Assignment WebWork3-Definition_of_Rings due 02/05/2024 at 11:59pm EST

Problem 1. (9 points)

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Below are given the multiplication and addition tables for a commutative ring *S* with three elements.

+	r	n	c
r		_	r
n	c	r	
c			

*	r	n c
r	r	
n	n	_
c	_	

- a. Complete the tables.
- b. What element of *S* is the additive identity? ____
- c. What element of S is the multiplication identity?

Answer(s) submitted:

- r
- (
- n
- 1
- n
- n
- c
- r
- c
- 0
- c
- . .
- r

submitted: (correct) recorded: (correct)

Problem 2. (6 points)

Let $M_2(\mathbb{R})$ be the ring of 2×2 matrices with \mathbb{R} entries, using matrix addition and multiplication as the operations. Which of the following subsets of $M_2(\mathbb{R})$ are also rings?

For each subset $S \subset M_2(\mathbb{R})$ below, use matrix addition and multiplication for the addition and multiplication operations (respectively) on the subset. Don't forget to check that these operations on $M_2(\mathbb{R})$ actually are still operations on S.

a. $U_2(\mathbb{R})$ [?/Yes/No]

 $(U_2(\mathbb{R}))$ is the set of '2×2'upper-triangularmatrices with realent riellown [?/Yes/No]

 $(M_2(\mathbb{Q}) \text{ is the set of '}2\times2\text{'}matrices with rational entrieb.)}D_2(\mathbb{R})$ [?/Yes/No]

 $(D_2(\mathbb{R})$ is the set of '2×2' diagonal matrices with real entrie **b.**) $D_2(\mathbb{N})$ [?/Yes/No]

 $(D_2(\mathbb{N}))$ is the set of '2×2' diagonal matrices with positive integer entriest [?/Yes/No]

 $(GL_2(\mathbb{Q}))$ is the set of '2×2' invertible matrices with rational entries.)

Answer(s) submitted:

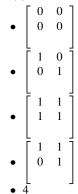
- Yes
- Yes
- Yes
- No
- No

submitted: (correct)
recorded: (correct)

Problem 3. (5 points) Let $M_2(\mathbb{Z}_2)$ be the ring of 2×2 matrices with entries in the ring \mathbb{Z}_2 . For this problem, we write the classes $[0]_2$ and $[1]_2$ as 0 and 1.

- **a.** The additive identity of $M_2(\mathbb{Z}_2)$ is: $\begin{bmatrix} & -- & -- \\ & & -- \end{bmatrix}$
- **c.** $A = \begin{bmatrix} ---- \\ --- \end{bmatrix}$ is a non-zero element of $M_2(\mathbb{Z}_2)$ with no multiplicative inverse.
- **d.** $B = \begin{bmatrix} --- \\ --- \end{bmatrix}$ is an element of $M_2(\mathbb{Z}_2)$ that is not the identity and has a multiplicative inverse.
- **e.** How many elements are in the set of 2×2 diagonal matrices \mathbb{Z}_2 ? ____

Answer(s) submitted:



submitted: (correct) recorded: (correct)

Problem 4. (10 points) Which of the following subsets of \mathbb{Z}_6 is a ring under the operations of \mathbb{Z}_6 ? Which is a subring? If it is a ring, what is the multiplicative identity? If it is not a ring leave the identity blank.

 \mathbb{Z}_6 :

Ring: [?/Yes/No] Subring: [?/Yes/No]

Identity: ___

 $\{0,2,4\}$:

Ring: [?/Yes/No]
Subring: [?/Yes/No]

Identity: ___

 $\{0\}$:

Ring: [?/Yes/No]

Subring: [?/Yes/No]

Identity: ___

 $\{0,1\}$:

Ring: [?/Yes/No]

Subring: [?/Yes/No]

Identity: ___

 $\{0,3\}$:

Ring: [?/Yes/No]

Subring: [?/Yes/No]

Identity: ___

Answer(s) submitted:

- Yes
- Yes
- 1
- Yes
- No
- 4
- Yes
- No
- 0
- No
- No
- Yes
- No
- 3

submitted: (correct)
recorded: (correct)