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Very well done!

Math 412

Quiz 10

Thursday, April 4

You have 15 minutes to complete the quiz. You may turn in corrections for up to half credit back by the beginning of the next class period.

Name: Qinlin Fan

1. **(3 points)** State the Orbit Stabilizer Theorem.

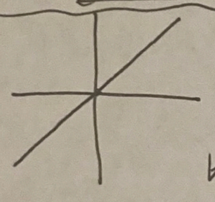
For a Group action $G \times X \to G$,
$\forall x \in X$, $|G| = |O(x)| \cdot |Stab(x)|$.

2. **(4 points)** Give an **explicit example** of a group action that is neither faithful nor trivial[1] (and briefly explain why your action is neither faithful nor trivial).

-2

You understand the material well, but an action on X can't leave X. Your op. isn't an action.

Consider: $G = D_4$ with natural action on $X = \{(x,y) \in \mathbb{R}^2 \mid y = x\}$



The group action is non trivial, but

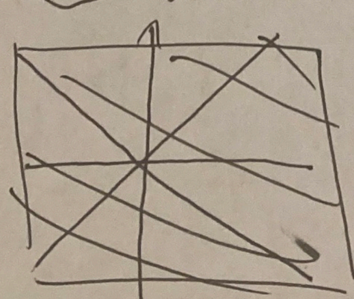
$\forall x \in X$, $\boxed{f_3 \cdot x = x}$, so it is not faithful.

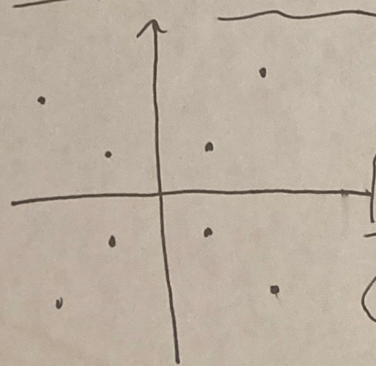
though f_3 is not $e(r_0)$ (along the $y=x$ line)

3. **(3 points)** Give an **explicit example** of a group action with exactly two orbits.

~~Still use the example in 2.~~

$G = D_4$, action on $X =$ ~~$\{(x,y) \in \mathbb{R}^2 \mid y = x\}$~~ $\{(1,1), (-1,1), (1,-1), (-1,-1), (2,2), (-2,2), (2,-2), (-2,-2)\}$ with natural





The only two orbits are:

$O((1,1)) = \{(1,1), (-1,-1), (1,-1), (-1,1)\}$

$O((2,2)) = \{(2,2), (-2,-2), (-2,2), (2,-2)\}$

[1]Recall, a group action of G on X is called *trivial* if for all $x \in X$ and all $g \in G$, $g \cdot x = x$

2 Consider the group $\mathbb{Z}_4 = \{0, 1, 2, 3\}$

And let it act on $\mathbb{Z}_2$.

$$g \cdot x = [g]_2 + x$$

This is a well-defined group action since

① $\forall x \in \mathbb{Z}_2$, $0 \cdot x = x \cdot 0 = x$

② $\forall x \in \mathbb{Z}_2$ and $g_1, g_2 \in \mathbb{Z}_4$, $g_1 \cdot (g_2 \cdot x) = g_1 \cdot ([g_2]_2 + x)$

$$= [g_1]_2 + [g_2]_2 + x$$

$$= [g_1 + g_2]_2 + x$$

$$= (g_1 + g_2) \cdot x$$

this action is not trivial,

but it is not faithful since

$\forall x$, $2 \cdot x = x \cdot 2 = x$ but 2 is not 0 in $\mathbb{Z}_4$.