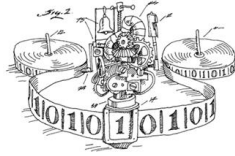


EECS 376: Foundations of Computer Science

Lecture 01 - Introduction



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Course Staff (Tentative)

- Instructor: Ali Movaghar (movaghar@umich.edu)
- GSI: Junghwan Kim (kimjhj@umich.edu)
- IA: Eric Khiu (erickhiu@umich.edu)
- **To be updated shortly.**

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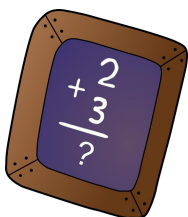
The Instructor

- **Ali Movaghar**, movaghar@umich.edu
 - B.S. in Electrical Engineering (EE), University of Tehran, 1977
 - M.S. and Ph.D. in Computer, Information, and Control Engineering (CICE), The University of Michigan, 1979 and 1985
 - Professor at Sharif University of Technology, 1994 -
 - Visiting Professor at The University of Michigan, 2023 -

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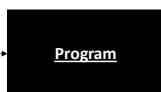
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What is this class about (practically)?



Given a “problem”, can I write a “program” to solve it on a “computer”?

problem instance (input)



answer to instance (output)

How “efficient” is my program? Can I make it more efficient?

What “algorithmic techniques” could I try?

How do I present/explain my program to other people and convince them it’s “correct”?

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Topics

Algorithms

Goal: Pass the coding interview
divide and conquer, DP, greedy

Computability

Goal: See limits of “computation”
Turing machines, undecidability

Complexity

Goal: Recognize “hard” problems
P vs NP, NP-Completeness

Advanced Algorithms

Goal: See what’s “out there”
Approximate/randomized algorithms, cryptography



At its heart, this course is about “ideas”,
the **art** of computer science.

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Course Outline

- Algorithm Design & Analysis (7 lectures)
- Computability theory (5 lectures)
- Complexity theory (6 lectures)
- Randomized algorithms (3 lectures)
- Cryptography (2 lectures)
- Review (1 lecture)
- **Total: 23 lectures + review**

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Administration

- **Website:** eeecs376.org (Syllabus, Schedule, links to the following)
- **Text:** <https://eeecs376.github.io/notes/>
 - Written by Amir Kamil for this course. Follows the lectures quite closely.
- **Canvas/Drive:** HWs, lecture slides+recordings, discussion material, OHs,...
- **Piazza:** questions (private post if sensitive)
- **Gradescope:** exam and HW submission
- **My office hours (student hours):**
 - Tuesday 3:00-5:00 pm (Dow 1031)
- **All office hours:** will be announced soon

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Administration

- 5 weekly HW assignments, due Thursdays at 8 pm (Eastern)
 - **late submissions until 9:59 pm with a 5% penalty.**
 - The lowest score will be dropped
 - Solutions published shortly after the deadline
- **Midterm:** Thursday, May 30, 2024, 7-9 pm
- **Final:** Monday, June 24, 7-9 pm

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Now onto the course content...

Topic 1: Algorithms

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Historical events in antiquity and medieval period

300BC: Euclid describes algorithms (for problems such as Greatest Common Divisor) that are still used today

TODAY

825: Muhammad ibn Musa al-Khwarizmi, often simply referred to as Al-Khwarizmi, was a Persian polymath who made significant contributions to the development of algebra and arithmetic during the Islamic Golden Age.

He is best known for his works that introduced the Hindu-Arabic numeral system to the Arab world and later to Europe.

In the 12th century, Latin-language translations of al-Khwarizmi's textbook on Indian arithmetic (*Algorithm de Numero Indorum*), which codified the various Indian numerals, introduced the decimal-based positional number system to the Western world.

Likewise, *Al-Jabr*, translated into Latin by the English scholar Robert of Chester in 1145, was used until the 16th century as the principal mathematical textbook of European universities.



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Historical events in Early Middle Ages

- Various mathematicians developed the algorithms for basic arithmetic operations within the decimal-based positional number system over time across different cultures.
- However, a significant milestone in this development is attributed to Al-Khwarizmi.
- His name gave rise to the English terms *algorithm* and *algorithm*; the Spanish, Italian, and Portuguese terms *algoritmo*; and the Spanish term *guarismo* and Portuguese term *algarismo*, both meaning "digit".

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Roman numerals

Roman numerals are a numeral system that originated in ancient Rome and remained the usual way of writing numbers throughout Europe well into the Late Middle Ages.

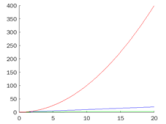
Individual decimal places			
	Thousands	Hundreds	Tens
1	M	C	X
2	MM	CC	XX
3	MMM	CCC	XXX
4		CD	XL
5		D	L
6		DC	LX
7		DCC	LXX
8		DCCC	LXXX
9		CM	XC

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Review: Running Time

- Measure of efficiency of algorithms: how worst-case running time scales with input size
- We express this asymptotically using Big-O notation: e.g., $O(\log n)$, $O(n)$, $O(n^2)$ where n is the input size.
- Common interpretations of input size:
 - size of array = # elements
 - size of graph = # vertices + # edges
 - size of integer = # digits = $O(\log(\text{size of integer}))$
 - Rule of thumb: size = # bits to represent input



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"Efficient": running time polynomial in input size

