

# STATS / DATA SCI 315

## Transformers

slides credit: Dr. Simon J. D. Prince

<https://udlbook.github.io/udlbook/>

# Natural language processing (NLP)

- Translation
- Question answering
- Summarizing
- Generating new text
- Correcting spelling and grammar
- Finding entities
- Classifying bodies of text
- Changing style etc.

# Transformers

- Motivation
- Dot-product self-attention
- Matrix form
- The transformer
- NLP pipeline
- Decoders
- Large Language models

# Motivation

Design neural network to encode and process text:

The restaurant refused to serve me a ham sandwich, because it only cooks vegetarian food. In the end, they just gave me two slices of bread. Their ambience was just as good as the food and service.

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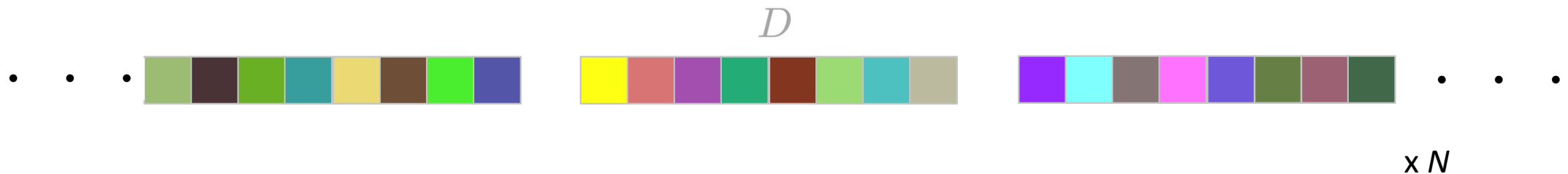


Word embeddings convert words into fixed dimensional vectors  
word2vec is a popular family of word embeddings

# Motivation

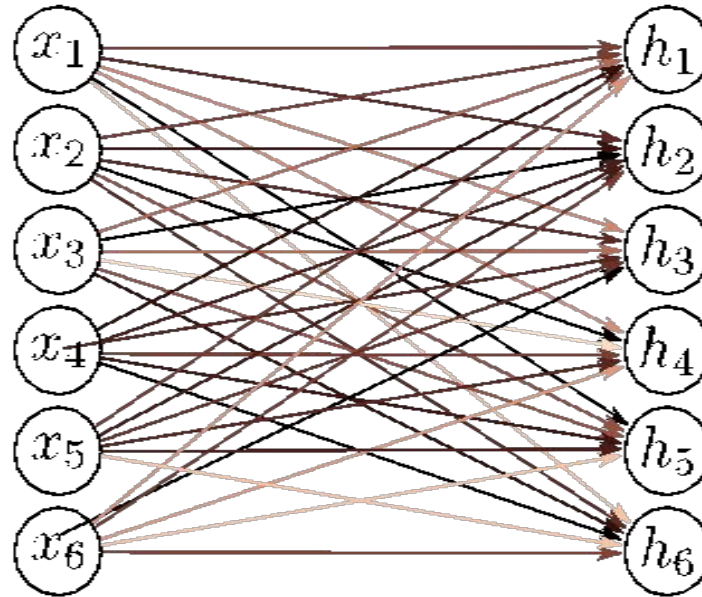
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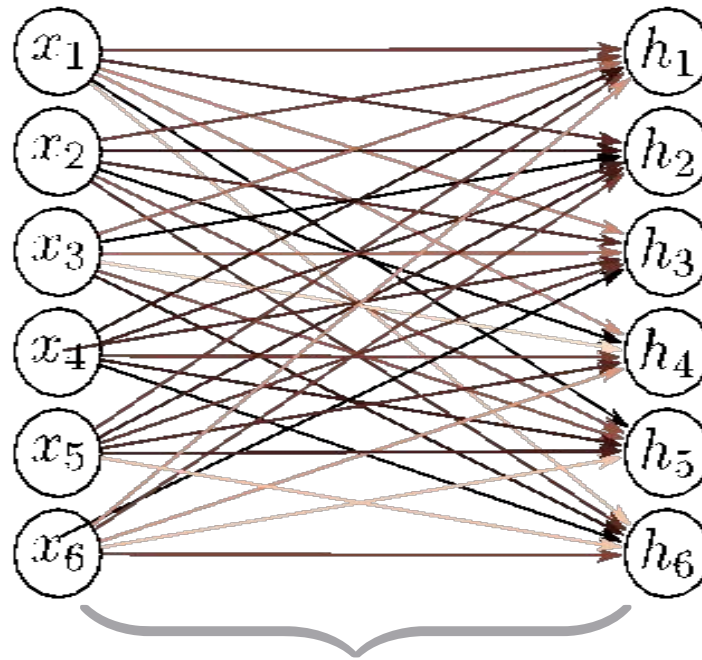
# Standard fully-connected layer

$$\mathbf{h} = \mathbf{a}[\boldsymbol{\beta} + \boldsymbol{\Omega}\mathbf{x}]$$



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$\Phi$  contains  
 $D^2$  connections



# Standard fully-connected layer

$$\mathbf{h} = \mathbf{a}[\boldsymbol{\beta} + \boldsymbol{\Omega}\mathbf{x}]$$

Problem:

- A very large number of parameters
- Can't cope with text of different lengths

Conclusion:

- We need a model where parameters don't increase with input length

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The word **their** must “attend to” the word **restaurant**.

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Conclusions:

- There must be connections between the words.
- The strength of these connections will depend on the words themselves.

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# Dot-product self attention

- Takes N inputs of size D<sub>x</sub>1 and returns N inputs of size D<sub>x</sub>1
- Computes N **values** (no ReLU)

$$\mathbf{v}_n = \beta_v + \Omega_v \mathbf{x}_n$$

# Dot-product self attention

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- $N$  outputs are weighted sums of these values

$$\mathbf{sa}[\mathbf{x}_n] = \sum_{m=1}^N a[\mathbf{x}_n, \mathbf{x}_m] \mathbf{v}_m$$

