Measuring Runtime Performance

Data Structures & Algorithms

Complexity Notation

- *n* = input size
- *f(n)* = max number of steps when input has size *n*
- ***O(f(n))*** = asymptotic upper bound

```
void f(int *out, const int *in, int size) {
  int product = 1;
  for (int i = 0; i < size; ++i)
    product *= in[i];
  for(int i = 0; i < size; ++i)
    out[i] = product / in[i];
} // f()
```

$f(n) = 1 + (2 + 2n) + 3n + (2 + 2n) + 4n = 11n + 5 = \boldsymbol{O(n)}$

Ways to measure complexity

- Analytically
  - Analysis of the code itself
  - Recognizing common algorithms/patterns
  - Based on a recurrence relation
- Empirically
  - Measure runtime programmatically
  - Measure runtime using external tools
  - Test on inputs of a variety of sizes

Measuring Time In C++11+

```
#include <chrono>

class Timer {
  std::chrono::time_point<std::chrono::system_clock> cur;
  std::chrono::duration<double> elap;
public:
  Timer() : cur(), elap(std::chrono::duration<double>::zero()) {}

  void start() {
    cur = std::chrono::system_clock::now();
  } // start()

  void stop() {
    elap += std::chrono::system_clock::now() - cur;
  } // stop()

  void reset() {
    elap = std::chrono::duration<double>::zero();
  } // reset()

  double seconds() {
    return elap.count();
  } // seconds()
}; // Timer{}
```

Note: Checking time too often will slow down your program!

Example

```
int main() {
  Timer t;
  t.start();
  doStuff1();
  t.stop();
  cout << "1: " << t.seconds()
       << "s" << endl;

  t.reset();
  t.start();
  doStuff2();
  t.stop();
  cout << "2: " << t.seconds()
       << "s" << endl;
  return 0;
} // main()
```

Credit: Amir Kamil

Let's try it!

Save the file to a folder you can access from a *NIX shell, and/or upload to CAEN

- Browser Download
  https://eecs281staff.github.io/search-demo/search.cpp
- Command Line Download

```
$ mkdir search-demo && cd search-demo
$ wget https://eecs281staff.github.io/search-demo/search.cpp
```

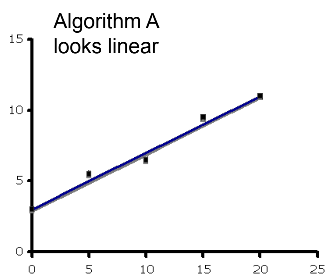
EECS 281: Search Demo

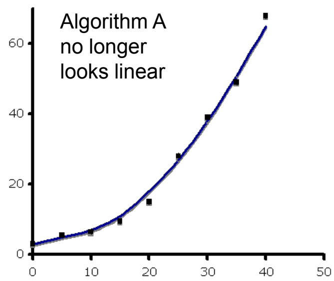
After Downloading

- Compile:
  `$ g++ -std=c++17 -O3 search.cpp -o search`
- Run a binary search, 1M items:
  `$ ./search b 1000000`
- Run a linear search, 1M items:
  `$ ./search l 1000000`
- Try with larger numbers!

Empirical Results

- Plot actual run time versus varying input sizes
- Include a large range to accurately display trend





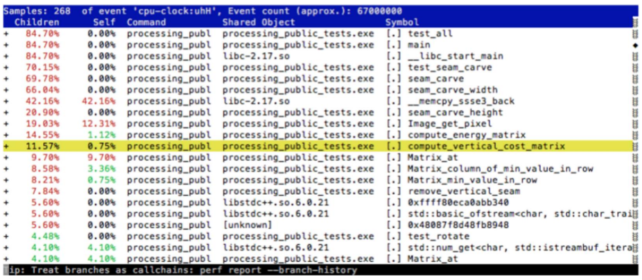
- Caveat: for small inputs, asymptotics may not play out yet

Prediction versus Experiment

- What if experimental results are *worse* than predictions?
  - Example: results are exponential when analysis is linear
  - Error in complexity analysis
  - Error in coding (check for extra loops, unintended operations, etc.)
- What if experimental results are *better* than predictions?
  - Example: results are linear when analysis is exponential
  - Experiment may not have fit worst case scenario
  - Error in complexity analysis
  - Error in analytical measurements
  - Incomplete algorithm implementation
  - Algorithm implemented is better than the one analyzed
- What if experimental data match asymptotic prediction but runs are too slow?
  - Performance bug?
  - Check compile options (e.g. use `-O3`)
  - Look for optimizations to improve the constant factors

