Problem 1. **(1 point)**

Let $A = \begin{bmatrix} 5 & 10 & 5 \\ 5 & 7 & 5 \\ -2 & -1 & 0 \end{bmatrix}$ and $b = \begin{bmatrix} 6 \\ 3 \\ 1 \end{bmatrix}$.

Denote the columns of A by a_1, a_2, a_3, and let $W = \text{span}\{a_1, a_2, a_3\}$.

$\boxed{?}$ 1. Determine if b is in W

$\boxed{?}$ 2. Determine if b is in $\{a_1, a_2, a_3\}$

How many vectors are in $\{a_1, a_2, a_3\}$? (For infinitely many, enter -1) ____

How many vectors are in W? (For infinitely many, enter -1) ____

Answer(s) submitted:

- `YES`
- `NO`
- 3
- -1

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Find the value of a for which

$$v = \begin{bmatrix} 3 \\ a \\ 12 \\ -1 \end{bmatrix}$$

is in the set

$$H = span\left\{ \begin{bmatrix} -3 \\ 1 \\ -3 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ -4 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ -1 \\ -2 \end{bmatrix} \right\}.$$

$a =$ ____

Answer(s) submitted:

- 5

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by

$$T(x,y) = (-5x - y, 4x - 4y, x + 4y).$$

Find a vector $\vec{w}$ that is **not** in the image of T.

$$\vec{w} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Answer(s) submitted:

- $\begin{bmatrix} 20 \\ 19 \\ 24 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Let

$$A = \begin{bmatrix} 1 & -2 & 5 & 5 & 4 & 4 \\ 0 & 0 & 1 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Describe all solutions of $A\vec{x} = \vec{0}$.

$$\vec{x} = x_2 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + x_4 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} + x_6 \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Answer(s) submitted:

- $\begin{bmatrix} 2 \\ 1 \\ -0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 10 \\ 0 \\ -3 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 19 \\ 0 \\ -3 \\ 0 \\ -2 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

Find a set of vectors $\{\vec{u},\vec{v}\}$ in $\mathbb{R}^4$ that spans the solution set of the equations

$$\begin{cases} w-x-2y+2z &= 0, \\ 5w+2x-y+z &= 0. \end{cases}$$

$$\vec{u} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}, \vec{v} = \begin{bmatrix} \_ \\ \_ \\ \_ \\ \_ \end{bmatrix}.$$

(The components of these vectors appear in alphabetical order: (w,x,y,z)).

Answer(s) submitted:

- $\begin{bmatrix} \frac{5}{7} \\ -\frac{9}{7} \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{5}{7} \\ \frac{9}{7} \\ 0 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Consider the subset W of $\mathbb{R}^3$ consisting of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $x+y+z \geq 1$.

Select all statements that are correct:

- A. W is closed under scalar multiplication.
- B. W contains the zero vector.
- C. W is closed under addition.
- D. There exists a 3×3 matrix B whose image is W.
- E. There exists a 3×3 matrix A whose kernel is W.
- F. W is a subspace of $\mathbb{R}^3$.

Answer(s) submitted:

- C

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

Let $\vec{v}_1 = \begin{bmatrix} -6 \\ 6 \\ -4 \end{bmatrix}$, $\vec{v}_2 = \begin{bmatrix} -4 \\ 3 \\ 3 \end{bmatrix}$, and $\vec{v}_3 = \begin{bmatrix} -28 \\ 24 \\ k \end{bmatrix}$. First find a value of k for which the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ are linearly dependent:
$k =$ __________.
Now find a value of k for which the vectors $\vec{v}_1$, $\vec{v}_2$, and $\vec{v}_3$ are linearly independent:
$k =$ __________.

Answer(s) submitted:

- 4
- 1

submitted: (correct)
recorded: (correct)

Problem 8. **(1 point)**

Let W_1 be the set: $\begin{bmatrix} 1 \\ -3 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 9 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 5 \end{bmatrix}$.

Determine if W_1 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_1 is not a basis because it is linearly dependent.
- B. W_1 is a basis.
- C. W_1 is not a basis because it does not span $\mathbb{R}^3$.

Let W_2 be the set: $\begin{bmatrix} -2 \\ 3 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ -1 \\ 5 \end{bmatrix}$.

Determine if W_2 is a basis for $\mathbb{R}^3$ and check the correct answer(s) below.

- A. W_2 is a basis.
- B. W_2 is not a basis because it does not span $\mathbb{R}^3$.
- C. W_2 is not a basis because it is linearly dependent.

Answer(s) submitted:

- A
- B

submitted: (correct)
recorded: (correct)

Problem 9. **(1 point)**

Find a basis for the column space of

$$A = \begin{bmatrix} 1 & -1 & 1 & 0 \\ 0 & 4 & 0 & -2 \\ 0 & 4 & 0 & -2 \end{bmatrix}.$$

$$\text{Basis} = \left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

Answer(s) submitted:

- $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 10. **(1 point)**

Let $A = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 \\ -5 \\ 1 \end{bmatrix}$, and $C = \begin{bmatrix} 4 \\ -8 \\ 3 \end{bmatrix}$

Are A,B and C linearly dependent, or are they linearly independent?

- Linearly independent
- Linearly dependent

If they are linearly dependent, determine a non-trivial linear relation. Otherwise, if the vectors are linearly independent, enter 0's for the coefficients, since that relationship **always** holds.

$\_\_\_\_A + \_\_\_\_B + \_\_\_\_C = 0.$

Answer(s) submitted:

- Linearly dependent
- $-\frac{1}{2}; -\frac{3}{2}; 1$

submitted: (correct)
recorded: (correct)