MATH 110, SPRING 2020 **Syllabus**

Vector Space Methods for Differential Equations

Instructor: Christian Brennecke (brennecke@math.harvard.edu), Science Ctr. 239
Office Hours: To be arranged

Class: Tuesday & Thursday 9:00AM – 10:15AM in Science Ctr. 507, weekly problem section to be arranged. In the weekly problem session you will work on challenging problems from homework and you will have time to ask questions on the course material.

Prerequisites: Mathematics 23a,b or 25a,b or Mathematics 19a,b or 21a,b plus any Mathematics course at the 100 level; or an equivalent background in Mathematics.

Course Homepage: https://canvas.harvard.edu/courses/67139 On the course homepage, you'll find announcements, homework assignments, information on the quizzes, project, and so on. Please check the course website regularly.

Course Objectives and Content:

Develops the theory of inner product spaces, both finite-dimensional and infinite-dimensional, and applies it to a variety of ordinary and partial differential equations. Topics: existence and uniqueness theorems, Sturm-Liouville systems, orthogonal polynomials, Fourier series, Fourier and Laplace transforms, eigenvalue problems, and solutions of Laplace's equation and the wave equation in the various coordinate systems.

Textbooks:
Required:

- Samuel Holland, *Applied Analysis by the Hilbert Space Method*

Recommended:

- John D'Angelo, *Linear and Complex Analysis*
  This book is available online from the Author's website.
- John D'Angelo, *Hermitian Analysis*
  This book is available online from the Author's website.
- Michael Stone and Paul Goldbart, *Mathematics for Physics*
  This book is available online from the Author's website.

Problem Sets: The problem sets are an important part of this course, so be sure to start the problems early and think carefully about your solutions. You are expected to write clearly and coherently using complete sentences. A good rule of thumb for the level of detail is that another student who has taken Math 110 should be able to easily understand your solution.

There will be weekly problem sets due every Tuesday. You should submit your solutions for each problem set to the mailbox of the course assistant (to be announced soon), preferably in PDF format. Most mathematical and scientific articles are typeset using LaTeX, and we encourage you to learn to typeset in LaTeX along the way. The first problem set contains some useful links to make yourself familiar with LaTeX.

It is recommended to work together on problem sets, but you must turn in your solutions written in your own words and citing any sources you use. You should be able to clearly explain any work you turn in. If your solution is too similar to your sources, then you will be asked to explain your solution to Christian or the course assistant in person. Please be sure to also acknowledge any assistance you get from other students.

Quizzes: There will be three quizzes, each about one of the roughly three main parts of the course. The precise dates will be announced soon on the course website.

Grading: Your class participation (A), problem sets (H) and quizzes (Q) all contribute to your final grade according to:

$$5\%\text{A} + 35\%\ \text{H} + 60\%\ \text{Q}$$

Schedule: The term has 14 weeks and a rough schedule for the class is as follows.

W1 Introduction & motivation, first order ODEs, variation of parameters

W2 Analytic solutions, review linear algebra, Wronskian, dimension of kernel

W3 Abel's formula, constant coefficient ODEs, Laplace transform

W4 Inverse Laplace transform, variation of parameters, Green's function, inner product spaces

W5 Fundamental Inequalities, Metric Spaces, Hilbert Spaces

W6 The space L^2, operators on L^2, compatibility conditions

W7 Eigenvalues & eigenfunctions

W8 **Spring Recess**

W9 Self-adjoint operators, some operator theory, boundary conditions

W10 Excursion to Fourier analysis, Sturm-Liouville theory, Laplacian in 1d

W11 Wave equation, Legendre operator and Legendre polynomials

W12 Hermite operator and Hermite polynomials, wave equation in other coordinates

W13 Bessel functions, Schrödinger equation

W14 Further optional topics from Fourier Analysis & PDE