Using a Profiling Tool

- This won't tell you the complexity of an algorithm, but it tells you where you program spends its time.
- Many different tools exist – you'll learn to use `perf` in lab.



```
Samples: 268  of event 'cpu-clock:uhH', Event count (approx.): 67000000
  Children      Self  Command          Shared Object                 Symbol
+   84.70%     0.00%  processing_publ  processing_public_tests.exe   [.] test_all
+   84.70%     0.00%  processing_publ  processing_public_tests.exe   [.] main
+   84.70%     0.00%  processing_publ  libc-2.17.so                  [.] __libc_start_main
+   70.15%     0.00%  processing_publ  processing_public_tests.exe   [.] test_seam_carve
+   69.78%     0.00%  processing_publ  processing_public_tests.exe   [.] seam_carve
+   66.04%     0.00%  processing_publ  processing_public_tests.exe   [.] seam_carve_width
+   42.16%    42.16%  processing_publ  libc-2.17.so                  [.] __memcpy_ssse3_back
+   20.90%     0.00%  processing_publ  processing_public_tests.exe   [.] seam_carve_height
+   19.03%    12.31%  processing_publ  processing_public_tests.exe   [.] Image_get_pixel
+   14.55%     1.12%  processing_publ  processing_public_tests.exe   [.] compute_energy_matrix
+   11.57%     0.75%  processing_publ  processing_public_tests.exe   [.] compute_vertical_cost_matrix
+    9.70%     9.70%  processing_publ  processing_public_tests.exe   [.] Matrix_at
+    8.58%     3.36%  processing_publ  processing_public_tests.exe   [.] Matrix_column_of_min_value_in_row
+    8.21%     0.75%  processing_publ  processing_public_tests.exe   [.] Matrix_min_value_in_row
+    7.84%     0.00%  processing_publ  processing_public_tests.exe   [.] remove_vertical_seam
+    5.60%     0.00%  processing_publ  libstdc++.so.6.0.21           [.] 0xffff80eca0abb340
+    5.60%     0.00%  processing_publ  libstdc++.so.6.0.21           [.] std::basic_ofstream<char, std::char_trai
+    5.60%     0.00%  processing_publ  [unknown]                     [.] 0x48087f8d48fb8948
+    4.48%     0.00%  processing_publ  processing_public_tests.exe   [.] test_rotate
+    4.10%     4.10%  processing_publ  libstdc++.so.6.0.21           [.] std::num_get<char, std::istreambuf_itera
+    4.10%     4.10%  processing_publ  processing_public_tests.exe   [.] Matrix_at
ip: Treat branches as callchains: perf report --branch-history
```

A snapshot of a `perf` report generated for EECS 280 Project 2.
Image credit: Alexandra Brown

Measuring Runtime on Linux

If you are launching a program using command

```
% progName -options args
```

Then

```
% /usr/bin/time progName -options args
```

will produce a runtime report

```
0.84user 0.00system 0:00.85elapsed 99%CPU
```

If you're timing a program in the current folder, use `./`

```
% /usr/bin/time ./progName -options args
```

Often, you can just type `time` rather than `/usr/bin/time`.

Measuring Runtime on Linux

- Example: this command just wastes time by copying zeros to the "bit bucket"

```
% time dd if=/dev/zero of=/dev/null
```

kill it with control-C

```
3151764+0 records in
3151764+0 records out
1613703168 bytes (1.6 GB) copied, 0.925958 s, 1.7 GB/s
Command terminated by signal 2
0.26user 0.65system 0:00.92elapsed 99%CPU
(0avgtext+0avgdata 3712maxresident)k
0inputs+0outputs (0major+285minor)pagefaults 0swaps
```


Measuring Runtime on Linux

`0.26user 0.65system 0:00.92elapsed 99%CPU`

- `user` time is spent by your program
- `system` time is spent by the OS on behalf of your program
- `elapsed` is wall clock time - time from start to finish of the call, including any time slices used by other processes
- `%CPU` Percentage of the CPU that this job got. This is just (`user` + `system`) / `elapsed`
- `man time` for more information

Using `valgrind`

- Suppose we want to check for memory leaks:

```
valgrind ./search b 1000000
```

- Force a leak!
  - Replace `return 0` with `exit(0)`, run valgrind using flags `--leak-check=full --show-leak-kinds=all`
- Who leaked that memory?
  - The memory address isn't very useful, we just know that `main()` called `operator new`
  - Recompile with `-g3` instead of `-O3` and run valgrind one more time

```
valgrind ./search_valgrind b 1000000
```


