Problem 1. **(1 point)**

Suppose that A is a matrix with 9 rows and 12 columns, and that there exist vectors $\vec{v}_1,\ldots,\vec{v}_5 \in \mathbb{R}^{12}$ for which $A\vec{v}_1,\ldots,A\vec{v}_5$ are linearly independent.
a) What is the minimum possible value of rank(A)?
rank$(A) \geq$ __________.
b) What is the maximum possible value of the nullity of A?
nullity$(A) \leq$ __________.

Answer(s) submitted:

- 5
- 7

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Let

$$A = \begin{bmatrix} -2 & 2 & 0 \\ 1 & -1 & 0 \\ -2 & 2 & 0 \\ -3 & 3 & 0 \end{bmatrix}.$$

Find dimensions of the kernel and image of $T(\vec{x}) = A\vec{x}$.
$\dim(\mathrm{Ker}(A)) =$ ___,
$\dim(\mathrm{Im}(A)) =$ ___.

Answer(s) submitted:

- 2
- 1

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Let

$$A = \begin{bmatrix} 6 & 4 & 4 & 6 \\ 9 & 6 & 6 & 9 \end{bmatrix}.$$

Find a basis of Ker(A).

$$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}.$$

Answer(s) submitted:

- $\begin{bmatrix} -\frac{2}{3} \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -\frac{2}{3} \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Let

$$A = \begin{bmatrix} 1 & 1 & -1 & 2 \\ 0 & 1 & 4 & 1 \\ 3 & 6 & 9 & 9 \end{bmatrix}.$$

Find a pair of vectors $\vec{u},\vec{v}$ in $\mathbb{R}^4$ that span the set of all $\vec{x} \in \mathbb{R}^4$ that are mapped into the zero vector by the transformation $\vec{x} \mapsto A\vec{x}$.

$$\vec{u} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}, \vec{v} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Answer(s) submitted:

- $\begin{bmatrix} 5 \\ -4 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

a) Find the value of k for which the matrix

$$A = \begin{bmatrix} 9 & 2 & 5 \\ -4 & -4 & 4 \\ -6 & 1 & k \end{bmatrix}$$

has rank 2.
$k =$ _____
b) For this value of k, find a basis of ker(A).

$$\text{Basis} = \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix}.$$

Answer(s) submitted:

- -8
- $\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Which of the following are vector spaces?

- A. The set of all diagonal 2×2 matrices.
- B. The set of non-invertible 2×2 matrices.
- C. The set $A, A^2, A^3, A^4 \ldots$, where $A = \begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$.
- D. The set of continuous functions $F(\mathbb{R},\mathbb{R})$.

Answer(s) submitted:

- AD

submitted: (correct)
recorded: (correct)

Problem 7. (1 point)

Find a basis for the space of 2×2 diagonal matrices.

Basis = $\left\{ \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \right\}$

Answer(s) submitted:

- $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 8. (1 point)

Which of the following subsets of $\mathbb{R}^{3\times3}$ are subspaces of $\mathbb{R}^{3\times3}$?

- A. The invertible 3×3 matrices
- B. The 3×3 matrices whose entries are all integers
- C. The 3×3 matrices with all zeros in the third row
- D. The diagonal 3×3 matrices
- E. The 3×3 matrices whose entries are all greater than or equal to 0
- F. The symmetric 3×3 matrices

Answer(s) submitted:

- CDF

submitted: (correct)
recorded: (correct)

Problem 9. (1 point)

Consider the vector space P_2 and the set

$$4 - 2t - 2t^2, 4 + 2t + 3t^2, 16 + kt^2.$$

For which $k \in \mathbb{R}$ do these three elements *fail* to be a basis of P_2?
$k =$ __________.

Answer(s) submitted:

- 2

submitted: (correct)
recorded: (correct)

Problem 10. (1 point)

Recall that $U^{2\times2}$ is the vector space of 2×2 upper triangular matrices.
Which of the following functions are isomorphisms?

- A. The function $T : U^{2\times2} \to P_2$ given by $T(\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}) = (a+b) + ct + (a+b)t^2$.
- B. The function $T : \mathbb{C} \to \mathbb{R}^2$ given by $T(a+bi) = \begin{bmatrix} a \\ a+b \end{bmatrix}$.
- C. The function $T : \mathbb{P}_2 \to U^{2\times2}$ given by $T(a+bt+ct^2) = \begin{bmatrix} a & ab \\ 0 & c \end{bmatrix}$.
- D. The function $T : P_1 \to \mathbb{R}^{2\times2}$ given by $T(a+bt) = \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$.

Answer(s) submitted:

- B

submitted: (correct)
recorded: (correct)