Chapter 10: Rotation

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April 29, 2021

General

Quantities

$$
\begin{aligned}
s &= \text{arc length} \\
r &= \text{radius} \\
\theta &= \text{angular position} \\
\omega &= \text{angular velocity} \\
\alpha &= \text{angular acceleration} \\
v &= \text{linear velocity} \\
a_t &= \text{tangential acceleration} \\
a_r &= \text{radial acceleration} \\
T &= \text{period of motion} \\
K &= \text{kinetic energy} \\
I &= \text{rotational inertia} \\
I_{\text{com}} &= \text{rotational inertia about an axis at the center of mass} \\
h &= \text{perpendicular distance between two axes} \\
\tau &= \text{torque} \\
\phi &= \text{angle between the force and the lever arm}
\end{aligned}
$$

1 Rotational Variables

Angular Position

$$\theta = \frac{s}{r} \tag{1}$$

Angular Velocity

$$\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt} \tag{2}$$

Angular Acceleration

$$\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt} \tag{3}$$

2 Rotation With Constant Angular Acceleration

$$\omega = \omega_0 + \alpha t \tag{4}$$

$$\theta - \theta_0 = \omega_0 t + \frac{1}{2}\alpha t^2 \tag{5}$$

$$\omega^2 = \omega_0^2 + 2\alpha\left(\theta - \theta_0\right) \tag{6}$$

$$\theta - \theta_0 = \frac{1}{2}\left(\omega_0 + \omega\right)t \tag{7}$$

$$\theta - \theta_0 = \omega t - \frac{1}{2}\alpha t^2 \tag{8}$$

3 Relating the Linear and Angular Variables

Position

$$s = \theta r \tag{9}$$

Speed

$$v = \omega r \tag{10}$$

Tangential Acceleration

$$a_t = \alpha r \tag{11}$$

Radial Acceleration

$$a_r = \frac{v^2}{r} = \omega^2 r \tag{12}$$

Radial acceleration is directed toward the center

Period of Motion

$$T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} \tag{13}$$

4 Kinetic Energy of Rotation

Rotation Inertia

$$I = \sum m_i r_i^2 \tag{14}$$

Rotational Kinetic Energy

$$K = \frac{1}{2}I\omega^2 \tag{15}$$

5 Calculating the Rotational Inertia

$$I = \int r^2 \, dm \tag{16}$$

Parallel-Axis Theorem

$$I = I_{\text{com}} + Mh^2 \tag{17}$$

6 Torque

$$\tau = (r)\left(F\sin\phi\right) \tag{18}$$

There are two (equivalent) ways to calculate torque:

$$\tau = (r)\left(F\sin\phi\right) = rF_t \tag{19}$$

$$\tau = \left(r\sin\phi\right)(F) = r_\perp F \tag{20}$$