Problem 1. **(1 point)**

If $f(x)$ and $g(x)$ are arbitrary polynomials of degree at most 2, then the mapping

$$\langle f,g\rangle = f(-3)g(-3)+f(0)g(0)+f(3)g(3)$$

defines an inner product in P_2. Use this inner product to find $\langle f,g\rangle$, $\|f\|$, $\|g\|$, and the angle $\alpha_{f,g}$ between $f(x)$ and $g(x)$ for

$$f(x) = 4x^2+5x+9 \ \text{ and } \ g(x) = 3x^2-5x-5.$$

$\langle f,g\rangle =$ ________,
$\|f\| =$ ________,
$\|g\| =$ ________,
$\alpha_{f,g} =$ ________.

Answer(s) submitted:

- 1485
- $\sqrt{4581}$
- $\sqrt{1443}$
- $\cos^{-1}\left(\frac{1485}{\sqrt{4581\cdot 1443}}\right)$

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

If

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \ \text{ and } \ B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$$

are arbitrary vectors in $\mathbb{R}^{2\times 2}$, then the mapping

$$\langle A,B\rangle = a_{11}b_{11}+a_{12}b_{12}+a_{21}b_{21}+a_{22}b_{22}$$

defines an inner product in $\mathbb{R}^{2\times 2}$. Use this inner product to determine $\langle A,B\rangle$, $\|A\|$, $\|B\|$, and the angle $\alpha_{A,B}$ between A and B for

$$A = \begin{bmatrix} 5 & -2 \\ -3 & -4 \end{bmatrix} \ \text{ and } \ B = \begin{bmatrix} 3 & 3 \\ -5 & -5 \end{bmatrix}.$$

$\langle A,B\rangle =$ ________,
$\|A\| =$ ________,
$\|B\| =$ ________,
$\alpha_{A,B} =$ ________.

Answer(s) submitted:

- 44
- $\sqrt{54}$
- $\sqrt{68}$
- $\cos^{-1}\left(\frac{44}{\sqrt{54\cdot 68}}\right)$

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Use the inner product

$$\langle f,g\rangle = \int_0^1 f(x)g(x)\,dx$$

in the vector space $C^0[0,1]$ of continuous functions on the domain $[0,1]$ to find $\langle f,g\rangle$, $\|f\|$, $\|g\|$, and the angle $\alpha_{f,g}$ between $f(x)$ and $g(x)$ for

$$f(x) = 5x^2-9 \ \text{ and } \ g(x) = 9x+3.$$

$\langle f,g\rangle =$ ________,
$\|f\| =$ ________,
$\|g\| =$ ________,
$\alpha_{f,g}$ ________.

Answer(s) submitted:

- $-\frac{205}{4}$
- $2\sqrt{14}$
- $3\sqrt{7}$
- $\cos^{-1}\left(\frac{-205}{4\cdot 6\cdot 7\sqrt{2}}\right)$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Let

$$M_1 = \begin{bmatrix} -1 & -1 \\ 1 & -1 \end{bmatrix} \ \text{ and } \ M_2 = \begin{bmatrix} -6 & -2 \\ 0 & 2 \end{bmatrix}.$$

Consider the inner product $\langle A,B\rangle = \text{trace}(A^TB)$ in the vector space $\mathbb{R}^{2\times 2}$ of 2×2 matrices. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $\mathbb{R}^{2\times 2}$ spanned by the matrices M_1 and M_2.

$$\left\{ \begin{bmatrix} \_\_\_ & \_\_ \\ \_\_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_\_ & \_\_ \\ \_\_\_ & \_\_ \end{bmatrix}, \right\}$$

Answer(s) submitted:

- $\begin{bmatrix} -0.5 & -0.5 \\ 0.5 & -0.5 \end{bmatrix}$
- $\begin{bmatrix} -\frac{9}{2\sqrt{35}} & -\frac{1}{2\sqrt{35}} \\ -\frac{3}{2\sqrt{35}} & \frac{7}{2\sqrt{35}} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

Let

$$f(x) = 3,\ g(x) = -4x+8 \ \text{ and } \ h(x) = -9x^2.$$

Consider the inner product

$$\langle p,q\rangle = \int_0^1 p(x)q(x)\,dx$$

in the vector space $C^0[0,1]$ of continuous functions on the domain $[0,1]$. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of $C^0[0,1]$ spanned by the functions $f(x)$, $g(x)$, and $h(x)$.

{ ______, ______, ______ }.

Answer(s) submitted:

- 1
- $-2\sqrt{3}x+\sqrt{3}$
- $\frac{20}{9}\left(3-9x^2-\frac{3}{2}\sqrt{3}\left(\sqrt{3}-2\sqrt{3}x\right)\right)$

submitted: (score 0.666667)
recorded: (score 0.666667)

Problem 6. **(1 point)**

Use the inner product

$$\langle f,g\rangle = \int_0^1 f(x)g(x)\,dx$$

in the vector space $C^0[0,1]$ of continuous functions on the domain $[0,1]$ to find the orthogonal projection of $f(x) = 6x^2-1$ onto the subspace V spanned by $g(x) = x$ and $h(x) = 1$. (Caution: x and 1 do not form an orthogonal basis of V.)

$\text{proj}_V(f) =$ ______.

Answer(s) submitted:

- $6x-2$

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

Let

$$f(x) = 3,\ g(x) = -2x-1 \ \text{ and } h(x) = 2x^2+5x+6.$$

Consider the inner product

$$\langle p,q\rangle = p(-1)q(-1)+p(0)q(0)+p(1)q(1)$$

in the vector space P_2 of polynomials of degree at most 2. Use the Gram-Schmidt process to determine an orthonormal basis for the subspace of P_2 spanned by the polynomials $f(x)$, $g(x)$ and $h(x)$.

{ ______, ______, ______ }.

Answer(s) submitted:

- $\frac{\sqrt{3}}{3}$
- $-\frac{\sqrt{2}x}{2}$
- $\frac{3x^2-2}{\sqrt{6}}$

submitted: (correct)
recorded: (correct)

Problem 8. **(1 point)**

If $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \text{k} & 0 & 1 \end{bmatrix}$ then $\det(A) =$ ____

Answer(s) submitted:

- 1

submitted: (correct)
recorded: (correct)

Problem 9. **(1 point)**

If $A = \begin{bmatrix} \text{-2} & 8 & \text{-4} & \text{-3} \\ 0 & \text{-4} & \text{-4} & 1 \\ 0 & 0 & 3 & \text{-6} \\ 0 & 0 & 0 & \text{-1} \end{bmatrix}$

then $\det(A) =$ ____

Answer(s) submitted:

- -24

submitted: (correct)
recorded: (correct)

Problem 10. **(1 point)**

Consider the following matrix

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 0 & -3 \\ 0 & 4 & 3 \end{bmatrix}.$$

(a) Find its determinant. ______

(b) Does the matrix have an inverse? [Choose/Yes/No]

Answer(s) submitted:

- 9
- Yes

submitted: (correct)
recorded: (correct)