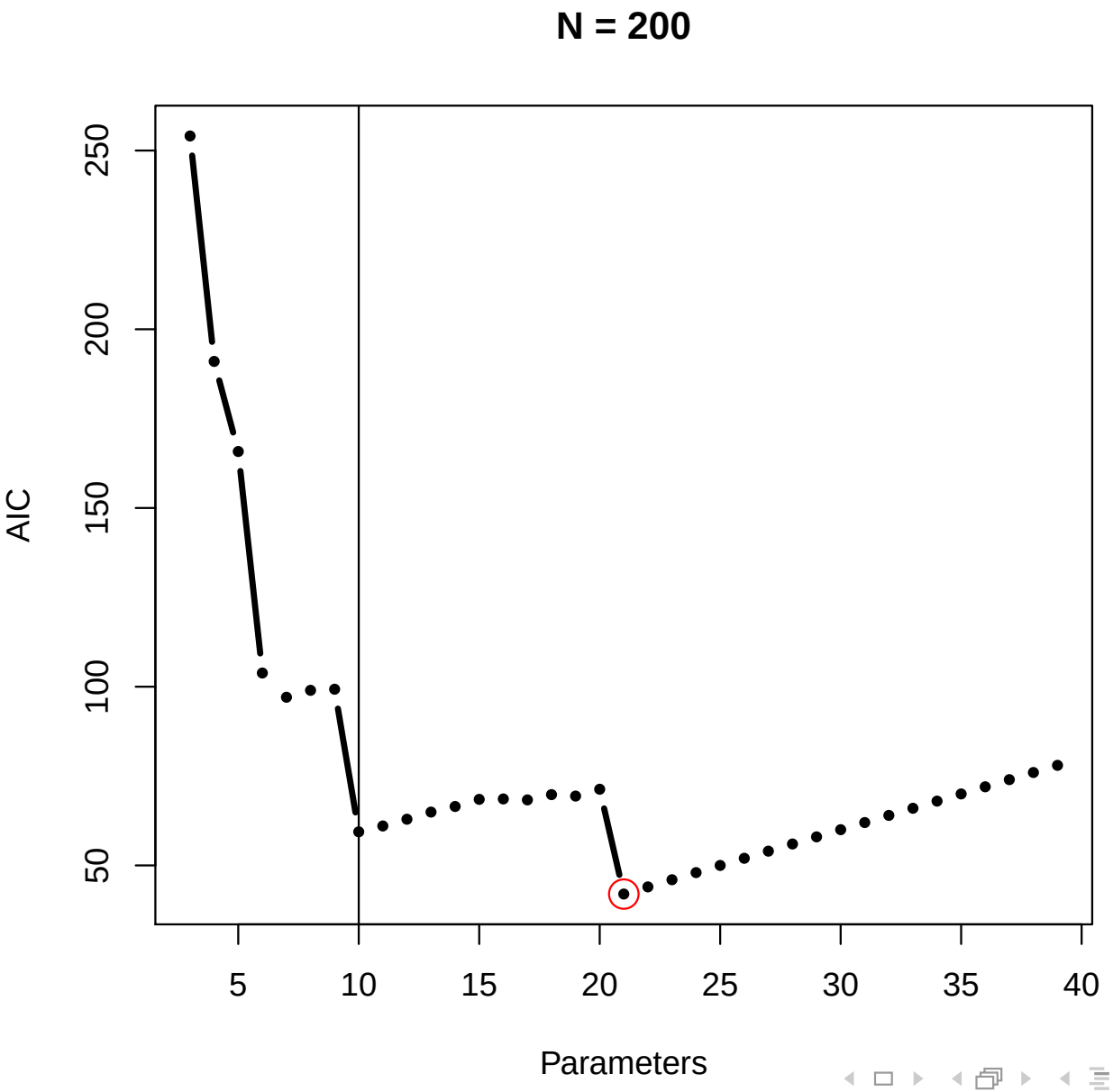
Parameters

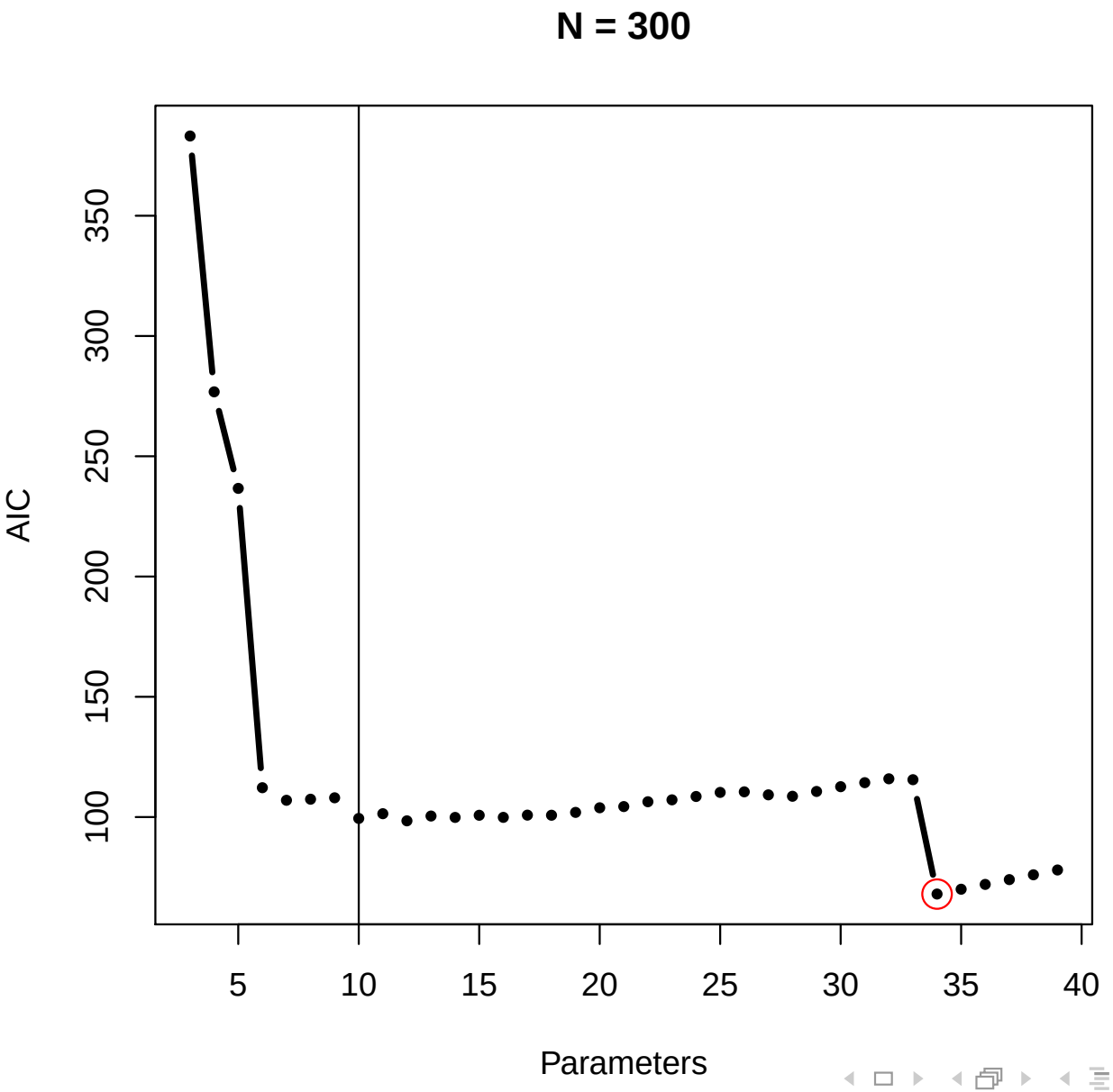
BIC or AIC?



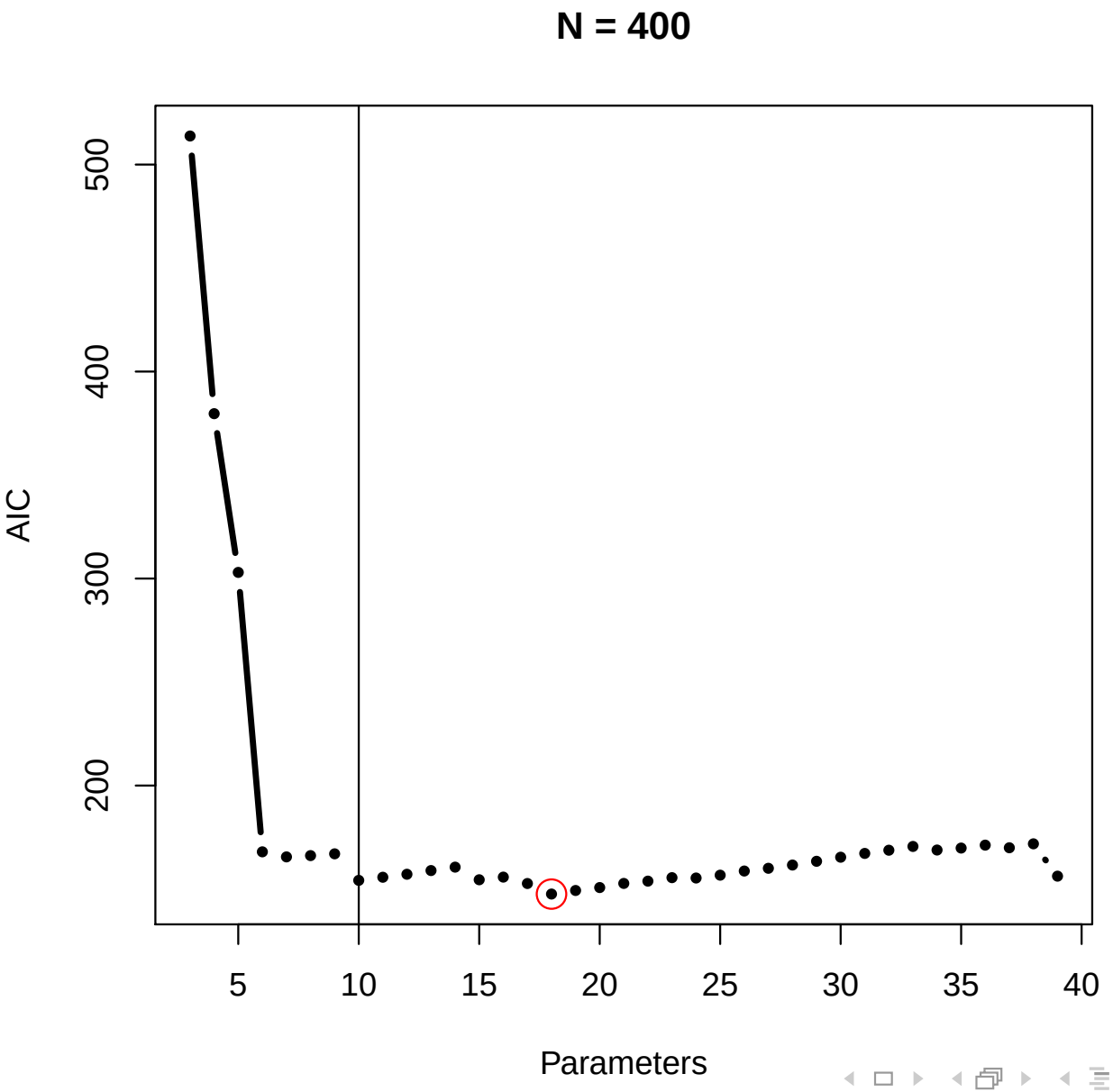
BIC or AIC?



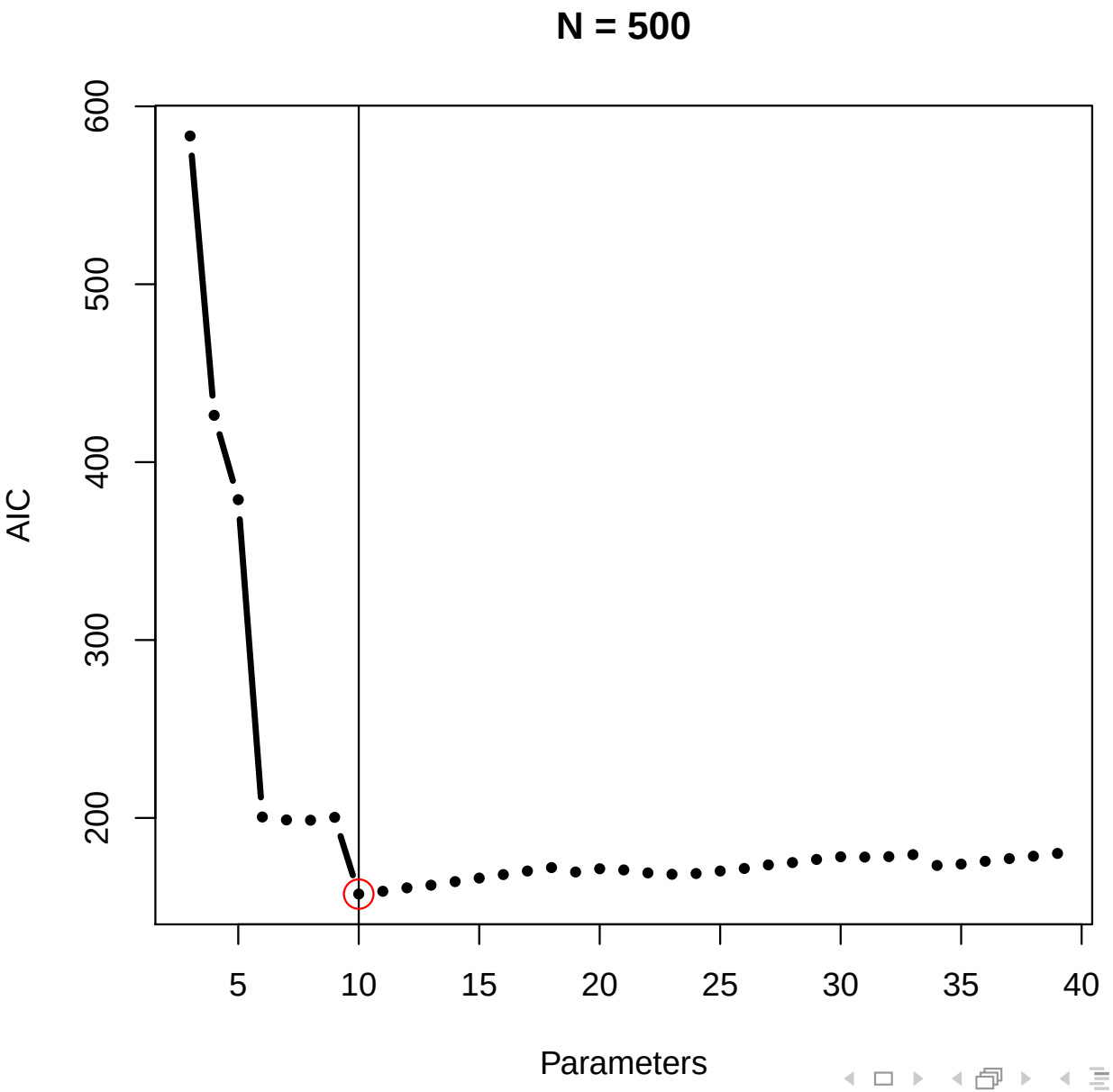
BIC or AIC?


