1 Potential Method (WN22 MCQ1)

Consider the following code. Note that the variables x and y are *real* numbers.

Input: $x > y > 0$ are *real* numbers // Hint: This actually matters...

```
function Foo376(x, y)
    if x ≤ 0 then return 1;
    z ← Foo376(x − log y, y)
    return (z + 1)
```

Which of the following is a valid potential function for the algorithm Foo376 (above)?

- ◯ $s = x + y$
- ◯ $s = e^y - x$
- ◯ $s = x$
- ◯ None of the above

2 Divide and Conquer (WS7 Review 2)

Describe an efficient divide and conquer algorithm to computer the value of 376^k. For simplicity, you may assume that k is a power of 2. Your solution should include a correctness and runtime analysis in terms of n (assuming multiplication takes constant time).

3 Dynamic Programming (WS7 Review 3)

Give a recurrence relation (including base cases) that is suitable for dynamic programming solutions to the following problem. You do not need to prove your correctness.

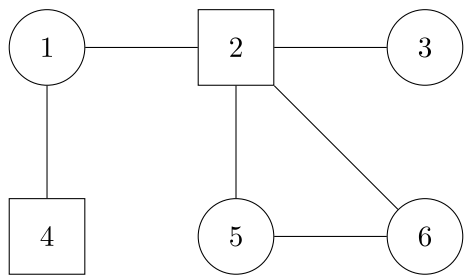
LONGEST-ARITHMETIC-SUBSEQUENCE(A, d): Given an array of integers A and a difference d, return the length of the longest arithmetic subsequence in A with difference d. That is, return the longest subsequence S such that $S[i+1] - S[i] = d$ for each i.

4 Greedy Algorithms (WN23 Short3)

A *dominating set* S in a graph G is a set of vertices for which every vertex of G either is in S, or is adjacent to some vertex in S.

We are interested in a *smallest* dominating set of a given graph, i.e., one that has the fewest possible vertices. (There may be more than one smallest dominating set.)

For example, the following graph has a smallest dominating set $S^* = \{2, 4\}$: every vertex other than 2 and 4 is adjacent to 2 or 4 (or both), and there is no dominating set consisting of a single vertex.



Consider the following greedy algorithm for finding a dominating set in a graph.

---

function GreedyDS(G)
    $S \leftarrow \emptyset$
    while G has at least one vertex **do**
        Select any vertex v in G that has largest degree (i.e., the most neighbors)
        Add v to S
        Remove v and all its neighbors, including all incident edges, from G
    return S

Give a small graph G on which the algorithm **might not return a *smallest* dominating set**.

Specifically, **give a sequence of vertices that the algorithm might choose** to make up its final output set, and **give an optimal dominating set of G that is smaller** than this output set.

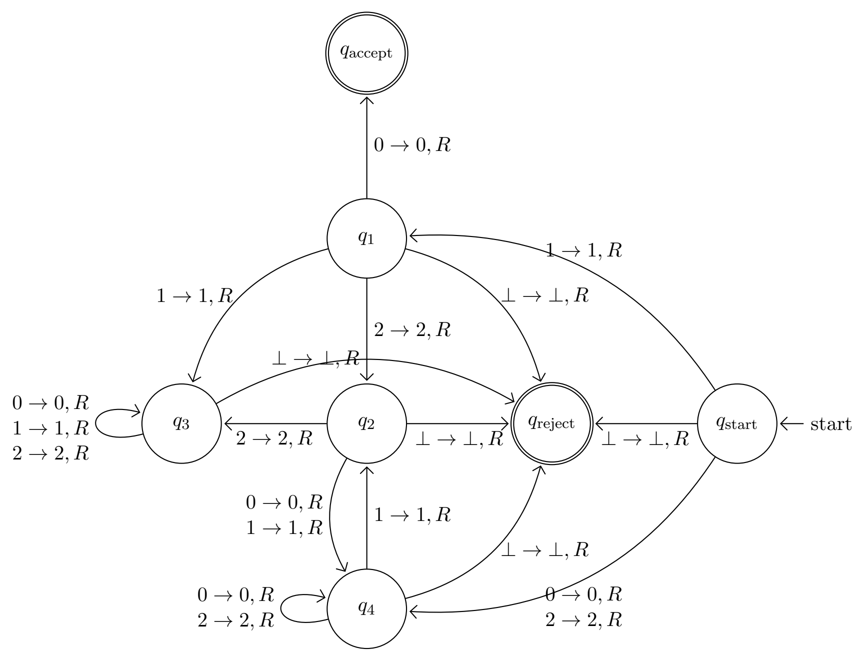
5 DFAs (WN22 9b)

Let $L \subseteq \{a, b, c\}^*$ be the set of all strings over the alphabet $\{a, b, c\}$ **except** those that contain both at least one b and at least one c. For example, aa, aba, cca are all in L, but abc is not as it contains both a b and a c.

Write a DFA over the alphabet $\{a, b, c\}$ that decides the language L.

6 Turing Machines (WS6 TM2)

Consider the Turing Machine whose state diagram is given below:



Which of the following statements is true about this Turing Machine?

- ◯ It accepts all strings that contain the substring "10."
- ◯ It loops on any input string that contains only 2s.
- ◯ It loops on any string that contains the substring "11" until it reaches $\perp$.
- ◯ None of the above