Invalid Logic, Equivalent Gains:

The Bizarreness of Reasoning in Language Model Prompting

Rylan Schaeffer * 1 Kateryna Pistunova * 2 Samar Khanna * 1 Sarthak Consul * Sanmi Koyejo 1

¹Computer Science, Stanford ²Physics, Stanford

Summary

- Language models can be prompted to reason through problems in a manner that improves performance
- Why such prompting improves performance is unclear
- Wang et al. [4] recently showed that *logically invalid* Chain-of-Thought (CoT) prompting [1, 5] improves performance almost as much as logically valid CoT prompting



- Critics responded Wang et al.'s finding was based on too few & too easily solved tasks to draw conclusions
- To resolve this dispute, we test whether *logically invalid* CoT prompts offer the same performance gains on the hardest tasks in BIG-Bench [2], termed BIG-Bench Hard (BBH) [3]
- Logically invalid CoT prompts DO achieve similar performance gains on BBH
- We also discover some CoT prompts used by previous works contain logical errors

Background: BIG-Bench [2] & BIG-Bench Hard (BBH) [3]

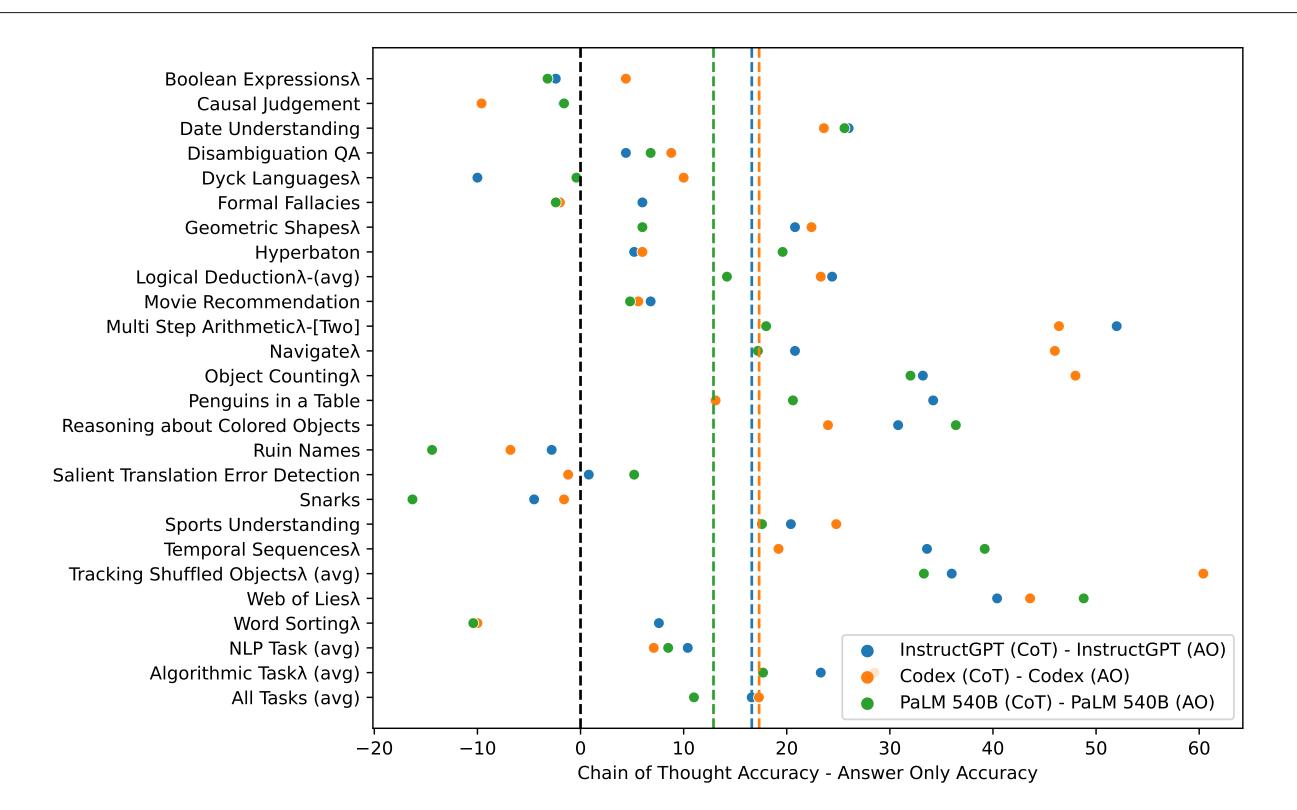
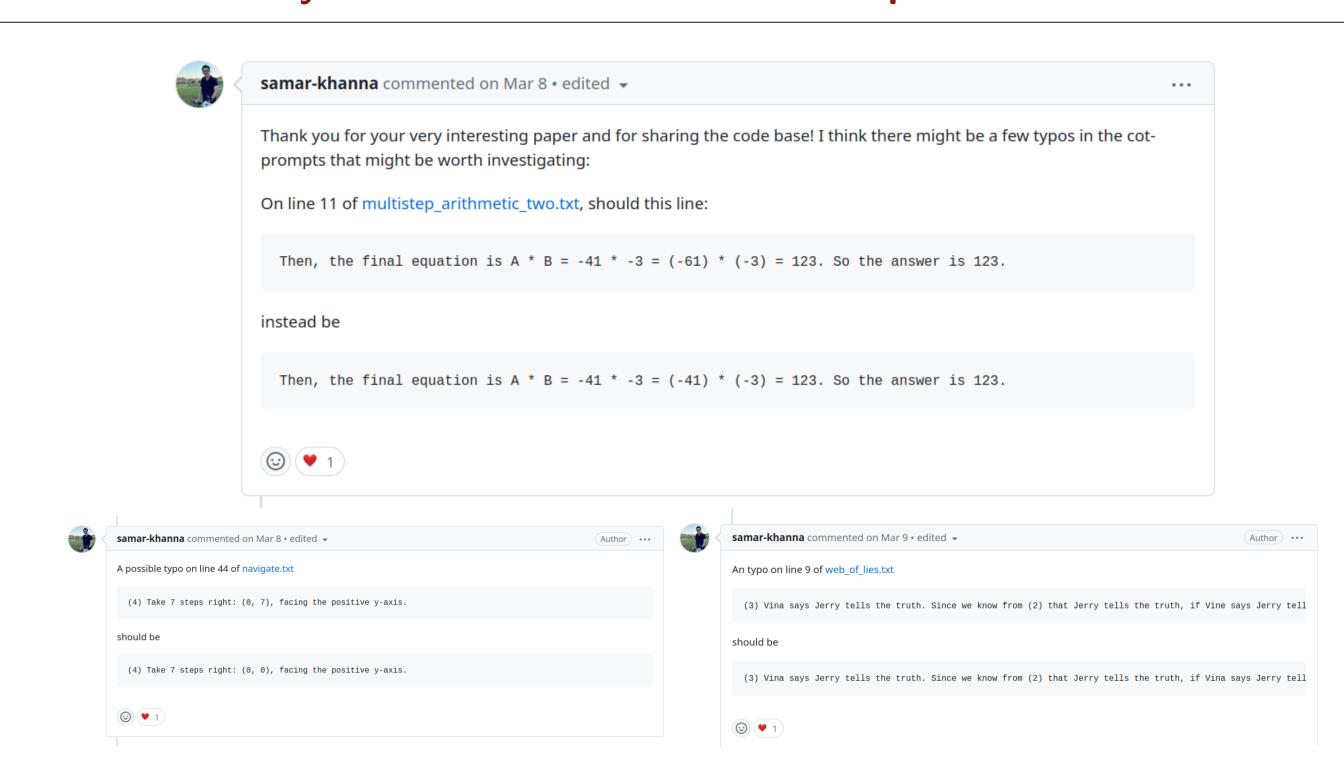
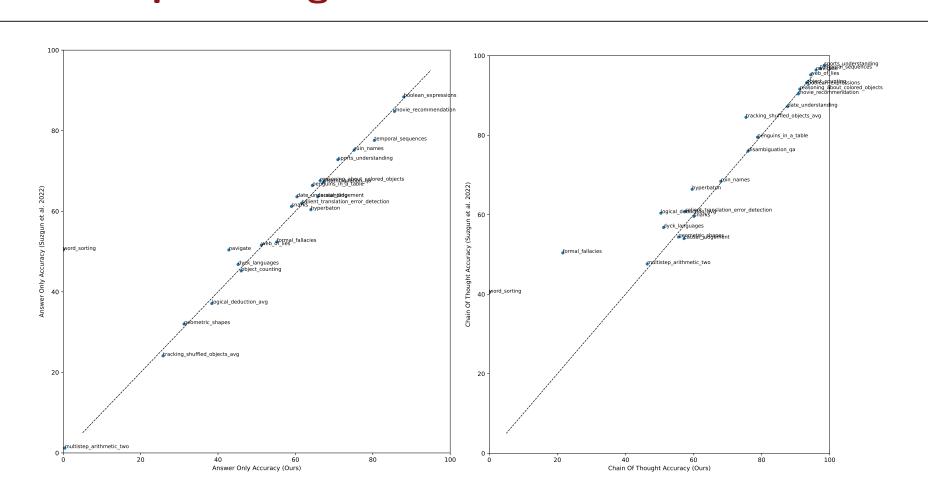


Figure 1. Chain-of-Thought (CoT) prompting significantly outperforms answer-only (AO) prompting on BIG-Bench Hard [3] = 23of the hardest tasks in Beyond the Imitation Game Benchmark [2].

