Obj: 1. Technical Vocab: logical equivalence; ① contrapositive 逆否; ② tautology 永真式(重言,废话); ③ contradiction 矛盾; ④ predicate, ⑤ quantifier
2. Key Logical equivalence rules: ① DeMorgan's, ② distributive law, ③ implication breakout.
3. the contrapositive of an if-then statement
4. Negate an if-then statement
5. A compound proposition is a tautology, a contradiction, or neither
6. Logical symbols for "there exists" and "for all" ($\exists$, $\forall$)

9/5/23

② Double Negation law: $\neg(\neg p) \equiv p$

Lecture 3 Handout: Logical Equivalence

p = "I pet my cat" h="my cat is happy"

Translate each English statement to logic, then complete the truth table for each.

1. *If I pet my cat then she is happy.* $p \to h$
2. *If my cat is unhappy then I didn't pet her.* $\neg h \to \neg p$
3. *I didn't pet my cat or she is happy.* $\neg p \vee h$

p	h	$\neg p$	$\neg h$	$p \to h$	$\neg h \to \neg p$	$\neg p \vee h$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	T	T
F	F	T	T	T	T	T

① $p \equiv q$

Logical Equivalence

① Two compound propositions A, B are **logically equivalent** if they have the <u>same truth value</u> for any instantiation 实例化 of their input truth values (i.e., if they have the <u>same truth table</u>)

Some notations for logical equivalence:
- "p, q are logically equivalent"
- p $\equiv$ q
- p $\leftrightarrow$ q
- "p if and only if q"
- "p is necessary and sufficient for q" (充要命题 即为 逻辑 等价)
- 即 "p iff q"

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DeMorgans Laws in Action (Words)

Negate each of the following:

Proposition 1: I will go to the store or I will go to the park. $s \vee p$

Negation: It's not true that I will go to S or I will go to P

Simplify: I will not go to S and I will not go to P.

③ DeMorgan's Law #1: $\neg(s \vee p) \equiv \neg s \wedge \neg p$

(negating an or statement)

Proposition 2: I am 18 years old and I live in West Quad. $y \wedge w$

Negation: It's not true that I 18 and I WQ

Simplify: I am not 18 and I don't live in WQ.

④ DeMorgan's Law #2: $\neg(y \wedge w) \equiv \neg y \vee \neg w$

(negating an and statement)

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Useful Logical Equivalence Rules

$p \vee F \equiv p$

$p \vee T \equiv T$

$p \wedge F \equiv F$

$p \wedge T \equiv p$

⑤ Identity Law

Distributive Laws:

⑥ Distributive Law #1: $p \vee (q \wedge r) \equiv (p \vee q) \wedge (q \vee r)$

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge q)$ ⑦ Distributive Law #2

DeMorgan's Laws:

$\neg(p \vee q) \equiv \neg p \wedge \neg p$

$\neg(p \wedge q) \equiv \neg p \vee \neg q$

"Implication breakout" rule: ⑩

$p \to q \equiv \neg p \vee q$

Contrapositive: ⑨

$p \to q \equiv \neg q \to \neg p$

Negating an "implies": ⑩

$\neg(p \to q) \equiv p \wedge \neg q$

Which of the following ALWAYS has the same truth value as $p \to q$? C

A) Converse: $q \to p$

B) Inverse: $\neg p \to \neg q$

C) Contrapositive: $\neg q \to \neg p$

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⑧ $(p \vee q) \vee r = p \vee (q \vee r)$

$(p \wedge q) \wedge r = p \wedge (q \wedge r)$

Contrapositives

Statement: "If p, then q" $p \to q$

Contrapositive of statement: "If not q, then not p" $\neg q \to \neg p$

$$\boldsymbol{p \to q \equiv \neg q \to \neg p}$$

Negate both sides and switch the order

Find the contrapositive of each statement:

- If it's Tuesday, then we have EECS 203 class. (easy)
- If you don't live in Michigan, then you don't live in Ann Arbor.
- If $not\ p$, then q. *(Here p, q can stand for any propositions.)*

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Exercise: Negating an "implies" statement

$\neg(p \to q) \equiv$

Original statement :
"If x^2 is an integer, then x is an integer."
$p \to q$

Find the negation of the statement, and express it in logic and in English.