N = 300
AIC
100
150
200
250
300
350
5
10
15
20
25
30
35
40
Parameters

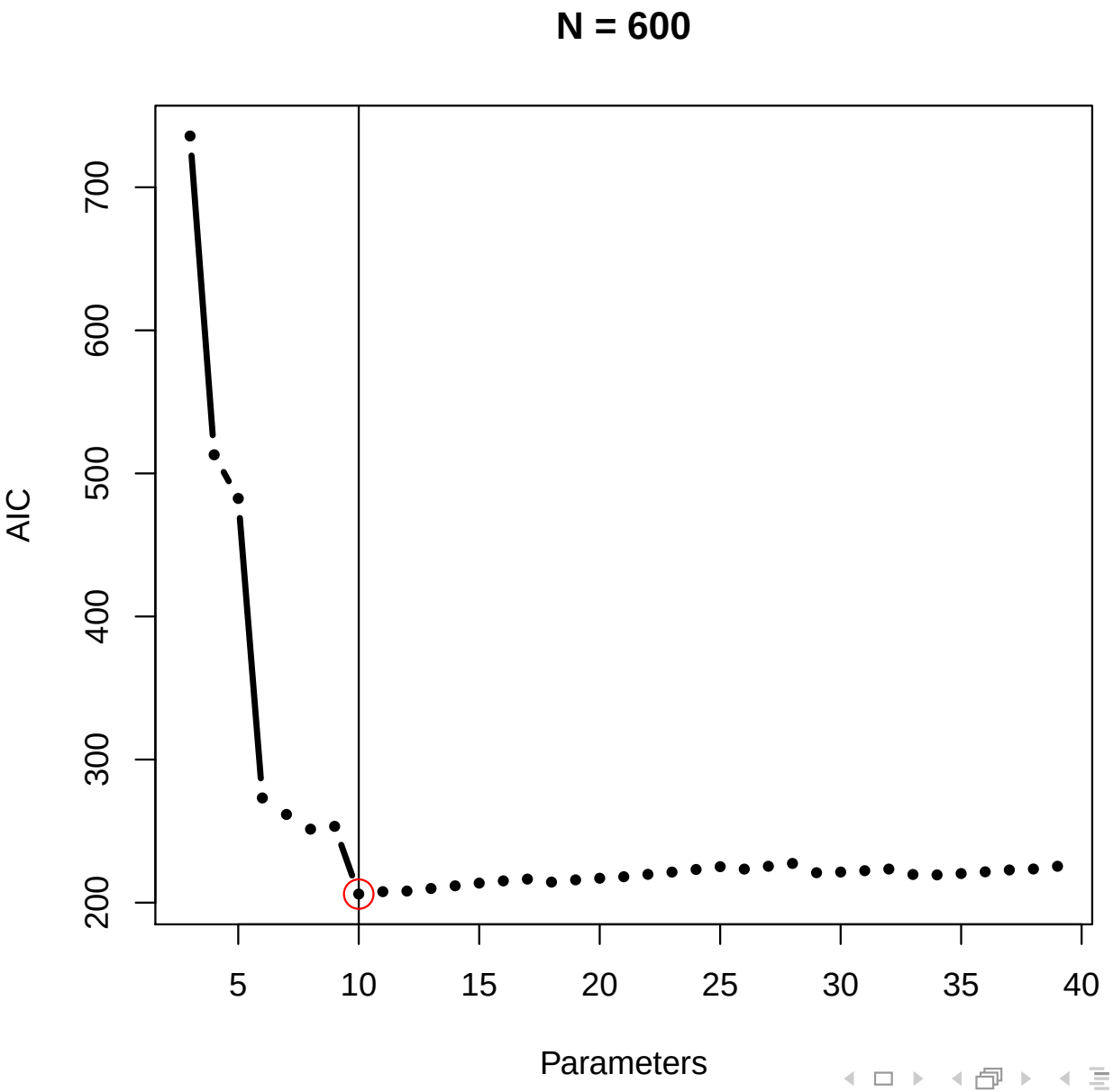
BIC or AIC?



BIC or AIC?



BIC or AIC?


N = 600
AIC
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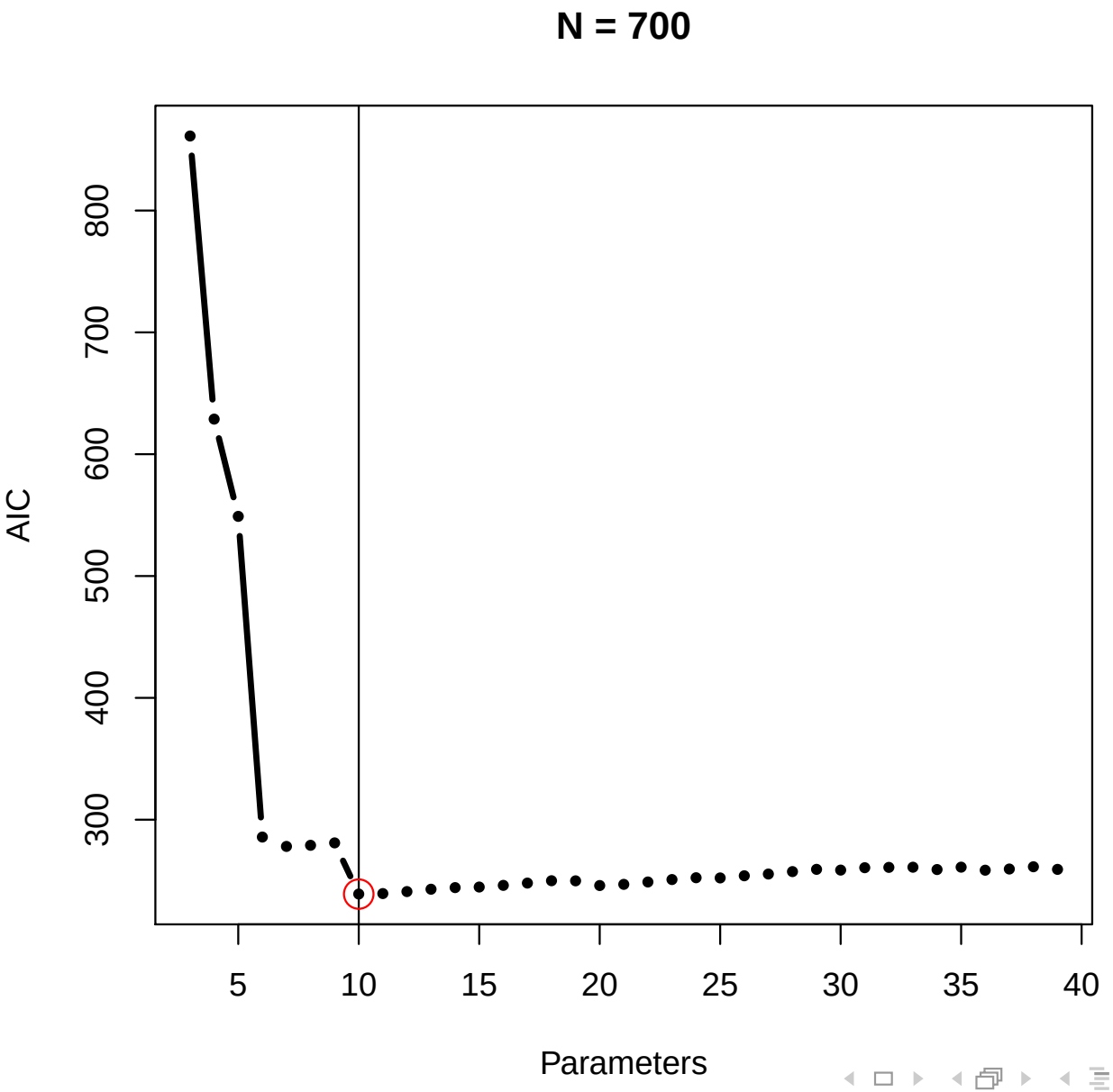
300
400
500
600
700
5
10
15
20
25
30
35
40
Parameters

BIC or AIC?


N = 700
AIC
300
400
500
600
700
800
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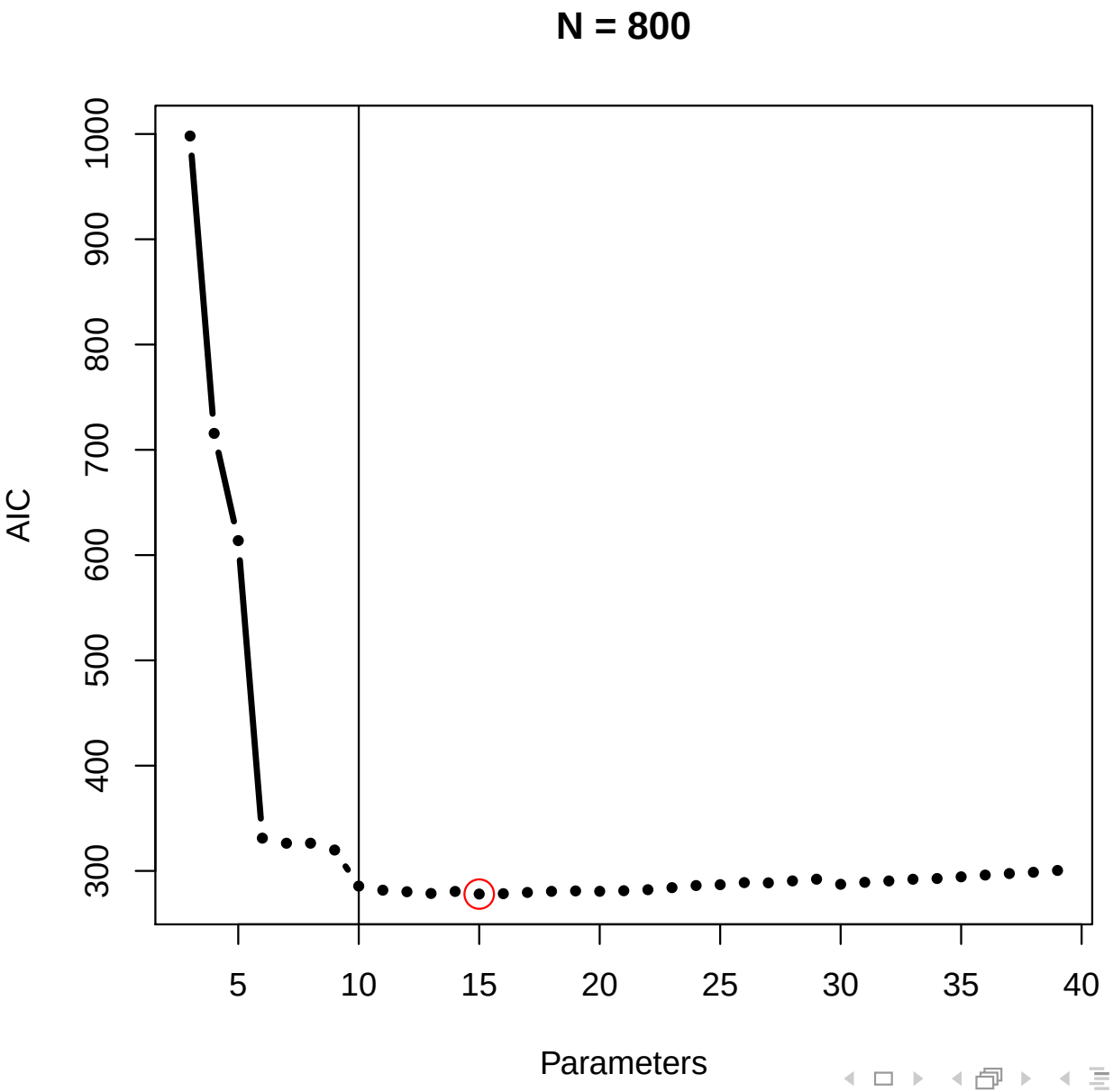
Parameters

BIC or AIC?