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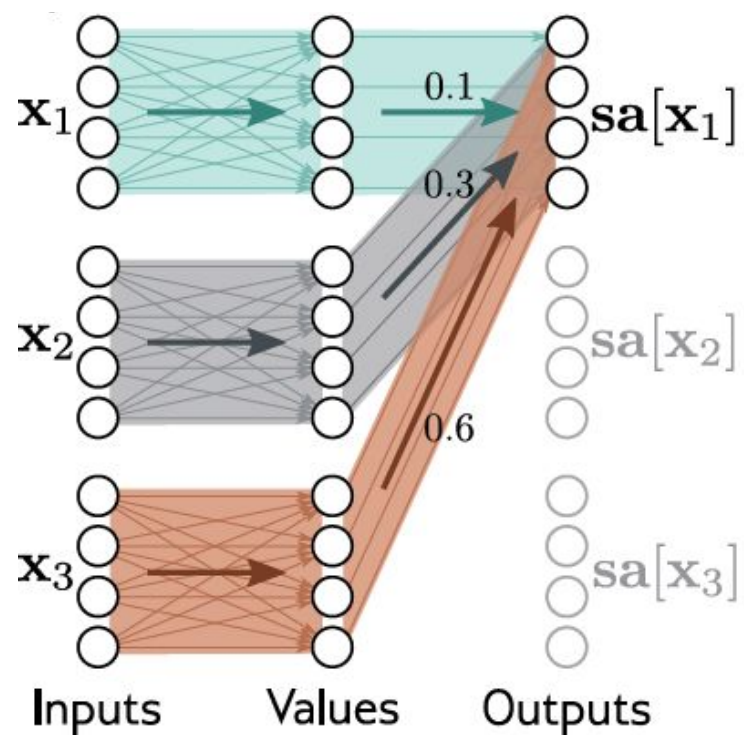
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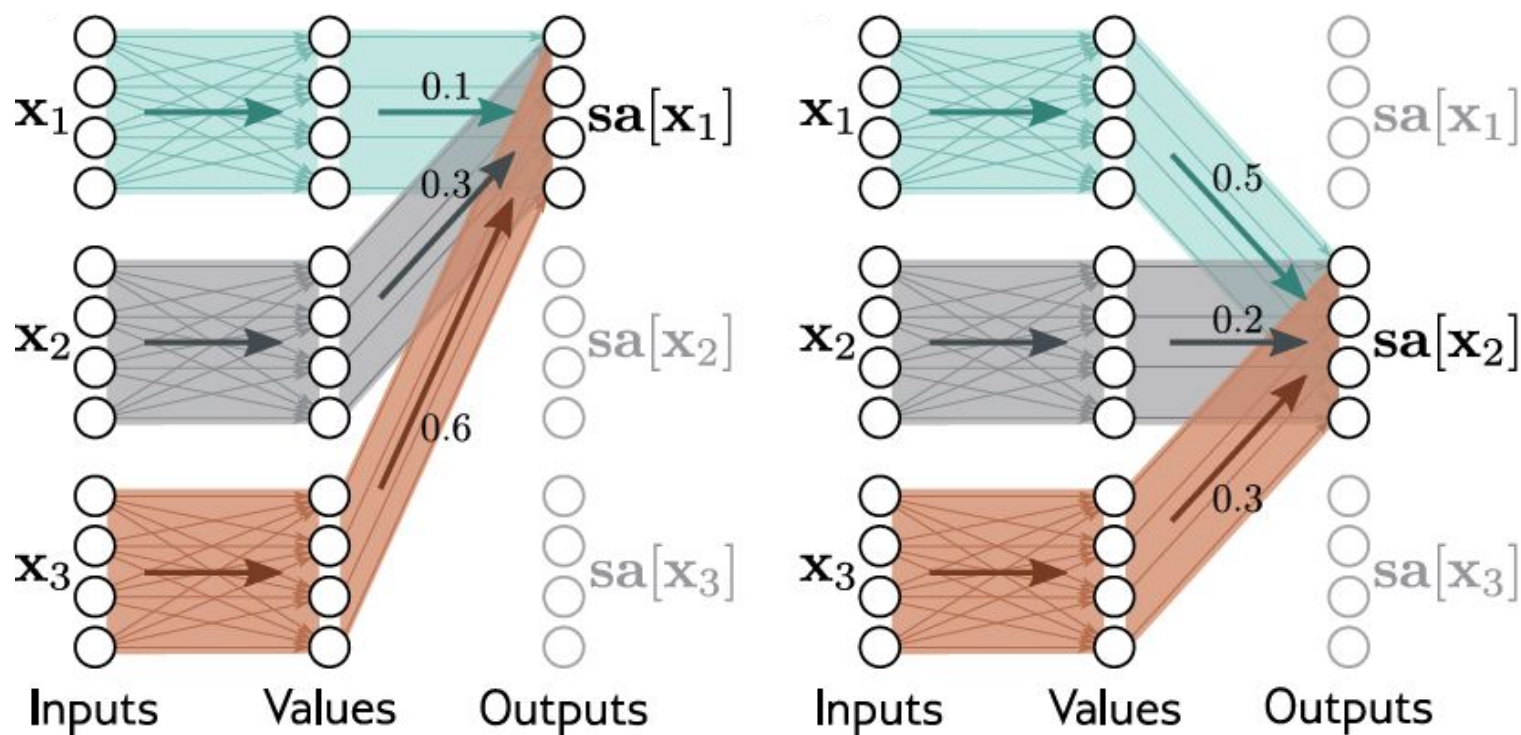
- Weights depend on the inputs themselves

# Attention as routing

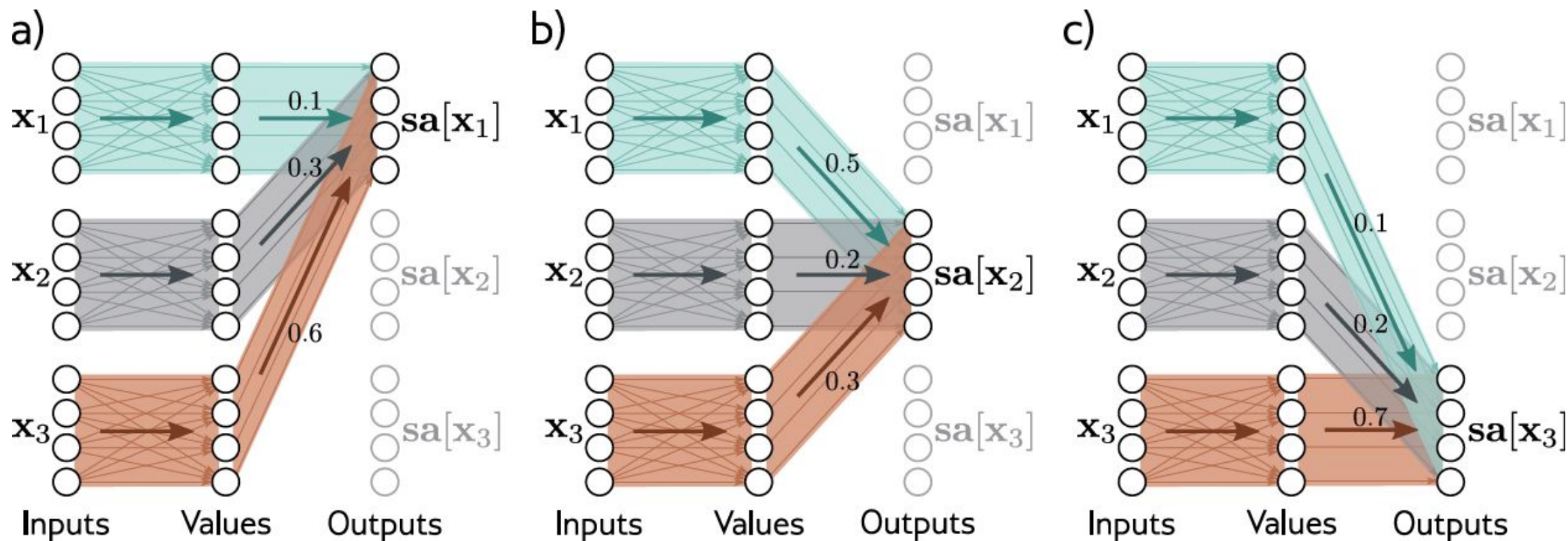




# Attention as routing



# Attention as routing



# Attention weights

- Compute N “queries” and N “keys” from input

$$\mathbf{q}_n = \beta_q + \Omega_q \mathbf{x}_n$$

$$\mathbf{k}_n = \beta_k + \Omega_k \mathbf{x}_n;$$

- Calculate similarity and pass through softmax:

$$\begin{aligned} a[\mathbf{x}_n, \mathbf{x}_m] &= \text{softmax}_m [\text{sim}[\mathbf{k}_m \mathbf{q}_n]] \\ &= \frac{\exp [\text{sim}[\mathbf{k}_m \mathbf{q}_n]]}{\sum_{m'=1}^N \exp [\text{sim}[\mathbf{k}'_m \mathbf{q}_n]]}; \end{aligned}$$

