Lecture 2 - Cryptographic Hash Functions

Rylan Schaeffer and Vincent Yang

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Note: This lecture is based on Princeton University's BTC-Tech: Bitcoin and Cryptocurrency Technologies Spring 2015 course.

Terms

- Set : group of objects represented as a unit. enumerate, or give way to determine membership in set

  Empty set = set with 0 members

  Cartesian Product or Cross Product (x): tuples of all possible pairs
- Alphabet : finite, non-empty set
- String : finite sequence of characters from common alphabet

  empty string ε (string of length zero)

  concatenation by joining or |, can use exponent for repeat concatenation
- Language : set of strings over common alphabet
- Kleene star (Kleene operator, Kleene closure) : either on sets of strings or sets of characters. Means "zero or more". More formally:

  $V_0 = \{\varepsilon\}$

  $V_1 = V$

  $V_{i+1} = \{wv : w \in V_i and v \in V\}$ for i > 0

  $V^* = \bigcup_{i \in \mathbb{N}} V_i$ =======
- Set : group of objects represented as a unit
- Alphabet : finite, non-empty set
- String : finite sequence of characters from common alphabet, including empty string ε
- Language : set of strings over common alphabet

Hash Functions

- $H : \{0,1\}^* \to \{0,1\}^k$, for fixed k e.g. 256
- Should be efficiently computable O(n)
- Examples: mod operator, separate chaining hash (draw)
- Homework 1 - *SHA-256*

Cryptographic Hash Function

Two additional properties

- Collision Resistant: Computationally infeasible to find x, y such that $x \neq y$ and $H(x) = H(y)$

  mod operator is not collision resistant

  collisions exist by pigeonhole principle - hence, computationally infeasible

  Avalanche Effect - when one letter changes, everything changes

  can also call "binding," since once hash is published, you cannot replace input value with another input value without modifying the hash output

- Hiding: Computationally infeasible to find x given H_{given} such that $H(x) = H_{given}$

  Frequently, cryptographic hash functions will be called one-way hash functions (trap door functions)

  Frequently, message space is too small. Append nonce (i.e. random value) r to grow message space such that computationally infeasible to find x such that $H(x|r) = H_{given}$

- Birthday paradox reduces difficulty of finding collisions
  - How many people do you need in a room for there to be a 50% chance that two have the same birthday? (A: 23)
  - Exponents aren't intuitive
  - With 23 people we have 253 pairs - $\frac{23 \cdot 22}{2} = 253$.
  - Chance of 2 people having diff. birthdays is $1 - \frac{1}{365} = \frac{364}{365} = .997260$.
  - If all pairs are different, this is like having 253 heads in a row. $Probability = (\frac{364}{365})^{253} = .4995$.
  - SHA-256: Let's say we have a "perfect" hash function with output size n, with p messages to hash. Probability of collision is $\frac{p^2}{2^{n+1}}$.
  - This means, if we have $n = 256$ as in SHA-256, and 1 billion messages ($p = 10^9$) then the probability is still only $4.3 \cdot 10^{-60}$.
  - A mass murdering space rock happens about once every 30 million years. The probability of it happening in the next second is about 10^{-15}. This is 45 times more likely than a SHA-256 collision.

Applications

- Message Digest

  Create summary (or "digest") of block of text

  Suppose I have msg and H is a cryptographic hash function. Then I know that $H(msg)$ or perhaps $H(msg|r)$ (where r is a random value and is needed because the message is predictable), will produce a hash value that no other block of text will.

  Example: cryptographic checksums

- Commitments

  Analogous to sealed envelope on the table

  Hiding ensures no one can "reverse engineer" the contents. Collision-resistant guarantees to the other party that you are bound to the value you initially put in.

