Ways we differ from the textbook

Math 412

University of Michigan

In this class, we assume every ring is a *ring with unity*. That is, when we say a set R is a ring, part of what we're saying is that R has a multiplicative ideantity 1_R. Our textbook does not make this assumption. This difference has a cascading effect, so that a few other definitions we use in this course will differ from the book. Here's a list of these definitions. We'll add more as we come across them.

1. *Rings* (Section 3.1, worksheet 6). For us, when we say "R is a ring," we include the hypothesis that there exists some element $1_R \in R$ such that $1_R r = r 1_R = r$ for all $r \in R$.

2. *Subrings* (Section 3.1, worksheet 6). For us, when we say "S is a subring of R", part of what we're saying is that $1_R \in S$.

3. *Ring homomorphism* (Section 3.3, worksheet 7). When we say "$\varphi : R \to S$ is a homomorphism of rings", part of what we're saying is that $\varphi(1_R) = 1_S$.

4. *Ideals* (Section 6.1, worksheet 10). We agree with the textbook on what it means for a subset I of a ring R to be an ideal. However, the book defines an ideal to be "a subring I of a ring R such that $ra \in I$ and $ar \in I$ whenever $r \in R$ and $a \in I$." *This is only a good definition if you don't define subrings of R to contain 1_R.* So we would say that a subset I of a ring R is an ideal provided that:

   (a) for all $a, b \in I$, $a + b \in I$, and

   (b) for all $r \in R$ and $a \in I$, $ar \in I$ and $ra \in I$.