$\neg(p \to q) \equiv \neg(\neg p \vee q)$ ⇐ Implication breakout

$\equiv p \wedge \neg q$ ⇐ DeMorgan's

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Tautologies and Contradictions

A compound proposition is a **tautology** iff ① it's always true regardless of the input values

Example: $p \vee \neg p$

A compound proposition is a **contradiction** iff ⑫ it's always fal regardless of the input value

Example: $p \wedge \neg p$

Additional Exercises: Determine whether each statement is a tautology.

1. $(\neg q \to \neg p) \to (p \to q)$
2. $(\neg p \to F) \to p$

$\equiv (p \to q) \to (p \to q)$

$\equiv T$,

∴ tautology

$\equiv (T \to p) \to p$

$\equiv T$ ∴ tautology

T	P	T→P	(T→P)→P
T	T	T	T
T	F	F	T

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Predicates & Quantifiers

⑬ A **predicate** is a statement with some variables unspecified

Ex. $P(x)$ = "x likes Bubble Tea", $Q(x)$ = "$2x = 7$"

Quantifiers:

⑭ ∀ means "for all", "for each", "for every" (like a big chain of ANDS)

⑮ ∃ means "there exists" (like a big chain of ORs)

	True iff …	False iff …
$\forall x\ P(x)$	P(x) is true for every x in the domain	P(x) is false for at least one x in the domain
$\exists x\ P(x)$	P(x) is true for at least one x in the domain	P(x) is false for every x in the domain

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P(x) = "x likes Bubble Tea"

$\forall x\ P(x)$: Everyone likes ~

$\exists x\ P(x)$: Someone likes ~.

总结:

1. De Morgan's And/Or Laws:

$\neg(p \land q) \equiv \neg p \lor \neg q$

$\neg(p \lor q) = \neg p \land \neg q$

2. Identity Laws

$p \land T \equiv p$

$p \lor F \equiv p$

3. Implication Breakout

$p \to q \equiv \neg p \lor q$

4 Commutative Laws

$p \land q \equiv q \land p$

$p \lor q \equiv q \lor p$

5 Associative Laws

$p \land (q \land r) \equiv (p \land q) \land r$

$p \lor (q \lor r) \equiv (p \lor q) \lor r$

6 Distributive Laws

$p \lor (q \land r) \equiv (p \lor q) \land (p \lor r)$

$p \land (q \lor r) \equiv (p \land q) \lor (p \land r)$

7 The Contrapositive

$p \to q \equiv \neg q \to \neg p$

8 Tautology : always T

Contradiction: always F

satisfiable: can be T

Eg1: Is this a tautology?

$$[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q$$

Sol 1. (Truth table)

Sol 2. There is only one situation where $A \rightarrow B \equiv F$,
that is, $A \equiv T$, $B \equiv F$.

Assume, $\neg q \equiv F$, then $q \equiv T$

then no matter $p \equiv T$ or $p \equiv F$, $(p \rightarrow q) \equiv T$

then if $p \equiv T$, $\neg p \equiv F$, then $[\neg p \wedge (p \rightarrow q)] \equiv F$,

$[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q \equiv (F \rightarrow F) \equiv T$

else if $p \equiv F$, $\neg p \equiv T$, then $[\neg p \wedge (p \rightarrow q)] \equiv T$,

then $[\neg p \wedge (p \rightarrow q)] \rightarrow \neg q \equiv (T \rightarrow F) \equiv F$

$\therefore$ not tautology

Eg2: Is it a tautology?

b $[p \wedge (p \rightarrow q)] \rightarrow q$

$\equiv p \wedge (\neg p \vee q)$

$\equiv (p \wedge \neg p) \vee (p \wedge q) \rightarrow q$

$F \vee (p \wedge q) \rightarrow q$

$\equiv (p \wedge q) \rightarrow q$

$\equiv T$, $\therefore$ tautology

Eg 3

$Q(x) = x+1 > 2x$, domain: all integers

a) $Q(-1) \equiv T$

b) $Q(1) \equiv F$

c) $\exists x Q(x) \equiv T$

d) $\forall x Q(x) \equiv F$

e) $\exists x \neg Q(x) \equiv T$

f) $\forall x \neg Q(x) \equiv F$