Text as Data

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Discovery and Measurement

What is the research process? (Grimmer, Roberts, and Stewart 2017)

- 1) Discovery: a hypothesis or view of the world
- 2) Measurement according to some organization
- 3) Causal Inference: effect of some intervention

Text as data methods assist at each stage of research process

Text as Data Methods for Discovery

Text as Data Methods for Discovery Goal: Automatically Discover Organization (Similar Groups)

Consider a document-term matrix

$$X = \begin{pmatrix} 1 & 2 & 0 & \dots & 0 \\ 0 & 0 & 3 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & 0 & \dots & 3 \end{pmatrix}$$

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Suppose documents live in a space \leadsto rich set of results from linear algebra

- Provides a geometry modify with word weighting
 - Natural notions of distance
 - Building block for clustering, supervised learning, and scaling

Doc1 =
$$(1, 1, 3, ..., 5)$$

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Doc2 = $(2, 0, 0, ..., 1)$

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= $1 \times 2 + 1 \times 0 + 3 \times 0 + ... + 5 \times 1$

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= 7

Length1.pdf



- Pythogorean Theorem: Side with length *a*

Length3.pdf

- Pythogorean Theorem: Side with length *a*
- Side with length *b* and right triangle

Length4.pdf

- Pythogorean Theorem: Side with length *a*
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- $c = \sqrt{a^2 + b^2}$

Length4.pdf

- Pythogorean Theorem: Side with length *a*
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$$- c = \sqrt{a^2 + b^2}$$

- This is generally true

Vector (Euclidean) Length

Definition

Suppose $\mathbf{v} \in \Re^J$. Then, we will define its length as

$$||\mathbf{v}|| = (\mathbf{v} \cdot \mathbf{v})^{1/2}$$

= $(v_1^2 + v_2^2 + v_3^2 + \dots + v_J^2)^{1/2}$

Initial guess \leadsto Distance metrics Properties of a metric: (distance function) $d(\cdot,\cdot)$. Consider arbitrary documents \boldsymbol{X}_i , \boldsymbol{X}_j , \boldsymbol{X}_k

Initial guess → Distance metrics

1)
$$d(\boldsymbol{X}_i, \boldsymbol{X}_j) \geq 0$$

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Explore distance functions to compare documents Do we want additional assumptions/properties?

Euclidean Distance

Doc1.pdf

Euclidean Distance

Doc2.pdf

Euclidean Distance

Doc3.pdf

Definition

The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Measuring the Distance Between Documents

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The Euclidean distance between documents X_i and X_j as

$$||X_i - X_j|| = \sqrt{\sum_{m=1}^{J} (x_{im} - x_{jm})^2}$$

Suppose $X_i = (1,4)$ and $X_j = (2,1)$. The distance between the documents is:

$$||(1,4) - (2,1)|| = \sqrt{(1-2)^2 + (4-1)^2}$$

= $\sqrt{10}$

What properties should similarity measure have?

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How should additional words be treated?

Measuring Similarity



Measure 1: Inner product

Measuring Similarity

Fig1.pdf

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$$(2,1)^{'} \cdot (1,4) = 6$$

Fig2.pdf

Fig2.pdf

Problem(?): length dependent

Fig2.pdf

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$$(4,2)^{'}(1,4) = 12$$

Fig3.pdf

Problem(?): length dependent

$$(4,2)'(1,4) = 12$$

 $a \cdot b = ||a|| \times ||b|| \times \cos \theta$

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(0.89, 0.45)'(0.24, 0.97) = 0.65$$



 $\cos\theta \colon$ removes document length from similarity measure



 $\cos\theta$: removes document length from similarity measure Projects texts to unit length representation \leadsto onto sphere



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How to generate weights?

