Problem 1. **(1 point)**

Determine if λ is an eigenvalue of the matrix A.

[?] 1. $A = \begin{bmatrix} 5 & 0 & 0 \\ -1 & 4 & 6 \\ -3 & -3 & -5 \end{bmatrix}, \lambda = 1$

[?] 2. $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -2 & 0 \\ 1 & 3 & 1 \end{bmatrix}, \lambda = 6$

[?] 3. $A = \begin{bmatrix} 0 & -3 & -6 \\ 6 & 3 & 0 \\ -3 & -3 & -3 \end{bmatrix}, \lambda = -5$

Answer(s) submitted:

- `YES`
- `NO`
- `NO`

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Let $A = \begin{bmatrix} -8 & -8 \\ 2 & k \end{bmatrix}$.
For A to have 0 as an eigenvalue, k must be ___
Answer(s) submitted:

- 2

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Give an example of a 2×2 matrix with no real eigenvalues.

$\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$
Answer(s) submitted:

- $\begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

The matrix
$$A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 0 & 2 \\ -4 & -1 & -3 \end{bmatrix}$$
has eigenvalues -2, -1, and 3. Find its eigenvectors.

The eigenvalue -2 has associated eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue -1 has associated eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

The eigenvalue 3 has associated eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Answer(s) submitted:

- $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$
- $\begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}$
- $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

The matrix
$$A = \begin{bmatrix} -2 & 1 & 0 \\ -1 & -4 & 0 \\ 1 & -1 & -1 \end{bmatrix}$$
has $\lambda = -3$ as an eigenvalue with algebraic multiplicity 2 and $\lambda = -1$ as an eigenvalue with algebraic multiplicity 1. Find the associated eigenvectors.

The eigenvalue -3 is associated with the eigenvector (___, ___, ___ $)^T$.

The eigenvalue -1 is associated with the eigenvector (___, ___, ___ $)^T$.
Answer(s) submitted:

- Eigenvectors $= (1,-1,-1),(1,1,2)$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Find the eigenvalues of the matrix A.

$$A = \begin{bmatrix} -18 & -10 & 0 & 0 \\ 15 & 7 & 0 & 0 \\ 0 & 0 & 6 & -3 \\ 0 & 0 & 0 & 3 \end{bmatrix}.$$

The eigenvalues are $\lambda_1 < \lambda_2 < \lambda_3 < \lambda_4$, where

$\lambda_1 =$ ___ has an eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\lambda_2 =$ ___ has an eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$,

$\lambda_3 =$ ___ has an eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$, $\lambda_4 =$ ___ has an eigenvector $\begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$.

Note: you may want to use a graphing calculator to estimate the roots of the polynomial which defines the eigenvalues.

Answer(s) submitted:

- -8
- $\begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$
- -3
- $\begin{bmatrix} -2 \\ 3 \\ 0 \\ 0 \end{bmatrix}$
- 3
- $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix}$
- 6
- $\begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

The matrix
$A = \begin{bmatrix} -20 & -5 \\ 29 & 4 \end{bmatrix}$
has complex eigenvalues $\lambda_1 = a + bi$ and $\lambda_2 = a - bi$ where $a =$ ___ and $b =$ ___.

The corresponding eigenvectors are $\vec{v} = \vec{c} \pm \vec{d}i$ where $\vec{c}^T = \left(\_\_, \_\_ \right)$ and $\vec{d}^T = \left(\_\_, \_\_ \right)$.

Answer(s) submitted:

- Eigenvalues $= (-8, 1), c = (-12, 29), d = (1, 0)$

submitted: (correct)
recorded: (correct)

Problem 8. **(1 point)**

Find all the eigenvalues (real and complex) of the matrix

$$M = \begin{bmatrix} -17 & 22 & 14 \\ -8 & 9 & 6 \\ -8 & 12 & 7 \end{bmatrix}.$$

The eigenvalues are ______________. (Enter your answers as a comma separated list.)

Answer(s) submitted:

- $-1, i\sqrt{7}, -i\sqrt{7}$

submitted: (correct)
recorded: (correct)

Problem 9. **(1 point)**

Find all the values of k for which the matrix

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & k-9 & -k+10 \end{bmatrix}$$

is not diagonalizable over $\mathbb{C}$.

$k =$ ________ (Enter your answers as a comma separated list.)

Answer(s) submitted:

- $8, 9$

submitted: (correct)
recorded: (correct)

Problem 10. **(1 point)**

Find the eigenvalues $\lambda_1 < \lambda_2 < \lambda_3$ and associated unit eigenvectors $\vec{u}_1, \vec{u}_2, \vec{u}_3$ of the symmetric matrix

$$A = \begin{bmatrix} 0 & 0 & 7 \\ 0 & -2 & 0 \\ 7 & 0 & 0 \end{bmatrix}.$$

The eigenvalue $\lambda_1 =$ ____ has associated unit eigenvector $\vec{u}_1 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

The eigenvalue $\lambda_2 =$ ____ has associated unit eigenvector $\vec{u}_2 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

The eigenvalue $\lambda_3 =$ ____ has associated unit eigenvector $\vec{u}_3 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Note: The eigenvectors above form an orthonormal eigenbasis for A.

Answer(s) submitted:

- -7
- $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ -\frac{1}{\sqrt{2}} \end{bmatrix}$
- -2
- $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- 7
- $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ 0 \\ \frac{1}{\sqrt{2}} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 11. **(1 point)**

Find the eigenvalues $\lambda_1 < \lambda_2$ and associated orthonormal eigenvectors of the symmetric matrix

$$A = \begin{bmatrix} -1 & -3 & 0 & 0 \\ -3 & -1 & 0 & 0 \\ 0 & 0 & -1 & -3 \\ 0 & 0 & -3 & -1 \end{bmatrix}.$$

$\lambda_1 =$ ____ has associated orthonormal eigenvectors $\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$.

$\lambda_2 =$ ____ has associated orthonormal eigenvectors $\begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}$.

Note: The eigenvectors above form an orthonormal eigenbasis for A.

Answer(s) submitted:

- -4
- $\begin{bmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}$
- 2
- $\begin{bmatrix} 0 \\ 0 \\ -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{bmatrix}, \begin{bmatrix} -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \\ 0 \\ 0 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 12. **(1 point)**

Let L from $\mathbb{R}^3$ to $\mathbb{R}^3$ be the reflection about the line spanned by

$$\vec{v} = \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}.$$

*(**Note**: you must enter a value for all parts of this problem for the system to correctly check whether your answers are correct.)*

(a) Find an orthonormal eigenbasis $\mathfrak{B}$ for L.

$\vec{v}_1 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

$\vec{v}_2 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

$\vec{v}_3 = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$

(b) Find the matrix B of L with respect to $\mathfrak{B}$.

$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

(c) Find the matrix A of L with respect to the standard basis of $\mathbb{R}^3$.

$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} \frac{1}{\sqrt{10}} \\ 0 \\ -\frac{3}{\sqrt{10}} \end{bmatrix}$
- $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$
- $\begin{bmatrix} \frac{3}{\sqrt{10}} \\ 0 \\ \frac{1}{\sqrt{10}} \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$
- $\begin{bmatrix} -\frac{4}{5} & 0 & -\frac{3}{5} \\ 0 & -1 & 0 \\ -\frac{3}{5} & 0 & \frac{4}{5} \end{bmatrix}$

submitted: (correct)
recorded: (correct)