Problem 1. **(6 points)**

Which of the following rings have no non-zero zero divisors?

a. $\mathbb{Z}_{24}[x]$ [?/Yes/No]
b. $\mathbb{Z}_4[x]$ [?/Yes/No]
c. $M_2(\mathbb{Q})$ [?/Yes/No]
d. $\mathbb{Z}_{11}[x]$ [?/Yes/No]
e. $\mathbb{Z}_3[x]$ [?/Yes/No]

Answer(s) submitted:

- No
- No
- No
- Yes
- Yes

submitted: (correct)
recorded: (correct)

Problem 2. **(6 points)**

For each of the following maps, say whether or not it is a Ring Homomorphism (using our class's definition of a ring Homomorphism, not the book's!).

a. $\mathbb{Z}_8 \times \mathbb{Z}_8 \to \mathbb{Z}_8$
$([n]_8, [m]_8) \mapsto [m]_8$ [?/Yes/No]

b. $\mathbb{R}[x] \to \mathbb{R}$
$f(x) \mapsto f(\pi)$ [?/Yes/No]

c. $\mathbb{Z}_2 \to \mathbb{Z}_2 \times \mathbb{Z}_3$
n
$_2 \mapsto ([n]_2, [0]_3)$ [?/Yes/No]

d. $\mathbb{Z}_3 \to \mathbb{Z}_2 \times \mathbb{Z}_3$
n
$_3 \mapsto ([1]_2, [n]_3)$ [?/Yes/No]

e. The inclusion of $\{[0]_6, [3]_6\}$ in $\mathbb{Z}_6$ [?/Yes/No]

Answer(s) submitted:

- Yes
- Yes
- No
- No
- No

submitted: (correct)
recorded: (correct)

Problem 3. **(1 point)**

Which of the following are ring homomorphisms? Answer Yes if it is and No otherwise.

1. $\phi : \mathbb{Z}_7 \to \mathbb{Z}_{28}$ such that $\phi([c]_7) = [c]_{28}$ [?/Yes/No]

2. $\phi : \mathbb{Z}_{54} \to \mathbb{Z}_6$ such that $\phi([c]_{54}) = [c]_6$ [?/Yes/No]

3. $\phi : \mathbb{Z}_3 \to \mathbb{Z}_3[x]$ such that $\phi([c]_3) = [c]_3$ [?/Yes/No]

4. $\phi : \mathbb{Z}_{24}[x] \to \mathbb{Z}_{24}$ such that $\phi(f) = [f(2)]_{24}$ [?/Yes/No]

5. $\phi : \mathbb{Z}[x] \to \mathbb{Z}$ such that $\phi(f) = [f(2)]_8$ [?/Yes/No]

6. $\phi : \mathbb{Z}_{10} \to \mathbb{Z}_{10}[x]$ such that $\phi([c]_{10}) = [c]_{10}x$ [?/Yes/No]

Answer(s) submitted:

- No
- Yes
- Yes
- Yes
- No
- No

submitted: (correct)
recorded: (correct)

Problem 4. **(9 points)**

Recall that a reduced fraction is a fraction a/b where $\gcd(a,b) = 1$. It is a fact that

$$\mathbb{Z}_{(7)} := \{ \text{ all reduced fractions whose denominators are powers of 7} \}$$
$$\text{(including the zero-th power)}$$

is a subring of $\mathbb{Q}$, the ring of rational numbers.

For this problem we require that ring homomorphisms take the multiplicative identity to the multiplicative identity.

(a) Determine the number of ring homomorphisms $\mathbb{Z}_{(7)} \to \mathbb{Z}_7$.

____

(b) Determine the number of ring homomorphisms $\mathbb{Z}_7 \to \mathbb{Z}_{(7)}$.

____

Answer(s) submitted:

- 0
- 0

submitted: (correct)
recorded: (correct)

Problem 5. **(9 points)**

Determine the number of possible ring homomorphisms for each pair of rings:

(a) $\mathbb{Z}_{13} \to \mathbb{Z}_{13}$: ____

(b) $\mathbb{Z}_{80} \to \mathbb{Z}_{16}$: ____

(c) $\mathbb{Z}_{127} \to \mathbb{Z}_{21}$: ____

Note: Recall that ring homomorphisms take the multiplicative identity to multiplicative identity.

Answer(s) submitted:

- 1
- 1
- 0

submitted: (correct)
recorded: (correct)