N = 800
AIC
1000
900
800
700
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500
400
300
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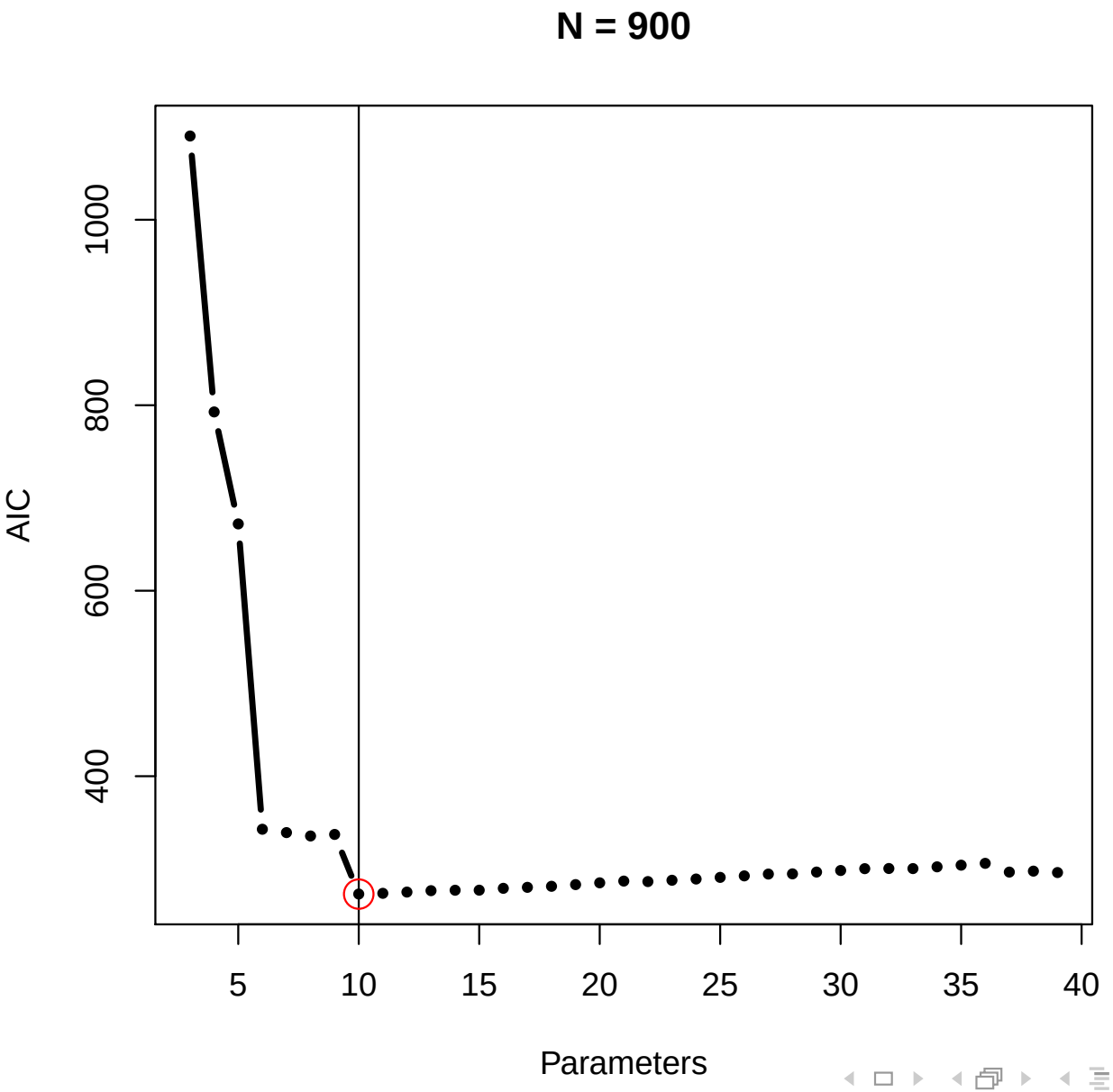
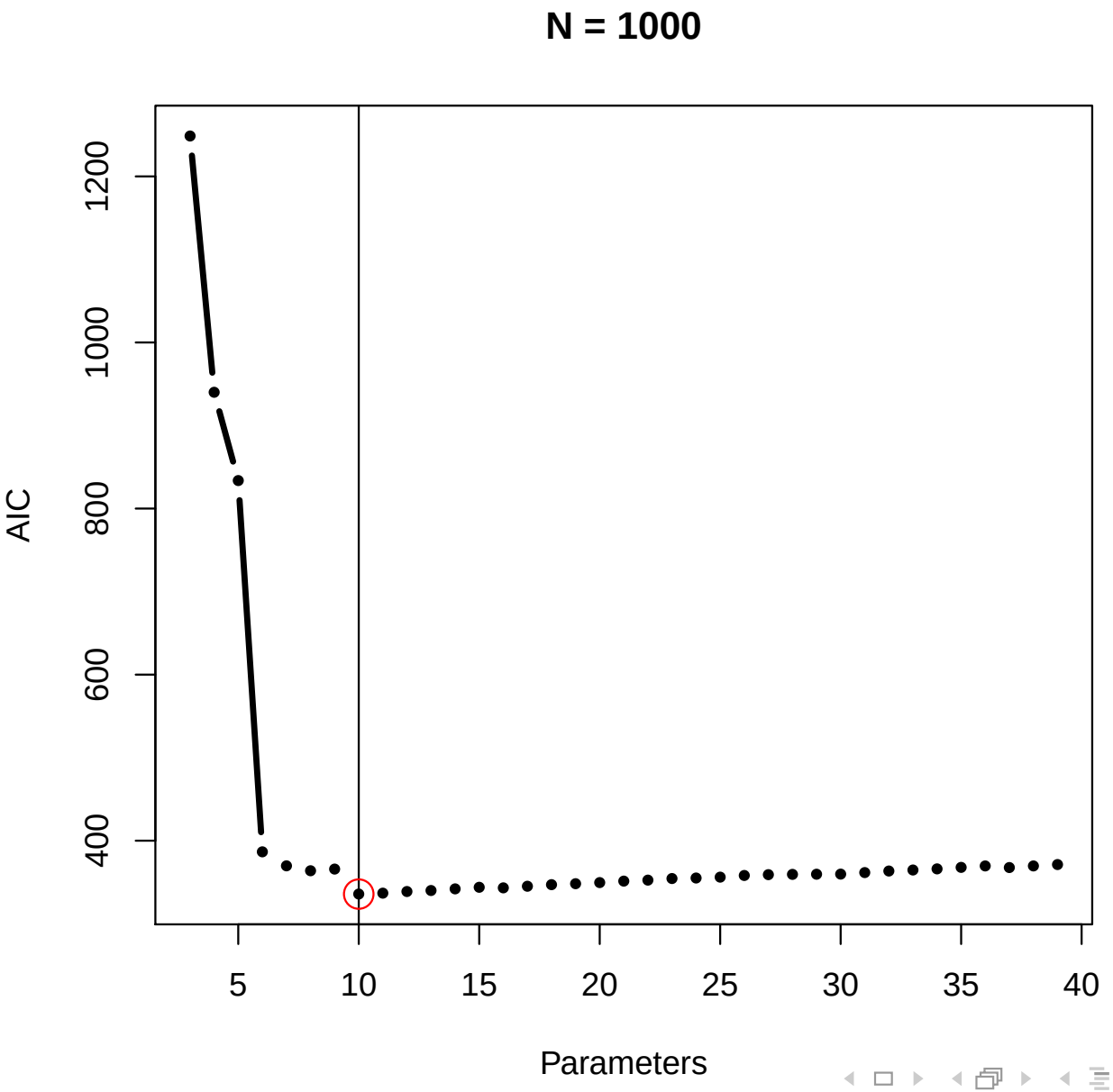
40
Parameters

BIC or AIC?



BIC or AIC?


N = 1000
AIC
400
600
800
1000
1200
5
10
15
20
25
30
35
40
Parameters

BIC or AIC?

- BIC
  - Asymptotically consistent if true model is in choice set
  - As $N \to \infty$ will choose correct model with probability 1 (if available)
  - Small samples$\rightsquigarrow$ overpenalize
- AIC
  - No asymptotic guarantees $\rightsquigarrow$ derivation doesn't require truth in set. (KL-criteria)
  - In large samples$\rightsquigarrow$ favors complexity
  - Small samples$\rightsquigarrow$ avoids over penalization

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

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- AIC : Akaka Information Criterion

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

Can work well, but...

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

Can work well, but...

- Rely on specific loss function

How Do We Select A Model?

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- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

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- Rely on asymptotic argument

How Do We Select A Model?

Analytic statistics for selection, include penalty for complexity

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- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

Can work well, but...

- Rely on specific loss function
- Rely on asymptotic argument
- Rely on estimate of number of parameters

How Do We Select A Model?

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- AIC : Akaka Information Criterion
- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

Can work well, but...

- Rely on specific loss function
- Rely on asymptotic argument
- Rely on estimate of number of parameters
- Extremely model dependent

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Analytic statistics for selection, include penalty for complexity

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- BIC: Bayesian Information Criterion
- DIC: Deviance Information Criterion

Can work well, but...

- Rely on specific loss function
- Rely on asymptotic argument
- Rely on estimate of number of parameters
- Extremely model dependent

Need: general tool for evaluating models, replicates decision problem

Cross-Validation: Some Intuition

Optimal division of data for prediction:

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

- Idea: use observations both in training and test sets

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

- Idea: use observations both in training and test sets
- Each step: use held out data to evaluate performance

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

- Idea: use observations both in training and test sets
- Each step: use held out data to evaluate performance
- Avoid overfitting and have context specific penalty

Cross-Validation: Some Intuition

Optimal division of data for prediction:

- Train: build model
- Validation: assess model
- Test: predict remaining data

K-fold Cross-validation idea: create many training and test sets.

- Idea: use observations both in training and test sets
- Each step: use held out data to evaluate performance
- Avoid overfitting and have context specific penalty

Estimates:

$$\text{Error} = \mathrm{E}\left[\mathrm{E}[L(\boldsymbol{Y}, f(\hat{\boldsymbol{\beta}}, \boldsymbol{X}))|\mathcal{T}]\right]$$

Cross-Validation: A How To Guide

Process:

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, . . ., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

  (Group 1, Group 2, Group3, ..., Group K)

- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

  (Group 1, Group 2, Group3, ..., Group K)

- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

  (Group 1, Group 2, Group3, ..., Group K)

- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.

  (Group 1, Group 2, Group3, ..., Group K)

- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮

Cross-Validation: A How To Guide

Process:

- Randomly partition data into K groups.
  (Group 1, Group 2, Group3, ..., Group K)
- Rotate through groups as follows

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
  - Mean square error, Absolute error, Prediction error, ...

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

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- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
  - Mean square error, Absolute error, Prediction error, ...

CV(ind. classification) $= \frac{1}{N}\sum_{i=1}^{N} L(\boldsymbol{Y}_i, f^{-k}(\boldsymbol{\beta}, \boldsymbol{X}_i))$

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
  - Mean square error, Absolute error, Prediction error, ...

CV(ind. classification) $= \frac{1}{N}\sum_{i=1}^{N} L(\boldsymbol{Y}_i, f^{-k}(\boldsymbol{\beta}, \boldsymbol{X}_i))$

CV(proportions) $=$
$\frac{1}{K}\sum_{j=1}^{K}$ Mean Square Error Proportions from Group j

Cross-Validation: A How To Guide

Step	Training	Validation ("Test")
1	Group2, Group3, Group 4, ..., Group K	Group 1
2	Group 1, Group3, Group 4, ..., Group K	Group 2
3	Group 1, Group 2, Group 4, ..., Group K	Group 3
⋮	⋮	⋮
K	Group 1, Group 2, Group 3, ..., Group K - 1	Group K

Strategy:

- Divide data into K groups
- Train data on $K-1$ groups. Estimate $\hat{f}^{-K}(\boldsymbol{\beta}, \boldsymbol{X})$
- Predict values for K^{th}
- Summarize performance with loss function: $L(\boldsymbol{Y}_i, \hat{f}^{-k}(\boldsymbol{\beta}, \boldsymbol{X}))$
  - Mean square error, Absolute error, Prediction error, ...

  CV(ind. classification) $= \frac{1}{N}\sum_{i=1}^{N} L(\boldsymbol{Y}_i, f^{-k}(\boldsymbol{\beta}, \boldsymbol{X}_i))$

  CV(proportions) $=$
  $\frac{1}{K}\sum_{j=1}^{K}$ Mean Square Error Proportions from Group j
- Final choice: model with highest CV score

How Do We Select K? (HTF, Section 7.10)

Common values of K

- $K = 5$: Five fold cross validation
- $K = 10$: Ten fold cross validation
- $K = N$: Leave one out cross validation

Considerations:

- How sensitive are inferences to number of coded documents? (HTF, pg 243-244)
- 200 labeled documents
  - $K = N \rightarrow 199$ documents to train,
  - $K = 10 \rightarrow 180$ documents to train
  - $K = 5 \rightarrow 160$ documents to train
- 50 labeled documents
  - $K = N \rightarrow 49$ documents to train,
  - $K = 10 \rightarrow 45$ documents to train
  - $K = 5 \rightarrow 40$ documents to train
- How long will it take to run models?
  - $K-$fold cross validation requires $K\times$ One model run
- What is the correct loss function?

