Chapter 9: Center of Mass and Linear Momentum

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General

Quantities

$$\begin{aligned}
x_{\text{com}} &= \text{center of mass} \\
m &= \text{mass} \\
M &= \text{total mass of a system} \\
\rho &= \text{density} \\
V &= \text{volume} \\
\vec{v} &= \text{velocity} \\
\vec{v}_{\text{com}} &= \text{velocity of the center of mass} \\
\vec{p} &= \text{linear momentum} \\
\vec{P} &= \text{linear momentum of a system} \\
\vec{F}_{\text{net}} &= \text{net force}
\end{aligned}$$

1 Center of Mass

Systems of Particles y_{com} and z_{com} can be calculated by replacing x_i with y_i and z_i respectively.

$$x_{\text{com}} = \frac{1}{M}\sum_{i=1}^{n} m_i x_i \tag{1}$$

Solid Bodies y_{com} and z_{com} can be calculated by replacing x with y and z respectively.

$$x_{\text{com}} = \frac{1}{M}\int x\,dm \tag{2}$$

To solve this, use the following relationship:

$$\rho = \frac{dm}{dV} = \frac{M}{V}$$

$$dm = \frac{M}{V}dV$$

Therefore, by substituting the above into (2) for dm:

$$x_{\text{com}} = \frac{1}{V}\int x\,dV \tag{3}$$

2 Linear Momentum

Single Particle

$$\vec{p} = m\vec{v} \tag{4}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{p}}{dt} \tag{5}$$

System of Particles

$$\vec{P} = M\vec{v}_{\text{com}} \tag{6}$$

$$\vec{F}_{\text{net}} = \frac{d\vec{P}}{dt} \tag{7}$$

3 Collision and Impulse

Impulse

$$\vec{J} = \Delta\vec{p} = \int_{t_i}^{t_f} \vec{F}(t)\,dt \tag{8}$$

4 Conservation of Linear Momentum

If no net external force acts on a system of particles, the total linear momentum $\vec{P}$ of the system cannot change.

5 Momentum and Kinetic Energy In Collisions

Elastic Collision The total kinetic energy of the system is *conserved* (it is the same before and after the collision).

Inelastic Collision Some energy is transferred from kinetic energy to other forms of energy, such as thermal energy.

Completely Inelastic Collision The bodies stick together resulting in the greatest amount of kinetic energy being lost.