- Assumptions about separating words
- Use training set to identify separating words (Monroe, Ideology measurement)

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 $idf = (idf_1, idf_2, ..., idf_J)$

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- Other functional forms are fine, embed assumptions about penalization of common use

$$\mathbf{X}_{i,\mathrm{idf}} \equiv \underbrace{\mathbf{X}_{i}}_{\mathrm{sf}} \times \mathrm{idf} = (X_{i1} \times \mathrm{idf}_{1}, X_{i2} \times \mathrm{idf}_{2}, \dots, X_{iJ} \times \mathrm{idf}_{J})$$

$$\begin{aligned} \mathbf{X}_{i,\mathrm{idf}} &\equiv \underbrace{\mathbf{X}_i}_{\mathrm{tf}} \times \mathrm{idf} &= (X_{i1} \times \mathrm{idf}_1, X_{i2} \times \mathrm{idf}_2, \dots, X_{iJ} \times \mathrm{idf}_J) \\ \mathbf{X}_{j,\mathrm{idf}} &\equiv \mathbf{X}_j \times \mathrm{idf} &= (X_{j1} \times \mathrm{idf}_1, X_{j2} \times \mathrm{idf}_2, \dots, X_{jJ} \times \mathrm{idf}_J) \end{aligned}$$

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How Does This Matter For Measuring Similarity/Dissimilarity? Inner Product

$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathrm{idf})' (\mathbf{X}_j \times \mathrm{idf})$$

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$$\mathbf{X}_{i,\mathrm{idf}} \cdot \mathbf{X}_{j,\mathrm{idf}} = (\mathbf{X}_i \times \mathbf{idf})'(\mathbf{X}_j \times \mathbf{idf})$$

$$= (\mathrm{idf}_1^2 \times X_{i1} \times X_{j1}) + (\mathrm{idf}_2^2 \times X_{i2} \times X_{j2}) + \dots + (\mathrm{idf}_J^2 \times X_{iJ} \times X_{jJ})$$

Define:

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If we use tf-idf for our documents, then

$$d_2(\boldsymbol{X}_i, \boldsymbol{X}_j) = \sqrt{\sum_{m=1}^{J} (x_{im,idf} - x_{jm,idf})^2}$$
$$= \sqrt{(\boldsymbol{X}_i - \boldsymbol{X}_j)' \boldsymbol{\Sigma} (\boldsymbol{X}_i - \boldsymbol{X}_j)}$$

Final Product

Applying some measure of distance, similarity (if symmetric) yields:

$$\mathbf{D} = \begin{pmatrix} 0 & d(1,2) & d(1,3) & \dots & d(1,N) \\ d(2,1) & 0 & d(2,3) & \dots & d(2,N) \\ d(3,1) & d(3,2) & 0 & \dots & d(3,N) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ d(N,1) & d(N,2) & d(N,3) & \dots & 0 \end{pmatrix}$$

Lower Triangle contains unique information N(N-1)/2

Clustering

Fully Automated Clustering

- 1) Distance metric when are documents close?
- 2) Objective function → how do we summarize distances?
- 3) Optimization method \leadsto how do we find optimal clustering?

THERE IS NO A PRIORI OPTIMAL METHOD Computer Assisted Clustering (Grimmer and King, 2011)

- crucial to combine human and computer insights

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$$\boldsymbol{\tau}_{i} = (0,0,\ldots,0,\underbrace{1}_{k^{th}},0,\ldots,0)$$

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Hard Assignment



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- Two trivial solutions
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 - Each observation in its own cluster

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Coordinate descent

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Change =
$$f(\mathbf{X}, \mathbf{T}^t, \mathbf{\Theta}^t) - f(\mathbf{X}, \mathbf{T}^{t-1}, \mathbf{\Theta}^{t-1})$$

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In words: Assign each document x_i to the closest center θ_m^t

$$f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta})_k = \sum_{i=1}^N \tau_{ik}^t \left(\sum_{j=1}^J (x_{ij} - \theta_{jk})^2 \right)$$

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 - Update change $f(\boldsymbol{X}, \boldsymbol{T}^t, \boldsymbol{\Theta}^t) f(\boldsymbol{X}, \boldsymbol{T}^{t-1}, \boldsymbol{\Theta}^{t-1})$

Visual Example



KMeans1.pdf

KMeans2.pdf

KMeans3.pdf

KMeans4.pdf

KMeans5.pdf

KMeans6.pdf

KMeans7.pdf

KMeans8.pdf

KMeans9.pdf

KMeans10.pdf

KMeans11.pdf

KMeansFinal.pdf

An Example: Jeff Flake

To the R Code!

Unsupervised methods

Unsupervised methods→ low startup costs, high post-model costs

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Think!

