Exam 3
QUESTIONS PACKET
EECS 203
Practice Exam 1

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

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*****MAKE SURE YOU HAVE PROBLEMS 1 - 18 IN THIS BOOKLET.*****

General Instructions

You have 120 minutes to complete this exam. You should have two exam packets.

- **Questions Packet:** Contains ALL of the questions for this exam, worth 100 points total. There are 8 Single-Answer Multiple Choice questions (4 points each), 4 Multiple-Answer Multiple Choice questions (4 points each), and 6 Free Response questions (52 points total). You may do scratch work on this part of the exam, but only work in the Answers Packet will be graded.
- **Answers Packet:** Write all of your answers in the Answers Packet, including your answers to multiple choice questions. **For free response questions, you must show your work! Answers alone will receive little or no credit.**
- You may bring **one** 8.5" by 11" note sheet, front and back, created by you.
- You may **NOT** use any other sources of information, including but not limited to electronic devices (including calculators), textbooks, or notes.
- After you complete the exam, sign the Honor Code Pledge on the front of the Answers Packet.
- You must turn in both parts of this exam.
- **You are not to discuss the exam until the solutions are published.**

Part A1: Single Answer Multiple Choice

Problem 1. (4 points)

An EECS 203 student eats dinner at Bursley Hall. For each meal, they choose:

- one of 3 main dish options (pizza, hamburger, or salad)
- one of 3 drink options (coffee, juice, or water)
- one of 2 dessert options (cookie or ice cream)

Additionally, if they choose a hamburger for their main dish, they will **not** have coffee to drink.

How many different meal combinations are possible?

(a) 7

(b) 8

(c) 16

(d) 17

(e) 18

Problem 2. (4 points)

Suppose you are given a compound proposition of the form

$$x_1 \vee x_2 \vee x_3 \vee \cdots \vee x_{n-1} \vee x_n.$$

The variables $x_1, \ldots, x_n$ are then independently and randomly set to either true or false with equal probability. What is the probability that the compound proposition is true?

(a) $\dfrac{1}{n}$

(b) $\dfrac{n-1}{n}$

(c) $\dfrac{1}{2^n}$

(d) $\dfrac{n}{2^n}$

(e) $\dfrac{2^n-1}{2^n}$

Problem 3. (4 points)

Consider a list of n answer choices for a multiple choice question, of which k are correct. How many ways could the list be rearranged such that the correct answers are the first k choices in the list?

For example, if the answer choices are {A, B, C, D, E} and A, C, and E are the correct answers, then (E, A, C, D, B) and (A, E, C, B, D) would be two such rearrangements.

(a) $n!$

(b) $n! - k!$

(c) $\binom{n}{k}$

(d) $k!$

(e) $k!(n-k)!$

Problem 4. (4 points)

You have 10 cards numbered 1 through 10. After shuffling the cards, what is the probability that the top 3 cards are in ascending order? Note that $C(n,k) = \binom{n}{k}$.

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{P(10,3)}{C(10,3)}$

(d) $\frac{1}{C(10,3)}$

(e) $\frac{3}{10}$

Problem 5. (4 points)

Professor Diaz rolls an unfair four-sided die and gets a number x, with probabilities

$$p(x=1) = \frac{1}{10}, \quad p(x=2) = \frac{2}{10}, \quad p(x=3) = \frac{3}{10}, \text{ and } p(x=4) = \frac{4}{10}.$$

- If x is odd, Professor Diaz will win x dollars (eg. rolling a 3 yields \$3).

- If x is even, she will win $2x$ dollars (eg. rolling a 4 yields \$8).

What is the expected value of Professor Diaz's winnings?

(a) 1.6

(b) 2.5

(c) 3

(d) 3.5

(e) 5

Problem 6. (4 points)

Regan flips a fair coin 10 times. What is the probability that at least 9 flips were heads?