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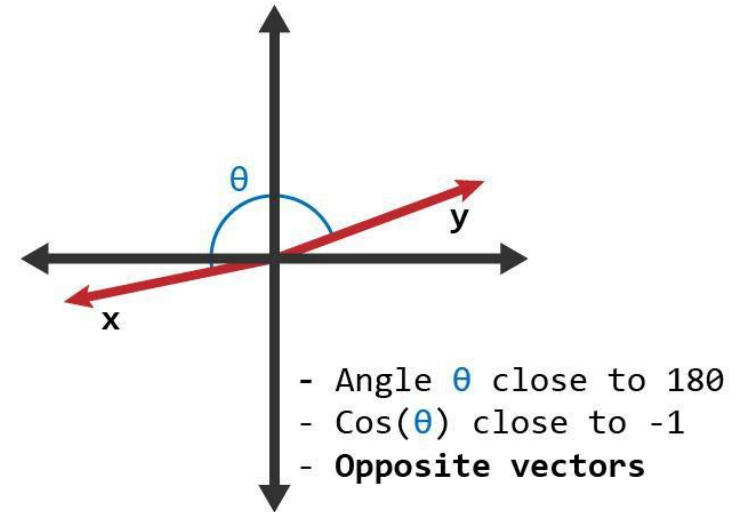
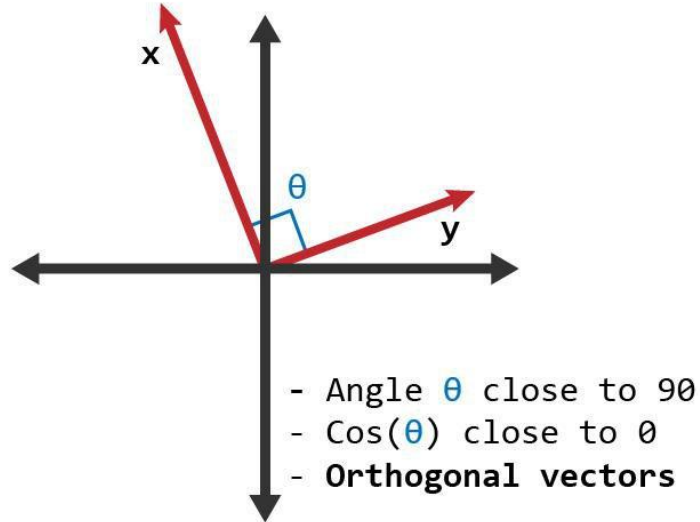
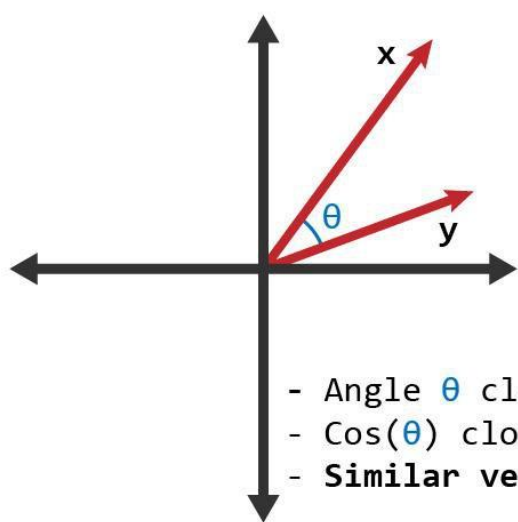
$$\mathbf{k}_n = \beta_k + \Omega_k \mathbf{x}_n;$$

- Take dot products and pass through softmax:

$$\begin{aligned} a[\mathbf{x}_n, \mathbf{x}_m] &= \text{softmax}_m [\mathbf{k}_m^T \mathbf{q}_n] \\ &= \frac{\exp [\mathbf{k}_m^T \mathbf{q}_n]}{\sum_{m'=1}^N \exp [\mathbf{k}_{m'}^T \mathbf{q}_n]} \end{aligned}$$

# Dot product = measure of similarity

$$\mathbf{x}^T \mathbf{y} = |\mathbf{x}| \cdot |\mathbf{y}| \cdot \cos(\theta)$$



# Motivation

Design neural network to encode and process text:

The restaurant refused to serve me a ham sandwich, because it only cooks vegetarian food. In the end, they just gave me two slices of bread. Their ambience was just as good as the food and service.

Conclusions:

- We need a model where parameters don't increase with input length

$$\phi = \{\beta_v, \Omega_v, \beta_q, \Omega_q, \beta_k, \Omega_k\}$$

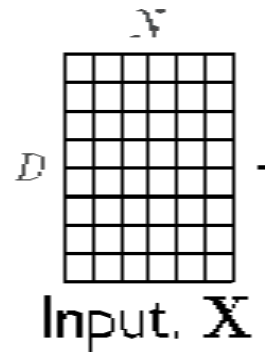
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# Matrix form

- Store N input vectors in matrix X



- Compute values, queries and keys (all dims are  $(D \times 1) \times (1 \times N) + D \times D \times (D \times N)$ ):

