Unsupervised Learning

COMP9417 Machine Learning and Data Mining

May 2, 2017

Acknowledgements

Material derived from slides for the book
"Elements of Statistical Learning (2nd Ed.)" by T. Hastie,
R. Tibshirani & J. Friedman. Springer (2009)
http://statweb.stanford.edu/~tibs/ElemStatLearn/

Material derived from slides for the book
"Machine Learning: A Probabilistic Perspective" by P. Murphy
MIT Press (2012)
http://www.cs.ubc.ca/~murphyk/MLbook

Material derived from slides for the book
"Machine Learning" by P. Flach
Cambridge University Press (2012)
http://cs.bris.ac.uk/~flach/mlbook

Material derived from slides for the book
"Bayesian Reasoning and Machine Learning" by D. Barber
Cambridge University Press (2012)
http://www.cs.ucl.ac.uk/staff/d.barber/brml

Material derived from figures for the book
"Python Data Science Handbook" by J. VanderPlas
O'Reilly Media (2017)
http://shop.oreilly.com/product/0636920034919.do

Material derived from slides for the course
"Machine Learning" by A. Srinivasan

BITS Pilani, Goa, India (2016)

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So: *unsupervised learning* methods address the problem of assigning instances to classes *given only observations about the instances*, i.e., without being given class "labels" for instances by a "teacher".

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- to learn generative models for images, text, video, speech, etc.

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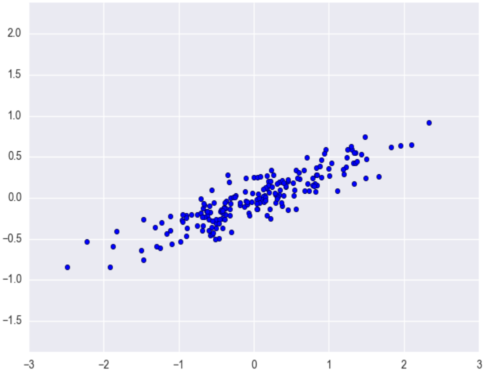
This suggests an approach: look into using the variance-covariance matrix we recall from correlation and regression

PCA Example



PCA looks for linear combinations of the original features. This dataset of 200 points seems to show such a relationship between two feature dimensions.

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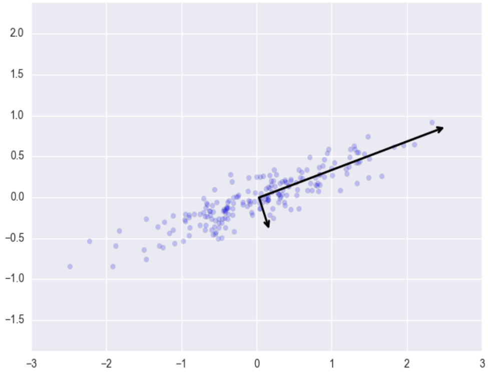

PCA finds two new features on which the original data can be projected, rotated and scaled. These explain respectively 0.75 and 0.02 of the variance.

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5. sort columns of $\mathbf{S}$ in decreasing order (decreasing variance)
6. remove columns of $\mathbf{S}$ below some minimum threshold

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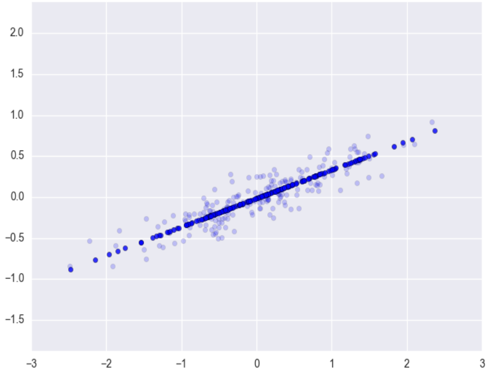

By rejecting the second component we reduce the dimensionality by 50% while preserving much of the original variance, seen by plotting the inverse transform of this component along with the original data.

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- many other methods use essentially the same matrix decomposition idea, such as finding “topics” in text using Latent Semantic Analysis (next slide), finding hidden “sub-groups” for recommender systems, and so on

Finding Topics in Text

$$\underset{t \times d}{\left[\ \tilde{\mathbf{X}}\ \right]} = \underset{t \times k}{\left[\ \mathbf{U}_k\ \right]} \underset{k \times k}{\begin{bmatrix} s_1 & & & \\ & s_2 & & \\ & & \ddots & \\ & & & s_k \end{bmatrix}} \underset{k \times d}{\left[\ \mathbf{V}_k^{\mathrm{T}}\ \right]}$$

Latent Semantic Analysis (LSA) factorizes a word count matrix for t terms from d documents using the Singular Value Decomposition (SVD) to find a number $k < d$ of topics. Here the diagonal matrix $\mathbf{S}$ has the topic "strength" sorted in decreasing order.

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- many alternatives, e.g., Random Projections, Independent Component Analysis, etc.

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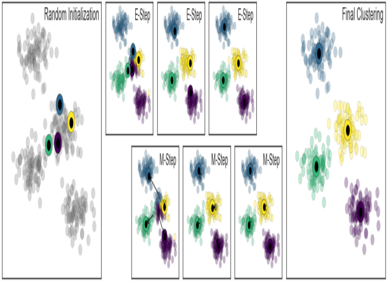
k-Means Clustering Example



Data.

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Summary: Unsupervised Learning

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- active learning
  - learning system acts to generate its own examples

Note: unsupervised learing an increasingly active research area, particularly in neural nets, e.g., Yann LeCun: "Unsupervised Learning: The Next Frontier in AI"
`https://www.youtube.com/watch?v=XTbLOjVF-y4`