If you cross validate, you really need to cross validate (Section 7.10.2, ESL)

- Use CV to estimate prediction error
- All supervised steps performed in cross-validation
- Underestimate prediction error
- Could lead to selecting lower performing model

Example from Facebook Data

What do people say to legislators? (Franco, Grimmer, and Lee 2017)

1) Example: estimating classification error
    a) Accuracy in legislator posts: 75%
    b) Accuracy in public posts: 66.25%

Credit Claiming (Back to Ridge/Lasso, Grimmer, Westwood, and Messing 2014)

```
library(glmnet)
set.seed(8675309) ##setting seed
folds<- sample(1:10, nrow(dtm), replace=T) ##assigning to fold
out_of_samp<- c() ##collecting the predictions
```

Credit Claiming (Back to Ridge/Lasso, Grimmer, Westwood, and Messing 2014)

```
for(z in 1:10){
train<- which(folds!=z) ##the observations we will use to train the model

test<- which(folds==z) ##the observations we will use to test the model
part1<- cv.glmnet(x = dtm[train,], y = credit[train], alpha = 1, family =
́binomial) ##fitting the LASSO model on the data.
## alpha = 1 -> LASSO
## alpha = 0 -> RIDGE
## 0<alpha<1 -> Elastic-Net
out_of_samp[test]<- predict(part1, newx= dtm[test,], s = part1$lambda.min,
type =́class) ##predicting the labels
print(z) ##printing the labels
}
conf_table<- table(out_of_samp, credit) ##calculating the confusion table
> round(sum(diag(conf_table))/len(credit), 3)
[1] 0.844
```

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In some special cases there are analytic solutions:

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&= \underbrace{\boldsymbol{X}\left(\boldsymbol{X}'\boldsymbol{X} + \lambda \boldsymbol{I}_J\right)^{-1} \boldsymbol{X}'}_{\text{Hat Matrix}} \boldsymbol{Y}
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\text{Cross Validation}(1) &= \frac{1}{N}\sum_{i=1}^{N}(Y_i - f(\boldsymbol{X}_{-i}, \boldsymbol{Y}_{-i}, \lambda, \hat{\boldsymbol{\beta}}))^2 \\
&= \frac{1}{N}\sum_{i=1}^{N}\left(\frac{Y_i - f(\boldsymbol{X}, \boldsymbol{Y}, \lambda, \hat{\boldsymbol{\beta}})}{1 - H_{ii}}\right)^2
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$$\begin{aligned} \Pr(\text{Correct Guess}|\text{Votes}) &= \Pr(3 \text{ correct}) + \Pr(2 \text{ correct}) \\ &= 0.75^3 + 3 \times (0.75^2 \times 0.25) \end{aligned}$$

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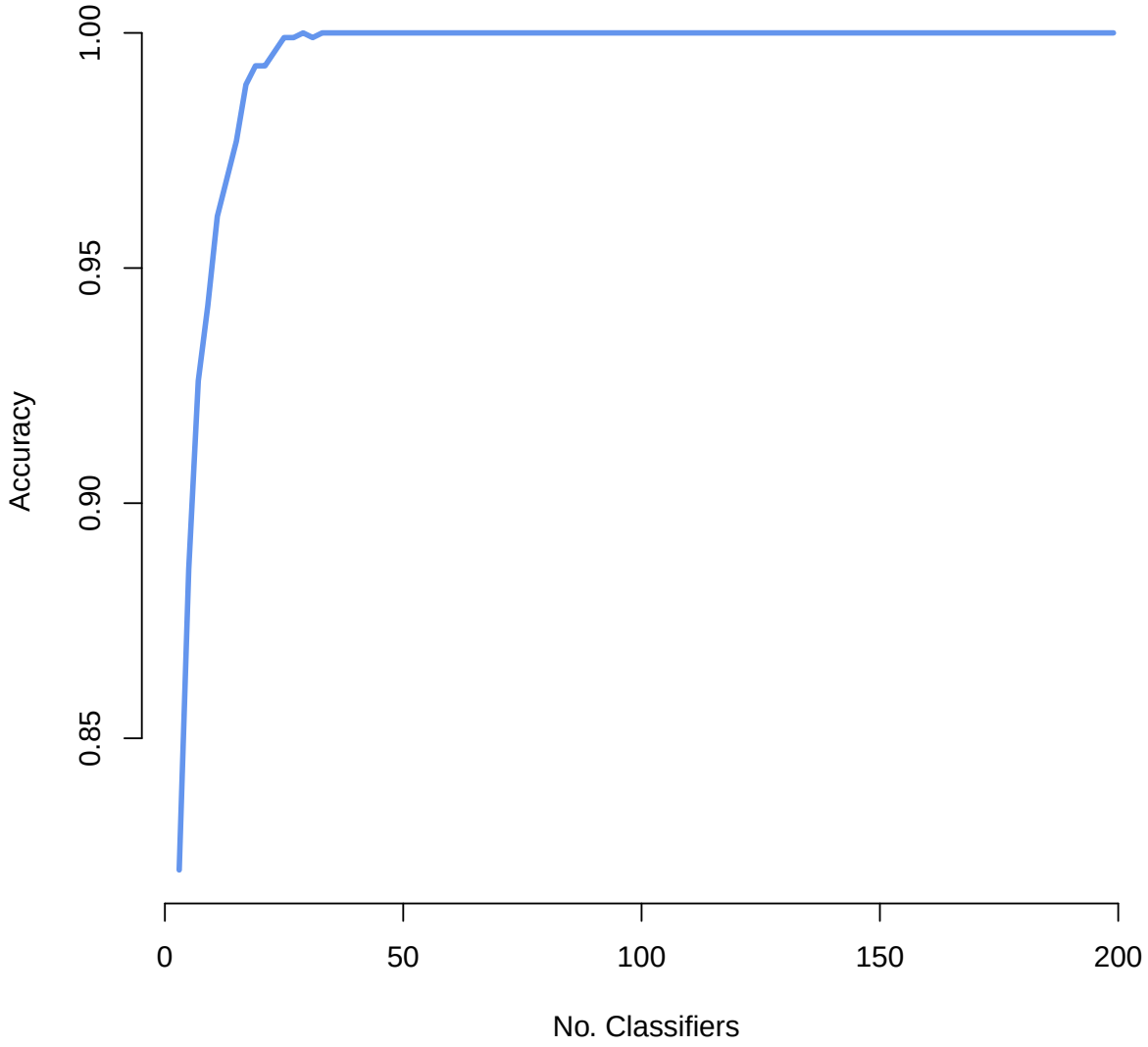

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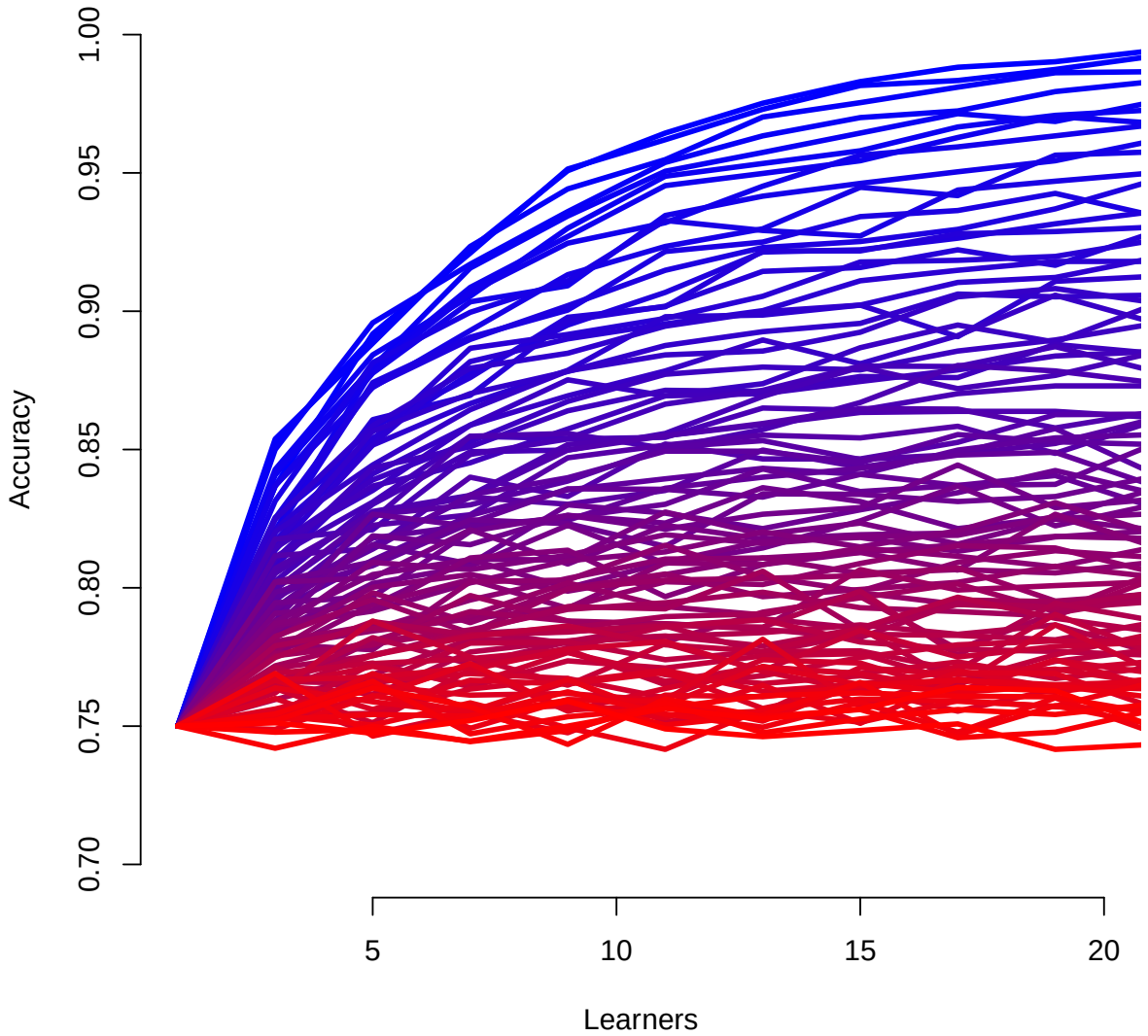
Diverse and Accurate matter.



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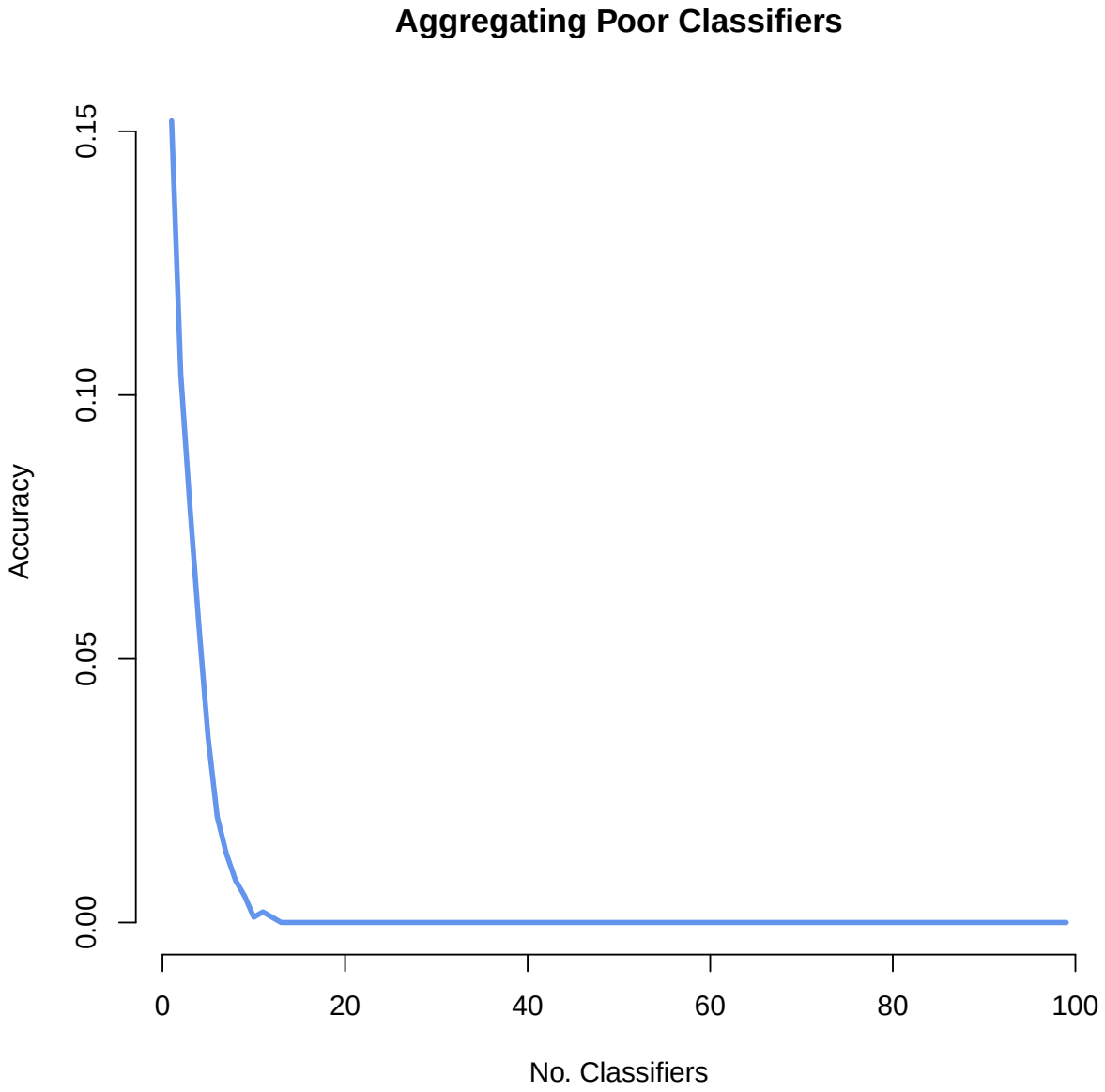

Wisdom of the Crowds:

Goal: estimate an observation's category $\rightsquigarrow$ $Y \in \{0, 1\}$

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$$\lim_{M \to \infty} P(\bar{B} > 0.5) = 1$$

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&= \underbrace{\rho\sigma^2}_{\text{Resolve with independence}} + \underbrace{\frac{1-\rho}{M}\sigma^2}_{\text{Resolve with }\uparrow\text{classifiers}}
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- Strong Correlation between classifiers (recall optimal division from previous slide)

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With no assumptions, stratify $\rightsquigarrow$ different mean for unique values of $\boldsymbol{x}_i$

- Within each strata p, compute average Y

$$\bar{Y}|\boldsymbol{x}_p = \sum_{i=1}^{N} \frac{I(\boldsymbol{x}_i = \boldsymbol{x}_p)Y_i}{\sum_{t=1}^{N} I(\boldsymbol{x}_t = \boldsymbol{x}_p)}$$

Implies that for test data we would fit:

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Curse of dimensionality(!!!)

