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此课将使用以下 symbols:

power set: $\underline{P(X) = \{A \mid A \subseteq X\}}$

indexed family: if I is a set;

对每个 $i \in I$, A_i 为一个 set;

则 $\{A_i \mid i \in I\}$ 为一个 indexed family of sets

indexed union: $\bigcup_{i\in I} A_i = \{x \mid x \in A_i \text{ for some } i \in I\}$

indexed intersection: $\bigcap_{i\in I} A_i = \{x \mid x \in A_i \text{ for all } i \in I\}$

relative complement: $A \setminus B = \{x \in A \mid x \notin B\}$

In this class, $0 \notin \mathbb{N}$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

$\mathbb{N}$: given by God

$\mathbb{C}$: "algebraically closed"

3种 approaches to foundamental issues:

(1) naive approach

(2) axiomatic approach

(3) constructive approach (set theory, 582)

Using constructive approach to build $\mathbb{N}$:

$0 = \emptyset$

$1 = \{0\} = \{\emptyset\}$

$2 = \{0,1\} = \{\emptyset, \{\emptyset\}\}$

$3 = \{0,1,2\} = \{\emptyset, \{\emptyset\}, \{\emptyset, \{\emptyset\}\}\}$

$\vdots$

$\mathbb{N} \subseteq \mathbb{Z} \subseteq \mathbb{Q} \subseteq \mathbb{R} \subseteq \mathbb{C}$

$\mathbb{Q}, \mathbb{R}$: ordered field

$\mathbb{Q}, \mathbb{R}, \mathbb{C}$: field

$\mathbb{R}$ is an ① order field with linear relation "<" (transitive, irreflexive, ② trichotomy)

and satisfies the completeness axiom ③

$(\forall S \subseteq \mathbb{R}\ (S \neq \emptyset),\ \sup S \in \mathbb{R})$

Def. $I \subseteq \mathbb{R}$ is $\boxed{\text{inductive}}$

if ① $1 \in I$

② $\forall x \in \mathbb{R}$, if $x \in I \Rightarrow x+1 \in I$

$\boxed{\mathbb{N} = \bigcap \{\text{all inductive subsets of } \mathbb{R}\}}$ (smallest inductive subset)

(Then $\mathbb{N} = \{1, 2, 3, \dots\}$)

Recall 一些中学知识:

① $\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$

② $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$

③ $\sum_{k=0}^{n} r^k = \frac{1-r^{n+1}}{1-r}$

④ $\binom{n+1}{k} = \binom{n}{k} + \binom{n}{k-1}$

⑤ Binomial Thm: $\forall a, b \in \mathbb{R}$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

Defs by induction

1. int powers of reals:

$(a \in \mathbb{R})$ ① $a^0 = 1$

② $a^n = a^{n-1} \cdot a$

2. factorial function:

① $0! = 1$

② $(n+1)! = (n+1) \cdot n!$

3. Summation & Product Notation

① $\sum_{k=1}^{1} a_k = \prod_{k=1}^{1} a_k = a_k$

② $\sum_{k=1}^{n+1} a_k = \sum_{k=1}^{n} a_k + a_{k+1}$, $\prod_{k=1}^{n+1} a_k = \left(\prod_{k=1}^{n} a_k\right) a_{k+1}$