Problem 1. **(9 points)**

Below are given the multiplication and addition tables for a commutative ring S with three elements.

+	**r**	**n**	**c**
r	___	___	r
n	c	r	___
c	___	___	___

$*$	**r**	**n**	**c**
r	r	___	___
n	n	___	___
c	___	___	___

a. Complete the tables.
b. What element of S is the additive identity? ___
c. What element of S is the multiplication identity? ___

Answer(s) submitted:

- n
- c
- n
- r
- n
- c
- n
- c
- r
- c
- c
- c
- c
- c
- r

submitted: (correct)
recorded: (correct)

Problem 2. **(6 points)**

Let $M_2(\mathbb{R})$ be the ring of 2×2 matrices with $\mathbb{R}$ entries, using matrix addition and multiplication as the operations. Which of the following subsets of $M_2(\mathbb{R})$ are also rings?

For each subset $S \subset M_2(\mathbb{R})$ below, use matrix addition and multiplication for the addition and multiplication operations (respectively) on the subset. Don't forget to check that these operations on $M_2(\mathbb{R})$ actually are still operations on S.

a. $U_2(\mathbb{R})$ [?/Yes/No]
($U_2(\mathbb{R})$ is the set of '2×2'$upper-triangularmatriceswithrealentrie$s.)b.
[?/Yes/No]
($M_2(\mathbb{Q})$ is the set of '2×2'$matriceswithrationalentrie$s.)c. $D_2(\mathbb{R})$
[?/Yes/No]
($D_2(\mathbb{R})$ is the set of '2×2'$diagonalmatriceswithrealentrie$s.)d. $D_2(\mathbb{N})$
[?/Yes/No]
($D_2(\mathbb{N})$ is the set of '2×2'$diagonalmatriceswithpositiveintegerentrie$
[?/Yes/No]
($GL_2(\mathbb{Q})$ is the set of '2×2'$invertiblematriceswithrationalentries.$)

Answer(s) submitted:

- Yes
- Yes
- Yes
- No
- No

submitted: (correct)
recorded: (correct)

Problem 3. **(5 points)** Let $M_2(\mathbb{Z}_2)$ be the ring of 2×2 matrices with entries in the ring $\mathbb{Z}_2$. For this problem, we write the classes $[0]_2$ and $[1]_2$ as 0 and 1.

a. The additive identity of $M_2(\mathbb{Z}_2)$ is: $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

b. The multiplicative identity of $M_2(\mathbb{Z}_2)$ is: $\begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$

c. $A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ is a non-zero element of $M_2(\mathbb{Z}_2)$ with no multiplicative inverse.

d. $B = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$ is an element of $M_2(\mathbb{Z}_2)$ that is not the identity and has a multiplicative inverse.

e. How many elements are in the set of 2×2 diagonal matrices $\mathbb{Z}_2$? ___

Answer(s) submitted:

- $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$
- $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$
- $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$
- 4

submitted: (correct)
recorded: (correct)

Problem 4. **(10 points)** Which of the following subsets of $\mathbb{Z}_6$ is a ring under the operations of $\mathbb{Z}_6$? Which is a subring? If it is a ring, what is the multiplicative identity? If it is not a ring leave the identity blank.

$\mathbb{Z}_6$:
Ring: [?/Yes/No]
Subring: [?/Yes/No]
Identity: ___

$\{0,2,4\}$:
Ring: [?/Yes/No]
Subring: [?/Yes/No]
Identity: ___

$\{0\}$:
Ring: [?/Yes/No]
Subring: [?/Yes/No]
Identity: ___

$\{0,1\}$:
Ring: [?/Yes/No]
Subring: [?/Yes/No]
Identity: ___

$\{0,3\}$:
Ring: [?/Yes/No]
Subring: [?/Yes/No]
Identity: ___

Answer(s) submitted:

- Yes
- Yes
- 1
- Yes
- No
- 4
- Yes
- No
- 0
- No
- No
-
- Yes
- No
- 3

submitted: (correct)
recorded: (correct)