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Curse of dimensionality(!!!)

Approximate with regions$\rightsquigarrow$ search for splits of data to approximate stratification

Classification and Regression Trees (CART): Objective function

Labels $\boldsymbol{Y}_i$ and documents $\boldsymbol{x}_i$

$$
\begin{aligned}
E[Y|\boldsymbol{x}_i] &= \widehat{f}(\boldsymbol{x}_i) \\
&= \sum_{p=1}^{P} c_p I(\boldsymbol{x}_i \in R_p)
\end{aligned}
$$

where:

- R_p describes a region $\rightsquigarrow$ node
- c_p describes values of Y_i for document in R_p

Classification and Regression Trees (CART): Optimization function

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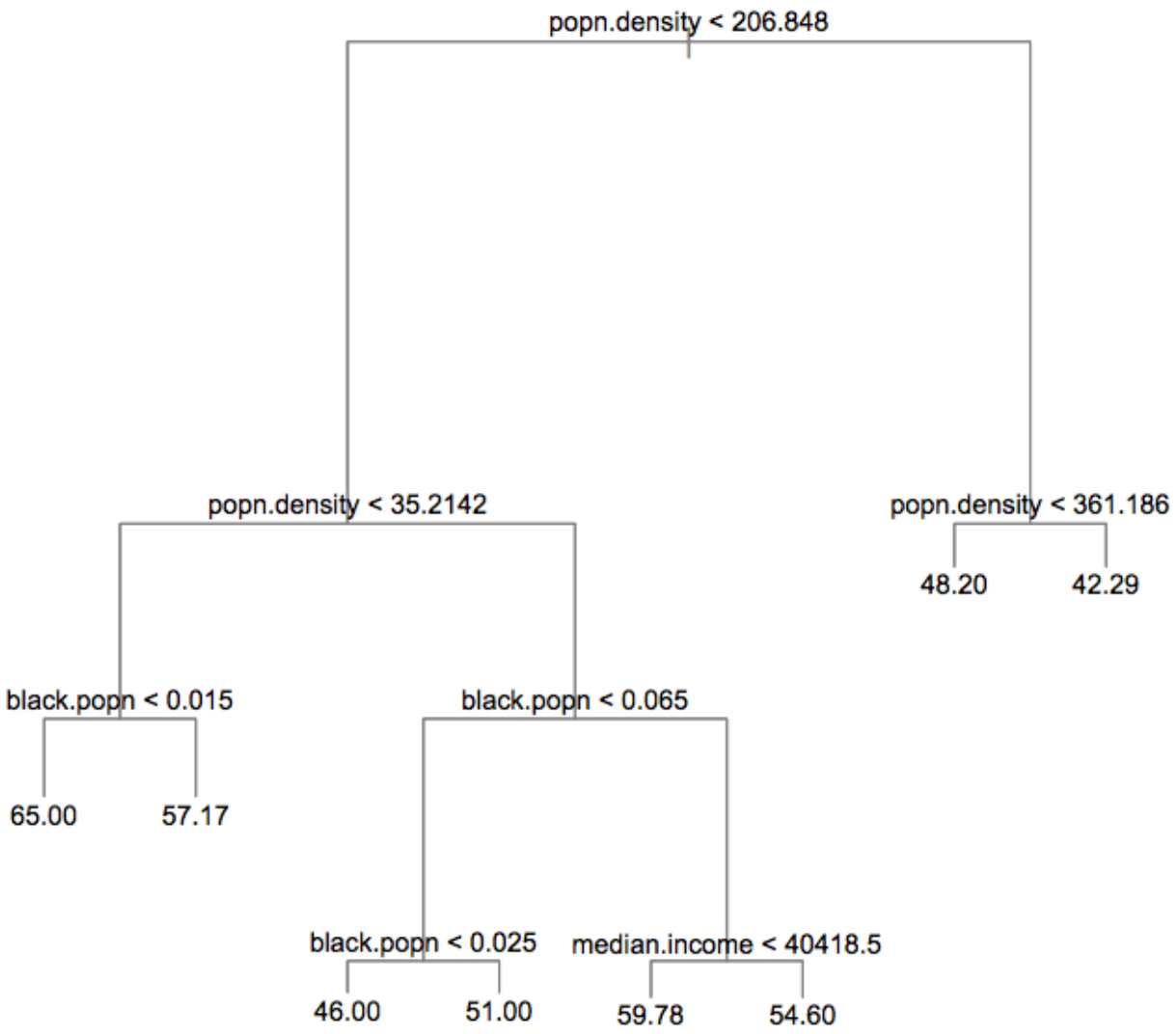
$$(j^*, s^*) = \arg\min_{j,s} \left[\underbrace{\min_{c_1} \sum_{i=1}^{N} I(x_{ij} < s)(Y_i - c_1)^2}_{\text{“cost” group 1}} + \underbrace{\min_{c_2} \sum_{i=1}^{N} I(x_{ij} > s)(Y_i - c_2)^2}_{\text{“cost” group 2}} \right]$$

Classification and Regression Trees (CART): Algorithm

- Start in Node
- Partition according to Greedy algorithm
- Continue until some stopping rule: number of observations per node

CART Picture (Spirling 2008)



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Recall: accurate (unbiased) and uncorrelated classifiers

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- Random forest $\rightsquigarrow$ introduce additional sampling to induce independence $\rightsquigarrow$ Only split on subset of variables

Random Forest Algorithm (ESL, 588)

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- Wager and Athey (2015): Random Forest for estimating heterogeneous effects

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Weighted ensemble: weights determined by (unique) out of sample predictive performance

Committee Methods:
Fit many methods, average with equal weights

- Voting (classification)
- Averaging (predictions)

Problem: many poor methods may overwhelm high quality fit (remember earlier figures)
Solution: learn weights via cross validation

Weighted Ensemble to Classify Documents

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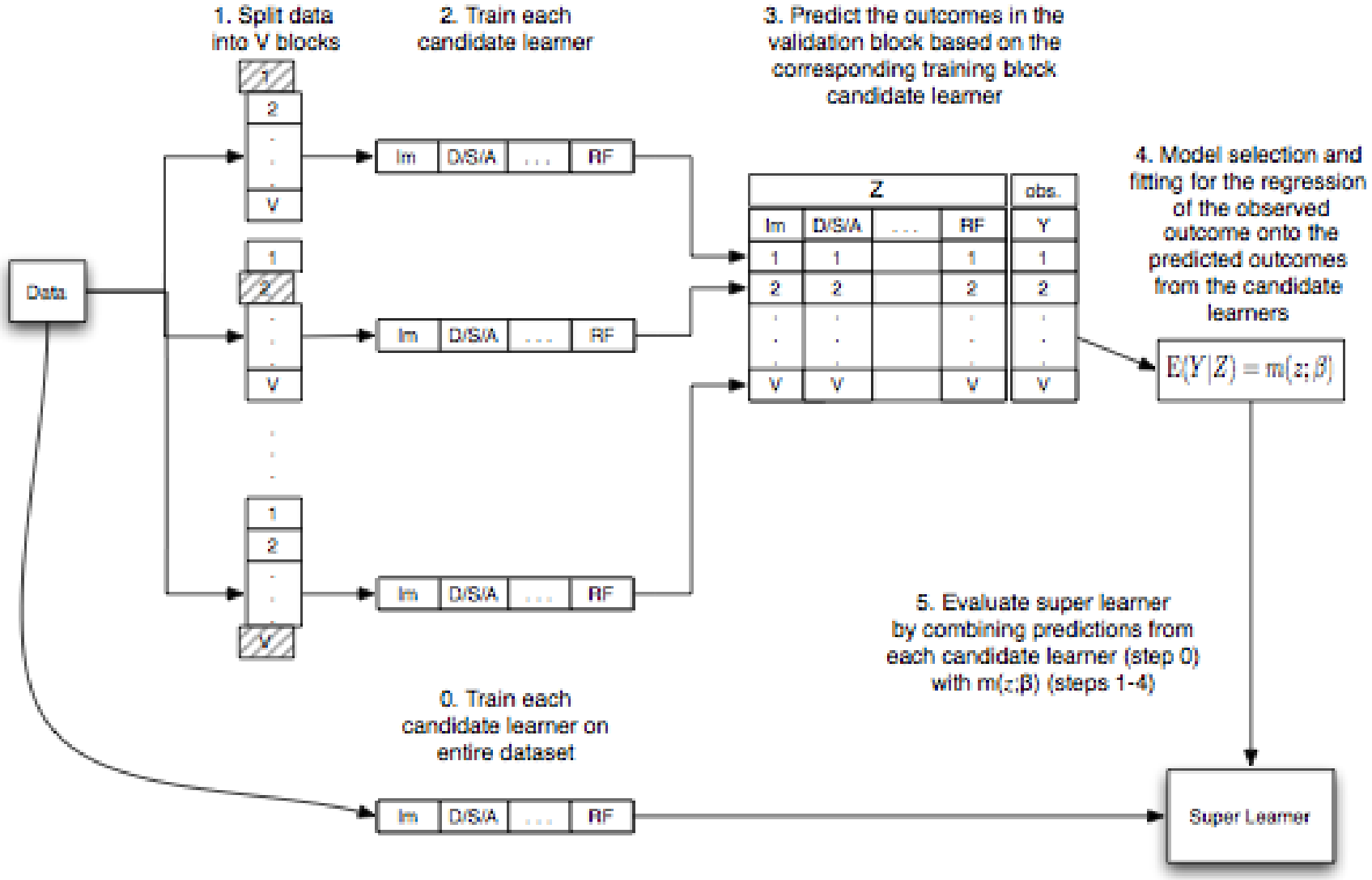

Figure 1: Flow Diagram for Super Learner

Why Super Learn?

van der Laan et al (2007) prove:

- Asymptotically: super learners will perform as well the best candidates for data
- Oracle: performs like the best possible method among candidate methods
    - Asymptotically outperforms constituent methods
    - Performs as well as optimal combinations of those methods

Practical questions:

- Final regression:
    - Logistic
    - Linear
    - Could super learn again!
- How Many Folds?
    - van der Laan et al's proofs rely on growing folds with N (but slowly)
    - Use 10-fold cross validation for simulations

