Step	Algorithm:
1a	
4	where
2	
3	while do
2,3	$\wedge$
5a	where
6	
8	
5b	
7	
2	
	endwhile
2,3	$\wedge \neg($ $)$
1b	

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<tr><th>Step</th><th>Algorithm: $[C] := \text{SYR2K\_LT\_UNB\_VAR3}(A, B, C)$</th></tr>
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Algorithm: $[C] := \text{SYR2K\_LT\_UNB\_VAR3}(A, B, C)$

$A \rightarrow \left(A_L \,\middle|\, A_R \right)$, $B \rightarrow \left(B_L \,\middle|\, B_R \right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$

where A_L has 0 columns, B_L has 0 columns, C_{TL} is 0×0

while $n(A_L) < n(A)$ **do**

$$\left(A_L \,\middle|\, A_R \right) \rightarrow \left(A_0 \,\middle|\, a_1 \,\middle|\, A_2 \right) , \left(B_L \,\middle|\, B_R \right) \rightarrow \left(B_0 \,\middle|\, b_1 \,\middle|\, B_2 \right) , \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$$

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$c_{10}^T := b_1^T A_0 + \widehat{c}_{10}^T$

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$$\left(A_L \,\middle|\, A_R \right) \leftarrow \left(A_0 \,\middle|\, a_1 \,\middle|\, A_2 \right) , \left(B_L \,\middle|\, B_R \right) \leftarrow \left(B_0 \,\middle|\, b_1 \,\middle|\, B_2 \right) , \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$$

endwhile

Step	Algorithm: $[C] := \textsc{Syr2k\_lt\_unb\_var3}(A, B, C)$
1a	$C = \widehat{C}$
4	where
2	
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6	
8	
5b	
7	
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	endwhile
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3	while do
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6	
8	
5b	
7	
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<tr><td>2,3</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge n(A_L) < n(A)$</td></tr>
<tr><td>5a</td><td>$\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} A_0 & a_1 & A_2 \end{array}\right)$, $\left(\begin{array}{c|c} B_L & B_R \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} B_0 & b_1 & B_2 \end{array}\right)$, $\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
where a_1 has 1 column, b_1 has 1 column, γ_{11} is 1×1</td></tr>
<tr><td>6</td><td>$\left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c|c|c} A_0^T B_0 + B_0^T A_0 + \widehat{C}_{00} & 0 & 0 \\ \hline a_1^T B_0 + \widehat{c}_{10}^T & \widehat{\gamma}_{11} & 0 \\ \hline A_2^T B_0 + \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right)$</td></tr>
<tr><td>8</td><td></td></tr>
<tr><td>5b</td><td>$\left(\begin{array}{c|c} A_L & A_R \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} A_0 & a_1 & A_2 \end{array}\right)$, $\left(\begin{array}{c|c} B_L & B_R \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} B_0 & b_1 & B_2 \end{array}\right)$, $\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$</td></tr>
<tr><td>7</td><td>$\left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c|c|c} A_0^T B_0 + B_0^T A_0 + \widehat{C}_{00} & 0 & 0 \\ \hline a_1^T B_0 + b_1^T A_0 + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & 0 \\ \hline A_2^T B_0 + \widehat{C}_{20} & A_2^T b_1 + \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right)$</td></tr>
<tr><td>2</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$</td></tr>
<tr><td></td><td>endwhile</td></tr>
<tr><td>2</td><td>$\left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(n(A_L) < n(A))$</td></tr>
<tr><td>1b</td><td>$[C] = \mathrm{Syr2k\_lt}(A, B, \widehat{C})$</td></tr>
</table>

