2022-CS355FZ-January-Solutions

1

▼ **(a) Give 5 elements of each following languages**

▼ $(1(01)^*)^+$

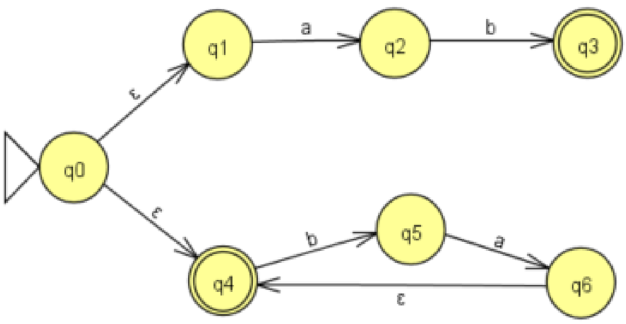
1, 101, 101101, 10101, 1010101

▼ $(^n)^n : n \in Ns$

(), (()), ((())), (((()))), ((((()))))

▼ **(b) Draw the state diagram of the non-deterministic finite automata that recognizes language** $L = \{w | w = ab \cup (\mathrm{ba})^*\}$

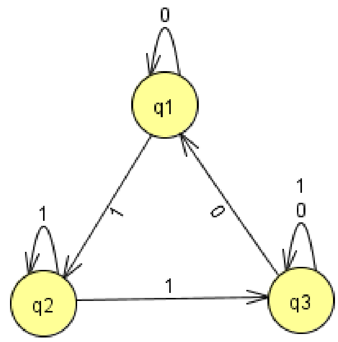
💡 防止整体出错，考试答案分步写。



▼ **(c) For a nondeterministic finite automata** M=(Q,Σ,δ,q,F), Q={q1,q2,q3}, Σ={0,1}, q0=q1, F={q2,q3}, **the transition function** δ **is given by**

δ	0	1
q_1	$\{q_1\}$	$\{q_2\}$
q_2	$\emptyset$	$\{q_2, q_3\}$
q_3	$\{q_1, q_3\}$	$\{ q_3\}$

Draw its state diagram.



2

▼ **(a) Give a context free grammar for the language $\{a^m b^m | m \geq 0\} \cup \{b^n a^n | n \geq 0\}$**

$S \rightarrow S_1 | S_2$

$S_1 \rightarrow 0S_1 1 | \varepsilon$

$S_2 \rightarrow 1S_2 0 | \varepsilon$

▼ **(b) Let Σ={1,2,3,4}, C = {*w*|*w* has equal numbers of 1s and 2s, and equal numbers of 3s and 4s}. Show that C is not context free.**

Suppose C is context free and there is a string $s = 1^p 3^p 2^p 4^p \in C$

$s = uvxyz$

To satisfy $|vy| \geqslant 0$ and $|vxy| \leqslant p$, there are following cases.

① vxy include one type of numbers

② **vxy include two types of numbers**

In any above cases, $uv^i xy^i z$ is not in C for every i.