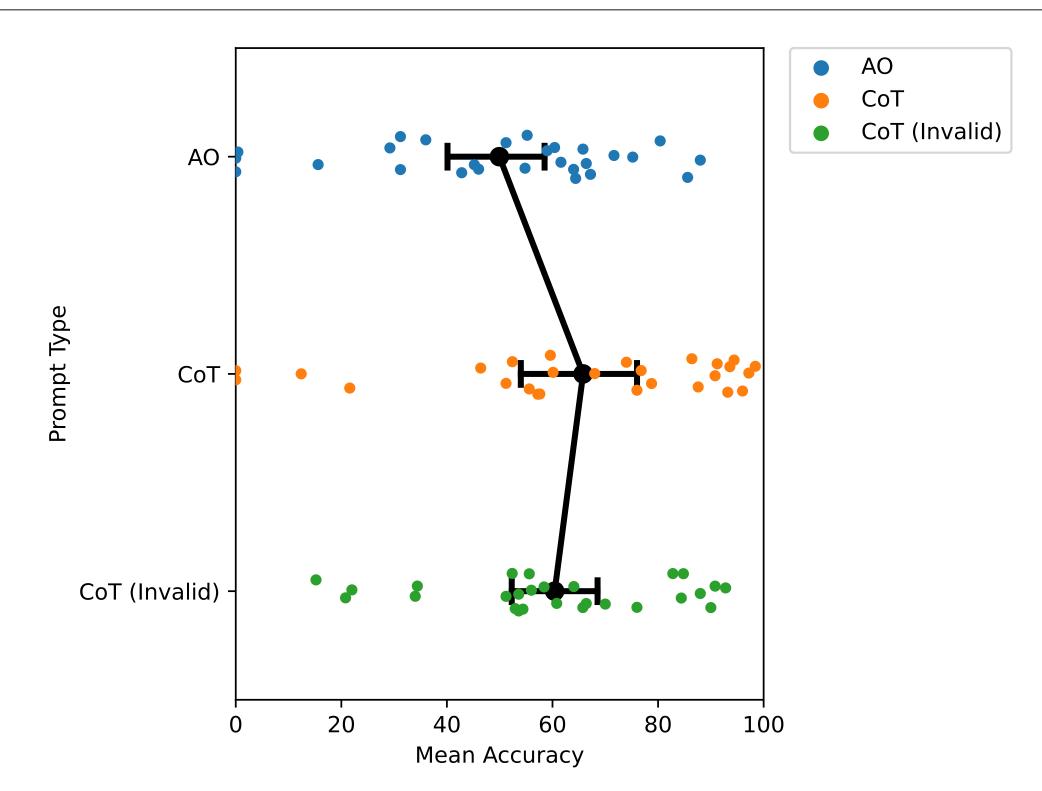
Discovery: BIG-Bench Hard's CoT Prompts Contain Errors!



Reproducing BIG-Bench Hard's Results



Logically Invalid CoT Prompting Almost Matches CoT on BBH



Note: We do not optimize logically invalid CoT prompts whatsoever. Write, run, done.

Example: Evaluate the result of a random Boolean expression. Q: not ((not not True)) is

