Problem 1. (1 point)

Consider the transformation $D(f) = f'$ from $P_3 \to P_3$, where P_3 is the space of all polynomials of degree ≤ 3. Find each of the following:

$\dim(P_3) =$ _____

$\text{nullity}(D) =$ _____

$\text{rank}(D) =$ _____

Answer(s) submitted:

- 4
- 1
- 3

submitted: (correct)

recorded: (correct)

Problem 2. (1 point)

Which of the following functions $T : \mathbb{R}^2 \to \mathbb{R}^2$ are isomorphisms?

- A. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 7x+7y \\ 7x+7y \end{bmatrix}$
- B. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 2x+y \\ x \end{bmatrix}$
- C. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x \\ -3 \end{bmatrix}$
- D. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} x^3 \\ y^3 \end{bmatrix}$
- E. $T(\begin{bmatrix} x \\ y \end{bmatrix}) = \begin{bmatrix} 7x \\ 7y \end{bmatrix}$

Answer(s) submitted:

- BE

submitted: (correct)

recorded: (correct)

Problem 3. (1 point)

Let V and W be vector spaces with $\dim(V) = \dim(W) = 3$. Consider a linear transformation $T : V \to W$ with $\ker(T) = \text{span}(f)$, with $f \neq 0$.

(a) Is T an isomorphism?

- ?
- Yes.
- No.
- Not enough information to tell.

(b) Are V and W isomorphic?

- ?
- Yes.
- No.
- Not enough information to tell.

(c) Does there exist an isomorphism $S : V \to W$?

- ?
- Yes.
- No.
- Not enough information to tell.

(d) Suppose that $T : P_k \to \mathbb{R}^4$, where P_k is the vector space of polynomials of degree $\leq k$. If $T(f(x)) = \begin{bmatrix} f(0) \\ f(1) \\ f(2) \\ f(3) \end{bmatrix}$, give a value of k so that T is an isomorphism.

$k =$ ___

Answer(s) submitted:

- No.
- Yes.
- Yes.
- 3

submitted: (correct)

recorded: (correct)