Big O Notation

Let f and g be functions $f, g: \mathcal{N} \rightarrow \mathcal{R}^+$. Say that $f(n) = O(g(n))$ if positive integers c and n_0 exist such that for every integer $n \geq n_0$,

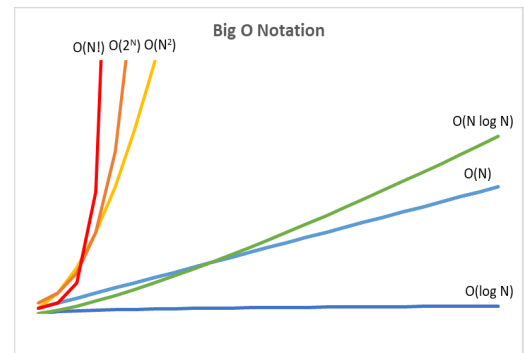
$$f(n) \leq c g(n).$$

When $f(n) = O(g(n))$, we say that $g(n)$ is an upper bound for $f(n)$, or more precisely, that $g(n)$ is an asymptotic upper bound for $f(n)$, to emphasize that we are suppressing constant factors.

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Examples: Big O Notation



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Exponential vs. Polynomial

	n=10	n=35	n=60	n=85
n^2	100 < 1 sec	1225 < 1 sec	3600 < 1 sec	7225 < 1 sec
n^3	1000 < 1 sec	43k < 1 sec	216k < 1 sec	614k < 1 sec
2^n	1024 < 1 sec	34×10^9 < 1 sec		

"Efficient": running time polynomial in input size

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Exponential vs. Polynomial

An experiment to try with a child (or adult):

If I pay you every day for a month, would you rather receive:

- On the 1st day 1 penny,
on the 2nd day 2 pennies,
on the 3rd day 4 pennies,
on the 4th day 8 pennies, etc.

OR

- \$100 per day
- $2^{30} - 1$ pennies = \$10,737,418.23
- \$3000

"Efficient": running time polynomial in input size

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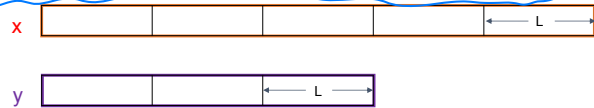
Now back to the oldest algorithm...

The Tiling problem (aka gcd)

Input: n-bit integers $x \geq y \geq 0$, but not both = 0.

Output: greatest common divisor $\gcd(x, y)$
largest integer L that divides both x and y

In other words: largest integer tile size for tiling both paths of length x and y



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Example: Compute these

- $\gcd(25, 15)$ 5
- $\gcd(12, 12)$ 12
- $\gcd(10, 13)$ 1
- $\gcd(10, 1)$ 1
- $\gcd(10, 0)$ 10

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What is the simplest brute force algorithm you can think of?

Algorithm:

```
Naïve(x, y):
1: for  $\ell = y, y - 1, \dots, 1$ :
2:   if  $\ell$  divides  $x$  and  $y$ :
3:     return  $\ell$ 
```

2 mod operations every time

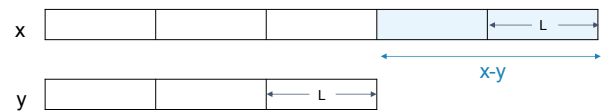
Worst-case running time in terms of n :

$O(y)$ (linear)

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Observation: We can reduce the size of problem

The answer is the same for x, y as it is for $x-y, y$



Lemma 5: $\forall m \in \mathbb{Z}, (x, y) = (x - my, y)$

Reason: (x, y) can be L-tiled $\Leftrightarrow (x-y, y)$ can be L-tiled

$\Rightarrow m = \lfloor \frac{x}{y} \rfloor$

- If L divides x and y , then L divides $x-y$ (and y , of course)
- If L divides $x-y$ and y , then L divides x (and y , of course)

$x - my = r \text{ mod } y$

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Euclid's idea! (actually it was known before Euclid)

The answer is the same for:

- x, y
- $x-y, y$
- $x-2y, y$
- $x-ky, y$ for any k such that $x-ky \geq 0$
- $x \text{ mod } y, y$

Now we have a smaller instance of the same problem.

What should we do?

Corollary: $(x, y) = (x \text{ mod } y, y)$

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Euclid's Algorithm

Euclid(x, y): // for integers $x \geq y \geq 0$ (since $\forall x, (x, 0) = x$)

Base case: If $y = 0$, return x

Recursive case: Else return Euclid($y, x \text{ mod } y$)

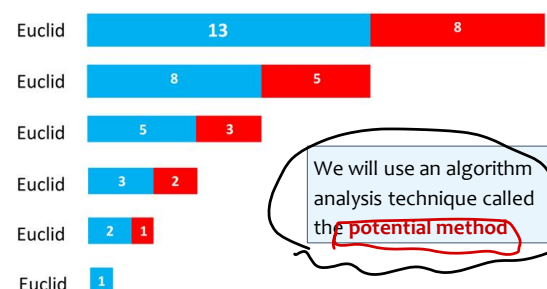
Next time: when x, y have n digits

- We will show this algo runs in $O(n)$ time
- instead of $2^{O(n)}$

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Next time: Analyzing Euclid's Algorithm

We will show that, in each recursive call to Euclid, the x, y arguments are collectively decreasing very quickly



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