

Math 451 001 Assignment 1

Due Jan 19th

Complete all of the following questions.

1. (10 pts) Prove, using the definition of a least upper bound, that a subset E in an ordered set S cannot have two different least upper bounds. *Hint: Suppose that both a and b were both least upper bounds of a set S . Show that $a = b$.*
2. (10 pts) Find the least upper bound and the greatest lower bound, if they exist, of the following subsets of \mathbb{Q} . Also decide which sets have a maximum or minimum. Recall that the maximum (resp. minimum) of an order set is the largest (resp. smallest) element in it.
 - (a) $\{1/n : n \in \mathbb{N}\}$.
 - (b) $\{1/n : n \in \mathbb{Z} \text{ and } n \neq 0\}$.
 - (c) $\{x : x = 0 \text{ or } x = 1/n \text{ for some } n \in \mathbb{N}\}$.
 - (d) $\{1/n + (-1)^n : n \in \mathbb{N}\}$.
3. (10 pts) Suppose that A and B are two nonempty subsets of an ordered set S such that $x \leq y$ for all $x \in A$ and $y \in B$.
 - (a) Prove that $\sup A \leq y$ for all y in B .
 - (b) Prove that $\sup A \leq \inf B$.