

1.4

b) First, I claim no cycles \Rightarrow original graph is chordal & not \tilde{K}_4

Proof by Contrapositive:

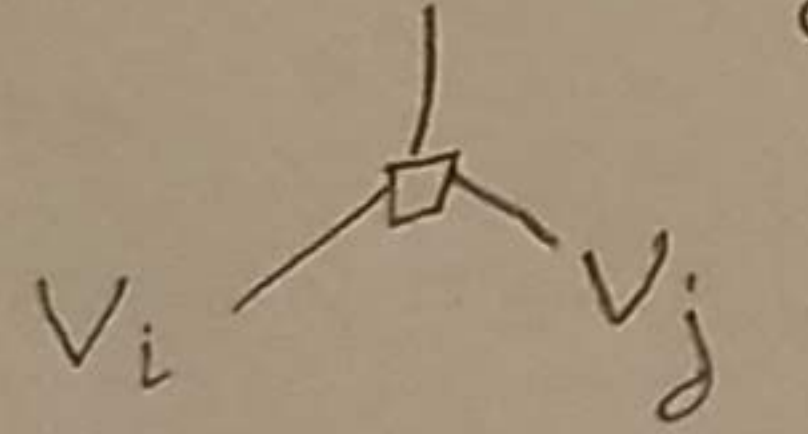
CP Claim: Not chordal or has $\tilde{K}_4 \Rightarrow$ cycle exists

Proof: Consider not chordal

$\Rightarrow \exists$ cycle w/ $\text{len} \geq 4$ s.t. no edge joins any 2 nonadj nodes. Say cycle is $x_1 - x_2 - \dots - x_N - x_1$ $N \geq 4$

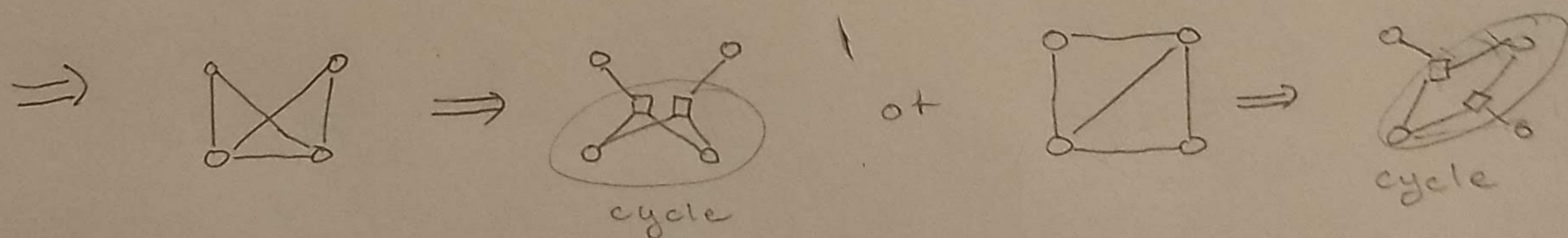
① Suppose $\forall i, j \in \{1, \dots, N\}$ have no other connections.

Then factor graph $V_i \text{---} V_j$ & cycle preserved \Rightarrow cycle exists

② Suppose $V_i - V_j$ belong to shared clique. Then
 & cycle preserved \Rightarrow cycle exists

③ Suppose $V_i - V_j$ belong to non-shared cliques. Then again
the factor graph $V_i \text{---} V_j$ & cycle preserved \Rightarrow cycle exists

Consider \tilde{K}_4



Algorithm: Check for chordality: $O(V)$

Check for \tilde{K}_4 : $O(V^4)$

If graph chordal & not \tilde{K}_4 , return True. Else return False

Total time $O(V^4 + V) = O(V^4) \Leftarrow$ polynomial time