$$\mathbf{V}[\mathbf{X}] = \beta_v \mathbf{1}^T + \Omega_v \mathbf{X}$$

$$\mathbf{Q}[\mathbf{X}] = \beta_q \mathbf{1}^T + \Omega_q \mathbf{X}$$

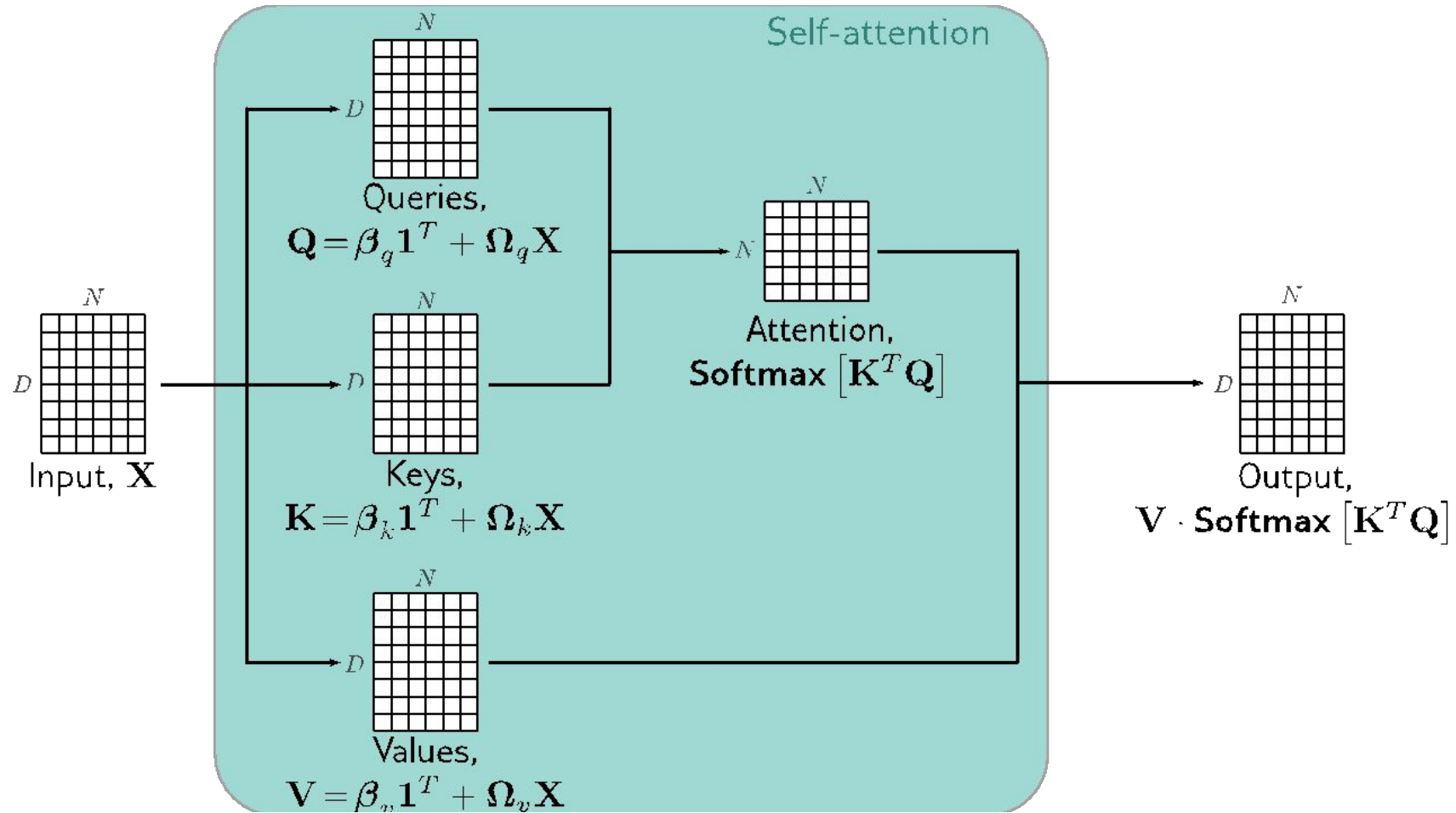
$$\mathbf{K}[\mathbf{X}] = \beta_k \mathbf{1}^T + \Omega_k \mathbf{X},$$

- Combine self-attentions ( $D \times N \times \text{ColSoftmax}(N \times D \times D \times N)$ )

$$\mathbf{Sa}[\mathbf{X}] = \mathbf{V}[\mathbf{X}] \cdot \text{Softmax} \left[ \mathbf{K}[\mathbf{X}]^T \mathbf{Q}[\mathbf{X}] \right]$$



# Matrix form



# Positional encoding

Self-attention is equivariant to permuting word order

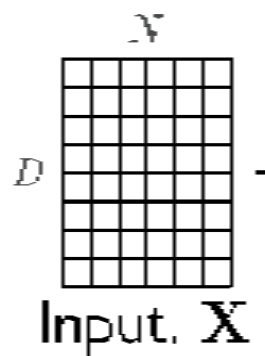
But word order is important in language:

The man ate the fish

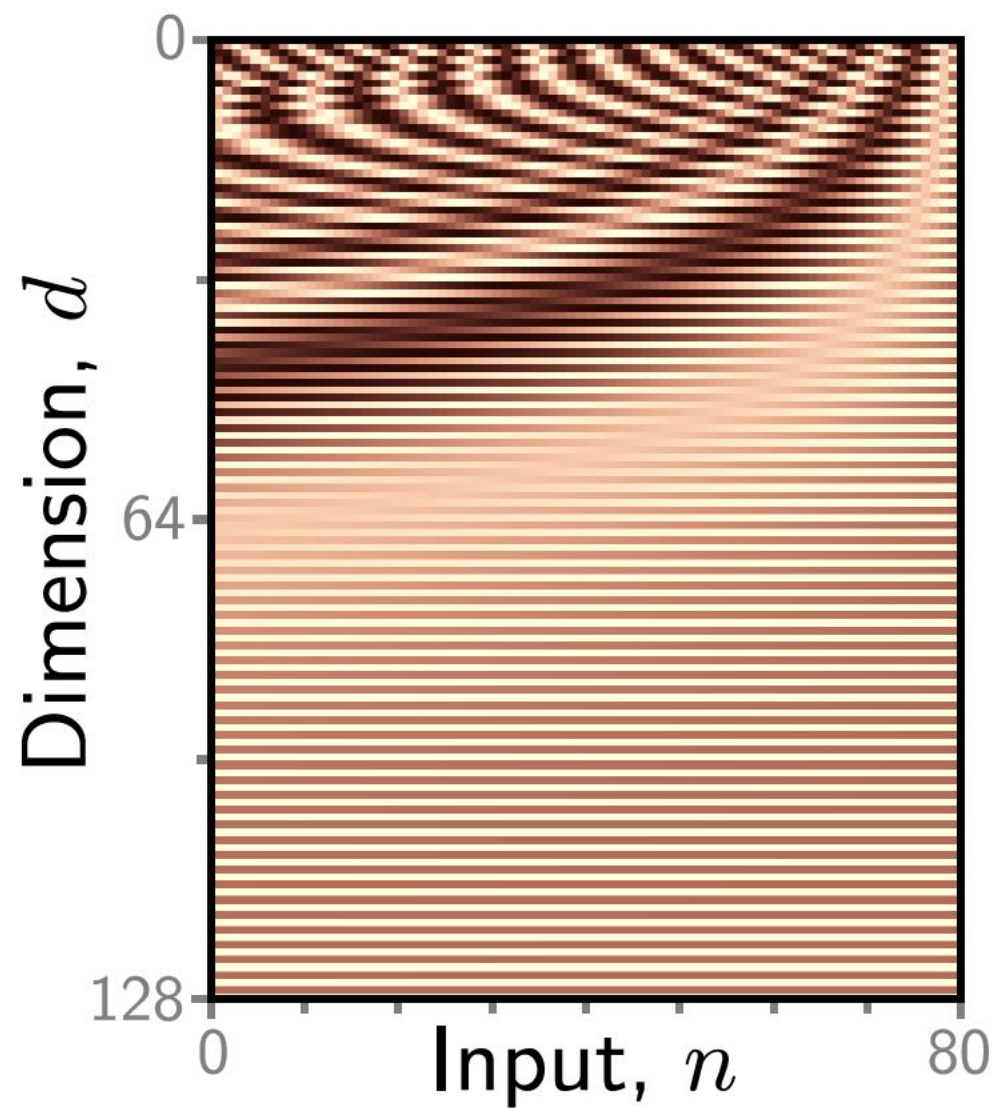
vs.