Impression of Influence

Estimate: $Y_i \in \{\text{Credit}, \text{Not Credit}\}$

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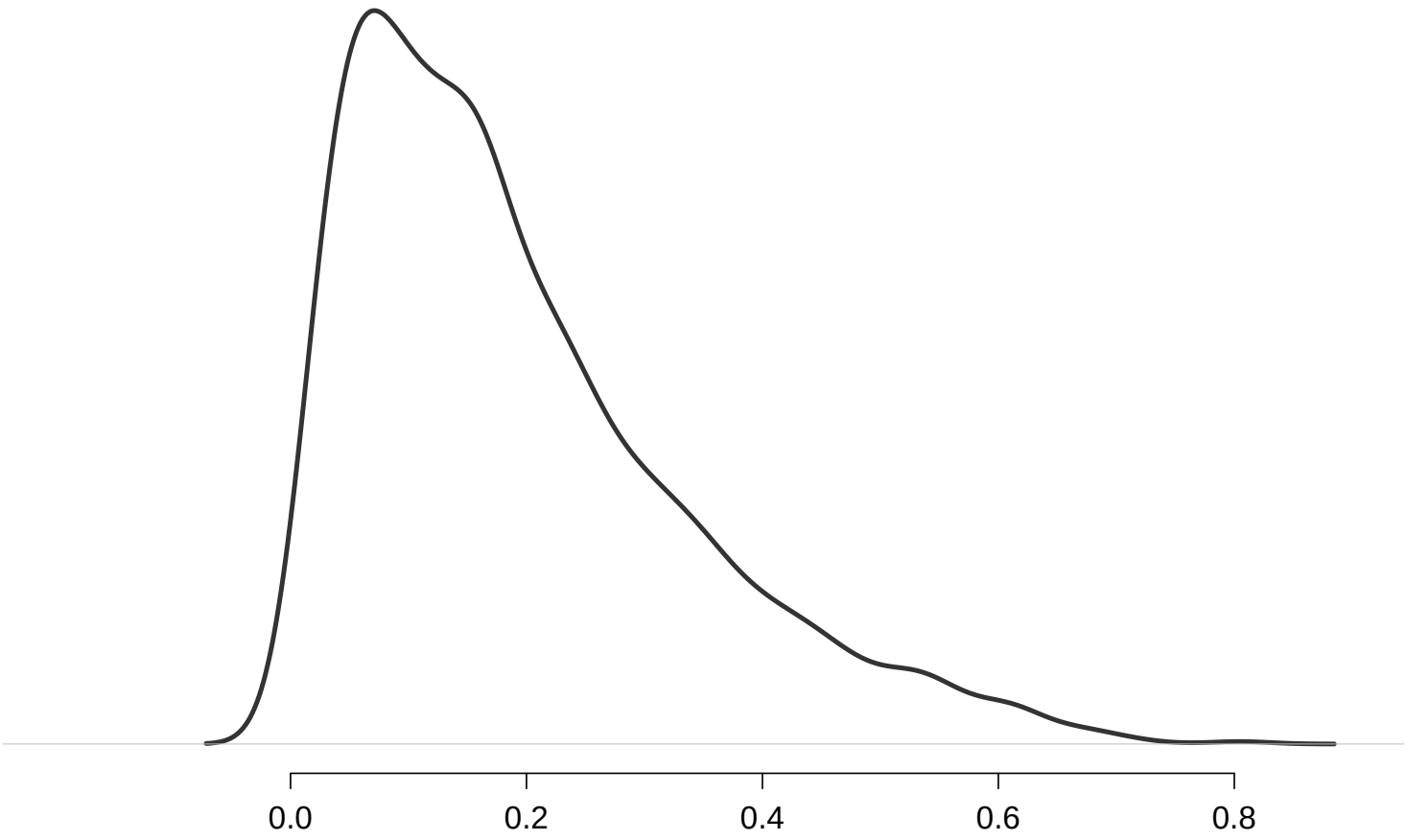
- LASSO 0
- Elastic-Net 23%
- Random Forest 61%
- A Support Vector Machine16%
- Kernel Regularized Least Squares (KRLS, Hainmueller and Hazlett 2014) 0

Strategic Credit Claiming to Build a Personal Vote



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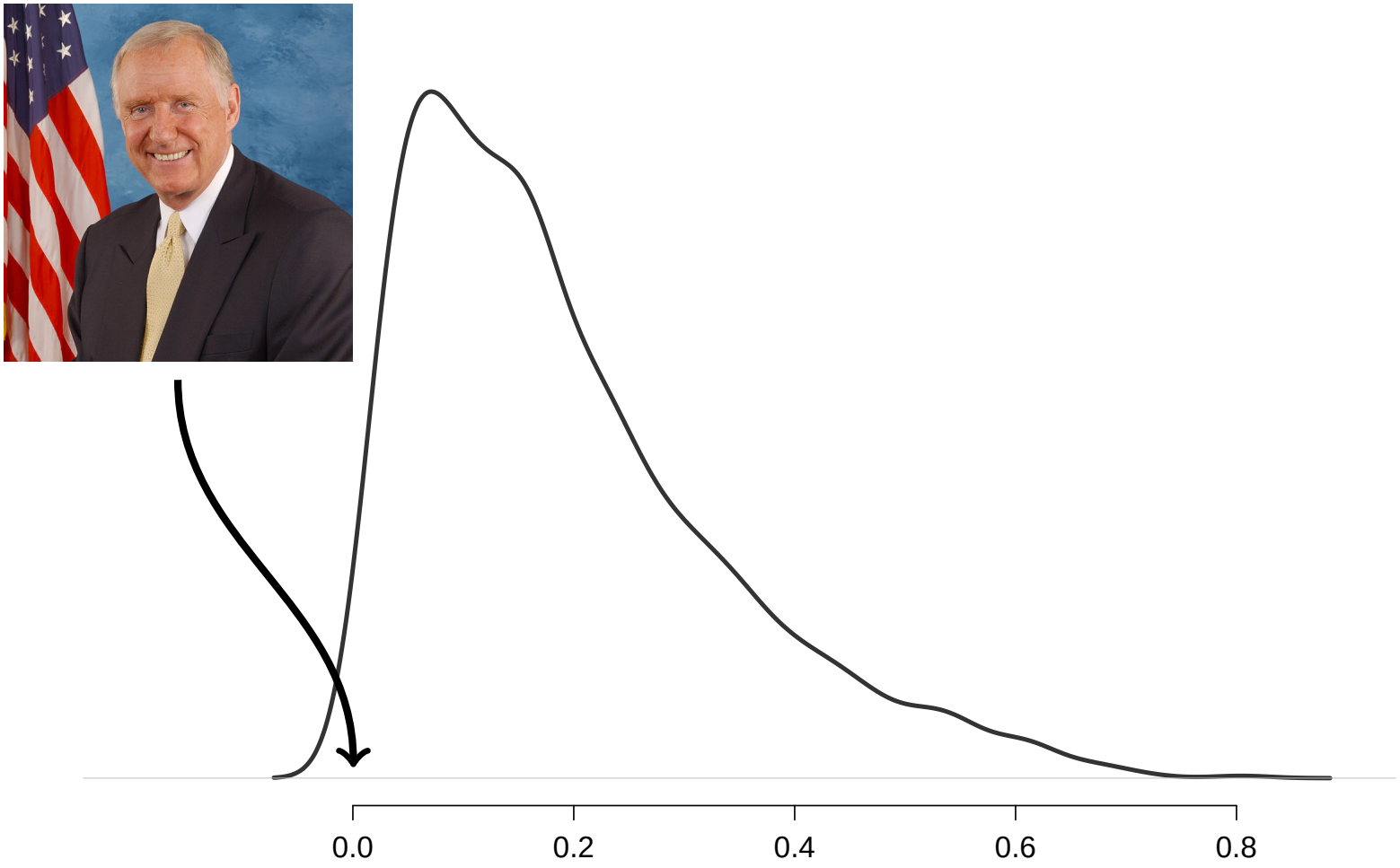

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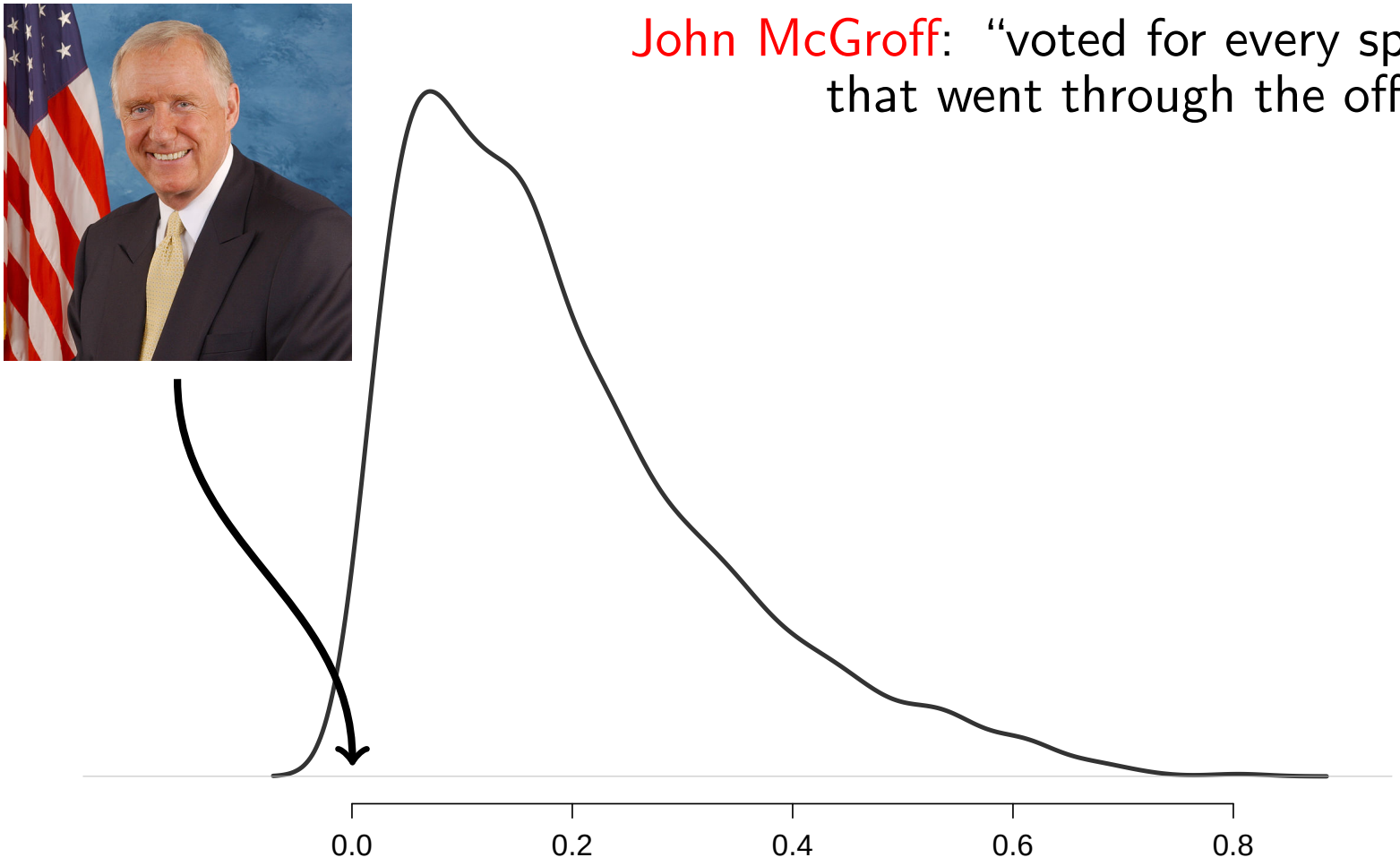

John McGroff: "voted for every spending bill " that went through the office"

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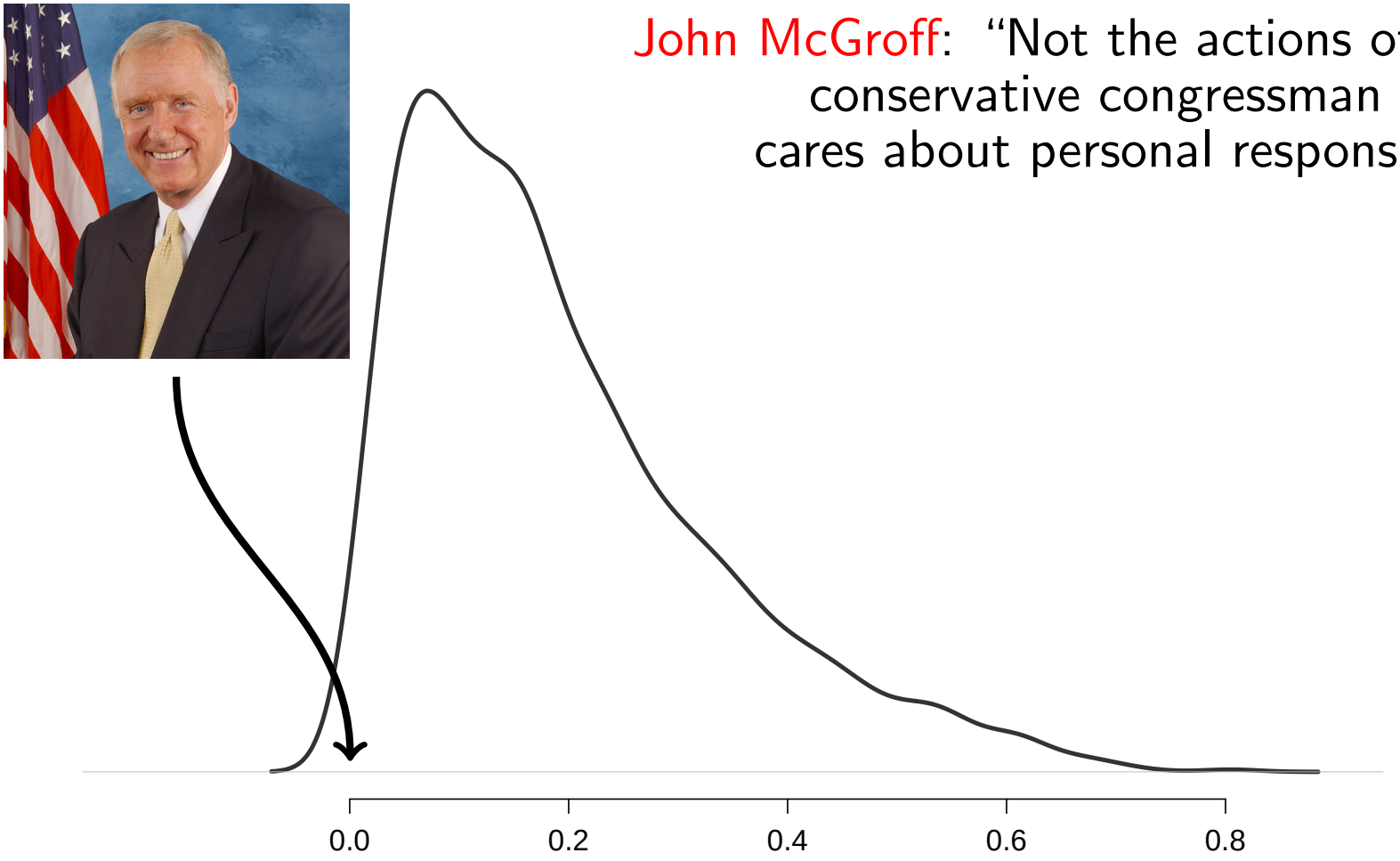