Job Interview Question

- Implement this function

```
// returns x^n
int power(int x, uint32_t n);
```

- The obvious solution using $n - 1$ multiplications is ***O*(*n*)**
  - $2^8 = 2*2* \ldots *2$
- Less obvious: ***O*(log *n*)** multiplications
  - Hint: $2^8 = ((2^2)^2)^2$
  - How does it work for 2^7?
- Write both solutions iteratively and recursively

Computing x^n

```
int power(int x, uint32_t n) {
  if (n == 0) {
    return 1;
  } // if

  int result = x;
  for (int i = 1; i < n; ++i) {
    result = result * x;
  } // for

  return result;
} // power()
```


Analyzing Solutions

- *Iterative* functions use loops
- A function is *recursive* if it calls itself
- What is the time complexity of each function?

Iterative

```
int power(int x, int n) {
  int result = 1;
  for (int i = 0; i < n; ++i) {
    result = result * x;
  } // for()

  return result;
} // power()
```

***Θ*(*n*)**
It's just a regular loop.

Recursive

```
int power(int x, int n) {
  if (n == 0) {
    return 1;
  } // if()
  return x * power(x, n - 1);
} // power()
```

???
We need another tool to analyze this.

Recurrence Relations

- A *recurrence relation* describes the way a problem depends on a subproblem.
  - A recurrence can be written for a computation:

$$x^n = \begin{cases} 1 & n == 0 \\ x * x^{n-1} & n > 0 \end{cases}$$

  - A recurrence can be written for the time taken:

$$T(n) = \begin{cases} c_0 & n == 0 \\ T(n-1) + c_1 & n > 0 \end{cases}$$

  - A recurrence can be written for the amount of memory used*:

$$M(n) = \begin{cases} c_0 & n == 0 \\ M(n-1) + c_1 & n > 0 \end{cases}$$

*Non-tail recursive

Solving Recurrences

- Substitution method
  1. Write out $T(n)$, $T(n - 1)$, $T(n - 2)$
  2. Substitute $T(n - 1)$, $T(n - 2)$ into $T(n)$
  3. Look for a pattern
  4. Use a summation formula
- Another way to solve recurrence equations is the Master Method (AKA Master Theorem)

Solving Recurrences: Linear

$$T(n) = \begin{cases} c_0 & n == 0 \\ T(n-1) + c_1 & n > 0 \end{cases}$$

```
int power(int x, int n) {
  if (n == 0)
    return 1;

  return x * power(x, n - 1);
} // power()
```

Recurrence: $T(n) = T(n - 1) + c$
Complexity: $\boldsymbol{\Theta(n)}$

$T(n) = n c_1 + c_0$

Solving Recurrences: Logarithmic

$$T(n) = \begin{cases} c_0 & n == 0 \\ T\left(\frac{n}{2}\right) + c_1 & n > 0 \end{cases}$$

```
int power(int x, int n) {
  if (n == 0)
    return 1;

  int result = power(x, n / 2);
  result *= result;
  if (n % 2 != 0)  // n is odd
    result *= x;

  return result;
} // power()
```

Recurrence: $T(n) = T(n / 2) + c$
Complexity: $\boldsymbol{\Theta(\log n)}$

$T(1) = c_0$
$T(2) = T(1) + c_1$
$T(4) = T(2) + c_1$
$\vdots$
$T(n) = T\left(\frac{n}{2}\right) + c_1$
$= c_1 \log_2 n + c_0 = O(\log n)$

A Logarithmic Recurrence Relation

$$T(n) = \begin{cases} c_0 & n == 0 \\ T\left(\frac{n}{2}\right) + c_1 & n > 0 \end{cases} \rightarrow \boldsymbol{\Theta(\log n)}$$

- Fits the logarithmic recursive implementation of `power()`
  - The power to be calculated is divided into two halves and combined with a single multiplication
- Also fits Binary Search
  - The search space is cut in half each time, and the function recurses into only one half

Recurrence Thought Exercises

- What if a recurrence cuts a problem into two subproblems, and both subproblems were recursively processed?
- What if a recurrence cuts a problem into three subproblems and…
  - Processes one piece
  - Processes two pieces
  - Processes three pieces
- What if there was additional, non-constant work after the recursion?

Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

- Binomial Coefficient – "*n* choose *k*"
- Write this function with pen and paper
- Compile and test what you've written
- Options
  - Iterative
  - Recursive
  - Tail recursive
- We'll come back to this at the end of the semester
- Analyze

Analyzing Recursion

Data Structures & Algorithms