Q) Obtain Fourier Expression for,

$$f(t) = \begin{cases} A; 0 < t < t_1 \\ 0; t_1 < t < 2t_1 \end{cases}$$

Answer

Trigonometric Fourier Series Formula,

$$f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos(nt) + b_n \sin(nt)]$$

Where,

$$a_n = \frac{1}{\pi} \int_T f(t) \cos(nt)\, dt$$

$$b_n = \frac{1}{\pi} \int_T f(t) \sin(nt)\, dt$$

$$a_0 = \frac{1}{\pi} \int_T f(t)\, dt$$

Therefore,

$$a_0 = \int_0^{2t_1} f(t)\, dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} 1\, dt + \int_{\pi}^{2\pi} 0\, dt \right]$$

$$a_0 = 1$$

$$a_n = \frac{1}{\pi} \int_0^{2t_1} f(t) \cos(nt)\, dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} 1. \cos(nt)\, dt + \int_{\pi}^{2\pi} 0\, dt \right]$$

$$a_n = \frac{1}{n\pi} \sin(n\pi)$$

But,

$$a_n = 0 \qquad \forall_n \in \mathbb{Z}$$

$$b_n = \frac{1}{\pi} \int_0^{2t_1} f(t) \sin(nt)\, dt$$

$$= \frac{1}{\pi} \left[\int_0^{\pi} 1. \sin(nt)\, dt + \int_{\pi}^{2\pi} 0\, dt \right]$$

$$b_n = \frac{1}{n\pi} [1 - \cos(nt_1)]$$

But,

$$b_n = \frac{2}{n\pi} \quad where, \qquad n \in 1, 3, 5, 7$$

Therefore,

$$f(t) = \frac{1}{2} + \sum_{n=1}^{\infty} \left[\frac{2}{n\pi} \sin\,(nt) \right]$$

$$f(t) = \frac{1}{2} + \left[\frac{2}{\pi} \sin(t) + \frac{2}{3\pi} \sin(3t) + \frac{2}{5\pi} \sin(5t) + \frac{2}{7\pi} \sin(7t) + \cdots \right]$$