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Mixture of von Mises-Fisher (vMF) distributions:

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- $\mu_k \rightsquigarrow$ Exemplar document for cluster k

A Motivating Clustering Model → Mixture of von Mises Fisher Distributions

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- $\blacksquare \pi = (\pi_1, \pi_2, \dots, \pi_K) \leadsto \text{Proportion of documents in each component}$
- $\mu_k \rightsquigarrow$ Exemplar document for cluster k

EM algorithm in slides appendix of Class 10 for my text as data course 🔗

How well does our model perform?

How well does our model perform?→ predict new documents?

How well does our model perform? → predict new documents? Problem

How well does our model perform? → predict new documents? Problem → in sample evaluation leads to overfit.

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$$\log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu},\boldsymbol{\pi},\boldsymbol{X}) = \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*,\tau_{ik}|\boldsymbol{\mu}_k,\boldsymbol{\pi},\boldsymbol{X})$$

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$$\begin{aligned} \log p(\boldsymbol{x}_{\text{out}}^*|\boldsymbol{\mu}, \boldsymbol{\pi}, \boldsymbol{X}) &= \log \sum_{k=1}^K p(\boldsymbol{x}_{\text{out}}^*, \tau_{ik} | \boldsymbol{\mu}_k, \boldsymbol{\pi}, \boldsymbol{X}) \\ &= \log \sum_{k=1}^K \left[\pi_k \exp(\kappa \boldsymbol{\mu}_k' \boldsymbol{x}_{\text{out}}^*) \right] \end{aligned}$$

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$$\begin{split} \log p(\pmb{x}_{\text{out}}^*|\pmb{\mu},\pmb{\pi},\pmb{X}) &= \log \sum_{k=1}^K p(\pmb{x}_{\text{out}}^*,\tau_{ik}|\pmb{\mu}_k,\pmb{\pi},\pmb{X}) \\ &= \log \sum_{k=1}^K \left[\pi_k \exp(\kappa \pmb{\mu}_k^{'} \pmb{x}_{\text{out}}^*)\right] \\ \text{Perplexity}_{\text{word}} &= \exp\left(-\log p(\pmb{x}_{\text{out}}^*|\pmb{\mu},\pmb{\pi})\right) \end{split}$$

Perplex1.pdf

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(Roberts, et al 2017

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Different strategy → measure quality in topics and clusters

- Statistics: measure cohesiveness and exclusivity (Roberts, et al 2017 Forthcoming)
- Experiments: measure topic and cluster quality

Mathematical approaches

Mathematical approaches → suppose we can capture quality with numbers assumes we're in the model → including text representation

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Humans → read texts

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Do humans think the model is performing well?

- 1) Topic Quality
- 2) Cluster Quality

- 1) Take *M* top words for a topic
- 2) Randomly select a top word from another topic
 - 2a) Sample the topic number from I from K-1 (uniform probability)
 - 2b) Sample word j from the M top words in topic l
 - 2c) Permute the words and randomly insert the intruder:
 - List:

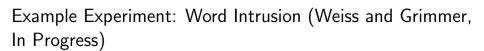
test =
$$(v_{k,3}, v_{k,1}, v_{l,j}, v_{k,2}, v_{k,4}, v_{k,5})$$

bowl, flooding, olympic, olympics, nfl, coach

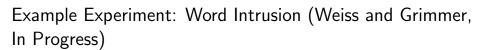
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stocks, investors, fed, guns, trading, earning

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Higher rate of intruder identification → more exclusive/cohesive topics



Higher rate of intruder identification → more exclusive/cohesive topics

Deploy on Mechanical Turk

Cluster Quality (Grimmer and King 2011)

Assessing Cluster Quality with experiments

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Design to assess cluster quality

- Estimate clusterings

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- Select clustering with highest cluster quality
- Can be used to compare any clusterings, regardless of source

How do we Choose K?

