

2.152 Nonlinear Control

Spring 2020

Problem Set #1

Issued : Thursday, 02/06/2020

Due : Thursday, 02/20/2020

Problem 1:

Exercise 3.1

Note: $\sup(A)$ is the smallest real number x such that $x \geq y$ for any y in the set A .

Problem 2:

Exercise 3.2

Hint: You may find these relations helpful: $|x| \geq |\sin(x)|$ and $|\sin^3(x) \cos^2(3x)| < 1$ for $x \neq 0$.

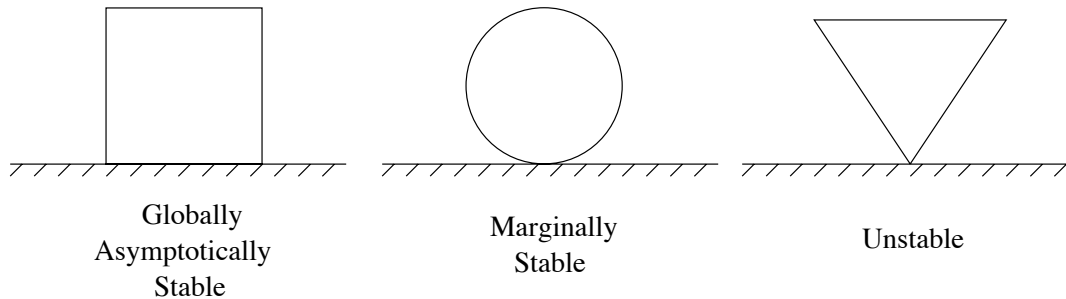
Problem 3:

Exercise 3.13

Problem 4:

Find a Lyapunov function to certify the stability of the system

$$\dot{\mathbf{x}} = \begin{bmatrix} -1 & 0 \\ 2 & -3 \end{bmatrix} \mathbf{x}$$

Problem 5:

Which label(s) are incorrect in the above drawings for the positions shown? Consider only *rotational* dynamics. State clearly any assumptions you make.

Problem 6:

Consider, for a given autonomous nonlinear system, a Lyapunov function V in a ball B_R , and let ϕ be a scalar, differentiable, strictly monotonically increasing function of its scalar argument. Show that $\phi(V) - \phi(0)$ is also a Lyapunov function for the system. Discuss stability and asymptotic stability as it is related to ϕ .

Problem 7:

Why does the definition of exponential stability imply stability?