

Step	Algorithm:
1a	
4	where
2	
3	while do
2,3	\wedge
5a	where
6	
8	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg(\quad)$
1b	

Step	Algorithm: $[y] := \text{SYMV_I_UNB_VAR4_UNB}(A, x, y)$
1a	$y = \hat{y}$
4	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ <p>where A_{TL} is 0×0, x_T has 0 rows, y_T has 0 rows</p>
2	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right)$
3	while $m(A_{TL}) < m(A)$ do
2,3	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right) \wedge m(A_{TL}) < m(A)$
5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ <p>where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row</p>
6	$\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} A_{00} + (a_{10}^T)^T \chi_1 + A_{20}^T x_2 + \hat{y}_0 \\ \hline a_{10}^T x_0 + \hat{\psi}_1 \\ \hline A_{20}x_0 + \hat{y}_2 \end{array} \right)$
8	$y_0 = y_0$ $\psi_1 = \alpha_{11}\chi_1 + a_{21}^T x_2 + \psi_1$ $y_2 = a_{21}\chi_1 + y_2$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$
7	$\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} A_{00} + a_{10}\chi_1 + A_{20}^T x_2 + \hat{y}_0 \\ \hline \alpha_{11}\chi_1 + a_{21}^T x_2 + a_{10}^T x_0 + \hat{\psi}_1 \\ \hline a_{21}\chi_1 + A_{20}x_0 + \hat{y}_2 \end{array} \right)$
2	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right)$
	endwhile
2,3	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$
1b	$y = Ax + \hat{y}$

Algorithm: $[y] := \text{SYMV_I_UNB_VAR4_UNB}(A, x, y)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$$

where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$y_0 = y_0$$

$$\psi_1 = \alpha_{11}\chi_1 + a_{21}^T x_2 + \psi_1$$

$$y_2 = a_{21}\chi_1 + y_2$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

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3	while do
2,3	$\left(\frac{y_T}{y_B}\right) = \left(\frac{A_{TL}x_T + A_{BL}^T x_B + \widehat{y}_T}{A_{BL}x_T + \widehat{y}_B}\right) \wedge$
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5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \chi_1 \\ x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \psi_1 \\ y_2 \end{array} \right)$
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5a	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row
6	$\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} A_{00} + (a_{10}^T)^T \chi_1 + A_{20}^T x_2 + \hat{y}_0 \\ \hline a_{10}^T x_0 + \hat{\psi}_1 \\ \hline A_{20}x_0 + \hat{y}_2 \end{array} \right)$
8	$y_0 = y_0$ $\psi_1 = \alpha_{11}\chi_1 + a_{21}^T x_2 + \psi_1$ $y_2 = a_{21}\chi_1 + y_2$
5b	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$
7	$\left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right) = \left(\begin{array}{c} A_{00} + a_{10}\chi_1 + A_{20}^T x_2 + \hat{y}_0 \\ \hline \alpha_{11}\chi_1 + a_{21}^T x_2 + a_{10}^T x_0 + \hat{\psi}_1 \\ \hline a_{21}\chi_1 + A_{20}x_0 + \hat{y}_2 \end{array} \right)$
2	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right)$
	endwhile
2,3	$\left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) = \left(\begin{array}{c} A_{TL}x_T + A_{BL}^T x_B + \hat{y}_T \\ \hline A_{BL}x_T + \hat{y}_B \end{array} \right) \wedge \neg(m(A_{TL}) < m(A))$
1b	$y = Ax + \hat{y}$

Step	Algorithm: $[y] := \text{SYMV_I_UNB_VAR4_UNB}(A, x, y)$
	$A \rightarrow \left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$ <p>where A_{TL} is 0×0, x_T has 0 rows, y_T has 0 rows</p>
	while $m(A_{TL}) < m(A)$ do
	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$ <p>where α_{11} is 1×1, χ_1 has 1 row, ψ_1 has 1 row</p>
	$y_0 = y_0$ $\psi_1 = \alpha_{11}\chi_1 + a_{21}^T x_2 + \psi_1$ $y_2 = a_{21}\chi_1 + y_2$
	$\left(\begin{array}{c c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c c c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$
	endwhile

Algorithm: $[y] := \text{SYMV_I_UNB_VAR4_UNB}(A, x, y)$

$$A \rightarrow \left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right), x \rightarrow \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right), y \rightarrow \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right)$$

where A_{TL} is 0×0 , x_T has 0 rows, y_T has 0 rows

while $m(A_{TL}) < m(A)$ **do**

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \rightarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \rightarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

where α_{11} is 1×1 , χ_1 has 1 row, ψ_1 has 1 row

$$y_0 = y_0$$

$$\psi_1 = \alpha_{11}\chi_1 + a_{21}^T x_2 + \psi_1$$

$$y_2 = a_{21}\chi_1 + y_2$$

$$\left(\begin{array}{c|c} A_{TL} & A_{TR} \\ \hline A_{BL} & A_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} A_{00} & a_{01} & A_{02} \\ \hline a_{10}^T & \alpha_{11} & a_{12}^T \\ \hline A_{20} & a_{21} & A_{22} \end{array} \right), \left(\begin{array}{c} x_T \\ \hline x_B \end{array} \right) \leftarrow \left(\begin{array}{c} x_0 \\ \hline \chi_1 \\ \hline x_2 \end{array} \right), \left(\begin{array}{c} y_T \\ \hline y_B \end{array} \right) \leftarrow \left(\begin{array}{c} y_0 \\ \hline \psi_1 \\ \hline y_2 \end{array} \right)$$

endwhile