$a(ck) = bc$ it follows that $a \mid bc$. **5.** If $a \mid b$ and $b \mid a$, there are integers c and d such that $b = ac$ and $a = bd$. Hence, $a = acd$. Because $a \neq 0$ it follows that $cd = 1$. Thus, either $c = d = 1$ or $c = d = -1$. Hence, either $a = b$ or $a = -b$. **7.** Because $ac \mid bc$ there is an integer k such

or $a = -b$. **7.** Because $ac \mid bc$ there is an integer k such
that $ack = bc$. Hence, $ak = b$, so $a \mid b$. **9.** It is given that

17. a) 10 **b)** 8 **c)** 0 **d)** 9 **e)** 6 **f)** 11 **19.** If $d \mid a$, then $a = md$

$(-a)$ **div** $d \neq -(a$ **div** $d)$. **21.** If a **mod** $m = b$ **mod** m, then a and b have the same remainder when divided by m. Hence, $a = q_1m + r$ and $b = q_2m + r$, where $0 \leq r < m$. It follows that $a - b = (q_1 - q_2)m$, so $m \mid (a - b)$. It follows that $a \equiv b \pmod{m}$. **23.** There is some b with $(b - 1)k < n \leq bk$.

$\lceil m/2 \rceil$ **27. a)** 1 **b)** 2 **c)** 3 **d)** 9 **29. a)** 1, 109 **b)** 40,

89 **c)** −31, 222 **d)** −21, 38259 **31. a)** −15 **b)** −7 **c)** 140

41. Let $m = tn$. Because $a \equiv b \pmod{m}$ there exists an integer s such that $a = b + sm$. Hence, $a = b + (st)n$, so $a \equiv b \pmod{n}$.

not of the form $4k + 3$. **47.** Because $a \equiv b \pmod{m}$, there exists an integer s such that $a = b + sm$, so $a - b = sm$. Then $a^k - b^k = (a - b)(a^{k-1} + a^{k-2}b + \cdots + ab^{k-2} + b^{k-1})$, $k \geq 2$, is also a multiple of m. It follows that $a^k \equiv b^k \pmod{m}$.