A system of linear equa.

$$\begin{vmatrix} x+2y+3z=39 \\ x+3y+2z=34 \\ 3x+2y+z=26 \end{vmatrix}$$

To solve for x,y,z, we need to transform the system into the form

$$\begin{vmatrix} x = \dots \\ y = \dots \\ z = \dots \end{vmatrix}$$

(in other words)

IDW, we need to

① Deliminate terms that are off the diagonal

② make the coefficients of the variables along the diagonal equal to 1

$$\begin{vmatrix} x+2y+3z=39 \\ x+3y+2z=34 \\ 3x+2y+z=26 \end{vmatrix}$$

To do it step by step:

$$\begin{vmatrix} x+2y+3z=39 \\ x+3y+2z=34 \\ 3x+2y+z=26 \end{vmatrix} \longrightarrow \text{-1st equation (简写为 -(1))} \begin{vmatrix} x+2y+3z=39 \\ y-z=-5 \\ 3x+2y+z=26 \end{vmatrix}$$

$$\begin{vmatrix} x+2y+3z=39 \\ y-z=-5 \\ 3x+2y+z=26 \end{vmatrix} \longrightarrow \text{(简写为 -3×(1))} \text{ -3×1st equation} \begin{vmatrix} x+2y+3z=39 \\ y-z=-5 \\ -4y-8z=-91 \end{vmatrix}$$

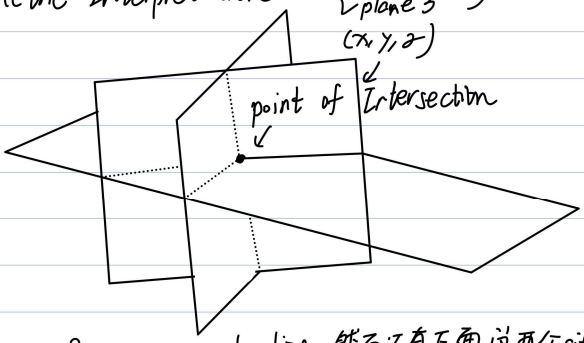
$$\begin{vmatrix} x+2y+3z=39 \\ y-z=-5 \\ -4y-8z=-91 \end{vmatrix} \text{ -2×2nd equation} \longrightarrow \text{+4×2nd equation} \begin{vmatrix} x+5z=49 \\ y-z=-5 \\ -12z=-111 \end{vmatrix}$$

$$\begin{vmatrix} x+5z=49 \\ y-z=-5 \\ -12z=-111 \end{vmatrix} \longrightarrow \div 12 \begin{vmatrix} x+5z=49 \\ y-z=-5 \\ z=9.25 \end{vmatrix}$$

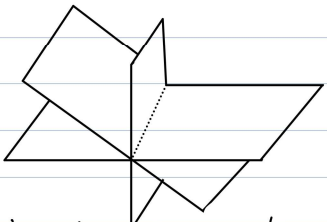
$$\begin{vmatrix} x+5z=49 \\ y-z=-5 \\ z=9.25 \end{vmatrix} \text{ -5×3rd equation + 3rd equation} \longrightarrow \begin{vmatrix} x=2.75 \\ y=4.25 \\ z=9.25 \end{vmatrix}$$

Finally we check the sol by substituting x,y,z into the original linear system. Happily in Linear Algebra it is easy to check.

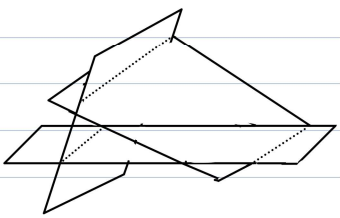
Geometric Interpretation { plane 1 ... plane 3 ⇒ (x,y,z)



这是 common situation. 然而还有下面这两个situation:



(b) 3 planes having a line in common



(c) 3 planes with no common intersection

A system with infinitely many sols. A system without sols.

ex $\begin{vmatrix} 2x+4y+6z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=6 \end{vmatrix}$

$\dots \longrightarrow \begin{vmatrix} x-z=2 \\ y+2z=-1 \end{vmatrix}$

$\longrightarrow \begin{vmatrix} x=z+2 \\ y=-2z-1 \end{vmatrix}$

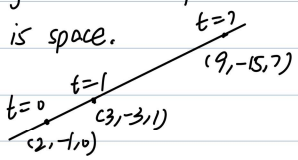
Generally, choose $z=t$, we get $x=t+2$ and $y=-2t-1$

⇒ The general solution is

$(x,y,z)=(t+2,-2t-1,t)$

$=(2,-1,0)+t(1,-2,1)$

The general sol represents a line is space.



ex $\begin{vmatrix} x+2y+3z=0 \\ 4x+5y+6z=3 \\ 7x+8y+9z=0 \end{vmatrix}$

$\dots \longrightarrow \begin{vmatrix} x-z=2 \\ y+2z=-1 \\ 0=-6 \end{vmatrix}$

Whatever value we choose, $0=-6$ cannot be satisfied.

⇒ This system is inconsistent

⇒ has no sol.

* Complement (Joy of sets): To say a set is close under some operation Δ, is to mean that $\begin{matrix} a\in S \\ b\in S \end{matrix} \Rightarrow a\Delta b\in S$