Problem 1. **(1 point)**

The set

$$B = \left\{ \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \end{bmatrix} \right\}$$

is a basis for $\mathbb{R}^2$. Find the coordinates of the vector $\vec{x} = \begin{bmatrix} 6 \\ 4 \end{bmatrix}$ relative to the basis B.

$[\vec{x}]_B = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Find the coordinate vector of $\vec{x} = \begin{bmatrix} -1 \\ 5 \\ 4 \end{bmatrix}$ with respect to the basis $B = \left\{ \begin{bmatrix} 1 \\ 7 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ -6 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$ for $\mathbb{R}^3$.

$[\vec{x}]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} -1 \\ 12 \\ 82 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be a linear transformation $T(\vec{x}) = A\vec{x}$, with

$$A = \begin{bmatrix} 2 & -2 & 1 \\ -3 & 1 & -2 \\ 1 & 3 & -1 \end{bmatrix}.$$

The set $\mathfrak{B} = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is a basis of $\mathbb{R}^3$. Find B, the $\mathfrak{B}$-matrix of T.

$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}.$

Answer(s) submitted:

- $\begin{bmatrix} 1 & -11 & 5 \\ 1 & -4 & -1 \\ -1 & 1 & 5 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Let $\vec{b}_1 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ and $\vec{b}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$; the set $\mathfrak{B} = \left\{\vec{b}_1, \vec{b}_2\right\}$ is a basis for $\mathbb{R}^2$. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\vec{b}_1) = 4\vec{b}_1 + 4\vec{b}_2$ and $T(\vec{b}_2) = 7\vec{b}_1 + 6\vec{b}_2$. Find B, the $\mathfrak{B}$-matrix of T.

$B = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}.$

Answer(s) submitted:

- $\begin{bmatrix} 4 & 7 \\ 4 & 6 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

Let $\vec{b}_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$ and $\vec{b}_2 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. The set $B = \left\{\vec{b}_1, \vec{b}_2\right\}$ is a basis for $\mathbb{R}^2$. Suppose that $T : \mathbb{R}^2 \to \mathbb{R}^2$ is a linear transformation whose $\mathfrak{B}$-matrix, B, is

$$B = \begin{bmatrix} 1 & -1 \\ 1 & -2 \end{bmatrix}.$$

Find the matrix A of T relative to the *standard* basis for $\mathbb{R}^2$.

$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Let B be the basis of $\mathbb{R}^2$ consisting of the vectors

$$\left\{ \begin{bmatrix} 5 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix} \right\},$$

and let C be the basis consisting of

$$\left\{ \begin{bmatrix} -2 \\ -1 \end{bmatrix}, \begin{bmatrix} -3 \\ -2 \end{bmatrix} \right\}.$$

Find a matrix P such that $[\vec{x}]_C = P[\vec{x}]_B$ for all $\vec{x}$ in $\mathbb{R}^2$.

$$P = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Answer(s) submitted:

- $\begin{bmatrix} -13 & 7 \\ 7 & -5 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

The set $B = \{4-3x^2,\ 20+x-15x^2,\ 24x^2-(30+2x)\}$ is a basis for P_2. Find the coordinates of $p(x) = 69x^2-(88+6x)$ relative to this basis:

$$[p(x)]_B = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

Answer(s) submitted:

- $\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 8. **(1 point)**

If $T : P_1 \to P_1$ is a linear transformation such that $T(1+5x) = -3+2x$ and $T(5+24x) = 2-4x$, then

$T(4-3x) =$ ____________.

Answer(s) submitted:

- $379-314x$

submitted: (correct)
recorded: (correct)

Problem 9. **(1 point)**

Let $T : P_2 \to P_2$ be a linear transformation such that

$T(2x^2) = 2x^2+3x,\quad T(0.5x+4) = -3x^2+3x+1,\quad T(3x^2-1) = 4x-4.$

Find $T(1)$, $T(x)$, $T(x^2)$, and $T(ax^2+bx+c)$, where a, b, and c are arbitrary real numbers.

$T(1) =$ ____________________,
$T(x) =$ ____________________,
$T(x^2) =$ ____________________,
$T(ax^2+bx+c) =$ ____________________.

Answer(s) submitted:

- $3x^2+0.5x+4$
- $-30x^2+2x-30$
- $x^2+1.5x$
- $(a-30b+3c)x^2+(1.5a+2b+0.5c)x+4c-30b$

submitted: (correct)
recorded: (correct)

Problem 10. **(1 point)**

Let $T : P_3 \to P_3$ be defined by

$$T(ax^2+bx+c) = (4a+b)x^2+(4a-2b+c)x-a.$$

Find the inverse of T.

$T^{-1}(ax^2+bx+c) =$ ____________________.

Answer(s) submitted:

- $-cx^2+(a+4c)x+2a+b+12c$

submitted: (correct)
recorded: (correct)