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Assignment webHW10 due 04/15/2024 at 11:59pm EDT

Problem 1. **(1 point)**

Given $\det\begin{bmatrix} \text{a} & \text{b} & \text{c} \\ \text{d} & \text{e} & \text{f} \\ \text{g} & \text{h} & \text{i} \end{bmatrix} = 4$, find the following determinants.

$\det\begin{bmatrix} \text{g} & \text{h} & \text{i} \\ \text{a} & \text{b} & \text{c} \\ \text{d} & \text{e} & \text{f} \end{bmatrix} =$ ___

$\det\begin{bmatrix} \text{a} & \text{b} & \text{c} \\ \text{7d + a} & \text{7e + b} & \text{7f + c} \\ \text{g} & \text{h} & \text{i} \end{bmatrix} =$ ___

$\det\begin{bmatrix} \text{7d + a} & \text{7e + b} & \text{7f + c} \\ \text{d} & \text{e} & \text{f} \\ \text{g} & \text{h} & \text{i} \end{bmatrix} =$ ___

Answer(s) submitted:

- 4
- 28
- 4

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

A and B are $n \times n$ matrices.

Check the true statements below:

- A. The determinant of A is the product of the pivots in the reduced row-echelon form U of A, multiplied by $(-1)^r$, where r is the number of row interchanges made during row reduction from A to U.
- B. $\det(A+B) = \det A + \det B$.
- C. If the columns of A are linearly dependent, then $\det A = 0$.
- D. Adding a multiple of one row to another does not affect the determinant of a matrix.

Answer(s) submitted:

- CD

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Use determinants to determine whether each of the following sets of vectors is linearly dependent or independent.

$\boxed{?}$ 1. $\begin{bmatrix} 1 \\ -3 \\ -2 \\ -5 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ -2 \\ -10 \end{bmatrix}, \begin{bmatrix} 5 \\ -13 \\ -6 \\ -25 \end{bmatrix}, \begin{bmatrix} 10 \\ -23 \\ -15 \\ -50 \end{bmatrix},$

$\boxed{?}$ 2. $\begin{bmatrix} -2 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ 1 \end{bmatrix}, \begin{bmatrix} -2 \\ 4 \\ -8 \end{bmatrix},$

$\boxed{?}$ 3. $\begin{bmatrix} 5 \\ -3 \\ 5 \end{bmatrix}, \begin{bmatrix} 15 \\ -11 \\ 12 \end{bmatrix}, \begin{bmatrix} -20 \\ 16 \\ -16 \end{bmatrix},$

$\boxed{?}$ 4. $\begin{bmatrix} 5 \\ 4 \\ -5 \\ -4 \end{bmatrix}, \begin{bmatrix} 4 \\ -3 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} -22 \\ 1 \\ 22 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ -3 \\ -3 \end{bmatrix},$

Answer(s) submitted:

- `LINEARLY DEPENDENT`
- `LINEARLY DEPENDENT`
- `LINEARLY INDEPENDENT`
- `LINEARLY DEPENDENT`

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Solve the system using Cramer's Rule.

$$\begin{array}{rcrcr} -9x & + & 4y & = & -4 \\ x & - & 7y & = & 3 \end{array}$$

The determinant of the coefficient matrix is: _____
$x =$ _____
$y =$ _____

Answer(s) submitted:

- 59
- $\frac{16}{59}$
- $-\frac{23}{59}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

Let $A = \begin{bmatrix} 2 & -2 & -3 \\ 0 & -1 & -3 \\ 3 & -2 & 2 \end{bmatrix}$

Find the following:

(a) $\det(A) =$ _____,

(b) $\operatorname{adj}(A) = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$,

(c) $A^{-1} = \begin{bmatrix} \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \\ \_\_\_ & \_\_\_ & \_\_\_ \end{bmatrix}$.

Answer(s) submitted:

- -7
- -8
- 10
- 3
- -9
- 13
- 6
- 3
- -2
- -2
- $\frac{8}{7}$
- $\frac{-10}{7}$
- $-\frac{3}{7}$
- $\frac{9}{7}$
- $\frac{13}{-7}$
- $\frac{-6}{7}$
- $-\frac{3}{7}$
- $\frac{2}{7}$
- $\frac{2}{7}$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Supppose A is an invertible $n \times n$ matrix and $\vec{v}$ is an eigenvector of A with associated eigenvalue 3. Convince yourself that $\vec{v}$ is an eigenvector of the following matrices, and find the associated eigenvalues.

(1) The matrix A^9 has an eigenvalue _____.

(2) The matrix A^{-1} has an eigenvalue _____.

(3) The matrix $A + 3I_n$ has an eigenvalue _____.

(4) The matrix $9A$ has an eigenvalue _____.

Answer(s) submitted:

- 19683
- $\frac{1}{3}$
- 6
- 27

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

If $\vec{v}_1 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}$ and $\vec{v}_2 = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$ are eigenvectors of a matrix A corresponding to the eigenvalues $\lambda_1 = 1$ and $\lambda_2 = -2$, respectively,

then $A(\vec{v}_1 + \vec{v}_2) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

and $A(2\vec{v}_1) = \begin{bmatrix} \_\_\_ \\ \_\_\_ \end{bmatrix}$

Answer(s) submitted:

- $\begin{bmatrix} 1 \\ 9 \end{bmatrix}$
- $\begin{bmatrix} -10 \\ -2 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 8. (1 point)

Let

$$\vec{v}_1 = \begin{bmatrix} 0 \\ -2 \\ 1 \end{bmatrix}, \vec{v}_2 = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}, \vec{v}_3 = \begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$$

be eigenvectors of the matrix A which correspond to the eigenvalues $\lambda_1 = -2$, $\lambda_2 = 0$, and $\lambda_3 = 4$, respectively, and let

$$\vec{x} = \begin{bmatrix} -1 \\ -6 \\ 0 \end{bmatrix}.$$

Express $\vec{x}$ as a linear combination of $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$, and find $A\vec{x}$.

$\vec{x} =$ ______ $\vec{v}_1 +$ ______ $\vec{v}_2 +$ ______ $\vec{v}_3$.

$$A\vec{x} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}$$

Answer(s) submitted:

- 1
- -2
- 1
- $\begin{bmatrix} 12 \\ 4 \\ -6 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 9. (1 point)

Find the characteristic polynomial of the matrix

$$A = \begin{bmatrix} 4 & -3 & 0 \\ 0 & -3 & 3 \\ -3 & 5 & 0 \end{bmatrix}.$$

$p(x) =$ ______________.

Answer(s) submitted:

- $-x^3 + x^2 + 27x - 33$

submitted: (correct)
recorded: (correct)

Problem 10. (1 point)

For which value of k does the matrix

$$A = \begin{bmatrix} -2 & k \\ 7 & -8 \end{bmatrix}$$

have one real eigenvalue of multiplicity 2?

$k =$ ________.

Answer(s) submitted:

- $-\frac{9}{7}$

submitted: (correct)
recorded: (correct)