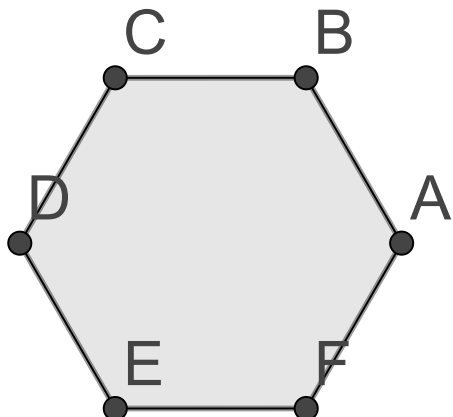


Problem 1. (5 points)

Consider the regular hexagon below.



Click on the image to render the hexagon.

Let X be the set of all triangles formed from the vertices of the regular hexagon; note that there are $\binom{6}{3} = 20$ such triangles. Consider the natural action of D_6 on the set X .

Find the cardinality of each of the indicated orbits:

a) $\text{Orbit}(\triangle FAB)$ ____

b) $\text{Orbit}(\triangle DBF)$ ____

c) $\text{Orbit}(\triangle EFB)$ ____

d) $\text{Orbit}(\triangle AFE)$ ____

e) $\text{Orbit}(\triangle CDA)$ ____

Answer(s) submitted:

- 6
- 2
- 12
- 6
- 12

submitted: (correct)

recorded: (correct)

Problem 2. (5 points)

Let S_5 act on the set X of all subsets of $\{1, 2, 3, 4, 5\}$ in the natural way. Note that X has 32 elements, each of which is a subset of $\{1, 2, 3, 4, 5\}$, of differing cardinalities.

Find $\{n : \exists x \in X \text{ such that } |O(x)| = n\}$. Enter this set as a comma separated list.

$n =$ ____

Answer(s) submitted:

- 1, 5, 10

submitted: (correct)

recorded: (correct)

Problem 3. (5 points)

Let $SL_3(\mathbb{Z}_2)$ be the set of 3×3 matrices with coefficients in \mathbb{Z}_2 which have determinant equal to 1.

Let $SL_3(\mathbb{Z}_2)$ act on the set \mathbb{Z}_2^3 of all 3×1 matrices (column vectors) by left multiplication. Find the cardinality of the orbit of each column vector below.

a. The orbit of $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ contains ____ elements.

b. The orbit of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ contains ____ elements.

c. The orbit of $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ contains ____ elements.

d. The orbit of $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ contains ____ elements.

e. The orbit of $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ contains ____ elements.

Answer(s) submitted:

- 1
- 7
- 7
- 7
- 7

submitted: (correct)

recorded: (correct)

Problem 4. (5 points)

For each of the following subgroups H of the respective groups G , determine if it is normal:

In the second row, $U(22) = \mathbb{Z}_{22}^\times$.

In the third and fourth rows, a is a counterclockwise rotation by $\frac{360}{7}$ degrees, and b is a flip along an axis of symmetry of a regular 7-gon.

G	H	Answer (Y/N)
\mathbb{Z}_{22}	$\langle 13 \pmod{22} \rangle$	_____
$U(22)$	$\langle 13 \pmod{22} \rangle$	_____
D_7	$\langle a^4 \rangle$	_____
D_7	$\langle a^4 b \rangle$	_____
$GL_2(\mathbb{Q})$	subgroup of upper triangular matrices	_____

Answer(s) submitted:

- Y
- Y
- Y
- N
- N

submitted: (correct)

recorded: (correct)

Problem 5. (5 points)

Let G be a finite group of order 35 acting on a finite set S of size 7. What are the possible values for the NUMBER of orbits of this G -action? Enter your answer as a comma-separated list.

Answer(s) submitted:

- 1,3,7

submitted: (correct)

recorded: (correct)