

Problem 1. (4 points)

Let U_n be the (multiplicative) group of units of \mathbb{Z}_n .

Find the values of a, b, c , in U_{10} such that the following map is an isomorphism.

$$U_5 \rightarrow U_{10}$$

$$1 \rightarrow a$$

$$2 \rightarrow 7$$

$$3 \rightarrow b$$

$$4 \rightarrow c$$

Then fill in the multiplication table for U_{10} with the correct numerical values in U_{10} , where the a, b and c in the table are the values in U_{10} you found above. You are given the multiplication table for U_5 , which you might consider using.

U_5	1	2	3	4
1	1	2	3	4
2	2	4	1	3
3	3	1	4	2
4	4	3	2	1

U_{10}	a	7	b	c
a	—	—	—	—
7	—	—	—	—
b	—	—	—	—
c	—	—	—	—

Answer(s) submitted:

- 1
- 7
- 3
- 9
- 7
- 9
- 1
- 3
- 3
- 1

- 9
- 7
- 9
- 3
- 7
- 1

submitted: (correct)

recorded: (correct)

Problem 2. (5 points)

Answer yes if the given map is a group homomorphism in the given groups and no otherwise.

$$a. \mathbb{Z}_4 \rightarrow \mathbb{Z}_8 \text{ [?/Yes/No]}$$

$$m \rightarrow 2m$$

$$b. \mathbb{Z} \rightarrow \mathbb{Q}^\times \text{ [?/Yes/No]}$$

$$n \rightarrow \frac{1}{2^n}$$

$$c. \mathbb{Z}_5 \rightarrow \mathbb{Z}_8 \text{ [?/Yes/No]}$$

$$[x]_5 \rightarrow [x]_8$$

$$d. \mathbb{R} \rightarrow \mathbb{R}^\times \text{ [?/Yes/No]}$$

$$\lambda \rightarrow 10^\lambda$$

$$e. U_{13} \rightarrow U_{13} \text{ [?/Yes/No]}$$

$$x \rightarrow -x$$

Answer(s) submitted:

- Yes
- Yes
- No
- Yes
- No

submitted: (correct)

recorded: (correct)

Problem 3. (4 points)

Find the number of elements in the kernel of the following map

$$\mathbb{Z}_{13}^\times \rightarrow \mathbb{Z}_{13}^\times$$

$$x \rightarrow x^2$$

Number of elements in kernel: —

Answer(s) submitted:

- 2

submitted: (correct)

recorded: (correct)

Problem 4. (4 points)

Let f and g be permutations on the set $\{1, 2, 3, 4, 5, 6\}$, defined as follows

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 3 & 2 & 1 & 6 \end{pmatrix}$$

$$g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 1 & 5 & 4 & 6 \end{pmatrix}$$

Write each of the following permutations as a product of disjoint cycles, separated by commas (e.g. $(1, 2), (3, 4, 5), \dots$). Do not include 1-cycles (e.g. (2)) in your answer.

(a) $fg =$ _____

(b) $f^{-1} =$ _____

(c) $fgf^{-1} =$ _____

Assume multiplication of permutations f, g obeys the rule $(fg)(x) = f(g(x))$ so $(1, 3)(1, 2) = (1, 2, 3)$ not $(1, 3, 2)$.

Answer(s) submitted:

- $(1, 3, 4), (2, 5)$
- $(5, 2, 4, 1)$
- $(3, 4), (1, 2)$

submitted: (correct)

recorded: (correct)

Problem 5. (4 points)

Let f be a permutation on the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9\}$, defined as follows

$$f = (72)(51724)$$

(a) Write the permutation f^6 as a product of disjoint cycles, separated by commas (e.g. $(1, 2), (3, 4, 5), \dots$). Do not include 1-cycles (e.g. (2)) in your answer.

$f^6 =$ _____

(b) Determine the order of f^6 . _____

Answer(s) submitted:

- $(2, 5), (4, 1)$
- 2

submitted: (correct)

recorded: (correct)