John McGroff: "Not the actions of a fiscally " conservative congressman who cares about personal responsibility"

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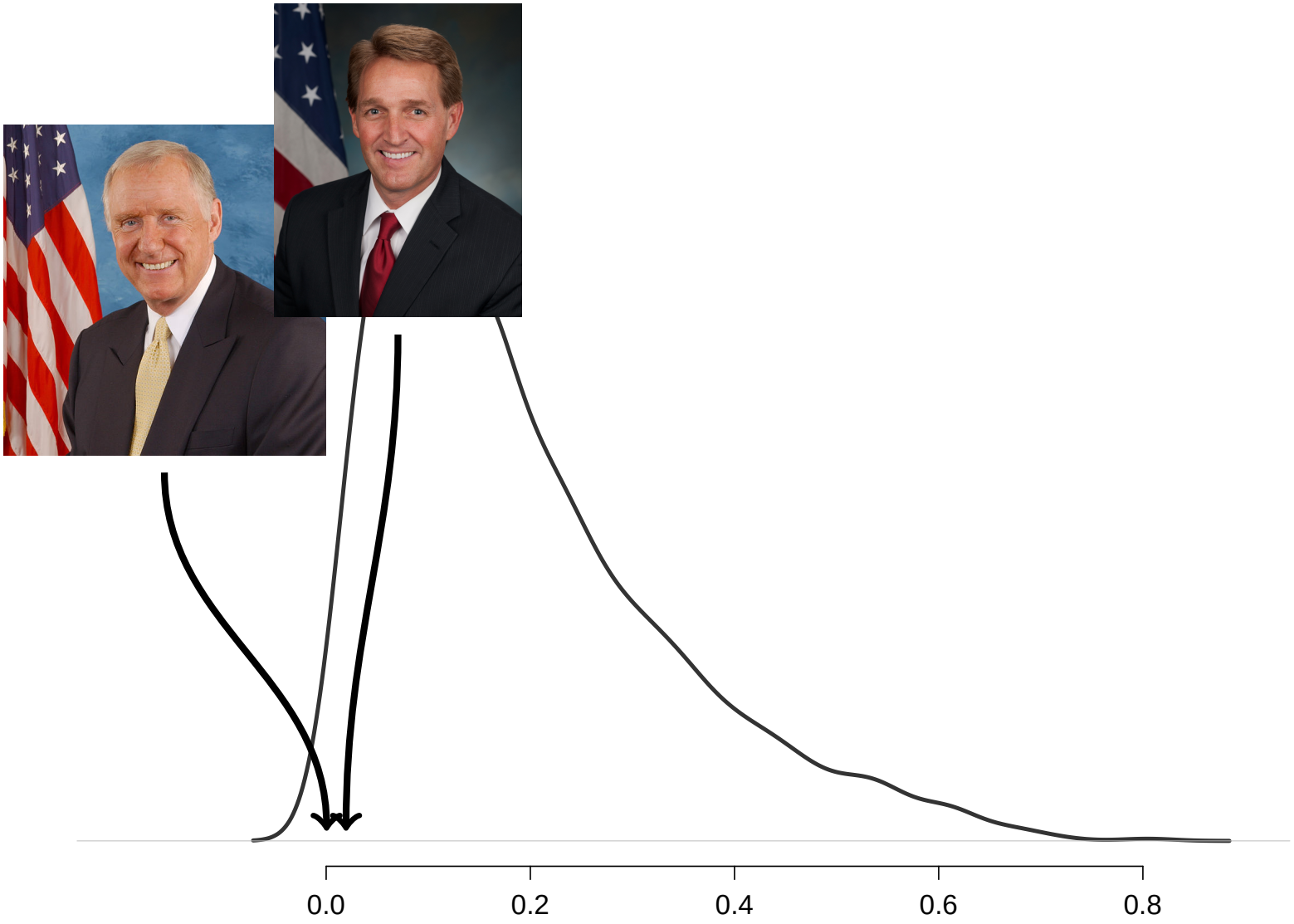

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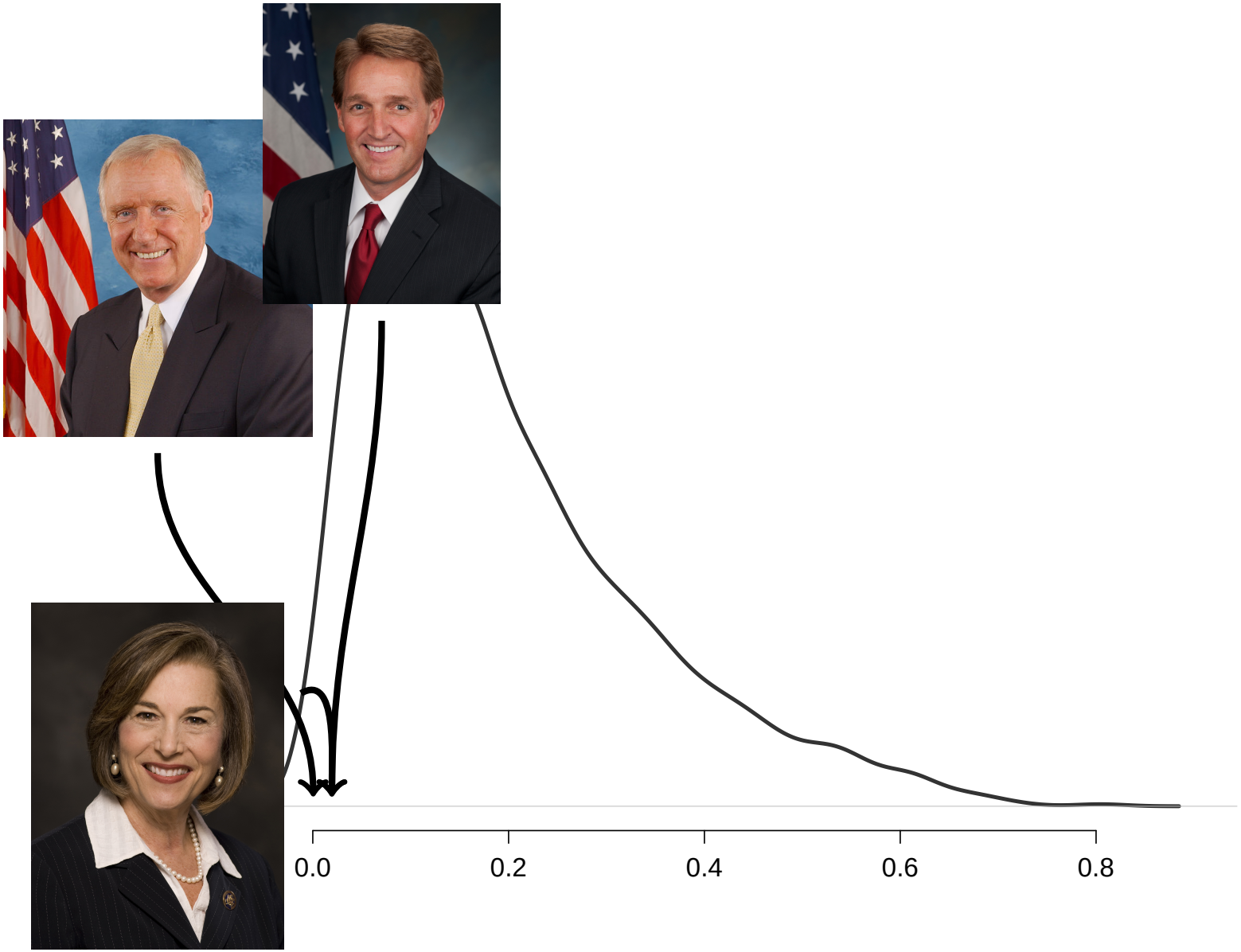

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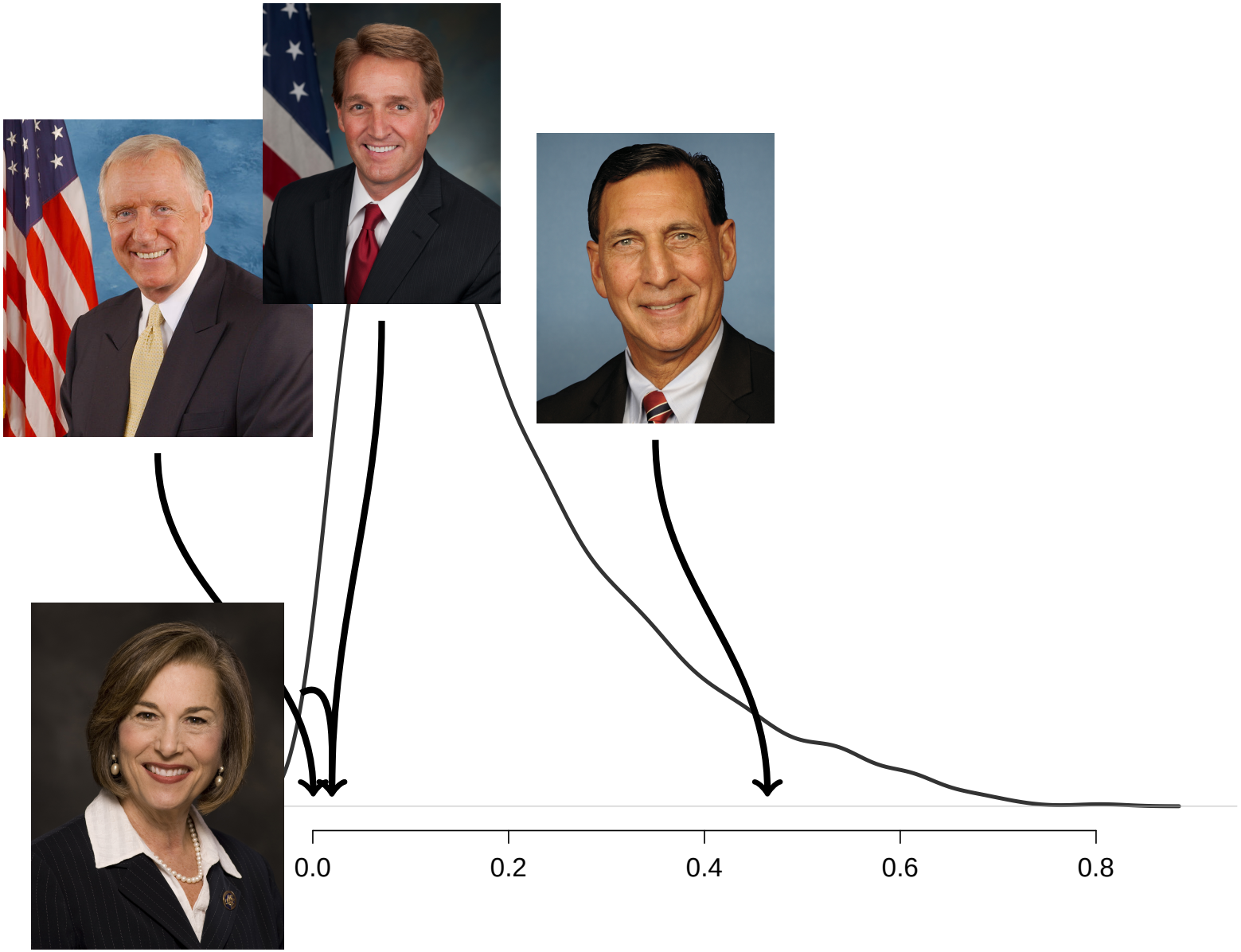