Puzzle Friendliness

- Search Puzzle

  Given H, target set Y, and value x

  Goal: find r such that $H(x|r) \in Y$

- Puzzle friendly if no solving strategy for puzzle other than trying random guesses at r

- Examples: $0|\{0,1\}^{k-1}$, $00|\{0,1\}^{k-1}$, $000|\{0,1\}^{k-1}$

  P(l leading zeroes) $= \frac{1}{2^l}$, can use geometric distribution's cumulative distribution function to model likelihood of observing a "hit" after a given number of failures

- Useful for mining, which we will get to later

Hash Structures

- Hash pointer : hash of data. Gives way to verify information hasn't changed, much like pointer gives a way to retrieve location of information

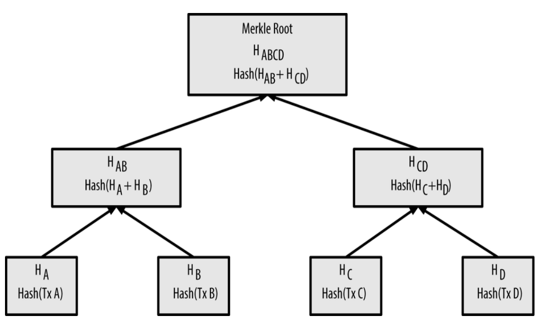
- Hash linked list (block chain) : Each block has hash of previous block plus new data. Head is hash of most recent block.

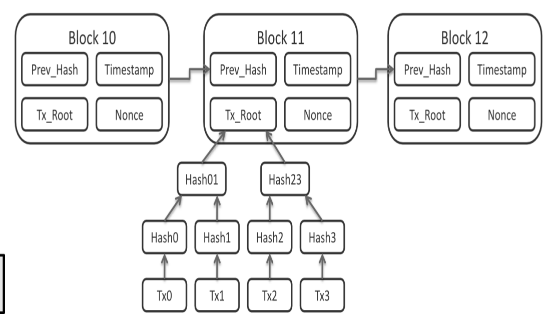
  Tamper-evident log

- Hash tree (Merkle Tree) : binary tree of data blocks. Proof of membership and proof of non-membership in $log(n)$, so faster than hash linked list. Can also sort.

  Verification in $O(log(n))$.

- Can combine. Block chain is usually hash linked list of hash trees (draw picture on board)





1 Actual Cryptographic Hash Functions

- Turns out, creating ideal cryptographic hash function is difficult

- Ron Rivest invented MD4 and MD5 in 1990 and 1991, collisions found

- SHA-1 designed by NSA in 1993, replaced in 1995. 2005 cryptanalysts found collisions, by 2010 organizations refuse to accept e.g. Google, Microsoft

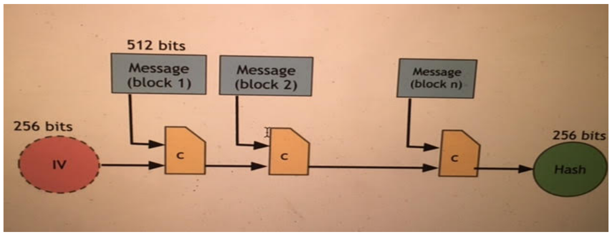
- SHA-2 designed by NSA. 2001. Family of functions with different possible digest lengths including 224, 256, 384, 512.

  SHA-256 uses Merkle-Damgard transform to turn fixed-length collision-resistant compression function into hash function that accepts arbitrary length inputs (same Merkle as Merkle trees!)

- SHA-3, released 2015.

SHA-256

- Merkle-Damgard Transform: change arbitrary length to fixed length - compression function
- Takes input length m and output length n.
  1. Split input into blocks of length $m - n$.
  2. Pass each block in with output of previous block
  3. Use IV (initialization vector) for first block
  4. Return last block's output
  5. Padding:
     (a) Append 1 to the end
     (b) Append k bits of 0 such that k is the smallest integer that fulfills $(l+1+k)\ mod\ 512 = 448$; l is length of message.
     (c) Add length $l < 2^{64}$ represented with 64 bits, added to the end.
     (d) Message is *always* padded.
- Feistel Network/Cipher (2 rounds; SHA is 8)
  1. F is the round function, and K0, K1, ... Kn are the keys for rounds 0, 1, ... n.
  2. Split plaintext to 2 blocks L0 and R0
  3. For reach round i = 0, 1, ..., n, compute:
     $$L_{i+1} = R_i$$
     $$R_{i+1} = R_{i+1} \oplus F(R_i, K_i).$$
  4. Ciphertext is (R_{n+1}, L_{n+1})
- After padding from above, split into smaller parts, XOR with each other, left bitshift.



Other Sources: (More SHA) `http://www.quadibloc.com/crypto/mi060501.htm`