1. **a)** Countably infinite, $-1, -2, -3, -4, \ldots$ **b)** Countably infinite, $0, 2, -2, 4, -4, \ldots$ **c)** Countably infinite, $99, 98, 97, \ldots$ **d)** Uncountable **e)** Finite **f)** Countably infinite, $0, 7, -7, 14, -14, \ldots$ **3.** **a)** Countable: match n with

Room $2^k(2j + 1)$. **11.** **a)** $A = [1, 2]$ (closed interval of real numbers from 1 to 2), $B = [3, 4]$ **b)** $A = [1, 2] \cup \mathbf{Z}^+$, $B = [3, 4] \cup \mathbf{Z}^+$ **c)** $A = [1, 3]$, $B = [2, 4]$ **13.** Suppose

uncountable, this is impossible. **17.** Assume that $A - B$ is countable. Then, because $A = (A - B) \cup (A \cap B)$, the elements of A can be listed in a sequence by alternating elements of $A - B$ and elements of $A \cap B$. This contradicts the uncountability of A. **19.** We are given bijections f from A to B and g

ability of A. **19.** We are given bijections f from A to B and g
from C to D. Then the function from $A \times C$ to $B \times D$ that sends (a, c) to $(f(a), g(c))$ is a bijection. **21.** By the definition of

(a, c) to $(f(a), g(c))$ is a bijection. **21.** By the definition of
$|A| \leq |B|$, there is a one-to-one function $f : A \to B$. Similarly, there is a one-to-one function $g : B \to C$. By Exercise 33 in Section 2.3, the composition $g \circ f : A \to C$ is one-to-one. Therefore, by definition $|A| \leq |C|$. **23.** Using the Axiom

$\frac{(x-1)x}{2} + 1 = f(1, x)$. **33.** By the Schröder-Bernstein theorem, it suffices to find one-to-one functions $f : (0, 1) \to [0, 1]$ and $g : [0, 1] \to (0, 1)$. Let $f(x) = x$ and $g(x) = (x + 1)/3$.