Step	Algorithm: $[C] := \textsc{Syr2k\_lt\_unb\_var3}(A, B, C)$
1a	$C = \widehat{C}$
4	$A \rightarrow \left(A_L \,\middle\|\, A_R\right)$, $B \rightarrow \left(B_L \,\middle\|\, B_R\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where A_L has 0 columns, B_L has 0 columns, C_{TL} is 0×0
2	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$
3	while $n(A_L) < n(A)$ do
2,3	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge n(A_L) < n(A)$
5a	$\left(A_L \,\middle\|\, A_R\right) \rightarrow \left(A_0 \,\middle\|\, a_1 \,\middle\|\, A_2\right)$, $\left(B_L \,\middle\|\, B_R\right) \rightarrow \left(B_0 \,\middle\|\, b_1 \,\middle\|\, B_2\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 column, b_1 has 1 column, γ_{11} is 1×1
6	$\left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} A_0^T B_0 + B_0^T A_0 + \widehat{C}_{00} & 0 & 0 \\ \hline a_1^T B_0 + \widehat{c}_{10}^T & \widehat{\gamma}_{11} & 0 \\ \hline A_2^T B_0 + \widehat{C}_{20} & \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right)$
8	$c_{10}^T := b_1^T A_0 + \widehat{c}_{10}^T$ $\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11}$ $c_{21} := A_2^T b_1 + \widehat{c}_{21}$
5b	$\left(A_L \,\middle\|\, A_R\right) \leftarrow \left(A_0 \,\middle\|\, a_1 \,\middle\|\, A_2\right)$, $\left(B_L \,\middle\|\, B_R\right) \leftarrow \left(B_0 \,\middle\|\, b_1 \,\middle\|\, B_2\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
7	$\left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right) = \left(\begin{array}{c\|c\|c} A_0^T B_0 + B_0^T A_0 + \widehat{C}_{00} & 0 & 0 \\ \hline a_1^T B_0 + b_1^T A_0 + \widehat{c}_{10}^T & a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11} & 0 \\ \hline A_2^T B_0 + \widehat{C}_{20} & A_2^T b_1 + \widehat{c}_{21} & \widehat{C}_{22} \end{array}\right)$
2	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right)$
	endwhile
2,3	$\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) = \left(\begin{array}{c\|c} A_L^T B_L + B_L^T A_L + \widehat{C}_{TL} & \widehat{C}_{TR} \\ \hline A_R^T B_L + \widehat{C}_{BL} & \widehat{C}_{BR} \end{array}\right) \wedge \neg(n(A_L) < n(A))$
1b	$[C] = \mathrm{Syr2k\_lt}(A, B, \widehat{C})$

Step	Algorithm: $[C] := \text{Syr2k\_lt\_unb\_var3}(A, B, C)$
	$A \rightarrow \left(A_L \middle\| A_R\right)$, $B \rightarrow \left(B_L \middle\| B_R\right)$, $C \rightarrow \left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right)$ where A_L has 0 columns, B_L has 0 columns, C_{TL} is 0×0
	while $n(A_L) < n(A)$ do
	$\left(A_L \middle\| A_R\right) \rightarrow \left(A_0 \middle\| a_1 \middle\| A_2\right)$, $\left(B_L \middle\| B_R\right) \rightarrow \left(B_0 \middle\| b_1 \middle\| B_2\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \rightarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$ where a_1 has 1 column, b_1 has 1 column, γ_{11} is 1×1
	$c_{10}^T := b_1^T A_0 + \widehat{c}_{10}^T$ $\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11}$ $c_{21} := A_2^T b_1 + \widehat{c}_{21}$
	$\left(A_L \middle\| A_R\right) \leftarrow \left(A_0 \middle\| a_1 \middle\| A_2\right)$, $\left(B_L \middle\| B_R\right) \leftarrow \left(B_0 \middle\| b_1 \middle\| B_2\right)$, $\left(\begin{array}{c\|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array}\right) \leftarrow \left(\begin{array}{c\|c\|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array}\right)$
	endwhile

Algorithm: $[C] := \textsc{Syr2k\_lt\_unb\_var3}(A, B, C)$

$A \rightarrow \left(A_L \,\middle|\, A_R \right)$, $B \rightarrow \left(B_L \,\middle|\, B_R \right)$, $C \rightarrow \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right)$

where A_L has 0 columns, B_L has 0 columns, C_{TL} is 0×0

while $n(A_L) < n(A)$ **do**

$$\left(A_L \,\middle|\, A_R \right) \rightarrow \left(A_0 \,\middle|\, a_1 \,\middle|\, A_2 \right), \left(B_L \,\middle|\, B_R \right) \rightarrow \left(B_0 \,\middle|\, b_1 \,\middle|\, B_2 \right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \rightarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$$

where a_1 has 1 column, b_1 has 1 column, γ_{11} is 1×1

$c_{10}^T := b_1^T A_0 + \widehat{c}_{10}^T$

$\gamma_{11} := a_1^T b_1 + b_1^T a_1 + \widehat{\gamma}_{11}$

$c_{21} := A_2^T b_1 + \widehat{c}_{21}$

$$\left(A_L \,\middle|\, A_R \right) \leftarrow \left(A_0 \,\middle|\, a_1 \,\middle|\, A_2 \right), \left(B_L \,\middle|\, B_R \right) \leftarrow \left(B_0 \,\middle|\, b_1 \,\middle|\, B_2 \right), \left(\begin{array}{c|c} C_{TL} & C_{TR} \\ \hline C_{BL} & C_{BR} \end{array} \right) \leftarrow \left(\begin{array}{c|c|c} C_{00} & c_{01} & C_{02} \\ \hline c_{10}^T & \gamma_{11} & c_{12}^T \\ \hline C_{20} & c_{21} & C_{22} \end{array} \right)$$

endwhile