
Problem 1. **(6 points)**

In this problem we work out step-by-step a complete set of left coset representatives of a subgroup. First, let us recall how to do this in general. Say you have a subgroup H of G. Remember that for any subgroup H of a general group G,

$(*)$ $xH = yH$ if and only if $x^{-1}y$ is in H.

In particular,

$(**)$ the identity e is a coset representative of the trivial coset H.

To find representatives for all left H-cosets we proceed ¡i¿inductively¡/i¿. Say we have already found representatives for a collection of H-cosets. Denote by S the union of the H-cosets for which we have already found representatives.

(i) If $S = G$ then we are done; if not,
(ii) pick an element x of G not in S
(iii) add to S the elements of xH, and then go back to step (i).

Let us now apply this to the subgroup $\langle 3\rangle$ of $U(56)$. To simplify the notation, we represent each congruence class by a positive integer < 56.

First, write down the elements of $\langle 3\rangle$:

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Now we look for coset representatives. For the trivial $\langle 3\rangle$-coset, condition $(**)$ says that

_____ is a coset represenatative for $\langle 3\rangle$ in $U(56)$.

We now apply the inductive algorithm above. The subgroup $\langle 3\rangle$ is a proper subgroup, so there are elements in $U(56)$ not in $\langle 3\rangle$. By the algorithm above, we need to look for x in $U(56)$ not in $\langle 3\rangle$. For a general group we just pick a random such element; in the case of $U(56)$ we can be more specific, by picking the ¡i¿smallest positive integer¡/i¿ x that represents a congruence class in $U(56) - \langle 3\rangle$. For the subgroup $\langle 3\rangle$ this integer x is

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We check that $S = \langle 3\rangle \cup x\langle 3\rangle$ together still do not fill up $U(56)$; to find the next coset representative we pick the smallest y in $U(n) - S$. Repeat this process until we fill up S as described in the algorithm above. In the end, this process the following list of coset representative:

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Answer(s) submitted:

- $1, 3, 9, 19, 25, 27$
- 1
- 5
- $1, 5, 11, 29$

submitted: (correct)
recorded: (correct)

Problem 2. **(3 points)**

Let H be a proper subgroup of a group G, and let K be a proper subgroup of H. If $|K| = 25$ and $|G| = 200$, what are the possible orders of H? Enter your answer as a comma separated list.

_____

Answer(s) submitted:

- $100, 50$

submitted: (correct)
recorded: (correct)

Problem 3. **(3 points)**

Let H, K be proper subgroups of a group G. If $|K| = 99$ and $|H| = 77$, what are the possible orders of $K \cap H$? Enter your answer as a comma separated list.

Answer: _______________

Answer(s) submitted:

- $1, 11$

submitted: (correct)
recorded: (correct)

Problem 6. **(4 points)**

Let $G = \mathbb{R}^\times$ act on $X = \mathbb{R}^2$ by the rule $\lambda \cdot (x,y) = (\lambda x, \lambda y)$. Which of the following subsets of $\mathbb{R}^2$ are orbits of the group action?

1. $\{(0,0)\}$ [?/Yes/No]
2. the x-axis [?/Yes/No]
3. the y axis with the origin removed [?/Yes/No]
4. the circle $\{(x,y) \| x^2 + y^2 = 1\}$ [?/Yes/No]
5. the hyperbola $\{(x,y) | xy = 1\}$ [?/Yes/No]
6. the line $y = x$ with the origin removed [?/Yes/No]

Answer(s) submitted:

- Yes
- No
- Yes
- No
- No
- Yes

submitted: (correct)
recorded: (correct)

Problem 7. **(4 points)**

Let $G = \mathbb{R}^\times$ act on $X = \mathbb{R}^2$ by the rule $\lambda \cdot (x,y) = (\lambda x, \lambda^{-1} y)$. Which of the following subsets of $\mathbb{R}^2$ are orbits of the group action?

1. $\{(0,0)\}$ [?/Yes/No]
2. the x-axis [?/Yes/No]
3. the y axis with the origin removed [?/Yes/No]
4. the circle $\{(x,y) \| x^2 + y^2 = 1\}$ [?/Yes/No]
5. the hyperbola $\{(x,y) | xy = 1\}$ [?/Yes/No]
6. the line $y = x$ with the origin removed [?/Yes/No]

Answer(s) submitted:

- Yes
- No
- Yes
- No
- Yes
- No

submitted: (correct)
recorded: (correct)

Problem 8. **(4 points)**

Let D_6, the group of symmetries of a hexagon $\mathcal{H}$, act on $\mathcal{H}$ (including the interior points) in the standard way. Find the set

$\{n : \exists h \in \mathcal{H} \text{ such that } |O(h)| = n\}$.

Here being "on a hexagon" is defined here as either on the edge or within the internal area of a hexagon. Enter your answer as a list of numbers separated by commas; thus, if the set above is $\{1,2,3\}$ you should enter 1, 2, 3 in the box below.

$n =$___

Answer(s) submitted:

- $1,6,12$

submitted: (correct)
recorded: (correct)