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6.438 Problem Set 1

1.3.

a) i. True. Any path connecting B-G must pass H. H is not observed & has V-structure i.e. $G \to H \leftarrow F$, blocking the path

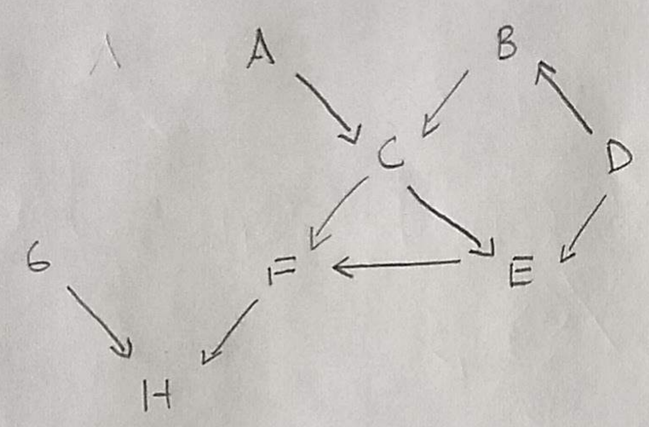
b) ii. False. Path $C \leftarrow B \to D$ and B is unobserved $\Rightarrow$ path unblocked $\Rightarrow$ not CI

iii False. Path $C \leftarrow B \to D$ and B is unobserved $\Rightarrow$ path unblocked $\Rightarrow$ not CI

iv True Path $B \to D \to E \leftarrow C \to F \to H$ has a $\to E \leftarrow$ but E is unobserved $\Rightarrow$ path is blocked

Paths $B \to C \to F \to H$, $B \to C \to E \to F \to H$, $B \to D \to E \to F \to H$ are all blocked by conditioning on C, F

b)



The only graph w/ same V-structures & skeleton as Fig 1.3.1 is w/ the B-D edge swapped

c) i. Necessary: Let G_1, G_2 be I-equivalent. For contradiction, add a directed edge to G_1. from $X_i \to X_j$. Then $X_{\Pi_j}^{G_1} \neq X_{\Pi_j}^{G_2}$ (i.e. node j has different parents). $\Rightarrow$

$$P(X_j^{G_1} \mid X_{\Pi_j}^{G_1}) \neq P(X_j^{G_2} \mid X_{\Pi_j}^{G_2})$$

$\Rightarrow G_1, G_2$ are not I-equivalent.

remove edge $\Rightarrow$ diff d-separation

Insufficient: $A \to B \leftarrow C$ and $A \to B \to C$ have same skeleton but different independencies i.e. $A \perp C \mid B$ on the right, but $A \not\perp C \mid B$ on the left