2.152 Advanced Control System Design
Spring 2020

Problem Set #2

Issued : 02/25/2020
Due : 03/10/2020

Problem 1:

Is the product of two

(a) *symmetric* matrices *symmetric*?

(b) *positive definite* matrices *positive definite*?

(c) *symmetric positive definite* matrices *positive definite*?

(d) *symmetric positive definite* matrices *symmetric positive definite*?

In each case show why or why not.

Problem 2:

Consider the following 2^{nd} order electrical system with nonlinear inductance, resistance, and capacitance

$$L(\dot{q})\ddot{q} + R(\dot{q}) + C(q) = 0$$

where q is the charge in the capacitance, $L(\dot{q})$ is strictly positive, and both $R(\dot{q})$ and $C(q)$ are of the same sign as the arguments. Find the equilibrium points and determine stability. Also indicate whether the stability is asymptotic, and under what conditions it is global.

Problem 3:

Exercise 3.9

continued on the next page...

Problem 4:

Exercise 3.10

Problem 5:

Exercise 3.11

Problem 6:

Exercise 4.8

Problem 7:

Exercise 4.9

Problem 8:

Consider a scalar, lower bounded, and twice differentiable function $V(t)$ for which

$$\forall\ t \geq 0 \quad \dot{V}(t) \leq 0$$

Show that

$$\dot{V}(t) = 0 \quad \Rightarrow \quad \ddot{V}(t) = 0$$

Problem 9:

The system

$$m\ddot{x} + (\alpha_1 + \alpha_2 \cos^2 x)|\dot{x}|\dot{x} = u + d$$

is a model of an underwater vehicle motion along one degree of freedom. In the viscous drag term, α_1 and α_2 are time-varying quantities such that $5 \leq \alpha_1 \leq 9$, $2 \leq \alpha_2 \leq 4$. The term d is a time-varying disturbance bounded by 1.

First, assume the mass is known, $m = 1$.

(a) Design a switching controller for this system.

(b) Assume that there is an unmodelled pendulum mode at 3 Hz. Design a sliding controller for this system.

(c) Implement your design in simple simulation.

The apparent mass m actually depends on the "added mass" created by the motion, and is time-varying. Assume that, in the range of operation, $1 \leq m \leq 3$.

(d) Design a sliding controller for this system.