Thm ① 1.3.1 (Number of sols in a linear system)

(a) A system of equas is said to be

$\boxed{\text{consistent}}$ if ≥ 1 sols.

$\boxed{\text{inconsistent}}$ if 0 sol.

(b) If a system of linear equas is consistent $\Rightarrow$ either inf many sols, or exactly 1 sol

Def ① 1.3.2 $\boxed{\text{Rank}}$ of a matri

Rank(A) = num of $\boxed{\text{leading 1's}}$ in $\boxed{\text{rref (A)}}$

Thm ② 1.3.3 Num of equas v.s. num of unknowns

(a) If a linear sys : $\boxed{\text{1 sol}}$

$\Rightarrow \boxed{m \leq n}$ (variables < equas)

(b) (contrapositive)

if $\boxed{n < m} \Rightarrow$ either $\boxed{\text{inf sol}}$ or $\boxed{\text{0 sol.}}$

Thm ④ 1.3.8 $\boxed{A\vec{x} \text{ by col vector}}$

$$A\vec{x} = \begin{bmatrix} | & | & & | \\ \vec{v_1} & \vec{v_2} & \cdots & \vec{v_m} \\ | & | & & | \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{bmatrix} = x_1\vec{v_1} + x_2\vec{v_2} + \dots + x_m\vec{v_m}$$

Def ⑤ 1.3.9 Linear combination

$\vec{b} \in \mathbb{R}^n$ is called a linear combi

of $\boxed{\vec{v_1}, \vec{v_2}, \dots, \vec{v_m}}$ $(\vec{v_i} \in \mathbb{R}^n)$

if $\exists$ $\boxed{\text{scalars}}$ $\boxed{x_1, \dots, x_m}$

s.t. $\boxed{\vec{b} = x_1\vec{v_1} + x_2\vec{v_2} + \dots + x_m\vec{v_m}}$

Thm ③ 1.3.4

A linear sys A (coeff) of n equas, n varis has a unique sol

iff rank(A) = n

Def ② 1.3.5

(a) Sum of matrices

$$A + B = \begin{bmatrix} a_{11}+b_{11} & \cdots & a_{1m}+b_{1m} \\ \vdots & & \\ a_{n1}+b_{n1} & \cdots & a_{nm}+b_{nm} \end{bmatrix}$$

(b) Scalar multip

$$kA = \begin{bmatrix} ka_{11} & \cdots & ka_{1m} \\ \vdots & & \\ ka_{n1} & \cdots & ka_{nm} \end{bmatrix}$$

Def ③ 1.3.6 Dot product of vectors ($\boxed{\text{same length}}$ $\boxed{\text{不论 same type}}$)

$\vec{v} \cdot \vec{w} = v_1 w_1 + \dots + v_n \cdot w_n$

Def ④ 1.3.7 $\boxed{A\vec{x}}$ $(A_{n\times m}, \vec{x} \in \mathbb{R}^m)$

$$A\vec{x} = \begin{bmatrix} - & \vec{w_1} & - \\ - & \vec{w_2} & - \\ & \vdots & \\ - & \vec{w_n} & - \end{bmatrix} \vec{x} = \begin{bmatrix} \vec{w_1}\cdot\vec{x} \\ \vec{w_2}\cdot\vec{x} \\ \vdots \\ \vec{w_n}\cdot\vec{x} \end{bmatrix}$$

Thm ⑤ 1.3.10 Algebriac Rules for $A\vec{x}$

A : $n \times m$ matrix

$x, y \in \mathbb{R}^m$

k : scalar

$\Rightarrow$ (a) $A(\vec{x}+\vec{y}) = A\vec{x} + A\vec{y}$

(b) $A(k\vec{x}) = k(A\vec{x})$

Thm ⑥ 1.3.11 Matrix form of linear sys

We can write augment matrix $[A \,\vdots\, \vec{b}]$ of a linear sys in matrix form

as $\boxed{A\vec{x} = \vec{b}}$