



Today's Discussion

- More on Binary: 2's complement & multiplication
- How to translate to LC2K
- How to write in LC2K

EECS 370

Lab 2: Project 1 - LC2K ISA



Negative Numbers Using 2's Complement



A way to represent integers using binary, both positive and negative

In 2's Complement, the most significant bit (MSB) is negative.

For example, in a 4-bit number, the MSB typically represents 2^3 , but in 2's Complement, the MSB **instead** represents -2^3

So, if the MSB is 0, the number will be positive (or zero)

And, if the MSB is 1, the number will be negative

The other bits remain the same, let's see how with an example:

Converting 3_{10} into -3_{10} with 4-bit Integers



With a 4-bit, 2's Complement, binary number, we represent 3 as

$$0011_2 \rightarrow 0 * (-2^3) + 0 * 2^2 + 1 * 2^1 + 1 * 2^0 = 3$$

Conversely, we represent -3 with the same constraints as

$$1101_2 \rightarrow 1 * (-2^3) + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 = -3$$

To convert quickly and easily, **flip the bits and add 1!**

$$3 = 0011 \xrightarrow{\text{flip}} 1100 \xrightarrow{\text{add 1}} 1101 = -3$$

Comparison of Representations



Consider this 8 bit number: $0b11110000$

Unsigned: $2^7 * 1 + 2^6 * 1 + 2^5 * 1 + 2^4 * 1 + 2^3 * 0 + 2^2 * 0 + 2^1 * 0 + 2^0 * 0$
 $= 128 + 64 + 32 + 16 = 240$

Signed: $-2^7 * 1 + 2^6 * 1 + 2^5 * 1 + 2^4 * 1 + 2^3 * 0 + 2^2 * 0 + 2^1 * 0 + 2^0 * 0 =$
 $-128 + 64 + 32 + 16 = -16$

Notice that $2^8 - 16 = 240$

To sign extend to a larger size, just copy the MSB:

$0b11111111110000$ is -16 in 16-bit signed representation.

Addition in 2's Complement



Consider $8 + 4 = 12 = 0b1100$ (unsigned 4-bit)

And, $-8 + 4 = -4 = 0b1100$ (signed 4-bit)

Notice the identical binary values.

Also, $-1 = 0b1111111111111111$ (signed 16-bit)

And, $2^{16} - 1 = 65535 = 0b1111111111111111$ (unsigned 16-bit)

This is why we use 2's complement! Addition is the same as normal binary

Multiplying in Decimal (Project 1m)



In decimal, we multiply by "shifting" each number as we go

Note the added 0 on the right when multiplying the 2

We can perform the same operation in binary to multiply

$$\begin{array}{r}
 104 \\
 \times 26 \\
 \hline
 624 \\
 + 2080 \\
 \hline
 2704
 \end{array}$$

Multiplying in Binary (Project 1m)



Since we always multiply by either 1 or 0, we either add to our running sum, or don't.

We perform one "shift" by adding a value to itself.

We get the value of each bit by masking with almost all 0's, and then checking if the result equals 0.

(Hint: Shift the mask also)

$$\begin{array}{r}
 1001 \\
 \times 1011 \\
 \hline
 1001 \\
 10010 \\
 000000 \\
 + 1001000 \\
 \hline
 110011
 \end{array}$$

The Add instruction

The add instruction allows us to do some bit shifting:

$$x + x == x * 2 == x \ll 1$$

By doing this multiple times, we get multiplication:

$$a * 2^b == a \ll b$$

The Nor Instruction

NOR in LC2K is bitwise: First do OR with $A \mid B$, then NOT with \sim result.

Recall from 270 or 203 that NOR is a universal gate; any gate can be built from it (AND, OR, etc.) https://en.wikipedia.org/wiki/NOR_logic

$$\sim A = \sim(A \mid A) = \text{NOR}(A, A)$$

Since $A = A \mid A$
 $\text{nor}(\text{nor}(A, B), \text{nor}(A, B))$

$$A \mid B = \sim(\sim(A \mid B)) = \sim(\text{NOR}(A, B))$$

Double Negation

$$A \& B = \sim(\sim A \mid \sim B) = \text{NOR}(\sim A, \sim B)$$

By DeMorgan's Laws

$$\text{nor}(\text{nor}(A, A), \text{nor}(B, B))$$

Combining Add and Nor

We can do subtraction by combining them:

$$A - B = A + (-B) = A + (\sim B + 1)$$

We can set or mask bitfields, like in Lab 1:

First shift with multiple add instructions, then do some nor operations.

How can you use this in your multiplication algorithm?

The Beq instruction

Beq always branches (sets PC) to $PC + 1 + \text{offset}$.

But if the offset is a label, it just branches to the label.

Review lec 4/5 & [P1 Walkthrough](#) for examples, and be comfortable with:

- If/Else statements
- While Loops
- Do-While Loops
- For Loops

The Beq instruction

Beq instructions implement conditional branches, but how do we do unconditional branches like in ARM?

When the 2 registers in beq are the same register, the branch is unconditional.

Always true (if true)

Example: `beq 1 1 offset` is unconditional since $r1 = r1$ always.

Testing Project 1a

There are 2^{10} (add, nor) + $3 \cdot 2^{22}$ (lw, sw, beq) + 2^6 (jalr) + 2 (noop, halt) = 12,584,002 possible LC2K instructions.

You *cannot* test all of these!

But, you can test every opcode, regA, regB, and destReg.

You should also test many types of offsets and fill values:

- Numbers (16-bit 2's complement)
- Absolutely-resolved labels for lw/sw/.fill
- Relatively-resolved labels for beq

Problem 3: LC2K Assembler Test Cases

Take some time to write some test cases for Project 1a

Guidelines:

- Also submit corresponding machine code
- Must not have errors
- Must catch 3 bugs
- Each test must be 5 lines or fewer

Pro tip: Use to debug your Project 1a

LC2K to Machine Code Conversion Example

From spec.as

lw 0 1 five load reg1 with 5 (symbolic address)

I-type instructions (lw , sw , beq)	bits 24-22: opcode
	bits 21-19: reg A
	bits 18-16: reg B
	bits 15-0: offsetField

0000000010000001000000000000111
= 0x00810007