(a) $\frac{9}{10}$

(b) $\left(\frac{1}{2}\right)^9$

(c) $11 \cdot \left(\frac{1}{2}\right)^{10}$

(d) $10 \cdot \left(\frac{1}{2}\right)^9$

(e) $\left(\frac{1}{2}\right)^9 + \left(\frac{1}{2}\right)^{10}$

Problem 7. (4 points)

What is the Big-Θ bound on the runtime of this algorithm?

```
procedure loopy(n):
    if n ≤ 1: return
    for i := 1 to 4:
        loopy(n/4)
    for i := 1 to n:
        for j := 1 to n:
            print 'I am feeling silly!'
    for i := 1 to 4:
        loopy(n/4)
```

(a) $\Theta(\log_4 n)$

(b) $\Theta(n)$

(c) $\Theta(n^{1.5})$

(d) $\Theta(n^2)$

(e) $\Theta(n^2 \log n)$

Problem 8. (4 points)

Reminders: A standard deck has 52 cards with 4 suits ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$) and 13 ranks (2, 3, ..., 10, Jack (J), Queen (Q), King (K), Ace (A)).

How many unique five-card hands have exactly 2 Queens and exactly 1 Heart? (The heart **is allowed** to be one of the queens.)

(a) $\binom{4}{2}\binom{13}{1}\binom{36}{2}$

(b) $\binom{4}{2}\binom{13}{1}\binom{36}{2} + \binom{4}{1}\binom{13}{2}\binom{36}{2}$

(c) $\binom{4}{2}\binom{36}{3} + \binom{3}{2}\binom{13}{1}\binom{36}{2}$

(d) $\binom{3}{1}\binom{36}{3} + \binom{3}{2}\binom{12}{1}\binom{36}{2}$

Part A2: Multiple Answer Multiple Choice

Problem 9. (4 points)

A simple undirected graph G has 5 vertices and 5 edges. Which of the following are possible?

(a) G contains a spanning tree

(b) G contains two or more cycles

(c) G is a complete graph

(d) G is a tree

(e) G is bipartite

Problem 10. (4 points)

Which of the following are scenarios are counted with $\binom{10}{5}$?

(a) Number of binary strings of 10 bits, 5 of which are 1's.

(b) Number of ways to give 1st through 5th awards to 10 racers

(c) Number of ways to choose a five digit PIN number

(d) Number of 5 digit numbers

(e) Number of ways to put 10 distinct balls into two identical boxes

Problem 11. (4 points)

Let f, g be functions where

- $f(n) = 5n^3 + 2n + \log n + 1$
- $g(n) = n! + n^4$

Which of the following are true?

(a) $f(n)$ is $O(g(n))$

(b) $f(n)$ is $\Theta(g(n))$

(c) $f(n)$ is $\Omega(g(n))$

(d) $\dfrac{g(n)}{f(n)}$ is $O(n!)$

(e) $g(n) \cdot f(n)$ is $O(n!)$

Problem 12. (4 points)

Suppose that there are independent events E and F in a sample space S. Both events have non-zero probability. Which of the following **must** be true?

(a) $p(E \cap F) = 0$

(b) $p(E|\overline{F}) = p(E)$

(c) $p(\overline{F}) = 1 - p(E \cap F)$

(d) $p(E|F) = p(F|E)$

(e) $p(E \cap F) = p(E|F) \cdot p(F|E)$

Problem 13. (4 points)

Ashu wants to name his child using only the letters from his name. More specifically, he wants to create a name with 3 A's, 2 S's, 2 H's and 3 U's. Which of the following represent the number of names that consist of exactly 3 A's, 2 S's, and 2 H's and 3 U's? Select all that apply.

(a) 4^{10}

(b) $\binom{10}{3} \cdot \binom{10}{2} \cdot \binom{10}{2} \cdot \binom{10}{3}$

(c) $\dfrac{10!}{3!{\cdot}2!{\cdot}2!{\cdot}3!}$

(d) $\binom{10}{3} \cdot \binom{7}{2} \cdot \binom{5}{2} \cdot \binom{3}{3}$

(e) $3! \cdot 2! \cdot 2! \cdot 3!$

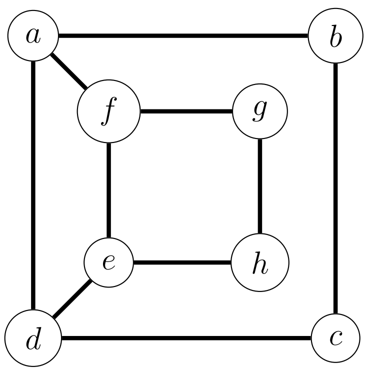
Part B: Short Answer

Problem 14. (6 points)

