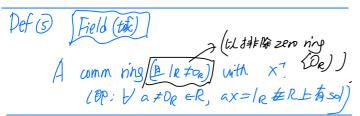


Countrapositive: a \$0, b \$0

= ab\$0.



Thm() Any operation 五分有 小 identity

Thm ② 3.1 (Product Ring) F(s,s) = F(s,s) + (r,s) = (r+r',s+s')def 其 x: (r,s) + (r',s') = (r+r',s+s') f(s,s) = (r+r',s+s')

Def® Subning (子环)

SER为ring R的力subning, if

(1) S is nonempty

(2) S 的十,×和R定义相同

(3) S有尺相同的0,1.

Thm (3) 3.2 [Subring 判定]

艺Risa ning, SSR non empty.

(1) OB IR ES (1) UK IR = (2) S is closed under to : a, b \(\in \) a thes

(3) S is closed under to : a \(\in \) = a \(\in \)

(4) S is closed under \(\in \); a \(\in \) \(\in \) \(\in \)

Dy =) S is a subring of R

(2)(3) 可合并为 chosed under substraction. (Thm 3.6))

Thm 4 3.3 ring \$,+1 unique.

(即 YXER, 3! YER 値x+y=OR)

(abo, x if exists is unique. 2要 uperation 是ALTO的 缝冠墓 63 inverse - 2 unique)

Thm 5 3.4,3.5 ring \$ 14 & elan a, b & ia \$ 120.

- (1) a. o = o a = or
- (2) 若a+b=a+c (cER) => b=c
- 13) 1-a)=a
- (4) (a+b) = -a b
- |5) (a-b) = -a + b
- (6) (-a) (-b) = ab
- (7) t/e). a =-a.

Def(D) Unit

元素a c ring R 为一个 unit f の在 R中有xT. (即 3 uER 使 ux=zu=IR)

unit a by x J denote by: a ? (金维斯hm:一个 field中任意 非 O 元素都为unit)

Def 图 Zeno divisor 零图子

非D元素aeing R为一女zeno divisor

if 目中的元素CeR使ac=O颜ca=O

eg: M2(R)4,

a为左0团乱 (台台)到

化煮石3色 matrix 都为左右0 图子。

Thm(B) 3.7 在integral domain中

if ab = ac A a ≠ 0e = b= c

Thm ①3.8, 任意 Field to integral domain 任意 finite integral domain to field. Def @ Homomorphism] # 8.

Let R-S be rings.

f:R->S to-t homomorphism if

 $\int f(a+b) = f(a) + f(b)$ $\int f(ab) = f(a) f(b)$

eq: zen map z:R-S given by z(r)=0s.

Def (Isomorphism) 1249

TPRES homomophism

(4: R-S& homomorphism & & bijective)

就像同一个环取不同名字。 isomorphic 环-定存在isomorphism.

Def (kemel A kernel of a ning homomorphism R上, S
是Source 的一分集) maps to Ds. R ker $P = (r \in R \mid P(r) = O_S)$

Thm & - A ring hom P:R-) Sto injective iff kery = LOR3

Thm (93.10(1)

低 hom D: P-JS

(1) f(0R) = 0

(2) faeR, f(-a) = -f(a)(3) f(a-b) = f(a) - f(b)

Thm (03,1002) 如果 ring hom \$\mathcal{Y}: R→S \$\times surjective} (R\$ 我们这X的 nhy 即任新)

> Whenever UER to trunit in R, fcw es & -t unit in s.

Corollary 10 3.11

若 yiR→Sおかかか hom = im(y) AS bb-4 subring