It is contraditcion and thus, C is not context free

$s = 1^p 2^p 3^p 4^p$ ❌

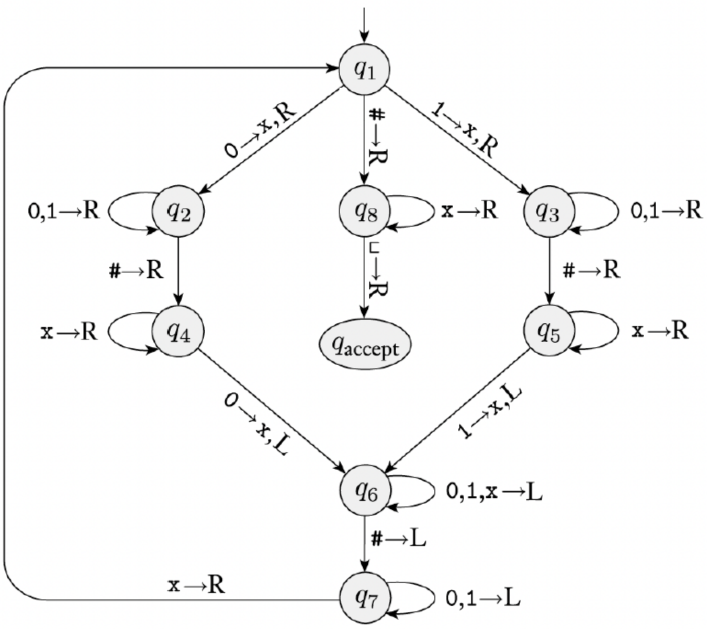
或者取具体的例子：$s = 13241324$ ✔️

反正1和2，3和4不要放在一起

▼ **(c) The state diagram of a Turing machine *M*2 is shown below (on page 2 of this paper). Give the sequence of configuration that Turing machine *M*2 (*Σ* = {0,1,#},*and* *Γ* = {0,1,#,x,␣}.) enters when started on the indicated input string 10#11.**

$$\rightarrow q_1 10\#11$$

$$\rightarrow Xq_3 0\#11$$



$\rightarrow X0q_3\#11$

$\rightarrow X0\#q_5 11$

$\rightarrow X0q_6\#X1$

$\rightarrow Xq_7 0\#X1$

$\rightarrow q_7 X0\#X1$

$\rightarrow Xq_1 0\#X1$

$\rightarrow XXq_2\#X1$

$\rightarrow XX\#q_4 X1$

$\rightarrow XX\#Xq_4 1$

$\rightarrow XX\#X1\sqcup$

q_{reject}

3

▼ (a) Let *D*= {(*i,j*)| *i,j*∈*N*}. Show that *D* is countable

(i,j)				
	(1,1)	(1,2)	(1,3)	…
	(2,1)	(2,2)	(2,3)	…
	(3,1)	(3,2)	(3,3)	…
	…	…	…	…

N	*D*
1	(1,1)
2	(2,1)
3	(2,2)
4	(1,2)
…	…

There is a one-to-one correspondence, so D has the same size as N and D is countable.

▼ Let *E*= {⟨*M*3⟩| *M*3 is a Turing machine that accepts w^R whenever it accepts *w*}. Show that *E* is undecidable.

▼ 法一

Assume that E is decidable by using TM M that decides A_{TM} .

Constrcut another TM M' as follows

M' = "On input x:

1. *If x ≠ 01 and x ≠ 10, reject.*
2. *If x = 01, accept.*
3. *If x = 10 simulate M on w.*
   a. *If M accepts w, accept. If M rejects, reject."*

Only when M is decidable, M' is decidable.

When M' is decidable, $L(M') = \{01, 10\} \in L(E)$, and E is decidable.

Therefore, A_{TM} `≤m E.`

In fact, A_{TM} is undecidable, so E is undecidable.

- 法二

Assume that E is decidable and there must exist a TM T that can decide E

Constrcut another TM M that decides A_{TM} as follows

M = "On input $\langle M_3, w\rangle$:

1. *Run T on input $\langle M_3, w\rangle$*
2. *If T accept, then run T on $\langle M_3, w^R\rangle$*
3. *If accpets, then accpet. Otherwise, reject."*

Clearly, if T decides E, then M decides A_{TM}.
Because A_{TM} is undecidable, T also must be undecidable.

A_{TM} `={<M,w>| M 是图灵机且接受串 w }`

M是否能停机的问题**取决于**M_3是否能停机，M 本身就是判定A_{TM}的（所以才用格式 *on input* $\langle M_3, w\rangle$ ）

因为已知A_{TM}不可判定，则M_3不可判定

另一个角度：$A_{TM} \leqslant_m E$

▼ **Describe P, NP, PSPACE, NPSPACE and EXPTIME, and their conjectured relationships**

P: Problems that can be solved in polynomial time

NP: Problems that can be verified in polynomial time

PSPACE: Problems that can be solved by a Turing machine using a polynomial space.

N*PSPACE: Problems that can be verified by a Turing machine using a polynomial space.*

EXPTIME: Problems that can be solvable by a deterministic Turing machine in exponential time

$\mathrm{P} \subseteq \mathrm{NP} \subseteq \mathrm{PSPACE} = \mathrm{NPSPACE} \subseteq \mathrm{EXPTIME}$