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Good job!

Math 412

Quiz 5

Thursday, February 15

You have 15 minutes to complete the quiz. You may turn in corrections for up to half credit back by the beginning of the next class period.

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1. (3 points) Give an example of each of the following (you do not need to justify your answer):

   (a) A ring that is not a domain

   $\mathbb{Z}_4$ $([2]\times[2]=[0])$

   (b) A domain that is not a field

   $\mathbb{Z}[x]$

2. (4 points)

   (a) Show that $x^3 + a$ is reducible in $\mathbb{Z}_3[x]$ for each $a \in \mathbb{Z}_3$

   Remember you just need to know it has a zero

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   ~~Let $a \in \mathbb{Z}_3$ be arbitrary~~

   ~~Since $\deg(x^3+a) = 3$ for each $a \in \mathbb{Z}_3$,~~

   ~~By the division algorithm, let $x^3+a =$~~

   There are only 3 elements in $\mathbb{Z}_3$: 1, 2, 0.

   $x^3 = x \cdot x \cdot x$

   $x^3+1 =$

   $x^3+2 =$

   (1) Take $x=0 \Rightarrow x^3=0$ so x is a factor of x^3 by the factor theorem

   (2) Take $x=2 \Rightarrow x^3 = 2\times2\times2 = 1\times2 = 2$ in $\mathbb{Z}_3$ $\Rightarrow x^3+1=0$ so $(x-2)$ is a factor of x^3+1 by the factor theorem

   (3) Take $x=1 \Rightarrow x^3=1 \Rightarrow x^3+2=0$ in $\mathbb{Z}_3$ so $(x-1)$ is a factor of x^3+2 by the factor theorem

   Therefore $\forall a \in \mathbb{Z}_3$, x^3+a can be factorized into polynomials of lower degree. $\Rightarrow$ reducible

   (b) Factor $x^4 - 4$ as a product of irreducibles in $\mathbb{Z}_5[x]$

   ~~Since difference of square rule is true in any ring~~

   ~~$x^4-4 = (x^2+2)(x^2-2)$~~

   Since difference of square rule is true in any ring

   $x^4-4 = (x^2+2)(x^2-2)$

   Since none of 0, 1, 2, 3, 4 is the root of (x^2-2) and (x^2+2) in $\mathbb{Z}_5[x]$, this is the final result.

3. (3 points) Let R be a ring and 1_R the multiplicative identity for R. Show that 1_R is the multiplicative identity for $R[x]$.

   pf Let m be an arbitrary element in $R[x]$

   so $m = a_1x^1 + a_2x^2 + \dots + a_nx^n$ for some integer $n \geq 0$ and $a_1, a_2, \dots, a_n \in R$

   $1_R \cdot m = 1_Ra_1x^1 + 1_Ra_2x^2 + \dots + 1_Ra_nx^n = m$

   $m \cdot 1_R = (a_1x^1)1_R + (a_2x^2)1_R + \dots + (a_nx^n)1_R = m$

   There for 1_R is the multiplicative identity be definition

There are some definition issues, but it'd be unfair to grade based on them.