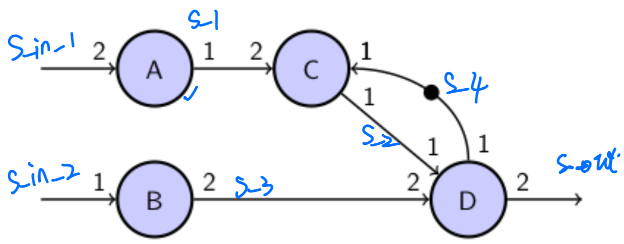
2.1 Data Flow Model of Computation

1. (**Mandatory**) Given is the following synchronous data flow graph.



(a) Give a mathematical representation of the SDF graph, and use this representation to derive a *periodic admissible sequential schedule* with the minimal buffer requirements, where you show each step of your calculation. What are the required buffer sizes for this schedule?

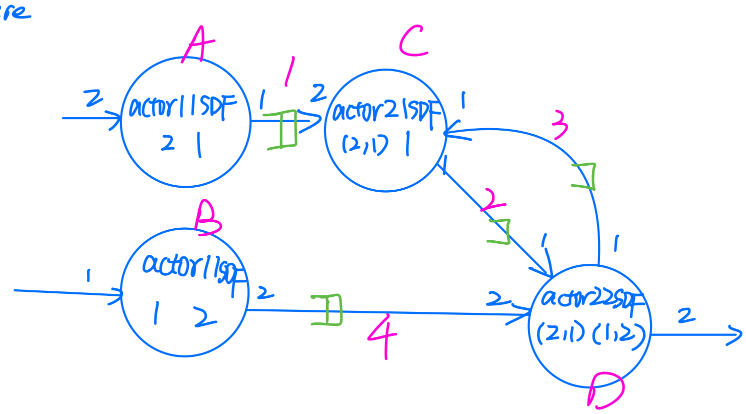
system (s_in_1, s_in_2) = s_out where

s_1 = A s_in_1

s_3 = B s_in_2

s_2 = C s_1

(s_out, s_4) = D s_2 s_3



$$\begin{array}{c|cccc} & A & B & C & D \\ \hline 1 & \\ 2 & \\ 3 & \\ 4 & \end{array}\quad \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 2 & 0 & -2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

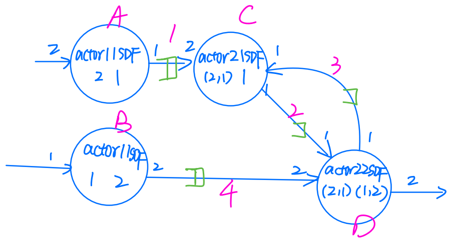
$$= \begin{bmatrix} 1 & 0 & -2 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

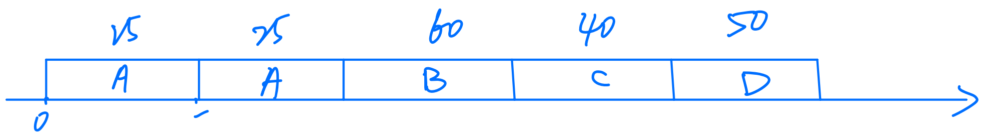
rank = 3 $\quad q = \begin{bmatrix} 2 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

maximum buffer size: $\begin{bmatrix} 2 \\ 1 \\ 1 \\ 2 \end{bmatrix}$

minimal buffer size: 3

schedule: A A C B D





(b) What is the maximum throughput for a single processor, defined as output tokens per time unit, given the following parameters:

- the execution times for the data flow actors are $C_A = 25$, $C_B = 60$, $C_C = 40$, and $C_D = 50$, and the
- communication time for sending tokens and receiving can be neglected.

the input rate is higher or lower than $R_{\text{single},i}$.

2 output tokens in 200 time unit

throughput $= \frac{2}{200} = 0.01$

(c) Give also the necessary input data rate $R_{\text{single},i}$, as tokens per time unit on input arc i, on b input arcs to be able to achieve the maximum throughput for both sub-tasks. What happens the input rate is higher or lower than $R_{\text{single},i}$.

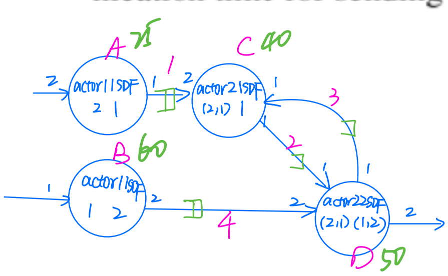
$R_{\text{single},A} = \frac{2}{200} = 0.01$

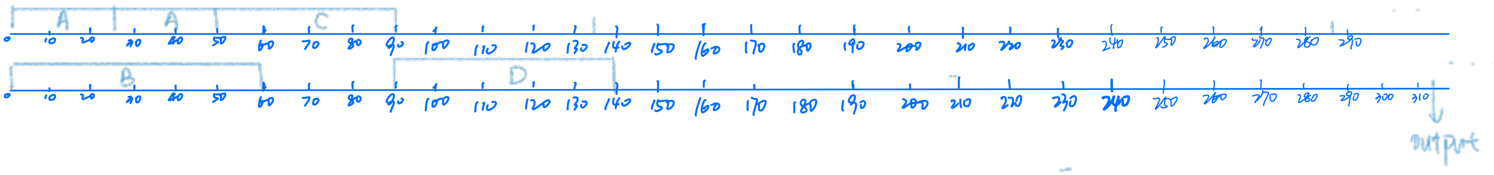
$R_{\text{single},B} = \frac{1}{200} = 0.005$

higher: buffer overflow

lower: latency

(d) Give a *periodic admissible parallel schedule* for an architecture with two processors aiming to maximise the throughput. Use the same execution times as above and neglect also the communication time for sending and receiving tokens.





(e) Which throughput can be achieved with this schedule for the two processor architecture? Give also the necessary input data rates $R_{\text{double},i}$ on both input arcs.

2 tokens per 140 time units. $\frac{2}{140} = \frac{1}{70}$

$R_{\text{double},A} = \frac{2}{140} = \frac{1}{70}$

$R_{\text{double},B} = \frac{1}{140} = \frac{1}{140}$