Problem 1. **(15 points)**

(a) Determine all elements in the ideal (8) of $\mathbb{Z}_{20}$. For each element of $\mathbb{Z}_N$, write it in terms of its smallest nonnegative element. (remember here we are abusing notation, in the answer you will write a number a, but really mean $[a]_N$)

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(b) Determine all elements in the ideal $(8,10)$ of $\mathbb{Z}_{20}$.

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(c) Determine all elements m of $\mathbb{Z}_{20}$ such that $(8,m)$ is a proper ideal of $\mathbb{Z}_{20}$.

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Answer(s) submitted:

- $0,4,8,12,16$
- $0,2,4,6,8,10,12,14,16,18$
- $0,2,4,6,8,10,12,14,16,18$

submitted: (correct)
recorded: (correct)

Problem 2. **(1 point)**

It is a fact that every ideal of $\mathbb{Z}_{40}$ is of the form (b) for some element b of $\mathbb{Z}_{40}$. A maximal ideal in a ring R is an ideal $I \subset R$ such that $I \neq R$, and the only ideal containing I is R.

(a) Determine all maximal ideals of $\mathbb{Z}_{40}$ containing the ideal (16). Enter a generator for each of these ideals. That is, if you think (16) is contained in the maximal ideals (a) and (b), enter a,b.

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Answer(s) submitted:

- 2

submitted: (correct)
recorded: (correct)

Problem 3. **(15 points)**

(a) Find all roots of $(x^2+6*x+8)$ in $\mathbb{Z}_{15}[x]$.

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(b) Find all factorizations of $(x^2+6*x+8)$ of the form $(x-A)(x-B)$ in the ring $\mathbb{Z}_{15}[x]$. There may be fewer than 5 distinct factorizations; enter your factorization(s) from the top row down, and leave blank row(s) below if you don't use all 5.

$$\begin{aligned}(x^2+6*x+8) &= (\;\_\_\;)(\;\_\_\;)\\ &= (\;\_\_\;)(\;\_\_\;)\\ &= (\;\_\_\;)(\;\_\_\;)\\ &= (\;\_\_\;)(\;\_\_\;)\\ &= (\;\_\_\;)(\;\_\_\;)\end{aligned}$$

Answer(s) submitted:

- $1,8,11,13$
- $x-1$
- $x-8$
- $x-11$
- $x-13$
-
-
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-
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submitted: (correct)
recorded: (correct)