Chapter 11: Rolling, Torque, and Angular Momentum

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April 28, 2021

General

Quantities

$$
\begin{aligned}
v &= \text{linear velocity} \\
\omega &= \text{angular velocity} \\
R &= \text{radius} \\
K &= \text{kinetic energy} \\
I &= \text{rotational inertia} \\
M &= \text{mass} \\
\tau &= \text{torque} \\
\vec{r} &= \text{lever arm} \\
\vec{F} &= \text{force} \\
\vec{\ell} &= \text{angular momentum} \\
\vec{L} &= \text{angular momentum of a system} \\
\vec{v} &= \text{linear momentum}
\end{aligned}
$$

1 Rolling As Translation and Rotation Combined

Center of Wheel

$$v_{\text{com}} = \omega R \tag{1}$$

Top of Wheel

$$v_{\text{top}} = 2v_{\text{com}} \tag{2}$$

Bottom of Wheel

$$v_{\text{bottom}} = 0 \tag{3}$$

2 Kinetic Energy of A Rolling Object

$$K = \frac{1}{2}I_{\text{com}}\omega^2 + \frac{1}{2}Mv_{\text{com}}^2 \tag{4}$$

3 Torque

$$\vec{\tau} = \vec{r} \times \vec{F} \tag{5}$$

4 Angular Momentum

$$\vec{\ell} = \vec{r} \times \vec{p} = m\left(\vec{r} \times \vec{v}\right) \tag{6}$$

Newton's Second Law in Angular Momentum The (vector) sum of all the torques acting on a particle is equal to the time rate of change of the angular momentum of that particle.

$$\vec{\tau}_{\text{net}} = \frac{d\vec{\ell}}{dt} \tag{7}$$

Angular Momentum of A Rigid Body

$$\vec{\tau} = \frac{d\vec{L}}{dt} \tag{8}$$

$$L = I\omega \tag{9}$$

5 Conservation of Angular Momentum

If the net external torque acting on a system is zero, the angular momentum $\vec{L}$ of the system remains constant, no matter what changes take place within the system.

$$\vec{L}_i = \vec{L}_f \tag{10}$$