The fish ate the man

# Positional encoding



$$PE_{(pos, 2i)} = \sin(pos/10000^{2i/d_{\text{model}}})$$
$$PE_{(pos, 2i+1)} = \cos(pos/10000^{2i/d_{\text{model}}})$$



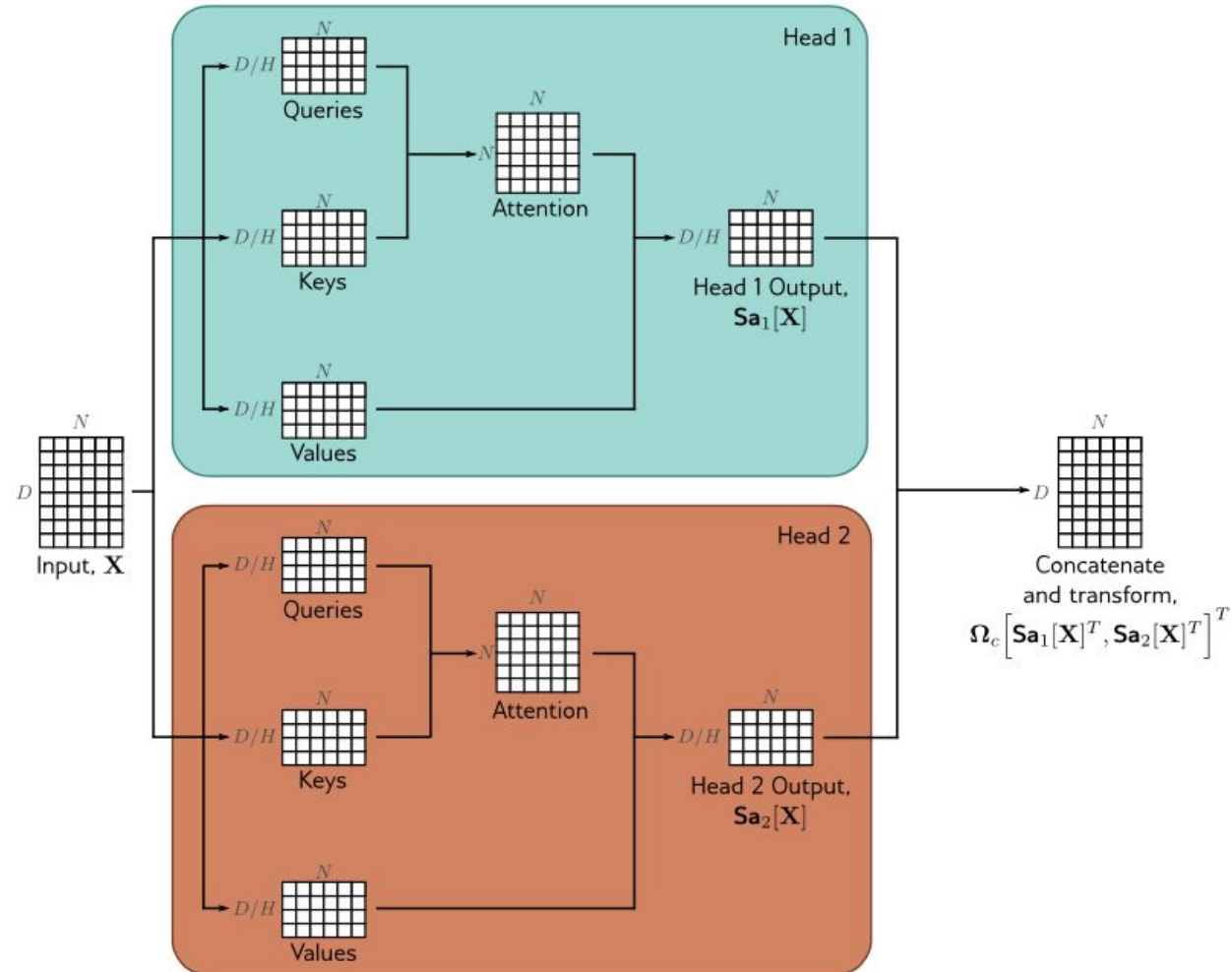
# Positional encoding

$$\mathbf{Sa}[\mathbf{X}] = \mathbf{V} \cdot \text{Softmax}[\mathbf{K}^T \mathbf{Q}]$$



$$\mathbf{Sa}[\mathbf{X}] = (\mathbf{V} + \mathbf{\Pi}) \cdot \text{Softmax}[(\mathbf{K} + \mathbf{\Pi})^T (\mathbf{Q} + \mathbf{\Pi})]$$

# Multi-head self-attention



# Why do we learn attention weights?

- Without learning attention weights, a self-attention layer is closely related to Nadaraya-Watson kernel regression
  - [https://d2l.ai/chapter\\_attention-mechanisms-and-transformers/attention-pooling.html](https://d2l.ai/chapter_attention-mechanisms-and-transformers/attention-pooling.html)
  - [https://en.wikipedia.org/wiki/Kernel\\_regression](https://en.wikipedia.org/wiki/Kernel_regression)
- Self-attention layer is more powerful because it can learn the attention weights as opposed to using a fixed hard-coded function (like the kernel in NW kernel regression)

# Transformers

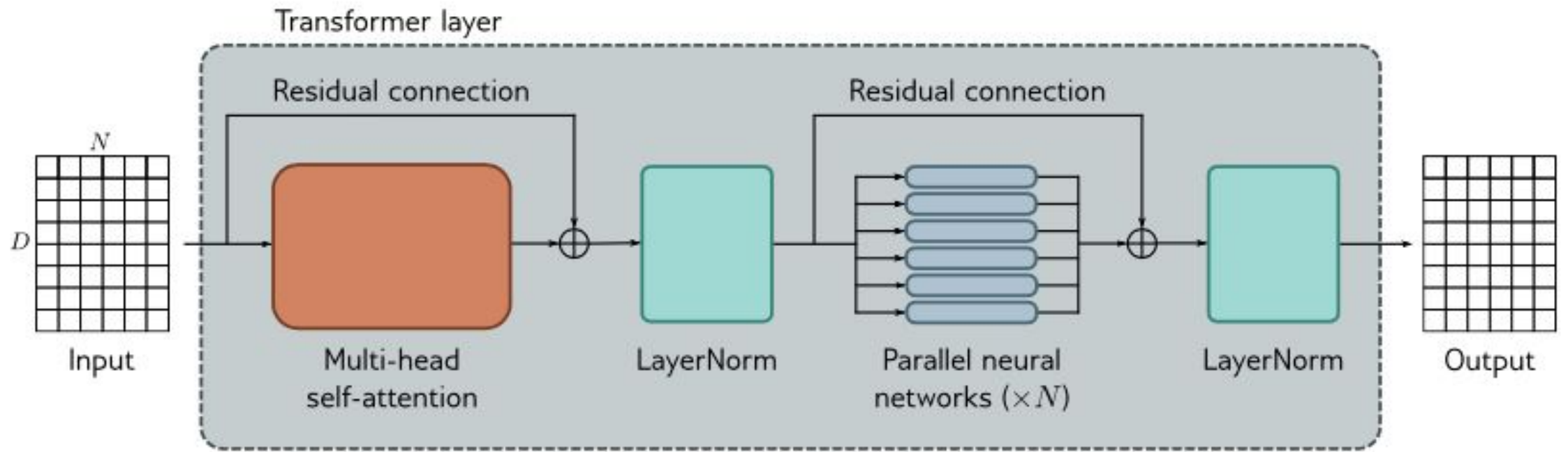
- Motivation
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- Matrix form
- The transformer
- NLP pipeline
- Encoders, decoders, and encoder-decoders
- Transformers for vision

