2.152 Nonlinear Control System Design
Spring 2020

Midterm

Problem 1: (20 points)

(a) (5 points)
Consider the system

$$\dot{x} = f(x),$$

where $x \in \mathbb{R}^n$ and $f(0) = 0$. Let $A = \frac{\partial f}{\partial x}|_{x=0}$ and suppose that A is Hurwitz. With this information, how would you build a quadratic Lypunov function (i.e., $V(x) = x^T P x$) to certify the stability of $x = 0$?

(b) (5 points) Let

$$\begin{aligned} \dot{x}_1(t) &= -x_1(t) + e^{at} x_2(t) \\ \dot{x}_2(t) &= b x_2(t), \end{aligned}$$

for what values of a, b is the system unstable, marginally stable, and asymptotically stable?

(c) (5 points)
Let $s(t) = (\frac{d}{dt} + \lambda)^{n-1} \tilde{x}(t)$. Suppose that $|s(t)| \leq \phi, \forall t \geq 0$ where ϕ is a positive constant. What upper bound can we guarantee on $\tilde{x}$ (neglecting the initial conditions of $\tilde{x}$)?

(d) (5 points) For a robotic arm equipped with an adaptive controller, is tracking a fixed set point a good method to learn the system parameters? Explain in words.

Problem 2: (35 points)

For the following systems, find the equilibrium points and determine their stability. If stable, indicate whether stability is asymptotic or exponential, and whether it is global. Briefly explain the reason and state properties of your Lyapunov-like functions clearly.

(a) (10 points)

$$\dot{x} + 4x^9 = 2x^3 \sin^6(x)$$

(b) (10 points)

$$(\dot{x}^4 + 1)\ddot{x} + \dot{x}^5(\cos(3\dot{x})^4 + 1) = -3x(\sin^4 x + 1)e^{-3x}$$

(c) (15 points)
Consider the system governed by the following differential equations

$$\begin{aligned} \dot{x}_1 &= (2\beta + 3)x_1 - 2.5x_2 \\ \dot{x}_2 &= 2.5x_1 + (-e^{\alpha} - \sin(t))x_2 \end{aligned}$$

Find a range of α and β which guarantees exponential convergence of $\mathbf{x} = [x_1, x_2]^T$ to the origin with a convergence rate ≥ 2
(*Hint: You may use a Lyapunov-like function which converges exponentially.*)

Problem 3: (20 points)

Consider the third-order system

$$\dddot{x} + \ddot{x} + (1 + b^3)\dot{x} + 2\cos(c)\cos(x)x^2 = u$$

where b and c are unknown and possibly time-varying parameters, and

$$|b| \leq 2$$

Design a switching controller to track a known time varying trajectory $x_d(t)$. Do not smooth the control signal.

Problem 4: (25 points)

(a) (15 points)
Consider the system

$$m\ddot{x} + b\cos(x)e^{-t}\dot{x}^9 + k(x - l) = u + d$$

where $m > 0$, b, k, l and d are unknown constants.

Design an adaptive tracking controller for this system. Explicitly show that the tracking error $\tilde{x}$ will converge to zero (state any extra conditions you may require).

(b) (10 points)
Now suppose the constant d is replaced by $d\sin(2t+\phi)$. The new system is given by

$$m\ddot{x} + b\cos(x)e^{-t}\dot{x}^9 + k(x-l) = u + d\sin(2t+\phi)$$

where $m > 0$, b, k, l, d, and ϕ are unknown constants.

Design an adaptive tracking controller for this new system (*hint: Recall the identity* $\sin(x+y) = \sin(x)\ \cos(y) + \cos(x)\ \sin(y)$).