11. Give a big-O estimate for the function f in Exercise 10 if f is an increasing function.

> **10.** Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) + 1$ with $f(1) = 1$.

13. Give a big-O estimate for the function f in Exercise 12 if f is an increasing function.

> **12.** Find $f(n)$ when $n = 3^k$, where f satisfies the recurrence relation $f(n) = 2f(n/3) + 4$ with $f(1) = 1$.

17. Suppose that the votes of n people for different candidates (where there can be more than two candidates) for a particular office are the elements of a sequence. A person wins the election if this person receives a majority of the votes.

a) Devise a divide-and-conquer algorithm that determines whether a candidate received a majority and, if so, determine who this candidate is. [*Hint:* Assume that n is even and split the sequence of votes into two sequences, each with $n/2$ elements. Note that a candidate could not have received a majority of votes without receiving a majority of votes in at least one of the two halves.]

b) Use the master theorem to give a big-O estimate for the number of comparisons needed by the algorithm you devised in part (a).

19. a) Set up a divide-and-conquer recurrence relation for the number of multiplications required to compute x^n, where x is a real number and n is a positive integer, using the recursive algorithm from Exercise 26 in Section 5.4.

b) Use the recurrence relation you found in part (a) to construct a big-O estimate for the number of multiplications used to compute x^n using the recursive algorithm.

21. Suppose that the function f satisfies the recurrence relation $f(n) = 2f(\sqrt{n}) + 1$ whenever n is a perfect square greater than 1 and $f(2) = 1$.

a) Find $f(16)$.

b) Give a big-O estimate for $f(n)$. [*Hint:* Make the substitution $m = \log n$.]

35. Give a big-O estimate for the function f in Exercise 34 if f is an increasing function.

34. Find $f(n)$ when $n = 4^k$, where f satisfies the recurrence relation $f(n) = 5f(n/4) + 6n$, with $f(1) = 1$.

37. Give a big-O estimate for the function f in Exercise 36 if f is an increasing function.

36. Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = 8f(n/2) + n^2$ with $f(1) = 1$.