Generate many candidate models

- 1) Assess using numerical values
- 2) Use experiments
- 3) Read
- 4) Final decision --> combination

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k-means, Mixture of multinomials, k-medoids, affinity propagation, agglomerative Hierarchical fuzzy k-means, trimmed k-means, k-Harmonic means, fuzzy k-medoids, fuzzy k modes, maximum entropy clustering, model based hierarchical (agglomerative), proximus, ROCK, divisive hierarchical, DISMEA, Fuzzy, QTClust, self-organizing map, self-organizing tree, unnormalized spectral, MS spectral, NJW Spectral, SM Spectral, Dirichlet Process Multinomial, Dirichlet Process Normal, Dirichlet Process von-mises Fisher, Mixture of von mises-Fisher (EM), Mixture of von Mises Fisher (VA), Mixture of normals, co-clustering mutual information, co-clustering SVD, LLAhclust, CLUES, bclust, c-shell, qtClustering, LDA, Express Agenda Model, Hierarchical Dirichlet process prior, multinomial, uniform process mulitinomial, Chinese Restaurant Distance Dirichlet process multinomial, Pitmann-Yor Process multinomial, LSA, ...

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Deep problem in cluster analysis literature: full automation requires more information

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- Our answer: a geography of clusterings

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- 8) (Or, our new strategy: represent entire Bell space directly; no need to examine document contents)

Crosas, Grimmer, King, and Stewart (2017) → Consilience

Consilience.com example (email me for assignment + access)

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- Apply our method (relying on many clustering algorithms)



Each point is a clustering Affinity Propagation-Cosine (Dueck and Frey 2007)





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Close to:

Mixture of von Mises-Fisher distributions (Banerjee et. al. 2005)

⇒ Similar clustering of documents



Space between methods:



Space between methods:



Space between methods: local cluster ensemble





Found a region with clusterings that all reveal the same important insight

Mixture:



Mixture:

0.39 Hclust-Canberra-McQuitty

0.13 Hclust-Correlation-Ward

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Credit Claiming, Pork:

"Sens. Frank R. Lautenberg (D-NJ) and Robert Menendez (D-NJ) announced that the U.S. Department of Commerce has awarded a \$100,000 grant to the South Jersey Economic Development District"



Credit Claiming, Legislation:

"As the Senate begins its recess, Senator Frank Lautenberg today pointed to a string of victories in Congress on his legislative agenda during this work period"



Advertising:

"Senate Adopts Lautenberg/Menendez Resolution Honoring Spelling Bee Champion from New Jersey"

Example Discovery: Partisan Taunting



Partisan Taunting:

"Republicans Selling Out Nation on Chemical Plant Security"

Important Concept Overlooked in Mayhew's (1974) typology

 "Senator Lautenberg Blasts Republicans as 'Chicken Hawks' " [Government Oversight]



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Definition: Explicit, public, and negative attacks on another political party or its members



Sen. Lautenberg on Senate Floor 4/29/04

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Consequences for representation: Deliberative, Polarization, Policy



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Over Time Tauting Rates in Speeches



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 - Best evaluation: An improbable experiment
 - Randomly assign incoming grad students to three conditions

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 - Observe group with most productivity 20-30 years later
 - To identify limits of methods, when to use which approach, need evaluations for the usefulness of conceptualizations

Stylometry Who Wrote Disputed Federalist Papers?

Federalist papers → Mosteller and Wallace (1963)

- Persuade citizens of New York State to adopt constitution
- Canonical texts in study of American politics
- 77 essays
 - Published from 1787-1788 in Newspapers
 - And under the name Publius, anonymously

Who Wrote the Federalist papers?

- Jay wrote essays 2, 3, 4,5, and 64
- Hamilton: wrote 43 papers
- Madison: wrote 12 papers

Disputed: Hamilton or Madison?

- Essays: 49-58, 62, and 63
- Joint Essays: 18-20

Task: identify authors of the disputed papers.

