5. Show that if $a \mid b$ and $b \mid a$, where a and b are integers, then $a = b$ or $a = -b$.

7. Show that if a, b, and c are integers, where $a \neq 0$ and $c \neq 0$, such that $ac \mid bc$, then $a \mid b$.

17. Suppose that a and b are integers, $a \equiv 4 \pmod{13}$, and $b \equiv 9 \pmod{13}$. Find the integer c with $0 \leq c \leq 12$ such that

a) $c \equiv 9a \pmod{13}$.
b) $c \equiv 11b \pmod{13}$.
c) $c \equiv a + b \pmod{13}$.
d) $c \equiv 2a + 3b \pmod{13}$.
e) $c \equiv a^2 + b^2 \pmod{13}$.
f) $c \equiv a^3 - b^3 \pmod{13}$.

21. Let m be a positive integer. Show that $a \equiv b \pmod{m}$ if a **mod** $m = b$ **mod** m.

27. Evaluate these quantities.

a) 13 **mod** 3
b) −97 **mod** 11
c) 155 **mod** 19
d) −221 **mod** 23

31. Find the integer a such that

a) $a \equiv -15 \pmod{27}$ and $-26 \leq a \leq 0$.
b) $a \equiv 24 \pmod{31}$ and $-15 \leq a \leq 15$.
c) $a \equiv 99 \pmod{41}$ and $100 \leq a \leq 140$.

41. Show that if $n \mid m$, where n and m are integers greater than 1, and if $a \equiv b \pmod{m}$, where a and b are integers, then $a \equiv b \pmod{n}$.

47. Show that if a, b, k, and m are integers such that $k \geq 1$, $m \geq 2$, and $a \equiv b \pmod{m}$, then $a^k \equiv b^k \pmod{m}$.

*HINT:

$$a^k - b^k = (a - b)(a^{k-1} + a^{k-2}b + a^{k-3}b^2 + \cdots + b^{k-1}).$$