# Transformers

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# The transformer



In Keras: [https://keras.io/api/layers/normalization\\_layers/layer\\_normalization/](https://keras.io/api/layers/normalization_layers/layer_normalization/)

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# Tokenizer

Goal: **Tokenizer** chooses input “units”

- Inevitably, some words (e.g., names) will not be in the vocabulary.
- It's not clear how to handle punctuation
- The vocabulary would need different tokens for versions of the same word with different suffixes (e.g., walk, walks, walked, walking) and there is no way to clarify that these variations are related

Solution: **Sub-word tokenization**

One particular algorithm: [Byte pair encoding](#)

a) a\_sailor\_went\_to\_sea\_sea\_sea\_  
 to\_see\_what\_he\_could\_see\_see\_see\_  
 but\_all\_that\_he\_could\_see\_see\_see\_  
 was\_the\_bottom\_of\_the\_deep\_blue\_sea\_sea\_sea\_

_	e	s	a	t	o	h	l	u	b	d	w	c	f	i	m	n	p	r
33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

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33	28	15	12	11	8	6	6	4	3	3	3	2	1	1	1	1	1	1

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33	15	13	12	11	8	6	6	4	3	3	3	2	2	1	1	1	1	1	1

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 to\_see\_what\_he\_could\_see\_see\_see\_  
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_	se	a	e	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

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_	se	a	e	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

⋮

⋮

- d) see\_sea\_e\_b\_l\_w\_a\_could\_hat\_he\_o\_t\_t\_the\_to\_u\_a\_d\_f\_m\_n\_p\_s\_sailor\_to  
7 6 4 3 3 3 3 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1

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21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

- 
- 
- 

d)

see_	sea_	e	b		w	a	could_	hat_	he_	o	t	t_	the_	to_	u	a_	d	f	m	n	p	s	sailor_	to
7	6	4	3	3	3	3	2	2	2	2	2	2	2	2	2	1	1	1	1	1	1	1	1	1

- 
- 
- 

[illegible]



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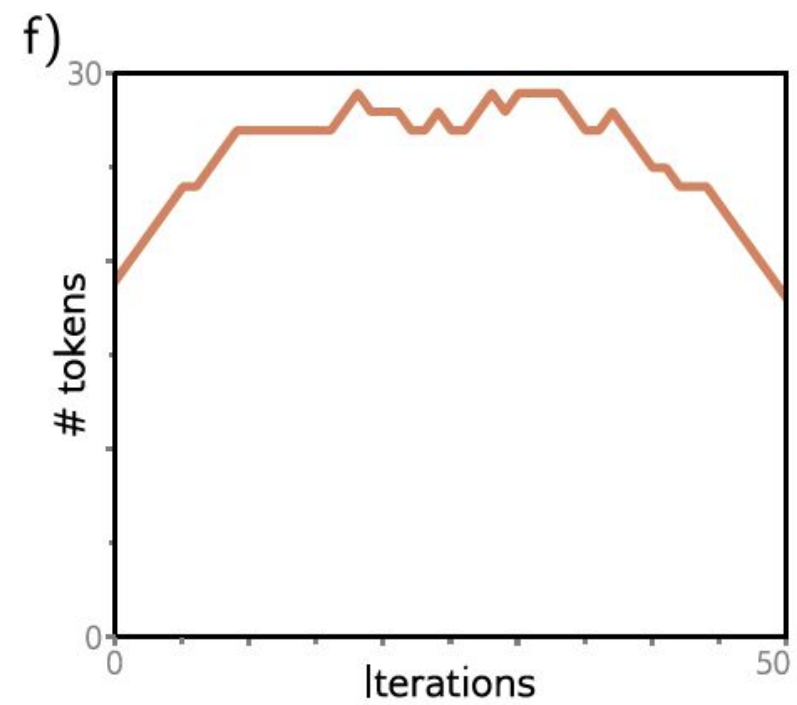
_	se	a	e	t	o	h	l	u	b	d	e	w	c	s	f	i	m	n	p	r
21	13	12	12	11	8	6	6	4	3	3	3	3	2	2	1	1	1	1	1	1

⋮ ⋮

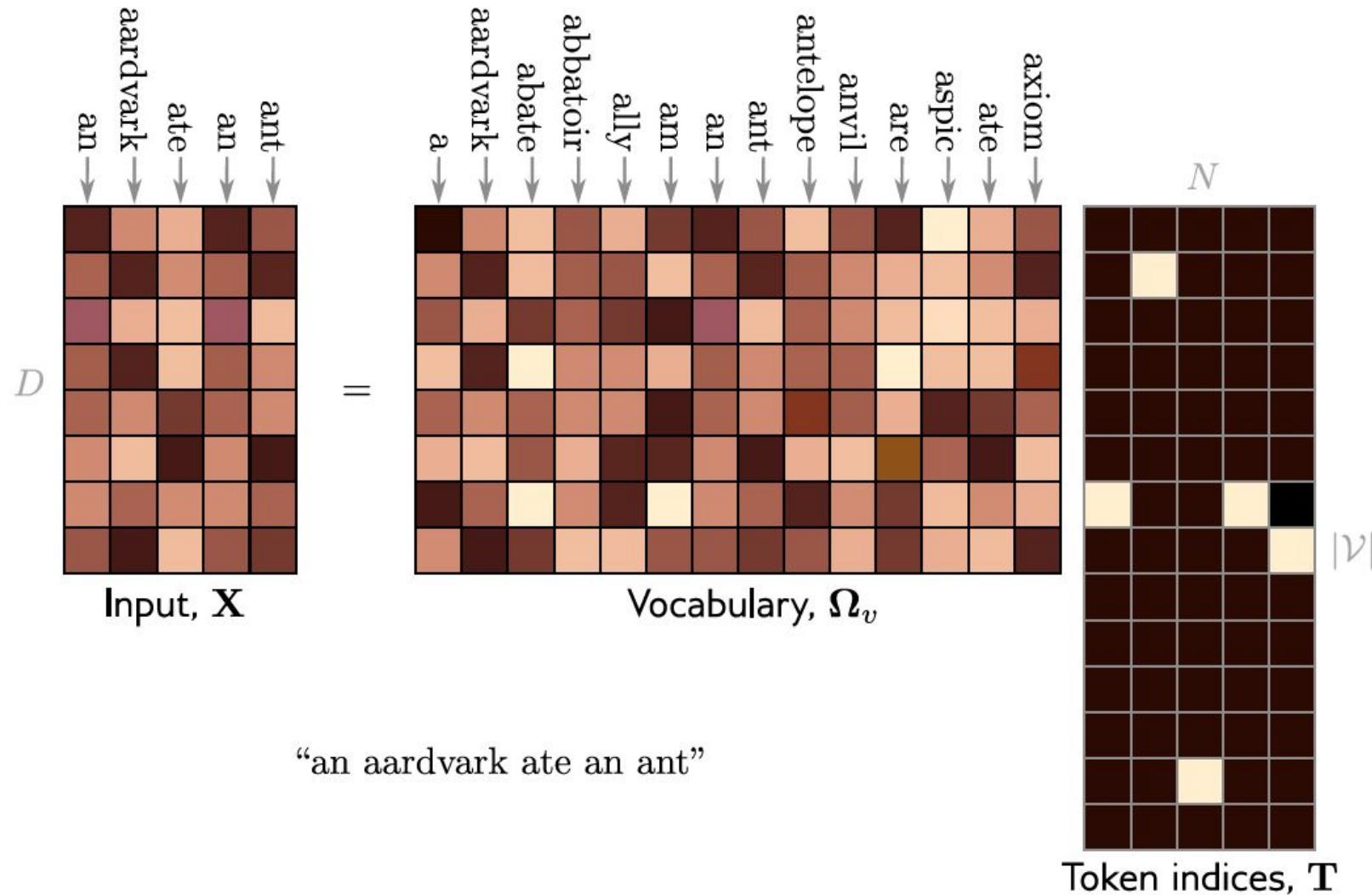
d) see\_sea\_e b l w a could\_hat he o t t the to u a d f m n p s sailor to  
 7 6 4 3 3 3 3 2 2 2 2 2 2 2 2 2 1 1 1 1 1 1 1 1 1 1

