2.152 Nonlinear Control System Design
Spring 2020

Problem Set #4

Issued : 04/07/2020
Due : 04/14/2020

Problem 1:

Consider a vehicle moving in the horizontal plane whose motion is described by

$$\begin{aligned}\dot{x} &= v\cos(\theta)\\ \dot{y} &= v\sin(\theta)\\ \dot{\theta} &= \omega\end{aligned}$$

where (x, y) gives the vehicle's position and θ is its heading angle off the x-axis. The control inputs are velocity v and turn rate ω. The outputs of interest are x and y.

(a) For a MIMO system with m outputs, the vector relative degree is given by $(r_1, ...r_m)$, where each r_i corresponds to the number of times output y_i must be differentiated before at least one of the inputs appears. The *total relative degree* is then just the sum of these r_i.

Find the total relative degree of this system.

(b) Write this system in the form given by equation (6.95) in the textbook. Is the decoupling matrix $\mathbf{E}(\mathbf{x})$ invertible?

(c) Now considered an extended version of this system model

$$\begin{aligned}\dot{x} &= v\cos(\theta)\\ \dot{y} &= v\sin(\theta)\\ \dot{\theta} &= \omega\\ \dot{v} &= a\end{aligned}$$

where the inputs are now ω and a and the outputs remain the same. This process of replacing a control input with its time derivative is known as *dynamic extension.*

Show that the total relative degree of the system with dynamic extension is 4.

Is the decoupling matrix $\mathbf{E}(\mathbf{x})$ invertible for this system?

(d) Use this $\mathbf{E}(\mathbf{x})$ to find control inputs ω and a that will track the desired path given by x_d and y_d. State any assumptions needed for the feedback linearization.

Problem 2:

Consider again the problem of adaptively controlling

$$J\ddot{x} + b\dot{x}|\dot{x}| + mgl\sin(x) = u$$

where b and mgl are known to be *positive* quantities. Show that temporarily stopping adaptation if it drives the parameter estimates below their know bounds (i.e., maintaining all parameter estimates positive or zero) can only be beneficial, in the sense it adds a negative term to $\dot{V}$.

Problem 3:

Consider the Examples 9.1 through 9.3 in the text. Simulate the P.D. and basic adaptive controller starting with zero initial conditions and no initial knowledge of the parameters.

(a) First simulate the P.D. controller responding to a step input to the desired final positions

$$q_{d1} = 1 \qquad\qquad q_{d2} = 2$$

(b) Next simulate the basic adaptive controller. Try desired trajectories of

$$q_{d1} = 1 - e^{-t} \qquad\qquad q_{d2} = 2(1 - e^{-t})$$

and

$$q_{d1} = 1 - \cos 2\pi t \qquad\qquad q_{d2} = 2(1 - \cos 2\pi t)$$

Also vary Γ between

$$\begin{aligned} \Gamma_1 &= \mathrm{diag}(0.03, 0.05, 0.1, 0.3) \\ \Gamma_2 &= 200\ \Gamma_1 \\ \Gamma_3 &= 0.1\ \Gamma_1 \end{aligned}$$

How do the different cases effect tracking and parameter convergence? Explain.