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6.438 Problem Set 1

(1)

1.1

a) Suppose $x \perp\!\!\!\perp y \mid z$. By defn of C.I.

$$p(x,y|z) = p(x|z)\,p(y|z)$$

$$\Rightarrow p(x,y,z) = \underbrace{p(x|z)}_{h(x,z)}\,\underbrace{p(y|z)p(z)}_{g(y,z)}$$

Suppose $p(x,y,z) = h(x,z)g(y,z)$. Define $h(x,z) = p(x|z)$ and $g(y,z) = p(y|z)p(z)$. By chain rule

$$p(x,y,z) = p(x|y,z)\,p(y|z)\,p(z)$$

$$= p(x|z)\,p(y|z)\,p(z)$$

$$\Rightarrow x \perp\!\!\!\perp y \mid z$$

b) We want pairwise but not mutual independence for 3 events A, B, C

$$P(A,B,C) = P(A)P(B|A)P(C|A,B) \quad \text{by chain rule}$$

$$= P(A)P(B)P(C|A,B) \quad \text{by pairwise indep.}$$

$$\neq P(A)P(B)P(C)$$

$$\Rightarrow P(C|A,B) \neq P(C)$$

Let $A, B \sim \text{Bern}(p=0.5)$. Define $C = A \text{ XOR } B$. Then

A	B	C
0	0	1
0	1	0
1	0	0
1	1	1

$\Rightarrow$

$$\left.\begin{array}{ll} P(C=0) = \frac{1}{2} & \text{and } P(C=1) = \frac{1}{2} \\ P(C=0|A) = \frac{1}{2} & \text{and } P(C=1|A) = \frac{1}{2} \\ P(C=0|B) = \frac{1}{2} & \text{and } P(C=1|B) = \frac{1}{2} \end{array}\right\} \Rightarrow \text{C is pairwise independent}$$

Undirected Graph: