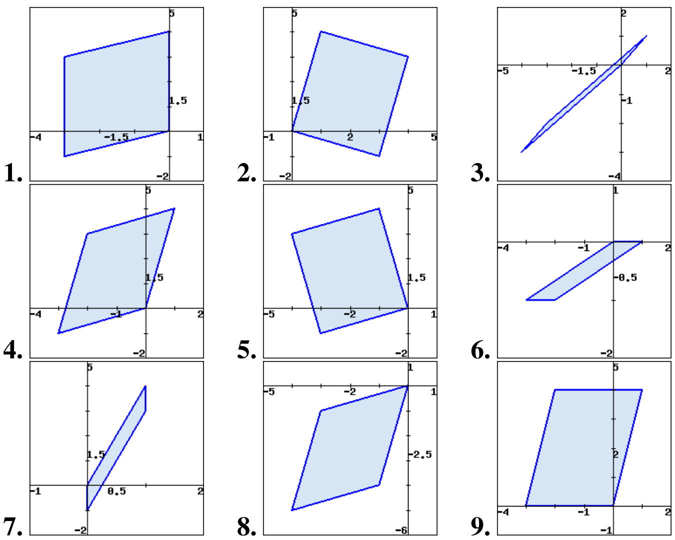
Qiulin Fan **ma217-w24**

Assignment webHW3 due 02/05/2024 at 11:59pm EST

Problem 1. **(1 point)**

Suppose that $T(\mathbf{x}) = A\mathbf{x}$ for $A = \begin{bmatrix} 1 & -3 \\ 4 & -1 \end{bmatrix}$. Sketch a graph of the image under T of the unit square in the first quadrant. Which of the following is the correct image?
graph [?/1/2/3/4/5/6/7/8/9]



Answer(s) submitted:

- 4

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Let L be the line through the origin and the vector $\begin{bmatrix} 6 \\ 8 \end{bmatrix}$. Find the matrix P of the transformation given by orthogonal projection onto L.

$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$.

Answer(s) submitted:

- $\begin{bmatrix} 0.36 & 0.48 \\ 0.48 & 0.64 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Let L be the line through the origin and the vector $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$. Find the reflection of the vector $\begin{bmatrix} -5 \\ 4 \end{bmatrix}$ about L.

$\mathrm{ref}_L\left(\begin{bmatrix} -5 \\ 4 \end{bmatrix}\right) = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$.

Answer(s) submitted:

- $\begin{bmatrix} -\frac{43}{17} \\ -\frac{100}{17} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation where $T(\vec{x})$ is obtained by first scaling $\vec{x}$ by a factor of 5 and then rotating the result *clockwise* by an angle of $\pi/6$. What is the matrix A of the transformation?

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} \frac{5\sqrt{3}}{2} & \frac{5}{2} \\ \frac{5}{-2} & \frac{5\sqrt{3}}{2} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

Consider the matrices

$A = \begin{bmatrix} 7 & -1 & -5 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ -5 \\ -3 \\ -3 \end{bmatrix}$, $C = \begin{bmatrix} 5 & 7 \\ -1 & -5 \\ -5 & -3 \end{bmatrix}$, and $D = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

Which of the following products are well-defined? Select all that are correct.

- A. AC
- B. BC
- C. CA
- D. AB
- E. AD
- F. DC
- G. BA
- H. DA
- I. CD

Answer(s) submitted:

- AFGI

submitted: (correct)
recorded: (correct)

Problem 6. (1 point)

Find the missing values a-f in the matrix equation

$$\begin{bmatrix} a & -4 \\ b & 3 \\ -3 & -1 \end{bmatrix} \begin{bmatrix} 4 & -1 & 1 \\ c & d & -2 \end{bmatrix} = \begin{bmatrix} -4 & -5 & e \\ -2 & 5 & -5 \\ f & 1 & -1 \end{bmatrix}.$$

$a =$ ____; $b =$ ____
$c =$ ____; $d =$ ____
$e =$ ____; $f =$ ____

Answer(s) submitted:

- -3
- 1
- -2
- 2
- 5
- -10

submitted: (correct)
recorded: (correct)

Problem 7. (1 point)

Let T_1 and T_2 be linear transformations given by

$$T_1\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 6y - x \\ -4y \end{bmatrix}$$

$$T_2\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 6x - 2y \\ -(6x+y) \end{bmatrix}$$

Find the matrix A such that

(a) $T_1(T_2(\mathbf{x})) = A\mathbf{x}$: $A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

(b) $T_2(T_1(\mathbf{x})) = A\mathbf{x}$: $A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

(c) $T_1(T_1(\mathbf{x})) = A\mathbf{x}$: $A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

(d) $T_2(T_2(\mathbf{x})) = A\mathbf{x}$: $A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} -42 & -4 \\ 24 & 4 \end{bmatrix}$
- $\begin{bmatrix} -6 & 44 \\ 6 & -32 \end{bmatrix}$
- $\begin{bmatrix} 1 & -30 \\ 0 & 16 \end{bmatrix}$
- $\begin{bmatrix} 48 & -10 \\ -30 & 13 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 8. (1 point)

Find two 3×3 matrices A and B such that $AB \neq BA$.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}; \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 9. (1 point)

Consider the block matrices

$$E = \begin{bmatrix} A & B \\ B & A \end{bmatrix} \text{ and } F = \begin{bmatrix} I_2 & I_2 \\ I_2 & I_2 \end{bmatrix},$$

where

$$A = \begin{bmatrix} 4 & -3 \\ -4 & 3 \end{bmatrix} \text{ and } B = \begin{bmatrix} 5 & -1 \\ 0 & 1 \end{bmatrix}.$$

Find the product EF.

$EF = \begin{bmatrix} \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ & \_\_ \end{bmatrix}.$

Answer(s) submitted:

- $\begin{bmatrix} 9 & -4 & 9 & -4 \\ -4 & 4 & -4 & 4 \\ 9 & -4 & 9 & -4 \\ -4 & 4 & -4 & 4 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 10. (1 point)

If

$$A = \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix},$$

then

$A^{-1} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$

Answer(s) submitted:

- $\begin{bmatrix} -1 & -1 & 1 \\ -1 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 11. (1 point)

a. The linear transformation $T_1 : R^2 \to R^2$ is given by:

$T_1(x,y) = (2x+7y, 10x+36y)$.

Find $T_1^{-1}(x,y)$.

$T_1^{-1}(x,y) =$ (___x + ___y, ___x + ___y)

b. The linear transformation $T_2 : R^3 \to R^3$ is given by:

$T_2(x,y,z) = (x+2z, 1x+y, 2y+z)$.

Find $T_2^{-1}(x,y,z)$.

$T_2^{-1}(x,y,z) =$ (___x + ___y + ___z, ___x + ___y + ___z, ___x + ___y + ___z)

c. Using T_1 from part a, it is given that:

$T_1(x,y) = (5,-4)$

Find x and y.

$x =$ ___ $y =$ ___

d. Using T_2 from part b, it is given that:

$T_2(x,y,z) = (4,-6,2)$

Find x, y, and z.

$x =$ ___ $y =$ ___ $z =$ ___

Answer(s) submitted:

- 18
- $-\frac{7}{2}$
- -5
- 1
- $\frac{1}{5}$
- $\frac{4}{5}$
- $-\frac{2}{5}$
- $-\frac{1}{5}$
- $\frac{1}{5}$
- $\frac{2}{5}$
- $\frac{2}{5}$
- $-\frac{2}{5}$
- $\frac{1}{5}$
- 104
- -29
- $\frac{-24}{5}$
- $-\frac{6}{5}$
- $\frac{22}{5}$

submitted: (correct)
recorded: (correct)

Problem 12. (1 point)

Consider the parallelogram with vertices at the origin, at $(-5,9)$, at $(6,2)$, and at $(1,11)$. Find its area.

Area = __________.

Answer(s) submitted:

- 64

submitted: (correct)
recorded: (correct)