Chapter 4: Motion In Two and Three Dimensions

Rylan Polster

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General

Quantities

$$\begin{aligned}
x &= \text{horizontal position} \\
\Delta x &= \text{change in horizontal position} \\
y &= \text{vertical position} \\
\Delta y &= \text{change in vertical position} \\
\vec{r} &= \text{position} \\
v_0 &= \text{launch speed} \\
\theta_0 &= \text{launch angle} \\
R &= \text{horizontal range} \\
t &= \text{time} \\
T &= \text{period} \\
\vec{v}_{\text{avg}} &= \text{average velocity} \\
\vec{v} &= \text{instantaneous velocity} \\
v_{\text{A,B}} &= \text{velocity of A relative to B} \\
\vec{a} &= \text{instantaneous acceleration} \\
\vec{a}_{\text{avg}} &= \text{average acceleration} \\
r &= \text{radius} \\
g &= \text{magnitude of free-fall acceleration}
\end{aligned}$$

Constants

$$g = 9.8\,\text{m/s}^2$$

1 Position and Displacement

$$\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k} \tag{1}$$

$$\Delta\vec{r} = \vec{r_1} - \vec{r_2} \tag{2}$$

2 Average Velocity and Instantaneous Velocity

Average Velocity

$$\vec{v}_{\text{avg}} = \frac{\Delta\vec{r}}{\Delta t} \tag{3}$$

Instantaneous Velocity

$$\vec{v} = \frac{d\vec{r}}{dt} \tag{4}$$

$$\vec{v} = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k} \tag{5}$$

$$v_x = \frac{dx}{dt}, \quad v_y = \frac{dy}{dt}, \quad v_z = \frac{dz}{dt} \tag{6}$$

3 Average Velocity and Acceleration Velocity

Average Acceleration

$$\vec{a}_{\text{avg}} = \frac{\Delta\vec{v}}{\Delta t} \tag{7}$$

Instantaneous Acceleration

$$\vec{a} = \frac{\vec{v_2} - \vec{v_1}}{\Delta t} = \frac{d\vec{v}}{dt} \tag{8}$$

$$\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k} \tag{9}$$

$$a_x = \frac{dv_x}{dt}, \quad a_y = \frac{dv_y}{dt}, \quad a_z = \frac{dv_z}{dt} \tag{10}$$

4 Projectile Motion

In projectile motion, the horizontal motion and the vertical motion are independent of ech other, that is, neither motion affects the other.

Horizontal Motion

$$\Delta x = (v_0 \cos\theta_0)\, t \tag{11}$$

Vertical Motion

$$\Delta y = (v_0 \sin\theta_0)\, t - \frac{1}{2}gt^2 \tag{12}$$

$$v_y = v_0 \sin\theta_0 - gt \tag{13}$$

$$v_y^2 = (v_0 \sin\theta_0)^2 - 2g\Delta y \tag{14}$$

Trajectory

$$y = (\tan\theta_0)\, x - \frac{gx^2}{2\,(v_0\cos\theta_0)^2} \tag{15}$$

Horizontal Range

$$R = \frac{v_0^2}{g}\sin 2\theta_0 \tag{16}$$

5 Uniform Circular Motion

Speed does not change. Direction, velocity, and acceleration change over time.

Centripetal Acceleration

$$a = \frac{v^2}{r} \tag{17}$$

Period

$$T = \frac{2\pi r}{v} \tag{18}$$

6 Relative Motion

$$\vec{r}_{\mathrm{P,A}} = \vec{r}_{\mathrm{P,B}} + \vec{r}_{\mathrm{B,A}} \tag{19}$$

$$\vec{v}_{\mathrm{P,A}} = \vec{v}_{\mathrm{P,B}} + \vec{v}_{\mathrm{B,A}} \tag{20}$$

$$\vec{a}_{\mathrm{P,A}} = \vec{a}_{\mathrm{P,B}} \tag{21}$$