Problem 1. **(1 point)**

Consider the transformation $T : U^{2\times 2} \to U^{2\times 2}$, where $U^{2\times 2}$ is the space of upper-triangular matrices. T is given by $T(M) = \begin{bmatrix} -3 & -3 \\ 0 & 1 \end{bmatrix} M$.

Find the matrix for this transformation relative to the standard basis $\mathfrak{U} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$.

$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Determine whether T is an isomorphism or not, and determine bases for $\ker(T)$ and $\operatorname{im}(T)$ select the correct answers below.
Select all of the following that are correct statements about T.

- A. T is an isomorphism.
- B. $\dim(\operatorname{im}(T)) = 3$ and $\dim(\ker(T)) = 0$
- C. $\dim(\operatorname{im}(T)) = 2$ and $\dim(\ker(T)) = 1$
- D. $\dim(\operatorname{im}(T)) = 0$ and $\dim(\ker(T)) = 3$
- E. T is not an isomorphism.

Answer(s) submitted:

- $\begin{bmatrix} -3 & 0 & 0 \\ 0 & -3 & -3 \\ 0 & 0 & 1 \end{bmatrix}$
- AB

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

Consider the transformation $T : \mathbb{P}_2 \to \mathbb{P}_2$, where $\mathbb{P}_2$ is the space of second-degree polynomials, given by $T(f) = f'' + 2f$.

Find the matrix for this transformation relative to the standard basis $\mathfrak{U} = (1, t, t^2)$.

$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$

Determine whether T is an isomorphism or not, and determine bases for $\ker(T)$ and $\operatorname{im}(T)$. Use these to answer the following.
Select all of the following that are correct statements about T.

- A. T is an isomorphism.
- B. $\operatorname{rank}(B) = 1$
- C. T is not an isomorphism.
- D. $\dim(\operatorname{im}(T)) = 3$ and $\dim(\ker(T)) = 0$
- E. $\dim(\operatorname{im}(T)) = 1$ and $\dim(\ker(T)) = 2$

Answer(s) submitted:

- $\begin{bmatrix} 2 & 0 & 2 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$
- AD

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Let $\mathcal{P}_2$ be the space of polynomials of degree at most 2 in the variable t, and consider the transformation $T : \mathcal{P}_2 \to \mathcal{P}_2$ given by $T(f) = f(3) + f'(3)\,(t-3)$.

Find the matrix for this transformation relative to the standard basis $\mathfrak{U} = (1, t, t^2)$.

$$B = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Determine whether T is an isomorphism or not, and determine bases for $\ker(T)$ and $\operatorname{im}(T)$ select the correct answers below.
Select all of the following that are correct statements about T.

- A. $\dim(\operatorname{im}(T)) = 2$ and $\dim(\ker(T)) = 1$
- B. T is not an isomorphism.
- C. $\dim(\operatorname{im}(T)) = 0$ and $\dim(\ker(T)) = 3$
- D. T is an isomorphism.
- E. $\dim(\operatorname{im}(T)) = 3$ and $\dim(\ker(T)) = 0$

The basis for $\operatorname{im}(T)$ suggested by the matrix B is { ________ }
(Enter the elements of the basis as a comma-separated list, and note that your basis should consist of vectors in $\mathcal{P}_2$.)
A basis for $\ker(T)$ is ________
(Note that your basis should consist of vectors in $\mathcal{P}_2$.)
Answer(s) submitted:

- $\begin{bmatrix} 1 & 0 & -9 \\ 0 & 1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$
- AB
- $1, t$
- $9 - 6t + t^2$

submitted: (correct)
recorded: (correct)

Problem 4. **(1 point)**

Consider the vector space $U^{2\times 2}$ of upper triangular 2×2 matrices, with two different bases: first the standard basis

$$\mathfrak{E} = \left(\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix} \right),$$

and also the non-standard basis

$$\mathfrak{B} = \left(\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & -2 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & -1 \end{bmatrix} \right).$$