⋮ ⋮ ⋮

e) see\_sea\_could\_he\_the\_a\_all\_blue\_bottom\_but\_deep\_of\_sailor\_that\_to\_was\_went\_what\_  
 7 6 2 2 2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1



# Learning vocabulary



"One hot encoding"

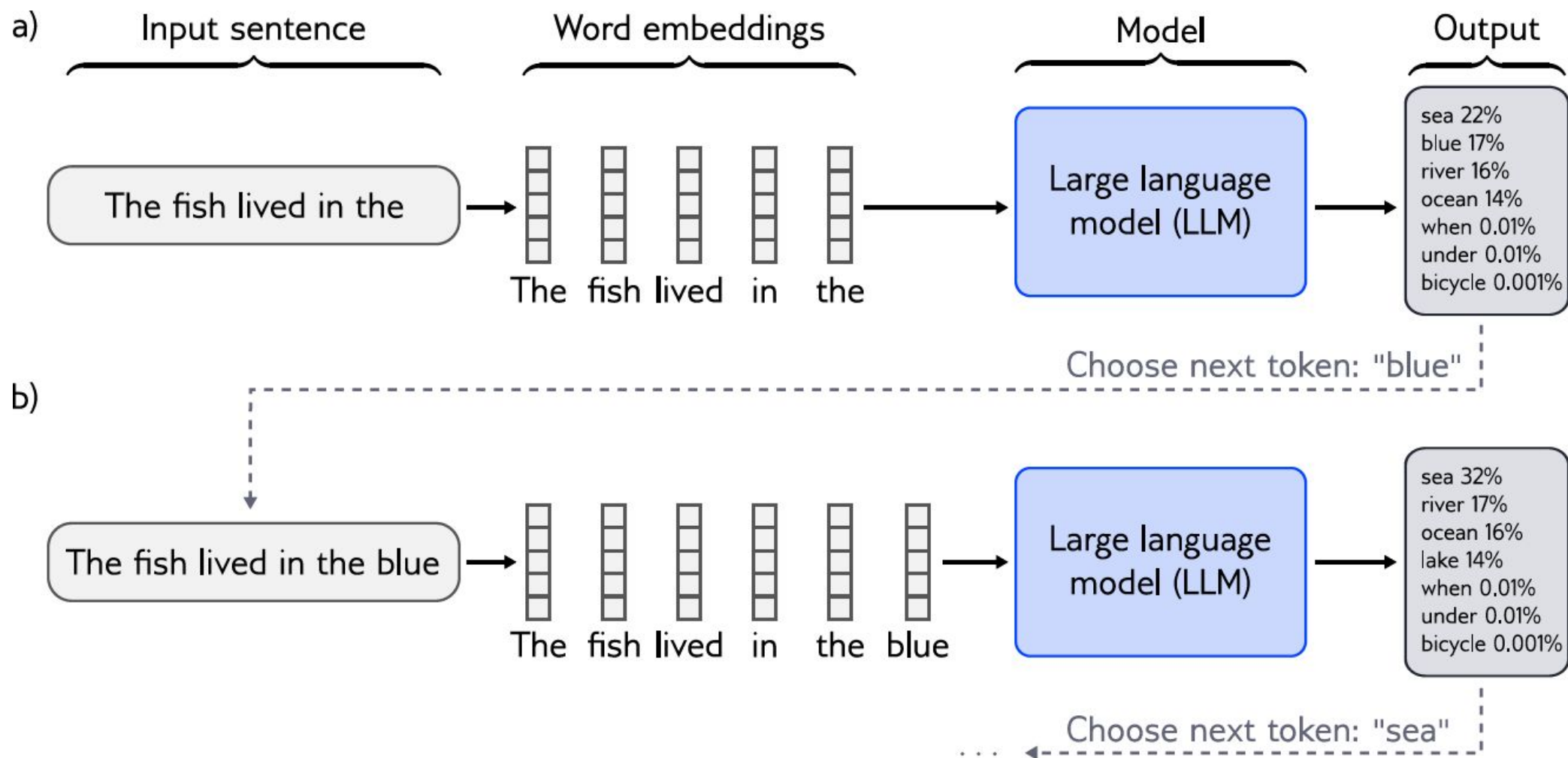
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# Three types of transformer layer

- Encoder (BERT)
- Decoder (GPT3) ← will only look at this today
- Encoder-decoder (Translation)

# Decoder model



# Decoder model: GPT3

- One job: predict the next word in a sequence
- More formally builds an **autoregressive** probability model

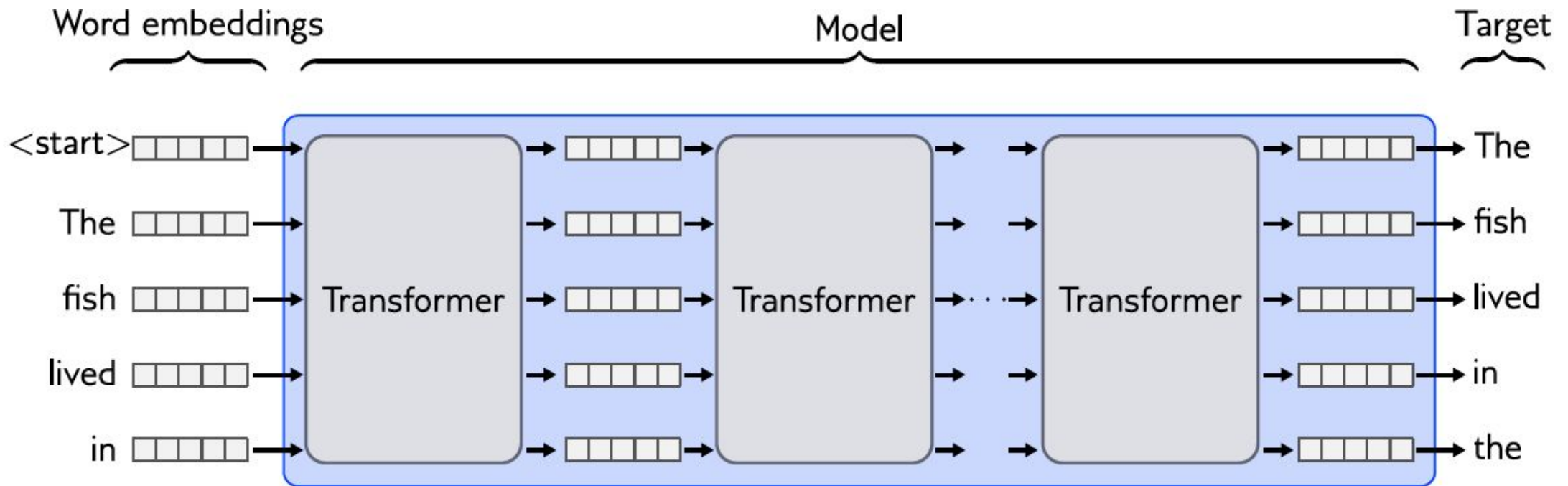
$$Pr(t_1, t_2, \dots t_N) = Pr(t_1) \prod_{n=2}^N Pr(t_n | t_1 \dots t_{n-1})$$