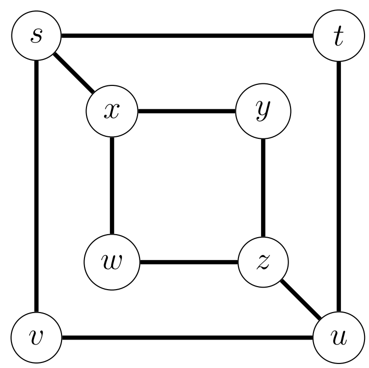
Are these graphs G and H isomorphic?

- If so, describe an isomorphism.
- If not, describe an invariant that one graph has that the other does not.

You do **not** have to prove that a named isomorphism is correct, or that G and H do/don't have the invariant that you name.



$\boxed{G}$



$\boxed{H}$

Problem 15. (7 points)

Is there a directed graph $G = (\{r, s, t, u, v\}, E)$ with the following properties? If so, draw the graph. If not, explain why no such graph can exist.

Note: for a node x in a directed graph, an

- "outgoing edge" is an edge that has arrow pointing away from x (x as its first node)
- "incoming edge" is an edge that has arrow pointing toward x (x as its second node)

(a) Properties of G :

- r, s have 1 incoming edge and 1 outgoing edge.
- t, u have 1 incoming edge and 0 outgoing edges.
- v has 1 incoming edge and 3 outgoing edges.

(b) Properties of G :

- r, s, t have 1 incoming edge and 1 outgoing edge.
- u, v have 1 incoming edge and 2 outgoing edges.

Part C: Free Response

Problem 16. (8 points)

Suzanne just started working for a new company. At this company, each employee receives one free gift per month. Each month there is a 40% chance of receiving a pen, a 35% chance of receiving socks, and a 25% chance of receiving a mug.

(a) Over the course of a year (12 months), what is the probabiltiy that Suzanne receives exactly 2 mugs?

(b) What is the expected number of months that Suzanne must work for the company to receive a mug?

(c) What is the expected number of mugs Suzanne recieves over the course of a 3 years?

Problem 17. (8 points)

Preeti has to teach discussion in a tiny classroom next semester. Her classroom has 8 different rows with 2 chairs in each row. She has 7 Right Handed People (RHP) and 3 Left Handed People (LHP) in her class. They must sit such that:

- Every RHP sits in one of the 8 chairs on the right
- Every LHP sits in one of the 8 chairs on the left
- Every row has at least one student.

How many ways can the students be seated?

Problem 18. (9 points)

Brian has a knapsack with 2 red, 3 yellow, and 4 green balls. Brian draws a random ball from the knapsack 50 times in a row, with replacement.

Consider the random variable:

$$D_i = \begin{cases} 1 & \text{if Brian draws a red ball in round } i \text{ and a yellow ball in round } i+1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $E[D_1]$.

(b) Are $D_1 = 1$ and $D_2 = 1$ independent events? Justify your answer.

(c) Let X be the total number of times Brian draws a red ball followed immediately by a yellow ball over all 50 draws. Find $E[X]$.

Problem 19. (9 points)

Xinhao wants to assign 9 IAs to ride 3 identical Blue Buses. Each Blue Bus should have at least 2 IAs assigned. How many different ways can he make this assignment?

Problem 20. (9 points)

Shubh's keyboard is broken such that when they press a key, the letter appears as normal with probability $\frac{3}{5}$, and otherwise nothing happens. They choose a string from the set {`an, can, tan`} at random and try to type that string.

- Let A be the event that Shubh tried to type `an`, with $p(A) = 1/2$
- Let C be the event that Shubh tried to type `can`, with $p(C) = 1/4$
- Let T be the event that Shubh tried to type `tan`, with $p(T) = 1/4$
- Let R be the event that the string `an` appears (with no other letter)

(a) Compute $p(R \mid C)$ and $p(R \cap C)$.

(b) If the string `an` appears, what is the probability that Shubh tried to type `can`?