1.7

a)

$$\Lambda = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing & 0 & 0 \\ \varnothing & \varnothing & \varnothing & 0 \\ 0 & \varnothing & \varnothing & \varnothing \\ 0 & 0 & \varnothing & \varnothing \end{bmatrix} \end{array}$$

$\varnothing$ means non-zero

(Conditional) Independencies

$1 \perp\!\!\!\perp 3$ $\quad$ $1 \perp\!\!\!\perp 4 \mid 2$ $\quad$ $2 \perp\!\!\!\perp 4 \mid 1$

$1 \perp\!\!\!\perp 4$ $\quad$ $1 \perp\!\!\!\perp 4 \mid 3$

$2 \perp\!\!\!\perp 4$ $\quad$ $1 \perp\!\!\!\perp 3 \mid 4$

Testing other conditional independencies. Let $\Lambda_{A,B|C}$ denote covariance of conditional distribution $p(x_A, x_B \mid x_C)$

$$\Lambda_{1,2|3} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{c} 3 \end{array} \\ \begin{bmatrix} \varnothing \\ \varnothing \end{bmatrix} \end{array} 3 \begin{bmatrix} \varnothing \end{bmatrix}^{-1} 3 \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \end{bmatrix} \end{array} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 2 \mid 3$$

$$\Lambda_{1,2|4} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{c} 4 \end{array} \\ \begin{bmatrix} \\ \end{bmatrix} \end{array} 4 \begin{bmatrix} \varnothing \end{bmatrix}^{-1} \begin{bmatrix} \quad \end{bmatrix} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 2 \mid 4$$

By symmetry, we also have $4 \not\perp\!\!\!\perp 3 \mid 2$ and $4 \not\perp\!\!\!\perp 3 \mid 1$

$$\Lambda_{2,3|1} = \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{c} 1 \end{array} \\ \begin{bmatrix} \varnothing \\ \varnothing \end{bmatrix} \end{array} 1 \begin{bmatrix} \varnothing \end{bmatrix}^{-1} 1 \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \end{bmatrix} \end{array} \Rightarrow \text{Not Diag} \Rightarrow 2 \not\perp\!\!\!\perp 3 \mid 1$$

By symmetry, we also have $2 \not\perp\!\!\!\perp 3 \mid 4$

$$\Lambda_{1,3|2} = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{c} 2 \end{array} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array} 2 \begin{bmatrix} \varnothing \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \end{bmatrix} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 3 \mid 2$$

$$\Lambda_{1,4|2} = \begin{array}{c} \\ 1 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 4 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 4 \end{array} \begin{array}{c} \begin{array}{c} 2 \end{array} \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array} 2 \begin{bmatrix} \varnothing \end{bmatrix}^{-1} \begin{bmatrix} 0 \end{bmatrix} \Rightarrow \text{Diag} \Rightarrow 1 \perp\!\!\!\perp 4 \mid 2$$

Not shown: $1 \perp\!\!\!\perp 3 \mid 4$ & $2 \perp\!\!\!\perp 4 \mid 1$

By symmetry, we also have $2 \not\perp\!\!\!\perp 4 \mid 3$ and $1 \perp\!\!\!\perp 4 \mid 3$

$$\Lambda_{1,2|3,4} = \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} 0 & 0 \\ \varnothing & 0 \end{bmatrix} \end{array} \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 3 & 4 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array}^{-1} \begin{array}{c} \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0 & \varnothing \end{bmatrix} \end{array} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 2 \mid 3,4$$

$$\Rightarrow 3 \not\perp\!\!\!\perp 4 \mid 1,2$$

$$\Lambda_{1,3|2,4} = \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 4 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ \varnothing & \varnothing \end{bmatrix} \end{array} \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 4 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array}^{-1} \begin{array}{c} \\ 2 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 3 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ 0 & \varnothing \end{bmatrix} \end{array} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 3 \mid 2,4$$

$$\Rightarrow 2 \not\perp\!\!\!\perp 4 \mid 1,3$$

$$\Lambda_{1,4|2,3} = \begin{array}{c} \\ 1 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} - \begin{array}{c} \\ 1 \\ 4 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 2 & 3 \end{array} \\ \begin{bmatrix} \varnothing & \varnothing \\ \varnothing & \varnothing \end{bmatrix} \end{array}^{-1} \begin{array}{c} \\ 2 \\ 3 \end{array} \begin{array}{c} \begin{array}{cc} 1 & 4 \end{array} \\ \begin{bmatrix} \varnothing & 0 \\ 0 & \varnothing \end{bmatrix} \end{array} \Rightarrow \text{Not Diag} \Rightarrow 1 \not\perp\!\!\!\perp 4 \mid 2,3$$

Not shown: $2 \not\perp\!\!\!\perp 3 \mid 1,4$