$k^2 + 2k + 1 - k^2 = 2k + 1 = n$. **9.** Suppose that r is rational and i is irrational and $s = r + i$ is rational. Then by Example 8, $s + (-r) = i$ is rational, which is a contradiction.

11. Because $\sqrt{2} \cdot \sqrt{2} = 2$ is rational and $\sqrt{2}$ is irrational, the product of two irrational numbers is not necessarily irrational.

Thus, by definition, x is rational. **15.** Assume that $\sqrt{x}$ were rational. Then, because the product of two rational numbers is rational, $\left(\sqrt{x}\right)^2 = x$ is also rational. This contradicts the hypothesis that x is irrational. **17.** Assume that it is not true

be on the same day of the week. **27.** Suppose by way of contradiction that a/b is a rational root, where a and b are integers and this fraction is in lowest terms (that is, a and b have no common divisor greater than 1). Plug this proposed root into the equation to obtain $a^3/b^3 + a/b + 1 = 0$. Multiply through by b^3 to obtain $a^3 + ab^2 + b^3 = 0$. If a and b are both odd, then the left-hand side is the sum of three odd numbers and therefore must be odd. If a is odd and b is even, then the left-hand side is odd+even+even, which is again odd. Similarly, if a is even and b is odd, then the left-hand side is even+even+odd, which is again odd. Because the fraction a/b is in simplest terms, it cannot happen that both a and b are even. Thus, in all cases, the left-hand side is odd, and therefore cannot equal 0. This contradiction shows that no such root exists. **29.** First,

$5n+6$ is odd. **31.** This proposition is true. Suppose that m is neither 1 nor -1. Then mn has a factor m larger than 1. On the other hand, $mn = 1$, and 1 has no such factor. Hence, $m = 1$ or $m = -1$. In the first case $n = 1$, and in the second case $n = -1$, because $n = 1/m$. **33.** We prove that all these are equivalent proved in Example 1. **35.** We give proofs by contraposition of $(i) \to (ii)$, $(ii) \to (i)$, $(i) \to (iii)$, and $(iii) \to (i)$. For the first of these, suppose that $3x + 2$ is rational, namely, equal to p/q for some integers p and q with $q \neq 0$. Then we can write $x = ((p/q) - 2)/3 = (p - 2q)/(3q)$, where $3q \neq 0$. This shows that x is rational. For the second conditional statement, suppose that x is rational, namely, equal to p/q for some integers p and q with $q \neq 0$. Then we can write $3x + 2 = (3p + 2q)/q$, where $q \neq 0$. This shows that $3x + 2$ is rational. For the third conditional statement, suppose that $x/2$ is rational, namely, equal to p/q for some integers p and q with $q \neq 0$. Then we can write $x = 2p/q$, where $q \neq 0$. This shows that x is rational. And for the fourth conditional statement, suppose that x is rational, namely, equal to p/q for some integers p and q with $q \neq 0$. Then we can write $x/2 = p/(2q)$, where $2q \neq 0$. This shows that $x/2$ is rational. **37.** No

43. We will show that the four statements are equivalent by showing that (i) implies (ii), (ii) implies (iii), (iii) implies (iv), and (iv) implies (i). First, assume that n is even. Then $n = 2k$ for some integer k. Then $n + 1 = 2k + 1$, so $n + 1$ is odd. This shows that (i) implies (ii). Next, suppose that $n + 1$ is odd, so $n + 1 = 2k + 1$ for some integer k. Then $3n + 1 = 2n + (n + 1) = 2(n + k) + 1$, which shows that $3n + 1$ is odd, showing that (ii) implies (iii). Next, suppose that $3n + 1$ is odd, so $3n + 1 = 2k + 1$ for some integer k. Then $3n = (2k + 1) - 1 = 2k$, so $3n$ is even. This shows that (iii) implies (iv). Finally, suppose that n is not even. Then n is odd, so $n = 2k + 1$ for some integer k. Then $3n = 3(2k + 1) = 6k + 3 = 2(3k + 1) + 1$, so $3n$ is odd. This completes a proof by contraposition that (iv) implies (i).