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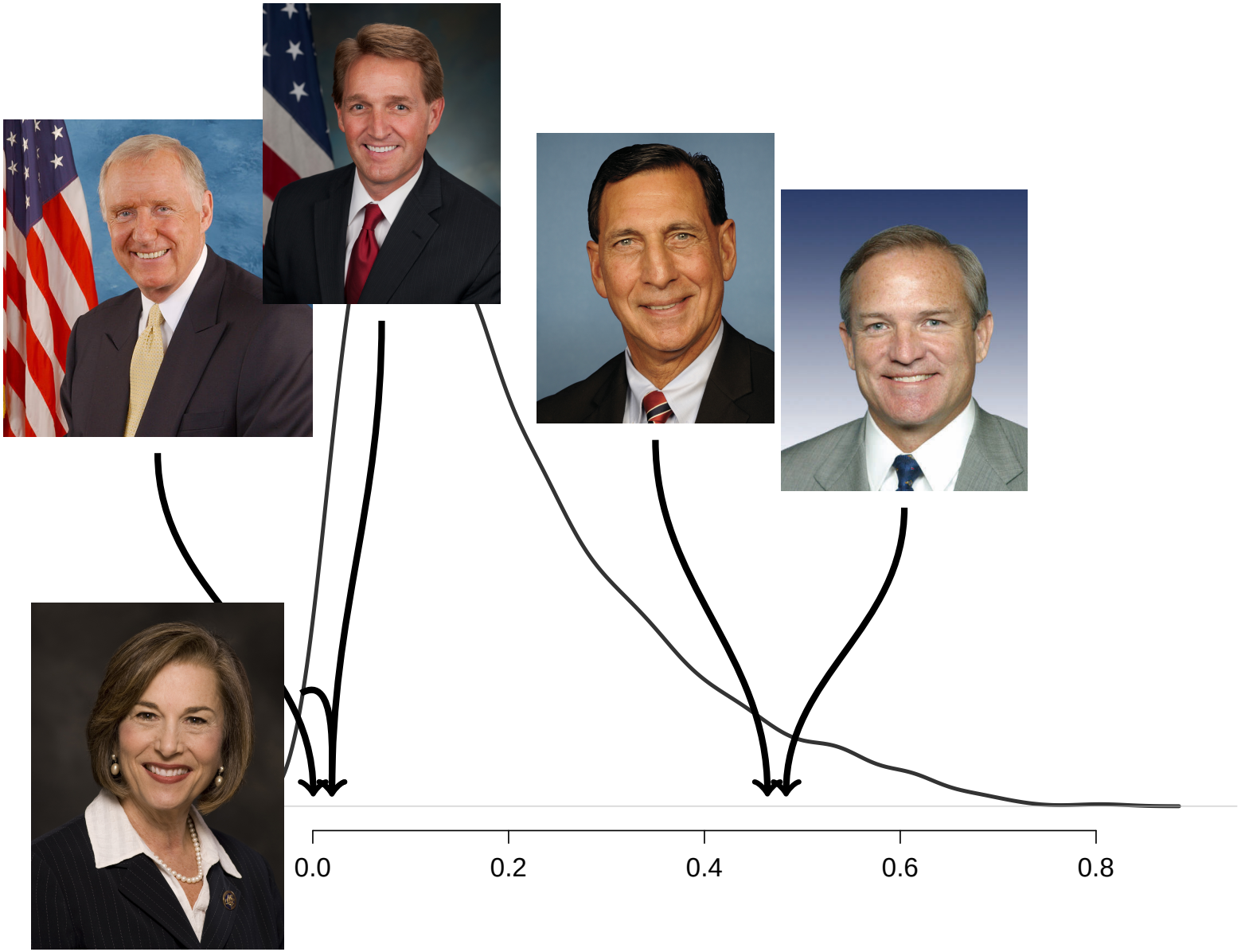

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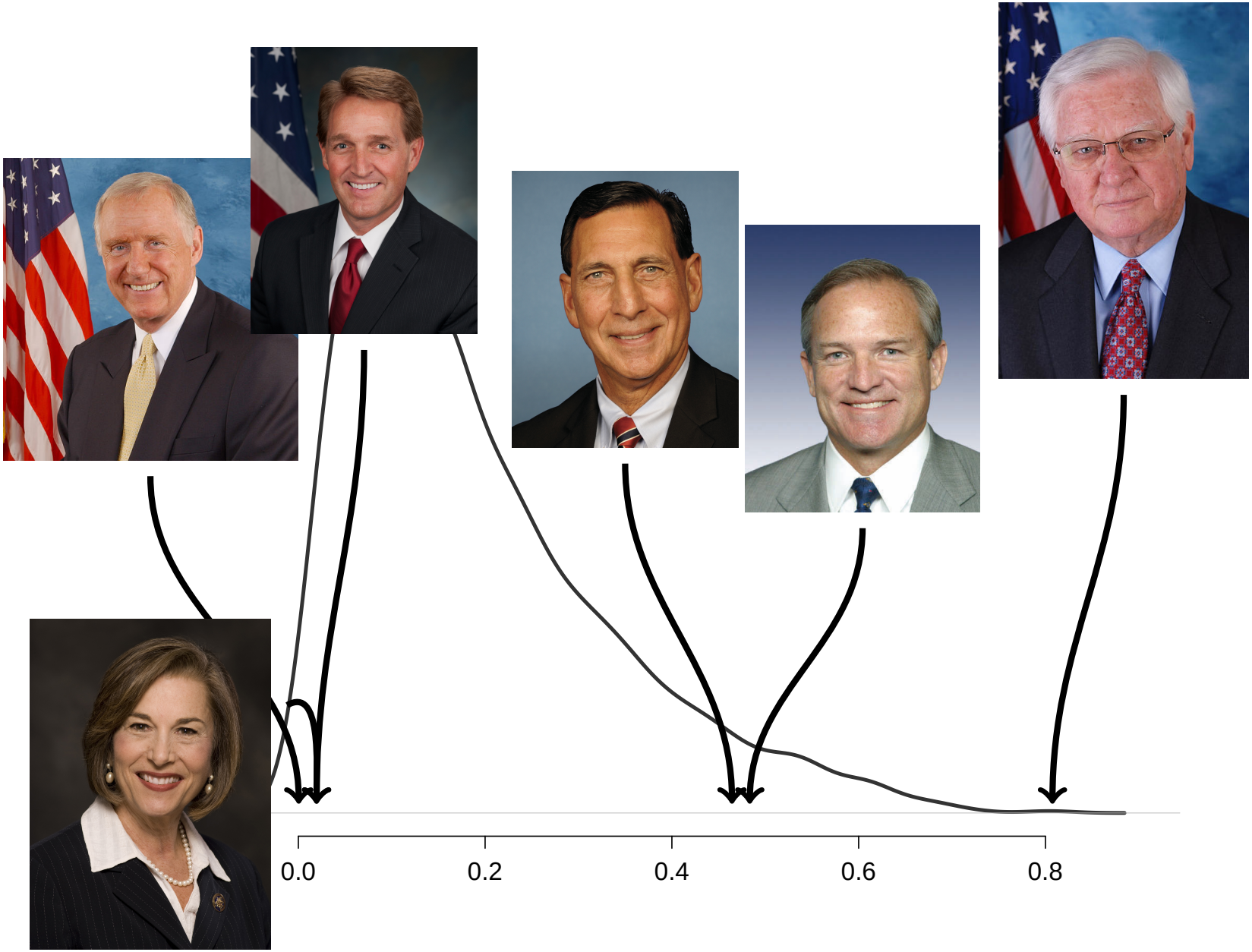
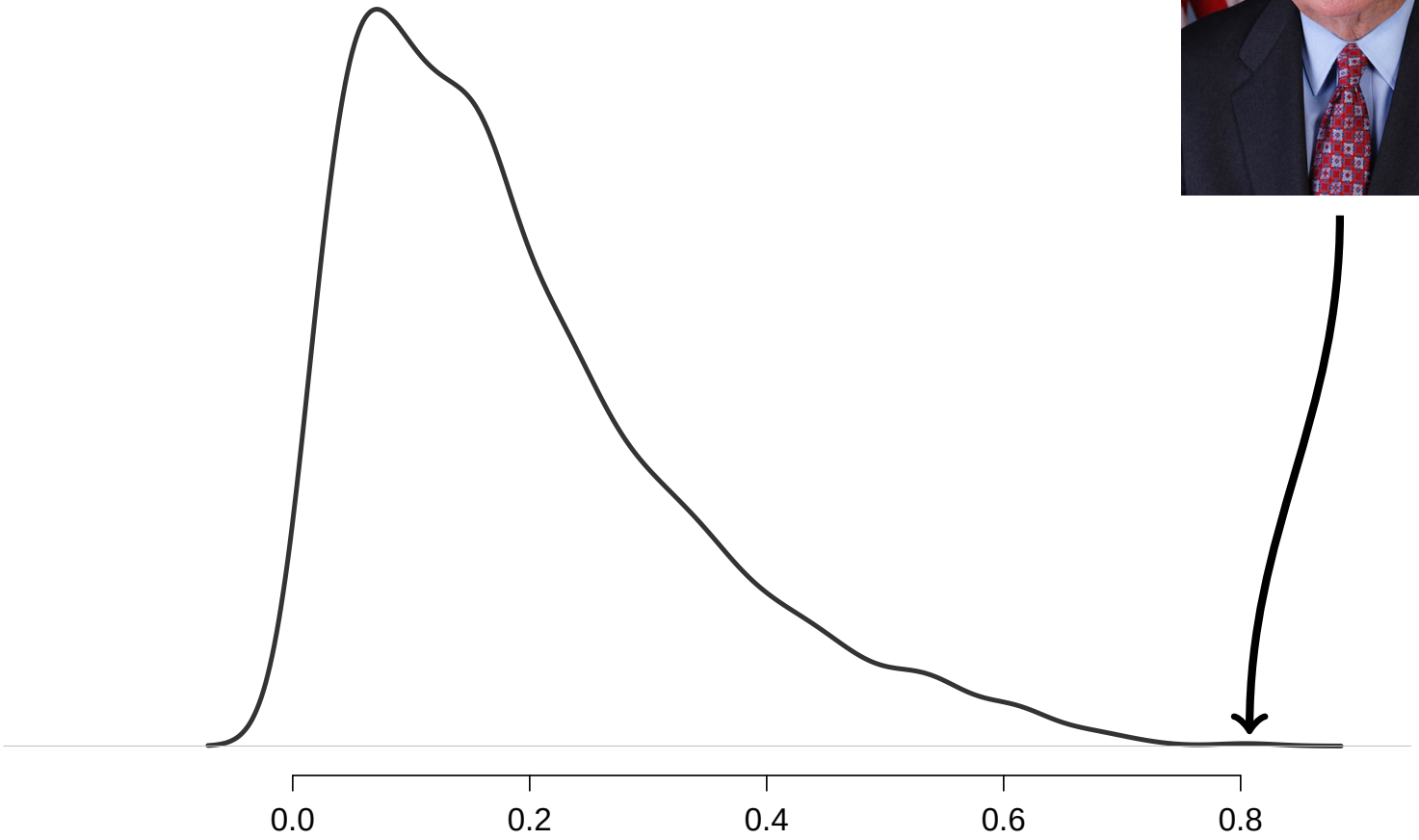


Strategic Credit Claiming to Build a Personal Vote

"We just can't afford luxuries like ideology"



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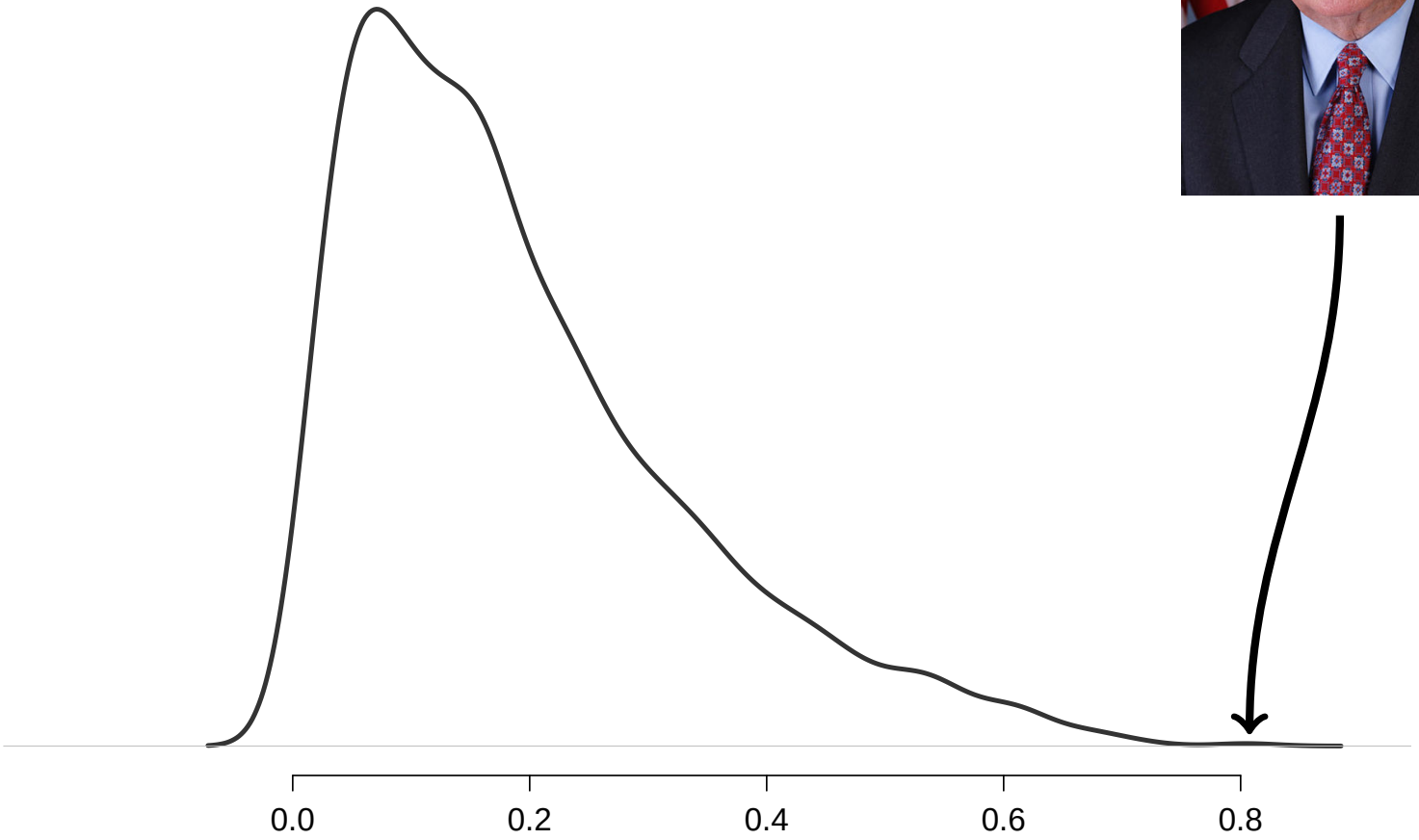
Lexington Herald-Leader: Prince of Pork



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