7

Good job! Just be careful when operating with quotient rings.

Math 412

Quiz 6

Thursday, February 22

You have 15 minutes to complete the quiz. You may turn in corrections for up to half credit back by the beginning of the next class period.

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1. (3 points) Let R be a ring and let I be an ideal of R. What does R/I mean?

$R/I = \{ y + I \mid y \in R \}$ is the set of all congruent classes modulo I on R

2. (4 points) Circle all the subsets that are ideals of the given ring.

a) The subset of $\mathbb{Z}$ of all even numbers.

b) The subset of $\mathbb{Z}$ of all odd numbers. ×

c) Matrices in $M_2(\mathbb{Z})$ with even entries.

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} e & f \\ g & h \end{bmatrix}$

$\begin{bmatrix} ae+bg & af+bh \\ ce+dg & cf+dh \end{bmatrix}$

d) All linear combinations of 4 and 10 in $\mathbb{Z}$.

e) $\{0\} \subseteq \mathbb{R}[x]$.

f) The set $\underbrace{\{r + r \mid r \in \mathbb{Z}_5\}}_{S}$ in $\mathbb{Z}_5$. ×

$0+0=0$

$1+1=2$

$2+2=4$

$3+3=1$

$4+4=3$ in $\mathbb{Z}_5$

So actually $S = \mathbb{Z}_5$. Therefore S is an ideal of $\mathbb{Z}_5$ since ring itself is an ideal of itself as we have prove.

−1

(5) is also a correct answer.

3. (3 points) An element a of a ring R is said to be idempotent if $a^2 = a$. Find at least 3 idempotents in the quotient ring $\mathbb{Q}[x]/(x^4 + x^2)$.

① $\boxed{1+I} \in \mathbb{Q}[x]/(x^4+x^2)$ ✓

$(1+I)^2 = (1+I)(1+I) = 1+I$

② $\boxed{0+I} \in \mathbb{Q}[x]/(x^4+x^2)$ ✓

$(0+I)^2 = (0+I)(0+I) = 0+I$

③ $\boxed{x^2+1+I} \in \mathbb{Q}[x]/(x^4+x^2)$

$(x^2+1+I)^2 = x^4+2x^2+1+I$

$= x^2+1+I$

③ ~~$x^4+x^2+1+I \in \mathbb{Q}[x]/x^4+x^2$~~

~~$(x^4+x^2+1+I)^2 = (1+I)(1+I) = 1+I$~~

③ $\boxed{x^4+I} \in \mathbb{Q}[x]/(x^4+x^2)$

$(x^4+I)^2 = x^8+I = x^4(x^4+x^2) - x^6 + I$

$= -x^6+I = -x^2(x^4+x^2)+x^4+I = x^4+I$

−2

$[x^4+1]_I = [-x^2+1]_I$ isn't idempotent.