$10 \geq 8 = 2^3$; $4^2 + 1 = 17 \geq 16 = 2^4$ **3.** We must show that for all positive integers x it is not true that $x^3 = 100$. *Case (i):* If $x \leq 4$, then $x^3 \leq 64$, so $x^3 \neq 100$. *Case (ii):* If $x \geq 5$, then $x^3 \geq 125$, so $x^3 \neq 100$. **5.** If $x \leq y$, then

If $x \geq 5$, then $x^3 \geq 125$, so $x^3 \neq 100$. **5.** If $x \leq y$, then
$\max(x, y) + \min(x, y) = y + x = x + y$. If $x \geq y$, then $\max(x, y) + \min(x, y) = x + y$. Because these are the only two cases, the equality always holds. **7.** Because $|x-y| = |y-x|$,

P true. **19.** The equation $|a-c| = |b-c|$ is equivalent to the disjunction of two equations: $a - c = b - c$ or $a - c = -b + c$. The first of these is equivalent to $a = b$, which contradicts the assumptions made in this problem, so the original equation is equivalent to $a - c = -b + c$. By adding $b + c$ to both sides and dividing by 2, we see that this equation is equivalent to $c = (a + b)/2$. Thus, there is a unique solution. Furthermore, this c is an integer, because the sum of the odd integers a and b is even. **21.** We are being asked to solve $n = (k-2)+(k+3)$

Because n is odd, $n - 1$ is even, so k is an integer. **23.** If x is itself an integer, then we can take $n = x$ and $\epsilon = 0$. No other solution is possible in this case, because if the integer n is greater than x, then n is at least $x + 1$, which would make $\epsilon \geq 1$. If x is not an integer, then round it up to the next integer, and call that integer n. Let $\epsilon = n-x$. Clearly $0 \leq \epsilon < 1$; this is the only ϵ that will work with this n, and n cannot be any larger, because ϵ is constrained to be less than 1. **25.** The harmonic

1, respectively, so it is always a 0, 1, 5, or 6. **31.** Because $n^3 > 100$ for all $n > 4$, we need only note that $n = 1$, $n = 2$, $n = 3$, and $n = 4$ do not satisfy $n^2 + n^3 = 100$. **33.** Because