Task: Classify papers as Hamilton or Madison using dictionary methods

Setting up the Analysis

Training → papers Hamilton, Madison are known to have authored Test → unlabeled papers Preprocessing:

- Hamilton/Madison both discuss similar issues
- Differ in extent they use stop words
- Focus analysis on the stop words

Setting up the Analysis

- $\mathbf{Y} = (Y_1, Y_2, ..., Y_N) = (Hamilton, Hamilton, Madison, ..., Hamilton)$ $N \times 1$ matrix with author labels
- Define the number of words in federalist paper i as num $_i$

$$\mathbf{X} = \begin{pmatrix} \frac{1}{\mathsf{num}_1} & \frac{2}{\mathsf{num}_1} & \frac{0}{\mathsf{num}_1} & \cdots & \frac{3}{\mathsf{num}_1} \\ \frac{0}{\mathsf{num}_2} & \frac{1}{\mathsf{num}_2} & \frac{0}{\mathsf{num}_2} & \cdots & \frac{0}{\mathsf{num}_2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{0}{\mathsf{num}_N} & \frac{0}{\mathsf{num}_N} & \frac{1}{\mathsf{num}_N} & \cdots & \frac{0}{\mathsf{num}_N} \end{pmatrix}$$

 $N \times J$ counting stop word usage rate

-
$$\theta = (\theta_1, \theta_2, \dots, \theta_J)$$

Word weights.

Objective Function

Heuristically: find $\theta^* = (\theta_1^*, \theta_2^*, \dots, \theta_J^*)$ used to create score

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

that maximally discriminates between categories

LDALine.pdf

Objective Function

Define:

$$oldsymbol{\mu}_{\mathsf{Madison}} = rac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) oldsymbol{\chi}_i$$
 $oldsymbol{\mu}_{\mathsf{Hamilton}} = rac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) oldsymbol{\chi}_i$

Objective Function

We can then define functions that describe the "projected" mean and variance for each author

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}) = \frac{1}{N_{\mathsf{Madison}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Madison}}$$

$$g(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) = \frac{1}{N_{\mathsf{Hamilton}}} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) \boldsymbol{\theta}' \boldsymbol{X}_i = \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Hamilton}}$$

$$s(\theta, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison}) = \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) (\boldsymbol{\theta}' \boldsymbol{X}_i - \boldsymbol{\theta}' \boldsymbol{\mu}_{\mathsf{Madison}})^2$$

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Objective Function --> Optimization

$$\begin{split} f(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}) &= \frac{\left(g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) - g(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison})\right)^2}{s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Hamilton}) + s(\boldsymbol{\theta}, \boldsymbol{X}, \boldsymbol{Y}, \mathsf{Madison})} \\ &= \frac{\left(\boldsymbol{\theta}'(\boldsymbol{\mu}_{\mathsf{Hamilton}} - \boldsymbol{\mu}_{\mathsf{Madison}})\right)^2}{\mathsf{Scatter}_{\mathsf{Hamilton}} + \mathsf{Scatter}_{\mathsf{Madison}}} \end{split}$$

Optimization \rightsquigarrow find θ^* to maximize $f(\theta, X, Y)$, assuming independence across dimensions.

(Fisher's) Linear Discriminant Analysis

Optimization >>> Word Weights

For each word j, construct weight θ_j^* ,

$$\begin{array}{ll} \mu_{j,\mathsf{Hamilton}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Hamilton}) X_{ij}} \\ \mu_{j,\mathsf{Madison}} & = & \frac{\sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}}{\sum_{j=1}^{J} \sum_{i=1}^{N} I(Y_i = \mathsf{Madison}) X_{ij}} \\ \sigma_{j,\mathsf{Hamilton}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Hamilton}) \\ \sigma_{j,\mathsf{Madison}}^2 & = & \mathsf{Var}(X_{i,j} | \mathsf{Madison}) \end{array}$$

We can then generate weight θ_i^* as

$$\theta_{j}^{*} = \frac{\mu_{j}, \text{Hamilton} - \mu_{j}, \text{Madison}}{\sigma_{j}^{2}, \text{Hamilton} + \sigma_{j}^{2}, \text{Madison}}$$

Optimization \sim Trimming the Dictionary

- Trimming weights: Focus on discriminating words (very simple regularization)
- Cut off: For all $|\theta_i^*| < 0.025$ set $\theta_i^* = 0$.

Classification → Determining Authorship

For each disputed document i, compute discrimination statistic

$$p_i = \sum_{j=1}^J \theta_j^* X_{ij}$$

 $p_i \rightsquigarrow \text{classification (linear discriminator)}$

- Above midpoint in training set \rightarrow Hamilton text
- Below midpoint in training set \rightarrow Madison text

Findings: Madison is the author of the disputed federalist papers.