Find the change of basis matrix $S_{\mathfrak{E}\to\mathfrak{B}}$ from $\mathfrak{E}$ to $\mathfrak{B}$. *(Read carefully!)*

$$S_{\mathfrak{E}\to\mathfrak{B}} = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Answer(s) submitted:

- $\begin{bmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 5 & -2 & -1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 5. **(1 point)**

$\mathfrak{B} = (-1 + 2t, 2 - 5t)$ is a basis for P_1. Suppose that $T : P_1 \to P_1$ is a linear transformation whose $\mathfrak{B}$-matrix, B, is

$$B = \begin{bmatrix} -2 & 1 \\ 1 & 0 \end{bmatrix}.$$

Find the matrix A of T relative to the standard basis $(1, t)$ for P_1.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Answer(s) submitted:

- $\begin{bmatrix} -18 & -7 \\ 41 & 16 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 6. **(1 point)**

Find the missing coordinates such that the three vectors form an orthonormal basis for $\mathbb{R}^3$:

$$\begin{bmatrix} 0.6 \\ -0.8 \\ \_\_ \end{bmatrix}, \begin{bmatrix} \_\_ \\ \_\_ \\ -1 \end{bmatrix}, \begin{bmatrix} \_\_ \\ -0.6 \\ \_\_ \end{bmatrix}.$$

Answer(s) submitted:

- 0
- 0
- 0
- -0.8
- 0

submitted: (correct)
recorded: (correct)

Problem 7. (1 point)

Compute the orthogonal projection of $\vec{v} = \begin{bmatrix} 2 \\ 1 \\ -7 \end{bmatrix}$ onto the line L through $\begin{bmatrix} 3 \\ 6 \\ -4 \end{bmatrix}$ and the origin.

$\mathrm{proj}_L(\vec{v}) = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Answer(s) submitted:

- $\begin{bmatrix} \frac{120}{61} \\ \frac{240}{61} \\ -\frac{160}{61} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 8. (1 point)

Let $\vec{y} = \begin{bmatrix} -9 \\ 1 \\ 5 \end{bmatrix}$ and $\vec{u} = \begin{bmatrix} 7 \\ -4 \\ -1 \end{bmatrix}$. Write $\vec{y}$ as the sum of two orthogonal vectors, $\vec{x}_1$ in $Span\{u\}$ and $\vec{x}_2$ orthogonal to $\vec{u}$.

$\vec{x}_1 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}, \vec{x}_2 = \begin{bmatrix} \_\_\_\_ \\ \_\_\_\_ \\ \_\_\_\_ \end{bmatrix}$.

Answer(s) submitted:

- $\begin{bmatrix} -\frac{84}{11} \\ \frac{48}{11} \\ \frac{12}{11} \end{bmatrix}$
- $\begin{bmatrix} -\frac{15}{11} \\ -\frac{37}{11} \\ \frac{43}{11} \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 9. (1 point)

Let W be the set of all vectors $\begin{bmatrix} x \\ y \\ x+y \end{bmatrix}$ with x and y real. Find a basis of $W^{\perp}$.

$\left\{ \begin{bmatrix} \_\_ \\ \_\_ \\ \_\_ \end{bmatrix} \right\}$.

Answer(s) submitted:

- $\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}$

submitted: (correct)
recorded: (correct)

Problem 10. (1 point)

Consider the two vectors $\vec{v} = \begin{bmatrix} 3 \\ -2 \\ -1 \\ 1 \end{bmatrix}$ and $\vec{w} = \begin{bmatrix} 1 \\ 3 \\ 3 \\ 2 \end{bmatrix}$ in $\mathbb{R}^4$.

Compute the following:
$\vec{v} \cdot \vec{w} =$ ________.
$||v||\ ||w|| =$ ________.
The angle θ between $\vec{v}$ and $\vec{w}$, measured in *radians* = ________.

Answer(s) submitted:

- -4
- $\sqrt{345}$
- $\cos^{-1}\left(-\frac{4}{\sqrt{345}}\right)$

submitted: (correct)
recorded: (correct)