1.8

b) $L(\mu, \Lambda) = \prod_{i=1}^{n} p(x_i | \mu, \Lambda)$

$$\log L(\mu, \Lambda) = \sum_{i=1}^{n} \log\left(\frac{1}{\sqrt{2\pi|\Lambda|}} \exp\left(-\frac{1}{2}(x_i-\mu)^T \Lambda^{-1}(x_i-\mu)\right)\right)$$

$$= -\frac{n}{2} \log 2\pi|\Lambda| - \frac{1}{2} \sum_{i=1}^{n} (x_i-\mu)^T \Lambda^{-1} (x_i-\mu)$$

$$\nabla_\mu \log L(\mu, \Lambda) = 0 = -\sum_{i=1}^{n} \Lambda^{-1}(x_i-\mu)$$

$$n\mu = \sum_{i=1}^{n} x_i$$

$$\boxed{\hat{\mu} = \frac{1}{n} \sum_{i=1}^{n} x_i}$$

$$\nabla_{\Lambda^{-1}} \log L(\mu, \Lambda) = 0 = \frac{n}{2} \nabla_{\Lambda^{-1}} \log |\Lambda^{-1}| - \frac{1}{2} \sum_{i=1}^{n} \nabla_{\Lambda^{-1}} \mathrm{Tr}\left[(x_i-\mu)(x_i-\mu)^T \Lambda^{-1}\right]$$

$$n\Lambda = \sum_{i=1}^{n} (x_i-\mu)(x_i-\mu)^T$$

$$\boxed{\hat{\Lambda} = \frac{1}{n} \sum_{i=1}^{n} (x_i-\mu)(x_i-\mu)^T}$$