Chapter 3: Vectors

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General

Quantities

$$\begin{aligned}
\vec{r} &= \text{resultant vector} \\
|\vec{a}| &= \text{magnitude of vector } \vec{a} \\
\vec{a} \cdot \vec{b} &= \text{dot product of vectors } \vec{a} \text{ and } \vec{b} \\
\vec{a} \times \vec{b} &= \text{cross product of vectors } \vec{a} \text{ and } \vec{b} \\
\theta &= \text{angle between a vector and the } +x \text{ axis} \\
\phi &= \text{smallest angle between two vectors}
\end{aligned}$$

1 Vectors and Their Components

Commutative Law For Adding Vectors

$$\vec{a} + \vec{b} = \vec{b} + \vec{a} \tag{1}$$

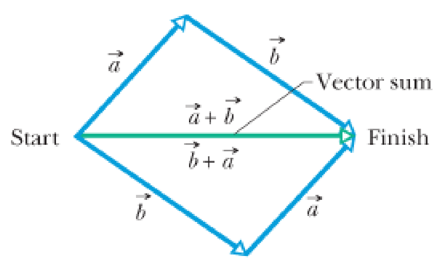

Figure 1: The two vectors $\vec{a}$ and $\vec{b}$ can be added in either order (see equation 1)

Associative Law For Adding Vectors

$$\left(\vec{a} + \vec{b}\right) + \vec{c} = \vec{a} + \left(\vec{b} + \vec{c}\right) \tag{2}$$

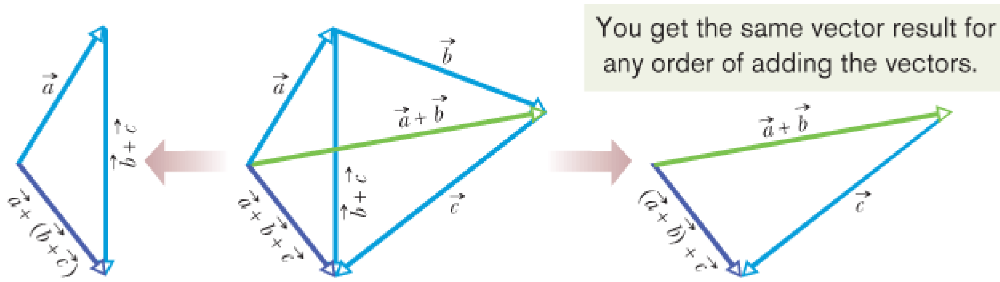

Figure 2: The three vectors $\vec{a}$, $\vec{b}$, and $\vec{c}$ can be grouped in any way as they are added (see equation 2)

Vector Components

$$a_x = a\cos\theta \quad \text{and} \quad a_y = a\sin\theta \tag{3}$$

$$a = \sqrt{a_x^2 + a_y^2} \quad \text{and} \quad \tan\theta = \frac{a_y}{a_x} \tag{4}$$

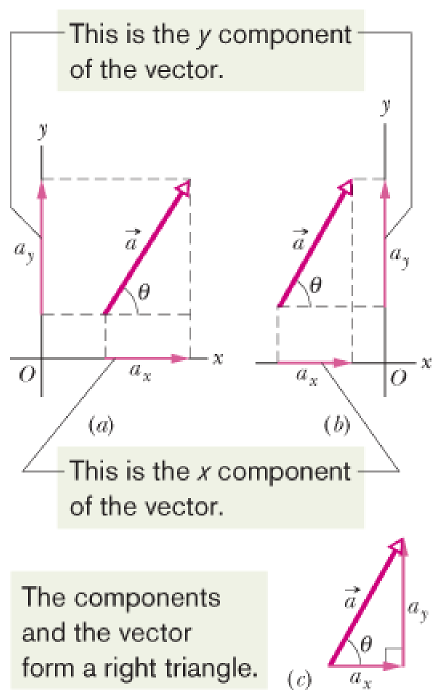

Figure 3: The vector components a_x and a_y are the projections of the vector $\vec{a}$ on the x and y axes (see equation 3)

2 Unit Vectors, Adding Vectors By Components

Unit Vectors

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \tag{5}$$

Adding Vectors by Components

$$\vec{r} = \vec{a} + \vec{b}$$

$$r_x = a_x + b_x \tag{6}$$
$$r_y = a_y + b_y \tag{7}$$
$$r_z = a_z + b_z \tag{8}$$

3 Multiplying Vectors

The Scalar/Dot Product

$$\vec{a} \cdot \vec{b} = ab \cos \phi \tag{9}$$
$$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z \tag{10}$$

If the angle ϕ between two vectors is $0°$, the component of one vector along the other is maximum, and so is the dot product of the vectors. If, instead, ϕ is $90°$, the component of one vector along the other is zero, and so it the dot product.

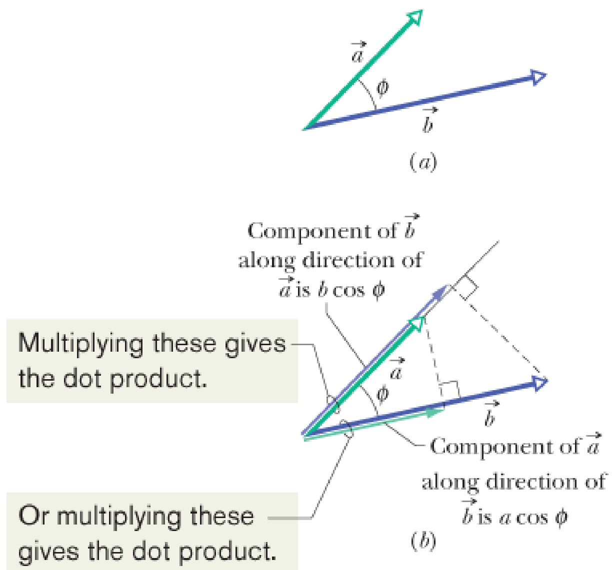

Figure 4: Two vectors $\vec{a}$ and $\vec{b}$, with an angle ϕ between them. Each vector has a component along the direction of the other vector (see equation 9)

The Vector/Cross Product

$$c = ab \sin \phi \tag{11}$$
$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = (a_y b_z - b_y a_z)\,\hat{\imath} + (a_z b_x - b_z a_x)\,\hat{\jmath} + (a_x b_y - b_x a_y)\,\hat{k} \tag{12}$$

If $\vec{a}$ and $\vec{b}$ are parallel or antiparallel, $\vec{a} \times \vec{b} = 0$. The magnitude of $\vec{a} \times \vec{b}$, which can be written as $veca \times \vec{b}|$, is maximum when $\vec{a}$ and $\vec{b}$ are perpendicular to each other.