

8.1

$$\begin{aligned}\frac{d[P]}{dt} &= k_3[ES] \\ \frac{d[S]}{dt} &= -k_1[E][S] + k_2[ES] \\ \frac{d[ES]}{dt} &= -k_3[ES] - k_2[ES] + k_1[E][S] \\ \frac{d[E]}{dt} &= -k_1[E][S] + k_2[ES] + k_3[ES]\end{aligned}$$

8.2

When

$$\begin{aligned}k_1 &= 100, k_2 = 600, k_3 = 150 \\ E(0) &= 1, S(0) = 10\end{aligned}$$

Solve the equations above.

For applying fourth-order Runge Kutta method, firstly convert equations into standard form.

$$\begin{aligned}[ES] + [E] &= E(0) \\ [ES] + [S] + [P] &= S(0) \\ \frac{d[E]}{dt} &= -k_1[E](S(0) - (E(0) - [E]) - [P]) + k_2(E(0) - [E]) + k_3(E(0) - [E]) \\ \frac{d[E]}{dt} &= -k_1[E](9 + [E] - [P]) + k_2(1 - [E]) + k_3(1 - [E])\end{aligned}$$

The method of expressing [P] by [E] did not occur to me, so I could not proceed. But the code has been written as follows.

The Matlab code for the RK4 function is listed below.

```
function y = RK4(fun, x, y0)

y = 0 * x;
y(1) = y0;
h = x(2) - x(1);
n = length(x);
for m = 1 : n-1
    k1 = fun(x(m), y(m));
    k2 = fun(x(m)+h/2, y(m)+h*k1/2);
    k3 = fun(x(m)+h/2, y(m)+h*k2/2);
    k4 = fun(x(m)+h, y(m)+h*k3);
    y(m+1) = y(m) + h*(k1 + 2*k2 + 2*k3 + k4) / 6;
end
End
```

The code for the solution is as follows

```
fun = @(x, y) (#Function#)
```

```
x = ;  
y0 = ;
```

The code for the solution is as follows

```
y = RK4(fun, x, y0);
```

```
fig = gcf;  
fig.Color = 'w';  
fig.Position = [250, 250, 960, 540];
```

```
p = plot(x, y);  
p.LineStyle = 'none';  
p.Marker = 'p';  
p.MarkerEdgeColor = 'r';  
p.MarkerFaceColor = 'b';  
p.MarkerSize = 8;
```

```
hold on, grid on  
syms y(x)  
equ = 2 * diff(y, x) == exp(-x) - y;  
cond = y(0) == 1/2;  
y = dsolve(equ, cond);
```

```
fplot(y, [0, 2])
```

```
xlabel('x', 'fontsize', 12);  
ylabel('y', 'fontsize', 12);  
title('RK4 for ODE', 'fontsize', 14);  
legend({'Numerical solutions', 'Symbol Solution'}, 'fontsize', 12);
```