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Assignment WebWork5-PolynomialRings due 02/19/2024 at 11:59pm EST

Problem 1. **(5 points)**

As in the text, if we are working in $\mathbb{Z}_n[x]$, we will drop brackets when writing coefficients. For example, we write $2x^2+7x-5$ for $[2]_n x^2+[7]_n x-[5]_n$. Answer "Yes" if a statement is true in $\mathbb{Z}_7$ and "No" otherwise.

a. $(2x+1)(3x-1) = -x^2+x+5$ [?/Yes/No]
b. $(x+4)-(2x+1) = 7x$ [?/Yes/No]
c. $-x^2-2 = 6x^2-23$ [?/Yes/No]
d. $(6x^2+5x+1)+(2x^2+3x+5) = (x^2+4x-1)$ [?/Yes/No]
e. $(x^2-2x+3)-(5x^2-6x+1) = 3x^2+4x+2$ [?/Yes/No]

Answer(s) submitted:

- No
- No
- Yes
- No
- Yes

submitted: (correct)
recorded: (correct)
Correct Answers:

- No
- No
- Yes
- No
- Yes

Problem 2. **(12 points)**

Let $f(x) = x^3+2x^2-3x-3$ and $g(x) = 3x^2+x-2$ be polynomials in $\mathbb{Z}_5[x]$. Find $a, b \in \mathbb{Z}_5[x]$ such that $af+bg=1$. Enter the coefficients of a and b below. If you don't need a term, please enter a 0.

$a =$___ x^5+___x^4+___x^3+___x^2+___$x+$___

$b =$___x^5+___x^4+___x^3+___x^2+___$x+$___

Answer(s) submitted:

- 0; 0; 0; 0; 4; 0; 0; 0; 0; 2; 0; 2

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0; 1; 2; -3; -3; 0; 3; 1; -2; 1; 2; 3

Problem 3. **(4 points)**

Compute the gcd of the polynomials $(x^2-16)(x^3-36x)(x-5)(x-2)$ and $(x^2-10x+24)(x-5)(x-1)$ in $\mathbb{Q}[x]$. Write the coefficients below. If you do not need a term of a certain order, write 0.

___x^3+___x^2+___$x+$___

Answer(s) submitted:

- 1
- -15
- 74
- -120

submitted: (correct)
recorded: (correct)
Correct Answers:

- 1
- -15
- 74
- -120

Problem 4. **(3 points)**
Let $f(x) = 2x^4 + 6x^3 + 3x^2 + 9x + 2$,
$g(x) = x - 2$,
$h(x) = x - 4$,
$k(x) = x + 1$

be polynomials with coefficients in $\mathbb{R}$

For $g(x)$, $h(x)$, and $k(x)$ compute r_g, r_h, r_k such that: $f(x) = q_g(x)g(x) + r_g(x)$, $f(x) = q_h(x)h(x) + r_h(x)$, $f(x) = q_k(x)k(x) + r_k(x)$.

Hint: Look at Theorem 4.15 (The Remainder Theorem) in the book.

$r_g(x) =$ ___

$r_h(x) =$ ___

$r_k(x) =$ ___

Answer(s) submitted:

- 112
- 982
- -8

submitted: (correct)
recorded: (correct)
Correct Answers:

- 112
- 982
- -8

Problem 5. **(4 points)**
Let $f(x), g(x), h(x) \in \mathbb{Z}_5[x]$ and $f(x) = x^4 + 2x^3 - 4x^2 + 3x + 2$,
$g(x) = x^3 + 0x^2 + 1x + 3$
$h(x) = x^2 - 2x + 1$

For $g(x)$ and $h(x)$compute q_g, r_g,q_h, and $r_h$such that: $f(x) = q_g(x)g(x) + r_g(x)$,
$f(x) = q_h(x)h(x) + r_h(x)$.

Please enter only the symbols 0, 1, 2, 3, 4 as coefficients; do not leave any blank or choice other representatives of the congruence classes

$q_g(x)$= ___ x^3+___ x^2+___ x+___
$r_g(x)$ = ___ x^3+___ x^2+___ x+___
$q_h(x)$= ___ x^3+___ x^2+___ x+___
$r_h(x)$ = ___ x^3+___ x^2+___ x+___

Answer(s) submitted:

- 0
- 0
- 1
- 2
- 0
- 0
- 3
- 1
- 0
- 1
- 4
- 3
- 0
- 0
- 0
- 4

submitted: (correct)
recorded: (correct)
Correct Answers:

- 0
- 0
- 1
- 2
- 0
- 0
- 3
- 1
- 0
- 1
- 4
- 3
- 0
- 0
- 0
- 4

Problem 6. **(4 points)**

Let $f(x)$, $d_1(x)$, and $d_2(x)$ be polynomials in $\mathbb{Q}[x]$ such that $f(x) = 2x^4+2x^3+2x^2+x+1$, $d_1(x) = x^2+1$, and $d_2(x) = x^3+2x+1$.

For each polynomial d, compute the unique quotient and remainder over $\mathbb{Q}[x]$ such that $f(x) = q(x)\cdot d(x)+r(x)$

For d_1: q_1 = ___ r_1 = ___

For d_2: q_2 = ___ r_2 = ___

Answer(s) submitted:

- $2x^2+2x$
- $-x+1$
- $2x+2$
- $-2x^2-5x-1$

submitted: (correct)
recorded: (correct)
Correct Answers:

- x^2+1
- x^2+1
- x^2+1
- x^2+1