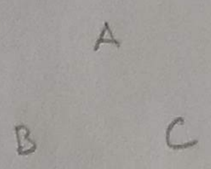
1.1

(2)

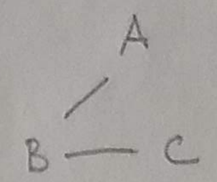
b cont) Removing all edges $\Rightarrow$ all independent

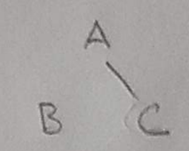
WRONG



Removing 1 or 2 edges breaks symmetry

WRONG





1.2

a) Symmetry: $P(x_A, x_B | x_C) = P(x_A | x_C) P(x_B | x_C)$

$= P(x_B | x_C) P(x_A | x_C)$

$= P(x_B, x_A | x_C) \quad \Rightarrow \quad x_B \perp\!\!\!\perp x_A \mid x_C$

b) Decomposition: $P(x_A, x_B, x_D | x_C) = P(x_A | x_C) P(x_B, x_D | x_C)$

$\sum_{x_D}$ $= \sum_{x_D}$

$P(x_A, x_B | x_C) = P(x_A | x_C) \sum_{x_D} P(x_B, x_D | x_C)$

$= P(x_A | x_C) P(x_B | x_C) \Rightarrow x_A \perp\!\!\!\perp x_B \mid x_C$

Redo instead marginalizing over $x_B \Rightarrow x_A \perp\!\!\!\perp x_D \mid x_C$

c) Weak Union: $P(x_A, x_B | x_C, x_D) = P(x_A | x_B, x_C, x_D) P(x_B | x_C, x_D)$ (Chain Rule)

$= P(x_A | x_C, x_D) P(x_B | x_C, x_D)$ (CI from (b))

$\Rightarrow x_A \perp\!\!\!\perp x_B \mid x_C, x_D$

d) Contraction: $P(x_A, x_B, x_C | x_D) = P(x_A | x_B | x_D) P(x_C | x_A, x_B, x_D)$ (Chain Rule)

$= P(x_A | x_D) P(x_B | x_D) P(x_C | x_A, x_B, x_D)$ CI 1

$= P(x_A | x_D) P(x_B | x_D) P(x_C | x_B, x_D)$ CI 2

$= P(x_A | x_D) P(x_{B \cup C} | x_D)$

$\Rightarrow x_A \perp\!\!\!\perp x_{B \cup C} \mid x_D$