$2^{n+1} - 1$. **3.** **a)** $a_n = 2a_{n-1} + a_{n-5}$ for $n \geq 5$ **b)** $a_0 = 1$, $a_1 = 2$, $a_2 = 4$, $a_3 = 8$, $a_4 = 16$ **c)** 1217 **5.** 9494

4.

We'll approach this problem working backwards. Consider the situation where the bill costs $n \geq 100$ pesos. Then we can break into cases based on the last denomination in the payment: they could have finished their payment with a 1 peso coin, a 2 peso coin, etc. Let a_n be the number of ways to pay n pesos given the denominations listed. In each of these cases, if the last denomination they paid was worth p pesos, then the number of ways they could have paid n pesos is a_{n-p}. Thus, summing the terms from each of the cases, we have $a_n = a_{n-1} + a_{n-2} + 2a_{n-5} + 2a_{n-10} + a_{n-20} + a_{n-50} + a_{n-100}$.

7. **a)** $a_n = a_{n-1} + a_{n-2} + 2^{n-2}$ for $n \geq 2$ **b)** $a_0 = 0$, $a_1 = 0$ **c)** 94

9. **a)** $a_n = a_{n-1} + a_{n-2} + a_{n-3}$ for $n \geq 3$ **b)** $a_0 = 1, a_1 = 2, a_2 = 4$
c) 81 **11.** **a)** $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 1$

c) 81 **11.** **a)** $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 1$
c) 34 **13.** **a)** $a_n = 2a_{n-1} + 2a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 3$

c) 34 **13.** **a)** $a_n = 2a_{n-1} + 2a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 3$
c) 448 **15.** **a)** $a_n = 2a_{n-1} + a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 3$

c) 448 **15.** **a)** $a_n = 2a_{n-1} + a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1, a_1 = 3$
c) 239 **17.** **a)** $a_n = 2a_{n-1}$ for $n \geq 2$ **b)** $a_1 = 3$ **c)** 96

17. **a)** $a_n = 2a_{n-1}$ for $n \geq 2$ **b)** $a_1 = 3$ **c)** 96

19. **a)** $a_n = a_{n-1} + a_{n-2}$ for $n \geq 2$ **b)** $a_0 = 1$, $a_1 = 1$
c) 89 **21.** **a)** $R_n = n + R_{n-1}$, $R_0 = 1$ **b)** $R_n = n(n+1)/2 + 1$