Lec 2 Handout: Propositions & Symbolic Logic

Proposition: statement about the world that has a truth value

Which of the following are propositions? For those that are propositions, what is their truth value?

1. Paris is the capital of Michigan ✓F
2. 2+3=5 ✓T
3. 5+7=10 ✓F
4. x+2=11 X unspecified
5. Answer this question X not a declaration
6. Emily's house is painted blue ✓ (unknown to me)
7. Everyone in this room likes Bubble Tea (unknown to me)

Why might sth not be a proposition? eg: Go Blue! (Q why...?
① It's not a declaration about the world
② It's paradoxical or not well defined.
eg: colorless green ideas sleep furiously
③ Some parts is left unspecified ④
eg: x+5≠10

Negation Practice

The **negation** of a proposition *p* is a statement that has the opposite truth value of *p*.

Original Proposition	Negation of the Proposition
1. Paris is the capital of Michigan	Paris is not ~
2. $2 + 3 = 5$	$2+3 \neq 5$
3. $7 + 10 < 12$	$7+10 \geq 12$
4. Everyone in this room likes Bubble Tea	At least one person ~ doesn't ~
5. Someone in this room likes Bubble Tea	No one ~ likes
6. No one in this room likes Bubble Tea	There is at least one person ~ likes
7. Someone in this room does not like Bubble Tea	Everyone ~ likes

Truth Tables

p	$\neg p$ not p
T	F
F	T

p	q	$p \wedge q$ p and q
T	T	T
T	F	F
F	T	F
F	F	F

p	q	$p \vee q$ p or q
T	T	T
T	F	T
F	T	T
F	F	F

The ONLY time that "if p, then q" is **false** is:

p ≡ T and q ≡ F

p	q	$p \rightarrow q$ if p then q
T	T	T
T	F	F
F	T	T (F) X
F	F	T

p	q	$p \leftrightarrow q$ p if and only if q
T	T	T
T	F	F
F	T	F
F	F	T

Exercises

Complete the truth tables.

$\neg (p \wedge q)$

p	q	$p \wedge q$	$\neg (p \wedge q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	F	T

$\neg q \wedge \neg (p \vee \neg q)$

p	q	$\neg q$	$p \vee \neg q$	$\neg (p \vee \neg q)$	$\neg q \wedge \neg (p \vee \neg q)$
T	T	F	T	F	F
T	F	T	T	F	F
F	T	F	F	T	F
F	F	T	T	F	F

Logic Symbols

Symbol	"Main" Words	Some alternate words you might see (incomplete list)
$\neg p$	"not p"	
$p \wedge q$	"p and q"	"p but q"
$p \vee q$	"p or q"	
$p \rightarrow q$	"if p, then q"	"p implies q" "p only if q" "q if p" "p is sufficient for q" "q is necessary for p"
$p \leftrightarrow q$ $p \equiv q$	"p if and only if q"	"p is equivalent to q" "p iff q" "p is necessary and sufficient for q"

Translating English to Logic

r = “it rains”
w = “I’ll watch a movie”

p = “I’ll eat popcorn”
c = “I’ll eat chocolate”

Symbol	Words
¬p	“not p”
p∧q	“p and q”
p∨q	“p or q (or both)”
p→q	“if p then q”

English	**Logic**
• If I don’t eat popcorn, I’ll eat chocolate.	¬p → c
• If it rains, I’ll watch a movie and eat popcorn.	r → (w ∧ p)
• I’ll eat chocolate if it doesn’t rain.	¬r → c

Translating Logic to English

F: the fox can catch the hare

L: the lynx can catch the hare

A: the hare is alert

Q: the hare is quick

Which of these is:
"Neither the fox nor the lynx can catch the hare if the hare is alert and quick"?

Translate:

(A) ¬(F ∨ L) → (A ∧ Q) _____

(Easy)

(B) (A ∧ Q) → (¬F ∧ ¬L) _____

(C) ¬F ∧ ¬L ∧ A ∧ Q _____

(D) (¬A ∨ ¬Q) → (F ∨ L) _____

Logic Puzzle #2

Everyone is either always tells the truth (TT) or always lies (L). Who is which?

Q1: "Anand, are you a liar?"
Anand: "????"

Q2: "Blanca, did Anand say he's a liar?"
Blanca: "Yes."

Q3: "Is Blanca a liar?"
Carol: "Yes."

Q4: "Is Anand a liar?"
Carol: "Yes."

Anand	Blanca	Carol	Q1 (Anand)	Q2 (Blanca)	Q3 (Carol 1)	Q4 (Carol 2)
TT	TT	TT	No	No		
TT	TT	L		No		
TT	L	TT		Yes	Yes	No
TT	L	L		Yes	No	
L	TT	TT		No		
L	TT	L		No		
L	L	TT		Yes	Yes	Yes
L	L	L		Yes