Quiz 1

If $a|c$, $b|c$, $d=(a,b)$, show $ab|cd$

Pf Since $a|c$, $b|c$,

$c=ka=lb$ for some integer k,l.

Since $d=(a,b)$,

$a=dx$, $b=dy$ for some integer x,y.

AND $(x,y)=1$ since if x,y has some common divider $s>1$ then $\frac{x}{s}, \frac{y}{s} \in \mathbb{Z}$, and then $a=ds(\frac{x}{s})$, $b=ds(\frac{y}{s})$, then $(a,b)=ds$.

Hence $c=ka=lb=dkx=dly$

so $cd=d^2kx=d^2ly$, then $kx=ly$

and $ab=d^2xy$.

By FTA, x, y can be factorized into some primes $p_1,\dots p_\alpha$, $q_1,\dots q_\beta$ respectfully

Since $(x,y)=1$, there is no q_i, $1\le i\le\beta$ matches any of p_j, $1\le j\le\alpha$

Therefore all of q_i, $1\le i\le\beta$ must be in factors of k, that is, $k=wy$ for some $w\in\mathbb{Z}$

Hence $cd=wd^2xy=wab$. So $ab|cd$.