Quiz 1

If $a|c$, $b|c$, $d=(a,b)$, show $ab|cd$

Pf Since $a|c$, $b|c$,
$c=ka=lb$ for some integer k,l.
Since $d=(a,b)$,
$a=dx$, $b=dy$ for some integer x,y.
$\boxed{\text{AND } (x,y)=1}$ since if x,y has some common divider $s>1$ then $\frac{x}{s}, \frac{y}{s} \in \mathbb{Z}$,
and then $a=ds(\frac{x}{s})$, $b=ds(\frac{y}{s})$,
then $(a,b)=ds$.
Hence $c=ka=lb=dkx=dly$
so $\boxed{cd=d^2kx=d^2ly}$, then $\boxed{kx=ly}$
and $\boxed{ab=d^2xy.}$
By FTA, x, y can be faconized into some primes $p_1,\ldots p_\alpha$, $q_1,\ldots q_\beta$ respectfully
Since $(x,y)=1$, there is no q_i, $1\le i\le\beta$ matches any of p_j, $1\le j\le\alpha$
Therefore all of q_i, $1\le i\le\beta$ must be in factors of k, that is, $k=wy$ for some $w\in\mathbb{Z}$
Hence $cd=wd^2xy=wab$. So $ab|cd$.