# Decoder model: GPT3

- Builds autoregressive probability model
- E.g. “It takes great courage to let yourself appear weak”

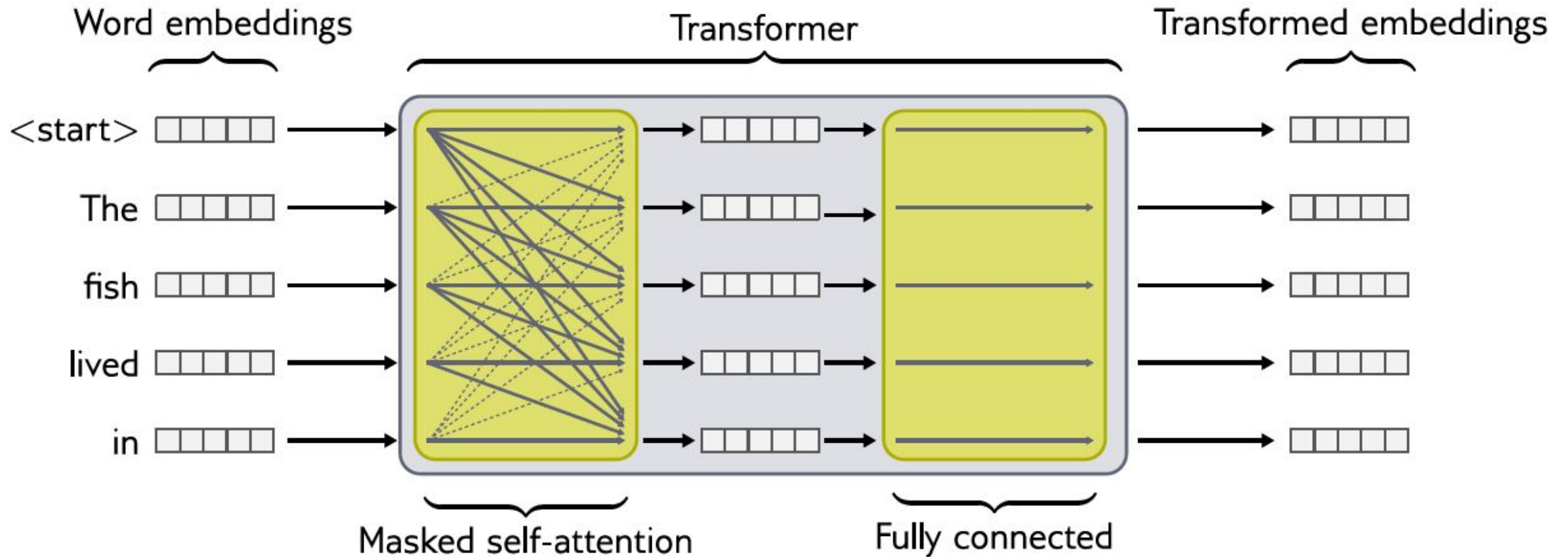
$$\begin{aligned} Pr(\text{It takes great personal courage to let yourself appear weak}) = \\ Pr(\text{It}) \times Pr(\text{takes}|\text{It}) \times Pr(\text{great}|\text{It takes}) \times Pr(\text{courage}|\text{It takes great}) \times \\ Pr(\text{to}|\text{It takes great courage}) \times Pr(\text{let}|\text{It takes great courage to}) \times \\ Pr(\text{yourself}|\text{It takes great courage to let}) \times \\ Pr(\text{appear}|\text{It takes great courage to let yourself}) \times \\ Pr(\text{weak}|\text{It takes great courage to let yourself appear}). \end{aligned}$$

# Predicting all next words simultaneously





# Masked self-attention



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# GPT3 (Brown et al. 2020)

- Sequence lengths are 2048 tokens long
- Batch size is 3.2 million tokens.
- 96 transformer layers (some of which implement a sparse version of attention), each of which processes a word embedding of size 12288.
- 96 heads in the self-attention layers and the value, query, and key dimension is 128.
- 300 billion tokens
- 175 billion parameters

## Large Language Models are becoming very large indeed



### Small models ( $\leq 100\text{b}$ parameters)



### Large models ( $>100\text{b}$ parameters)



# What does it learn?

- Syntax

“Tomorrow, let’s <VERB>...”

- General knowledge:

“The train pulled into the ...”

# Text completion

Understanding Deep Learning is a new textbook from MIT Press by Simon Prince that's designed to offer an accessible, broad introduction to the field. Deep learning is a branch of machine learning that is concerned with algorithms that learn from data that is unstructured or unlabeled. The book is divided into four sections:

- Introduction to deep learning
- Deep learning architecture
- Deep learning algorithms
- Applications of deep learning

The first section offers an introduction to deep learning, including its history and origins. The second section covers deep learning architecture, discussing various types of neural networks and their applications. The third section dives into deep learning algorithms, including supervised and unsupervised learning, reinforcement learning, and more. The fourth section applies deep learning to various domains, such as computer vision, natural language processing, and robotics.

# Few shot learning:

**Poor English input:** I eated the purple berries.

**Good English output:** I ate the purple berries.

**Poor English input:** Thank you for picking me as your designer. I'd appreciate it.

**Good English output:** Thank you for choosing me as your designer. I appreciate it.

**Poor English input:** The mentioned changes have done. or I did the alteration that you requested. or I changed things you wanted and did the modifications.

**Good English output:** The requested changes have been made. or I made the alteration that you requested. or I changed things you wanted and made the modifications.

**Poor English input:** I'd be more than happy to work with you in another project.

**Good English output:** I'd be more than happy to work with you on another project.

# ChatGPT

- GPT3.5 fine-tuned with human annotations
- Trained to predict the next word + be “helpful, honest, harmless”

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**Prompt:**

ELI5: What’s the cause of the "anxiety lump" in our chest during stressful or disheartening experiences?

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**Prompt:**

Serendipity means the occurrence and development of events by chance in a happy or beneficial way. Use the word in a sentence.

---



# ChatGPT

- GPT3.5 fine-tuned with human annotations
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**Prompt:**

ELI5: What’s the cause of the "anxiety lump" in our chest during stressful or disheartening experiences?

---

**Labeler demonstration**

The anxiety lump in your throat is caused by muscular tension keeping your glottis dilated to maximize airflow. The clenched chest or heartache feeling is caused by the vagus nerve which tells the organs to pump blood faster, stop digesting, and produce adrenaline and cortisol.

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**Prompt:**

Serendipity means the occurrence and development of events by chance in a happy or beneficial way. Use the word in a sentence.

---

**Labeler demonstration**

Running into Margaret and being introduced to Tom was a fortunate stroke of serendipity.