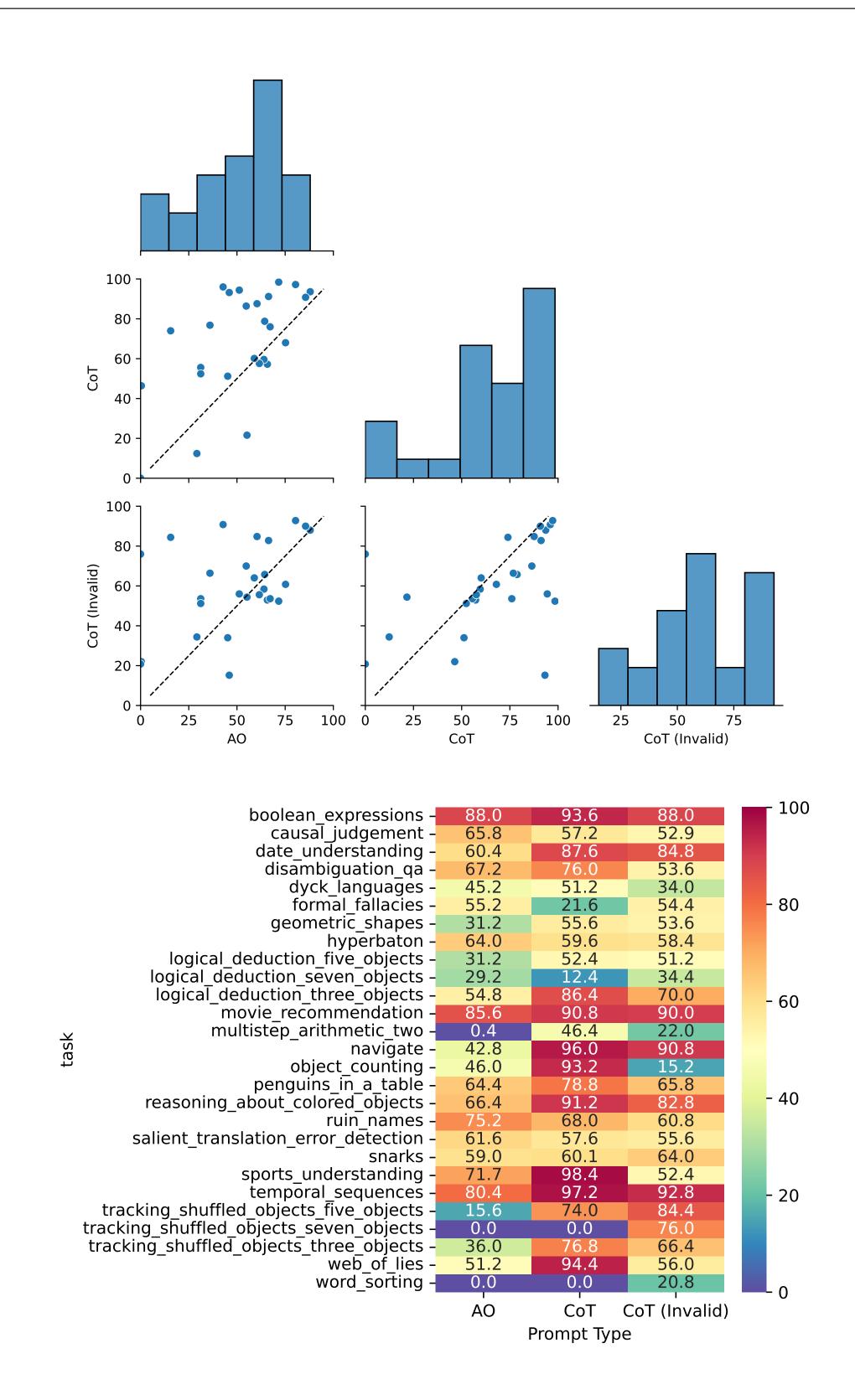
Answer Only (AO)	False
Scratchpad/Chain-of-Thought (CoT)	A: Let's think step by step. Remember that (i) expressions inside brackets are always evaluated first and that (ii) the order of operations from highest priority to lowest priority is "not", "and", "or", respectively. We first simplify this expression "Z" as follows: "Z = not ((not not True)) = not ((A))' where "A = not not True". Let's evaluate A: A = not not True = not (not True) = not False = True. Plugging in A, we get: Z = not ((A)) = not (True)) = not True = False. So the answer is False.
Logically-Invalid Chain-of-Thought (Invalid CoT)	A: Let's think step by step. Remember that (i) expressions inside brackets are always evaluated first and that (ii) the order of operations from highest priority to lowest priority is "not", "and", "or", respectively. Because English does not permit multiple negatives, the expression "(not not True)" evaluates

answer is False.

to "(not True)". The expression "not ((not not True))" therefore evaluates

to "not ((not True))". By the same logic, the expression "not ((not True

))" simplifies to "not True". In Boolean logic, "not True" is False. So the



References

- [1] M. Nye, A. J. Andreassen, G. Gur-Ari, H. Michalewski, J. Austin, D. Bieber, D. Dohan, A. Lewkowycz, M. Bosma, D. Luan, et al. Show your work: Scratchpads for intermediate computation with language models, 2021.
- [2] A. Srivastava, A. Rastogi, A. Rao, A. A. M. Shoeb, A. Abid, A. Fisch, A. R. Brown, A. Santoro, A. Gupta, A. Garriga-Alonso, et al. Beyond the imitation game: Quantifying and extrapolating the capabilities of language models, 2022.
- [3] M. Suzgun, N. Scales, N. Schärli, S. Gehrmann, Y. Tay, H. W. Chung, A. Chowdhery, Q. V. Le, E. H. Chi, D. Zhou, et al.
- Challenging big-bench tasks and whether chain-of-thought can solve them, 2022.
- [4] B. Wang, S. Min, X. Deng, J. Shen, Y. Wu, L. Zettlemoyer, and H. Sun. Towards understanding chain-of-thought prompting: An empirical study of what matters, 2022.
- [5] J. Wei, X. Wang, D. Schuurmans, M. Bosma, F. Xia, E. Chi, Q. V. Le, D. Zhou, et al. Chain-of-thought prompting elicits reasoning in large language models, 2022.