Problem 1. **(4 points)**

Perform the following congruence computations. Enter the smallest non-negative integer in the congruence class (that is, your entered answer should be ≥ 0 and $\leq N-1$, where N is the modulus of the congruence).

$6229 + 1237 \equiv$ ____ $\pmod{45}$
$6285 - 33 * 2649 \equiv$ ____ $\pmod{79}$
$67012+$ ____ $-451 \equiv 637 * 733 \pmod{41}$
$557 * (72360 - 645) * 6092 - 602 \equiv$ ____ $\pmod{79}$
$8544^2 \equiv$ ____ $\pmod{63}$

Answer(s) submitted:

- 41
- 1
- 36
- 50
- 9

submitted: (correct)
recorded: (correct)

Problem 2. **(4 points)**

Solve the following congruences. Enter for your answer the smallest non-negative integer in the correct congruence class (so, an integer ≥ 0 and $\leq N-1$, where N is the modulus of the congruence). If there is more than one solution, enter the answer as a list separated by commas.

$X_1 - 304 \equiv 287 \pmod{72}$
$X_1 =$ ____

$X_2 + 379 \equiv 151 \pmod{53}$
$X_2 =$ ____

$(X_3)^2 \equiv 12 \pmod{13}$
$X_3 =$ ____

$(X_4)^2 \equiv 1 \pmod{8}$
$X_4 =$ ____

Answer(s) submitted:

- 15
- 37
- 5,8
- 1,3,5,7

submitted: (correct)
recorded: (correct)

Problem 3. **(10 points)**

For n a nonnegative integer, either $n \equiv 0 \bmod 3$ or $n \equiv 1 \bmod 3$ or $n \equiv 2 \bmod 3$. In each case, fill out the following table with the canonical representatives modulo 3 of the expressions given:

$n \bmod 3$	$n^3 \bmod 3$	$2n \bmod 3$	$n^3 + 2n \bmod 3$
0	____	____	____
1	____	____	____
2	____	____	____

From this, we can conclude:

- A. Since $n^3 + 2n \not\equiv 0 \bmod 3$ for all n, we conclude that 3 does not necessarily divide $n^3 + 2n$ for all nonnegative integers n.
- B. Since $n^3 + 2n \equiv 0 \bmod 3$ for all n, we conclude that 3 divides $n^3 + 2n$ for any nonnegative integer n.

Answer(s) submitted:

- 0
- 0
- 0
- 1
- 2
- 0
- 2
- 1
- 0
- B

submitted: (correct)
recorded: (correct)

Problem 4. **(7 points)**

The goal of this exercise is to practice finding the inverse modulo m of some (relatively prime) integer n. We will find the inverse of 26 modulo 161, i.e., an integer c such that $26c \equiv 1 \pmod{161}$.

First we perform the Euclidean algorithm on 26 and 161:
$161 = 6*$ ________ + ________
________ $=$ ________ $*5 + 1$
[Note your answers on the second row should match the ones on the first row.]

Thus gcd(26,161)=1, i.e., 26 and 161 are relatively prime.

Now we run the Euclidean algorithm backwards to write $1 = 161s + 26t$ for suitable integers s, t.
$s =$ ________
$t =$ ________
when we look at the equation $161s + 26t \equiv 1$ (mod 161), the multiple of 161 becomes zero and so we get
$26t \equiv 1$ (mod 161). Hence the multiplicative inverse of 26 modulo 161 is ________

Answer(s) submitted:

- 26
- 5
- 26
- 5
- -5
- 31
- 31

submitted: (correct)
recorded: (correct)

Problem 5. **(2 points)**

Find the smallest positive integer x that solves the congruence:

$$21x \equiv 4 \pmod{152}$$

$x =$ _____

(Hint: From running the Euclidean algorithm forwards and backwards we get $1 = s(21) + t(152)$. Find s and use it to solve the congruence.)

Answer(s) submitted:

- 116

submitted: (correct)
recorded: (correct)

Problem 6. **(2 points)**

Solve each of the following congruences. Make sure that the number you enter is in the range $[0, M-1]$ where M is the modulus of the congruence. If there is more than one solution, enter the answer as a list separated by commas. If there is no answer, enter 2.

(a) $23x = 1 \pmod{255}$

$x =$ _____

(b) $129x = 27 \pmod{255}$

$x =$ _____

Answer(s) submitted:

- 122
- $18, 103, 188$

submitted: (correct)
recorded: (correct)

Problem 7. **(1 point)**

Find the smallest positive integer x such that:
$x \bmod 2 = 1$
$x \bmod 3 = 2$ and
$x \bmod 5 = 3$
___

What is the next integer with this property?
___

[You will have to do some trial and error, but thinking about divisiblity should lead you to some patterns.]

Answer(s) submitted:

- 23
- 53

submitted: (correct)
recorded: (correct)

Problem 8. (1 point)

There is exactly one integer x with $0 \leq x < 10$ that also is a solution to the two congruences below.

$$x \equiv 0 \pmod{2} \quad \text{and} \quad x \equiv 1 \pmod{5}$$

What is x?

$x =$ ____

(Hint: We know that for some integers s and t, we have $1 = 2s + 5t$. Find s and t first. Then find integers v and w such that $x = 2(vs) + 5(wt)$ solves these congruences.)

Answer(s) submitted:

- 6

submitted: (correct)
recorded: (correct)

Problem 9. (1 point)

For which of the integers $n \in \{45, 89, 33, 4, 85, 92, 73, 80\}$ is it true that if $x, y \in \mathbb{Z}_n$ satisfy $18x = 18y$ then $x = y$?

____

Answer(s) submitted:

- $89, 85, 73$

submitted: (correct)
recorded: (correct)