EECS 376 Midterm Exam

Multiple Choice (5 points each)

1. Consider the following algorithm:

```
function FUNC(A[1, 2, ..., n])
    if n = 1 then
        return A[1]
    x ← Func(A[1, 2, ..., 2n/3])
    y ← Func(A[n/3 + 1, n/3 + 2, ..., n])
    z ← Helper(A[1, 2, ..., n])
    return min(x, y, z)
```

   Suppose that $\text{Helper}(A[1,2,\ldots,n])$ takes $O(n^2)$ time. Which of the following is the tightest bound on the runtime complexity of $\text{Func}(A[1,2,\ldots,n])$?

   ◯ $O(n)$
   ◯ $O(n^{\log_{2/3} 2})$
   ◯ $O(n^2)$
   ◯ $O(n^2 \log n)$

2. Consider the following algorithm:

```
function ISSUMOFSQUARES(k (a positive integer))
    for a = 1, 2, ..., k do
        for b = 1, 2, ..., k do
            if a^2 + b^2 = k then
                return true
    return false
```

   This algorithm runs in polynomial time with respect to the size of the input k.

   ◯ True
   ◯ False

3. Suppose *Alg* is a bottom-up dynamic-programming algorithm that works on a one-dimensional table of size n when given an input of size n. Then *Alg* (◯ always / ◯ sometimes / ◯ never) has a runtime complexity of $O(n)$.

4. ~~Suppose a country is considering a set of coin denominations with values \$1, \$5, and \$k. Then for (◯ all / ◯ some / ◯ no) values of k ≤ 10, the greedy strategy for making change for n ≥ 1 dollars always results in the minimal number of coins.~~

5. Which one of the following sets is uncountable?

   ◯ The set of all recognizable languages
   ◯ The set of all finite languages
   ◯ The set of all irrational numbers
   ◯ The set Σ^* where Σ is the set of ASCII characters
   ◯ None of the sets are uncountable

6. ~~Which one of the following languages is decidable?~~
   - ◯ $L_{n\text{-HALT}} = \left\{ \begin{array}{ll} \langle M, n \rangle : & n \in \mathbb{N} \text{ and } M \text{ accepts some string of} \\ & \text{length } n \text{ in fewer than } n \text{ steps} \end{array} \right\}$
   - ◯ $\cancel{L_{\text{LOOPS}} = \{\langle M \rangle : M \text{ loops on the string ``LOOP''}\}}$
   - ◯ $\cancel{L_{\text{NEQ}} = \{(\langle M_1 \rangle, \langle M_2 \rangle) : L(M_1) \neq L(M_2)\}}$
   - ◯ $L_{\text{NOT-SMALL}} = \{\langle M \rangle : M \text{ rejects all strings of length less than } 376\}$
   - ◯ None of the languages are decidable

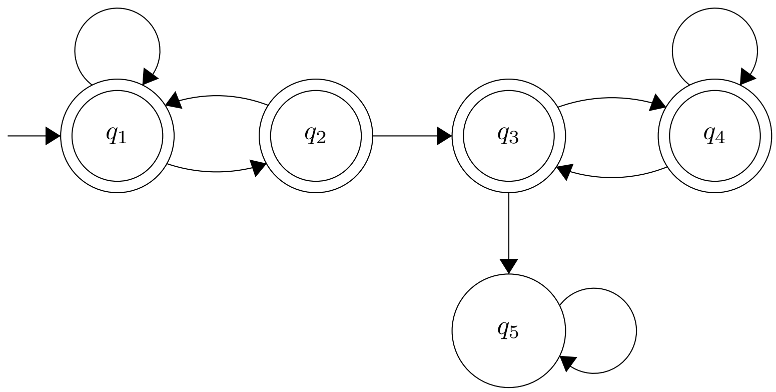
7. Let L_1 be an undecidable language. Then L_1 (◯ always / ◯ sometimes / ◯ never) has some strict subset $L_2 \subsetneq L_1$ that is also undecidable.

8. (◯ All / ◯ Some / ◯ No) languages that can be decided by a DFA can also be decided by a C++ program.

Written Answer (15 points each)

9. (a) Let $L_1 \subseteq \{0,1\}^*$ be the set of all binary strings that contain at most one occurrence of the substring "11". For example, 001010110 and 1100101 are both strings in L_1 but 0111001 is not, as the substring "11" occurs at both positions 1-2 and positions 2-3.

   Fill in the transitions of the following DFA over the alphabet $\{0,1\}$ so that the DFA decides the language L_1.



   (b) Let $L_2 \subseteq \{a,b,c\}^*$ be the set of all strings over the alphabet $\{a,b,c\}$ **except** those that contain both at least one b and at least one c. For example, aa, aba, cca are all in L_2, but abc is not as it contains both a b and a c.

   Write a DFA over the alphabet $\{a,b,c\}$ that decides the language L_2.

   Hint: The DFA needs at most four states to decide L_2.

10. Suppose input is a function that returns a user-specified positive integer. For each of the following programs, determine if the program halts for all possible valid inputs x.

    Either provide a proof of termination for all possible valid inputs x, or provide a specific input that causes the program to loop along with a brief explanation for why it loops on that input.

    Hint: Consider how the value of x changes after two iterations of the loop.

(a)

```
x ← input()
while x > 10 do
    if x is odd then
        x ← x + 3
    else
        x ← x/2
```

(b)

```
x ← input()
while x > 10 do
    if x is odd then
        x ← (x − 1)/2
    else
        x ← x + 2
```

11. ~~Consider the following language.~~

$$L_{\text{ALL-REJECT}} = \{\langle M \rangle : M \text{ is a Turing Machine and } M \text{ rejects all inputs}\}$$

~~Show that $L_{\text{ACC}} \leq_T L_{\text{ALL-REJECT}}$ or show that $L_{\text{HALT}} \leq_T L_{\text{ALL-REJECT}}$. (Do whichever one of the two you would prefer.)~~

12. You are organizing a trip for k students to attend the Rose Bowl, and you are looking to rent buses to take the students there. The bus company has n buses available, where bus i has $S(i)$ seats but costs $C(i)$ to rent. Your goal is to minimize the total cost to rent buses for the k students. (Each bus can only be used at most once.)

    Let $MB(i, j)$ be the minimum cost to rent buses for j students, allowing only buses $1, 2, \ldots, i$ to be rented. (Define $MB(i, j) = \infty$ for the cases where buses $1, 2, \ldots, i$ cannot accommodate j students.)

    (a) Provide a recurrence for $MB(i, j)$ (including base case(s)). Briefly justify your answer.