Groupwork

1. The Metaverse [14 points]

Suppose there is an island where there are exactly two types of people: truth-tellers and liars. A truth-teller always tells the truth, and a liar always lies.

Suppose a logician came across two inhabitants of this island, A and B. She asked A: "are you both truth-tellers?" A answered either yes or no. She stopped to think for a minute but could not determine what A and B were (truth-tellers or liars). She then asked "are you both of the same type?" (Same type means that they are either both truth-tellers or both liars.) A answered either yes or no, and then she knew what A and B were.

What are A and B?

Solution:

A	B	Q1	Q2
TT	TT	Yes	Yes
TT	L	No	No
L	TT	Yes	Yes
L	L	Yes	No

From the table we draw we can see that, for the logician to be uncertain after Q1, A must have answered yes. Else, the logician would have deduced what A and B were if A answered no.For Q2, there are 3 possible options: 2 where A answers yes and 1 where A answers no. Since the logician knew what A and B after this question, A must have answered no. That makes A and B both liars.

2. Majority Rules [10 points]

Consider the ternary logical connective # where $\#PQR$ takes on the value that the majority of P, Q and R take on. That is $\#PQR$ is true if at least two of P, Q or R is true and is false otherwise. Express $\#PQR$ using **only** the symbols: $P, Q, R, \wedge, \vee$ and parenthesis.

Solution:

$((P \vee Q) \wedge (P \vee R) \wedge (Q \vee R))$
At least 2 of P, Q, R need to be T if and only if #PQR is T. Therefore we can take all the 2-combinations out of 3, and as long as one of them is T, the #PQR is T. This can be proven true by true value table below. And to express it in symbols, it is $(P \vee Q) \wedge (P \vee R) \wedge (Q \vee R)$.

P	Q	R	$P \vee Q$	$P \vee R$	$Q \vee R$	$(P \vee Q) \wedge (P \vee R)$	$(P \vee Q) \wedge (P \vee R) \wedge (Q \vee R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	F	F
F	F	F	F	F	F	F	F

P	Q	R	$\#PQR$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F