2.152 Advanced Control System Design
Spring 2020

Problem Set #3

Issued : 03/10/2020
Due : 03/17/2020

Problem 1:

Consider the n-dimensional system

$$\dot{\mathbf{x}} = f(\mathbf{x})$$

Show that if $\mathbf{x} = \mathbf{0}$ is an equilibrium point, then one can write

$$\dot{\mathbf{x}} = A(\mathbf{x})\ \mathbf{x}$$

where $A(\mathbf{x})$ is an $n \times n$ matrix. (Adapted from [Vidyasagar, 1992].)

Problem 2:

Consider the linear system

$$\ddot{x} + \dot{x} + x = u + d$$

where d is an unknown constant disturbance. Design an adaptive tracking controller for this system. Write your control law explicitly in standard PID form (perhaps with some extra term).

Problem 3:

Consider the second-order system

$$a_1 \ddot{x} + a_2 \dot{x}^2 \sin x + a_3 \cos 2x = u$$

where $a_1 > 0$, a_2, and a_3 are unknown constants. Design an adaptive tracking controller for this system.

Also design an adaptive tracking controller for the system

$$a_1 \ddot{x} + a_2 \dot{x}^2 \sin x + a_3 \cos 2x + d(t) = u$$

where $d(t)$ is an unknown bounded disturbance, $|d(t)| \leq 1.5$.

Problem 4:

The dynamic equations of a robot manipulator can be written as

$$H(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + g(\mathbf{q}) = \tau$$

where the matrix $(\dot{H} - 2C)$ is skew-symmetric. Consider an adaptive tracking controller for such a robot with independent feedback gains for position and velocity errors. This corresponds the control law

$$\tau = \hat{H}(\mathbf{q})\ddot{\mathbf{q}}_{\mathbf{r}} + \hat{C}(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_{\mathbf{r}} + \hat{g}(\mathbf{q}) - K_d\dot{\tilde{\mathbf{q}}} - K_p\tilde{\mathbf{q}}$$

and the adaptation law

$$\dot{\hat{\mathbf{a}}} = -\Gamma Y^T \mathbf{s}$$

where

$$Y\mathbf{a} = H(\mathbf{q})\ddot{\mathbf{q}}_{\mathbf{r}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}}_{\mathbf{r}} + g(\mathbf{q})$$

and

$$\mathbf{s} = \dot{\mathbf{q}} - \dot{\mathbf{q}}_{\mathbf{r}} = \dot{\tilde{\mathbf{q}}} + \lambda\tilde{\mathbf{q}} \qquad \tilde{\mathbf{q}} = \mathbf{q} - \mathbf{q}_{\mathbf{d}}$$

The adaptation and controller gain matrices Γ, K_d and K_p are all symmetric positive definite.

Show that this controller will force the tracking error $\tilde{\mathbf{q}}$ to converge to zero. You may want to use the Lyapunov function candidate

$$V = \tfrac{1}{2}\mathbf{s}^T H \mathbf{s} + \tfrac{1}{2}\tilde{\mathbf{a}}^T \Gamma^{-1} \tilde{\mathbf{a}} + \tfrac{1}{2}\tilde{\mathbf{q}}^T (K_p + \lambda K_d)\tilde{\mathbf{q}} \ .$$