- **3.** How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?
- **9.** How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?
- 11. How many bit strings of length 10 contain
 - a) exactly four 1s?
 - **b**) at most four 1s?
 - c) at least four 1s?
 - d) an equal number of 0s and 1s?
- **17.** How many subsets with more than two elements does a set with 100 elements have?
- 21. How many permutations of the letters ABCDEFG contain
 - a) the string BCD?
 - **b**) the string *CFGA*?
 - c) the strings BA and GF?
 - **d**) the strings ABC and DE?
 - e) the strings ABC and CDE?
 - **f**) the strings *CBA* and *BED*?
- 23. How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

- 27. One hundred tickets, numbered 1, 2, 3, ..., 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if
 - a) there are no restrictions?
 - **b**) the person holding ticket 47 wins the grand prize?
 - c) the person holding ticket 47 wins one of the prizes?
 - d) the person holding ticket 47 does not win a prize?
 - e) the people holding tickets 19 and 47 both win prizes?
 - **f**) the people holding tickets 19, 47, and 73 all win prizes?
 - g) the people holding tickets 19, 47, 73, and 97 all win prizes?
 - **h)** none of the people holding tickets 19, 47, 73, and 97 wins a prize?
 - i) the grand prize winner is a person holding ticket 19, 47, 73, or 97?
 - **j**) the people holding tickets 19 and 47 win prizes, but the people holding tickets 73 and 97 do not win prizes?
- **29.** A club has 25 members.
 - **a)** How many ways are there to choose four members of the club to serve on an executive committee?
 - **b)** How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?
- **31.** How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers k, k + 1, k + 2, in the correct order
 - a) where these consecutive integers can perhaps be separated by other integers in the permutation?
 - **b)** where they are in consecutive positions in the permutation?
- **33.** The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
 - a) exactly one vowel?
 - **b)** exactly two vowels?
 - c) at least one vowel?
 - **d**) at least two vowels?

- **35.** Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?
- **41.** How many license plates consisting of three letters followed by three digits contain no letter or digit twice?

A **circular** r-permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.

- **43.** Find a formula for the number of circular r-permutations of n people.
- 47. There are six runners in the 100-yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)