

Chapter 3: Vectors

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General

Quantities

\vec{r} = resultant vector

$|\vec{a}|$ = magnitude of vector \vec{a}

$\vec{a} \cdot \vec{b}$ = dot product of vectors \vec{a} and \vec{b}

$\vec{a} \times \vec{b}$ = cross product of vectors \vec{a} and \vec{b}

θ = angle between a vector and the $+x$ axis

ϕ = smallest angle between two vectors

1 Vectors and Their Components

Commutative Law For Adding Vectors

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \quad (1)$$

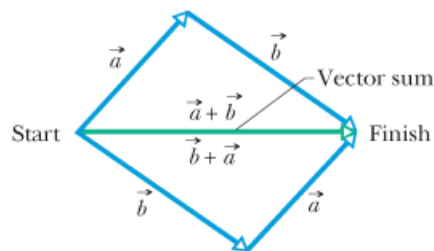


Figure 1: The two vectors \vec{a} and \vec{b} can be added in either order (see equation 1)

Associative Law For Adding Vectors

$$(\vec{a} + \vec{b}) + \vec{c} = \vec{a} + (\vec{b} + \vec{c}) \quad (2)$$

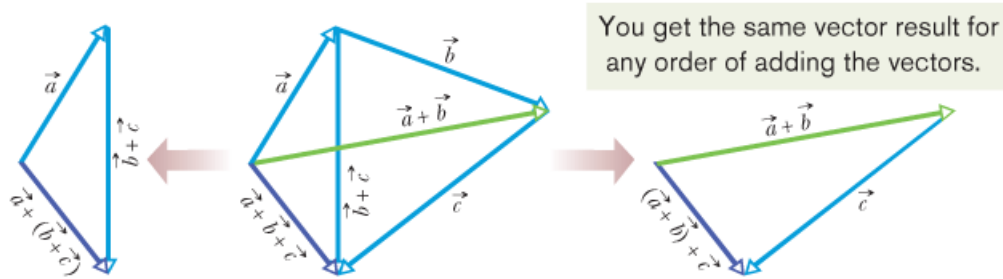


Figure 2: The three vectors \vec{a} , \vec{b} , and \vec{c} can be grouped in any way as they are added (see equation 2)

Vector Components

$$a_x = a \cos \theta \quad \text{and} \quad a_y = a \sin \theta \quad (3)$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan \theta = \frac{a_y}{a_x} \quad (4)$$

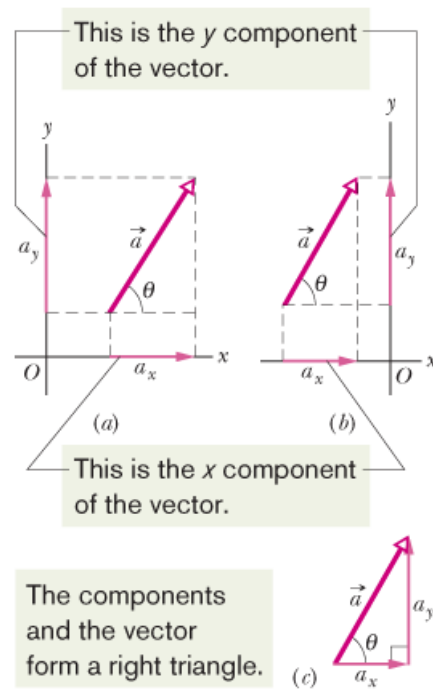


Figure 3: The vector components a_x and a_y are the projections of the vector \vec{a} on the x and y axes (see equation 3)

2 Unit Vectors, Adding Vectors By Components

Unit Vectors

$$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k} \quad (5)$$

Adding Vectors by Components

$$\vec{r} = \vec{a} + \vec{b}$$

$$r_x = a_x + b_x \quad (6)$$

$$r_y = a_y + b_y \quad (7)$$

$$r_z = a_z + b_z \quad (8)$$

3 Multiplying Vectors

The Scalar/Dot Product

$$\vec{a} \cdot \vec{b} = ab \cos \phi \quad (9)$$

$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \quad (10)$$

If the angle ϕ between two vectors is 0° , the component of one vector along the other is maximum, and so is the dot product of the vectors. If, instead, ϕ is 90° , the component of one vector along the other is zero, and so is the dot product.

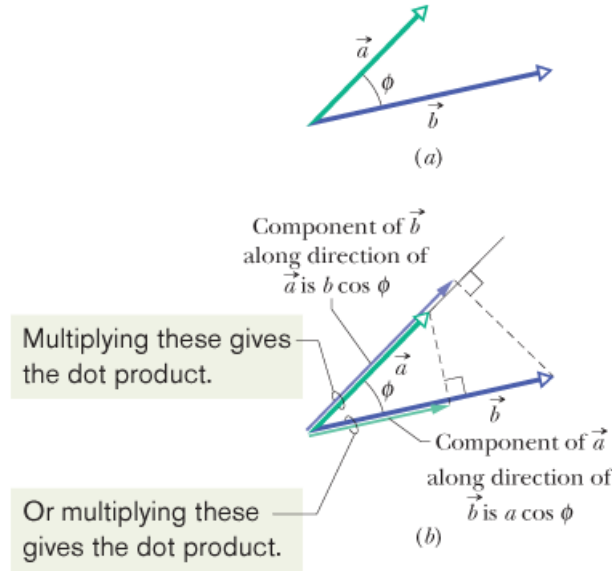


Figure 4: Two vectors \vec{a} and \vec{b} , with an angle ϕ between them. Each vector has a component along the direction of the other vector (see equation 9)

The Vector/Cross Product

$$c = ab \sin \phi \quad (11)$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - b_y a_z) \hat{i} + (a_z b_x - b_z a_x) \hat{j} + (a_x b_y - b_x a_y) \hat{k} \quad (12)$$

If \vec{a} and \vec{b} are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $|\vec{a} \times \vec{b}|$, is maximum when \vec{a} and \vec{b} are perpendicular to each other.