 ${\sf Classification} {\leadsto} \ {\sf Custom} \ {\sf Dictionaries}$

Classification → Custom Dictionaries

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Vague and Difficult to derive before hand

Congressional Press Releases and Floor Speeches

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- Partial answer: identify words that distinguish press releases and floor speeches

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Bigger mutual information \Rightarrow better discrimination

Objective function and optimization \leadsto estimate probabilities that we then place in mutual information

Formula for mutual information (based on ML estimates of probabilities)

```
n_p = Number Press Releases
  n_s = Number of Speeches
   D = n_p + n_s
  n_j = \sum_{i=1}^D X_{i,j} (No. docs X_j appears)
 n_{-i} = No. docs X_i does not appear
 n_{i,p} = No. press and X_i
 n_{i,s} = No. speech and X_i
n_{-i,p} = No. press and not X_i
n_{-i,s} = No. speech and not X_i
```

Formula for Mutual Information

$$MI(X_{j}) = \frac{n_{j,p}}{D} \log_{2} \frac{n_{j,p}D}{n_{j}n_{p}} + \frac{n_{j,s}}{D} \log_{2} \frac{n_{j,s}D}{n_{j}n_{s}} + \frac{n_{-j,p}}{D} \log_{2} \frac{n_{-j,p}D}{n_{-j}n_{p}} + \frac{n_{-j,s}}{D} \log_{2} \frac{n_{-j,s}D}{n_{-j}n_{s}}.$$











What's Different?

- Press Releases: Credit Claiming



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- Floor Speeches: Procedural Words



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What's Different About Press Releases



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Strategy Construct objective function on *proportions* (and then calculate log-odds)

Suppose we're interested in how a word separates partisan speech.

 $\mathbf{Y} = (Republican, Republican, Democrat, \dots, Republican)$

X =Unnormalized matrix of word counts $N \times J$ Define

$$\mathbf{x}_{\mathsf{Republican}} = (\sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i1}, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{i2}, \dots, \sum_{i=1}^{N} I(Y_i = \mathsf{Republican}) X_{iJ})$$

with $N_{Republican} = Total$ number of Republican words

 $\pi_{\mathsf{Republican}} \ \sim \ \mathsf{Dirichlet}(lpha)$

```
m{\pi}_{\mathsf{Republican}} \sim \mathsf{Dirichlet}(m{lpha}) \ m{x}_{\mathsf{Republican}} | m{\pi}_{\mathsf{Republican}} \sim \mathsf{Multinomial}(m{N}_{\mathsf{Republican}}, m{\pi}_{\mathsf{Republican}})
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 $p(\boldsymbol{\pi}|\boldsymbol{\alpha},\boldsymbol{X},\boldsymbol{Y})$ is a Dirichlet distribution:

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$$\pi_{\mathsf{Republican},j}^* \ = \ \frac{\mathit{x}_{\mathsf{Republican},j} + \alpha_j}{\mathit{N}_{\mathsf{Republican}} + \sum_{j=1}^J \alpha_j}$$

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Calculating Log Odds Ratio

Define log Odds Ratio; as

$$\log \mathsf{Odds} \; \mathsf{Ratio}_j \;\; = \;\; \log \left(\frac{\pi_{\mathsf{Republican},j}}{1 - \pi_{\mathsf{Republican},j}} \right) - \log \left(\frac{\pi_{\mathsf{Democratic},j}}{1 - \pi_{\mathsf{Democratic},j}} \right)$$

$$Var(\log Odds \ Ratio_j) \approx \frac{1}{x_{jD} + \alpha_j} + \frac{1}{x_{jR} + \alpha_j}$$

$$Std. \ Log \ Odds_j = \frac{\log Odds \ Ratio_j}{\sqrt{Var(\log Odds \ Ratio_j)}}$$

Applying the Model

https://gist.github.com/thiagomarzagao/5851207 How do Republicans and Democrats differ in debate? Condition on topic and examine word usage

- Press Releases (64,033)
- Topic Coded
- Given press release is about topic, what are the features that distinguish Republican and Democratic language?

Mutual Information, Standardized Log Odds



Mutual Information, Standardized Log Odds



Gentzkow, Shapiro, and Taddy (2017): Rhetorical Polarization

