1. The choices of C and k are not unique. **a)** $C = 1, k = 10$
b) $C = 4, k = 7$ **c)** No **d)** $C = 5, k = 1$ **e)** $C = 1, k = 0$
f) $C = 1, k = 2$ **3.** $x^4 + 9x^3 + 4x + 7 \leq 4x^4$ for all $x > 9$;

f) $C = 1, k = 2$ **3.** $x^4 + 9x^3 + 4x + 7 \leq 4x^4$ for all $x > 9$;
witnesses $C = 4, k = 9$ **5.** $(x^2+1)/(x+1) = x-1+2/(x+1) <$

witnesses $C = 4, k = 9$ **5.** $(x^2+1)/(x+1) = x-1+2/(x+1) <$
x for all $x > 1$; witnesses $C = 1, k = 1$ **7.** The choices of

x for all $x > 1$; witnesses $C = 1, k = 1$ **7.** The choices of
C and k are not unique. **a)** $n = 3, C = 3, k = 1$ **b)** $n = 3$, $C = 4, k = 1$ **c)** $n = 1, C = 2, k = 1$ **d)** $n = 0, C = 2, k = 1$

9. $x^2 + 4x + 17 \leq 3x^3$ for all $x > 17$, so $x^2 + 4x + 17$ is $O(x^3)$, with witnesses $C = 3, k = 17$. However, if x^3 were $O(x^2 + 4x + 17)$, then $x^3 \leq C(x^2 + 4x + 17) \leq 3Cx^2$ for some C, for all sufficiently large x, which implies that $x \leq 3C$ for all sufficiently large x, which is impossible. Hence, x^3 is
not $O(x^2 + 4x + 17)$. **11.** $3x^4 + 1 \leq 4x^4 = 8(x^4/2)$ for

not $O(x^2 + 4x + 17)$. **11.** $3x^4 + 1 \leq 4x^4 = 8(x^4/2)$ for
all $x > 1$, so $3x^4 + 1$ is $O(x^4/2)$, with witnesses $C = 8$, $k = 1$. Also $x^4/2 \leq 3x^4 + 1$ for all $x > 0$, so $x^4/2$ is
$O(3x^4 + 1)$, with witnesses $C = 1, k = 0$. **13.** Because

$O(3x^4 + 1)$, with witnesses $C = 1, k = 0$. **13.** Because
$2^n \leq 3^n$ for all $n > 0$, it follows that 2^n is $O(3^n)$, with witnesses $C = 1, k = 0$. However, if 3^n were $O(2^n)$, then for some C, $3^n \leq C \cdot 2^n$ for all sufficiently large n. This says that $C \geq (3/2)^n$ for all sufficiently large n, which is impossible. Hence, 3^n is not $O(2^n)$. **15.** All functions for which

sible. Hence, 3^n is not $O(2^n)$. **15.** All functions for which there exist real numbers k and C with $|f(x)| \le C$ for $x > k$. These are the functions $f(x)$ that are bounded for all sufficiently large x. **17.** There are constants C_1, C_2, k_1, and k_2

ciently large x. **17.** There are constants C_1, C_2, k_1, and k_2
such that $|f(x)| \le C_1|g(x)|$ for all $x > k_1$ and $|g(x)| \le C_2|h(x)|$ for all $x > k_2$. Hence, for $x > \max(k_1, k_2)$ it follows that $|f(x)| \le C_1|g(x)| \le C_1C_2|h(x)|$. This shows that $f(x)$ is $O(h(x))$. **19.** 2^{n+1} is $O(2^n)$; 2^{2n} is not. **21.** $1000 \log n$;

19. 2^{n+1} is $O(2^n)$; 2^{2n} is not.

$O(h(x))$. **19.** 2^{n+1} is $O(2^n)$; 2^{2n} is not. **21.** $1000 \log n$;
$\sqrt{n}$; $n \log n$; $n^2/1{,}000{,}000$; 2^n; 3^n; $2n!$ **23.** The algo-

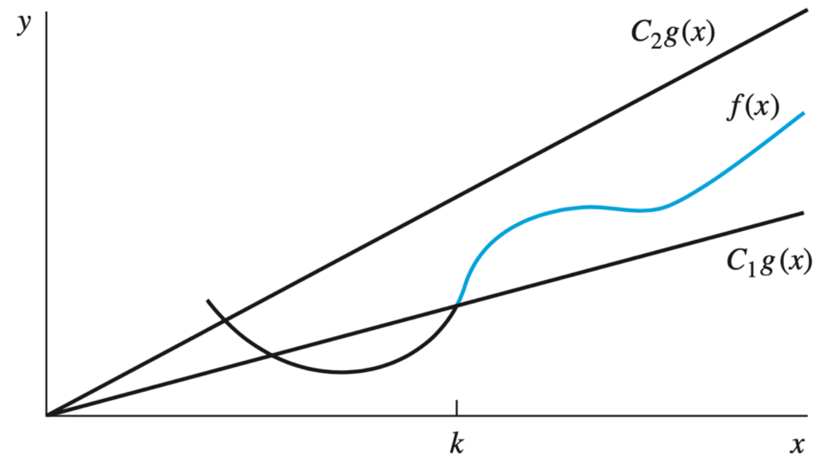
$\sqrt{n}$; $n \log n$; $n^2/1{,}000{,}000$; 2^n; 3^n; $2n!$ **23.** The algo-
rithm that uses $n \log n$ operations **25. a)** $O(n^3)$ **b)** $O(n^5)$

rithm that uses $n \log n$ operations **25. a)** $O(n^3)$ **b)** $O(n^5)$
c) $O(n^3 \cdot n!)$ **27. a)** $O(n^2 \log n)$ **b)** $O(n^2(\log n)^2)$ **c)** $O(n^{2^n})$

c) $O(n^3 \cdot n!)$ **27. a)** $O(n^2 \log n)$ **b)** $O(n^2(\log n)^2)$ **c)** $O(n^{2^n})$

29. a) Neither $\Theta(x^2)$ nor $\Omega(x^2)$ **b)** $\Theta(x^2)$ and $\Omega(x^2)$ **c)** Neither $\Theta(x^2)$ nor $\Omega(x^2)$ **d)** $\Omega(x^2)$, but not $\Theta(x^2)$ **e)** $\Omega(x^2)$, but not $\Theta(x^2)$ **f)** $\Omega(x^2)$ and $\Theta(x^2)$ **31.** If $f(x)$ is $\Theta(g(x))$, then there

35.



37. If $f(x)$ is $\Theta(1)$, then $|f(x)|$ is bounded between positive constants C_1 and C_2. In other words, $f(x)$ cannot grow larger than a fixed bound or smaller than the negative of this bound and must not get closer to 0 than some fixed bound.

41. Because $f(x)$ and $g(x)$ are increasing and unbounded, we can assume $f(x) \geq 1$ and $g(x) \geq 1$ for sufficiently large x. There are constants C and k with $f(x) \leq Cg(x)$ for $x > k$. This implies that $\log f(x) \leq \log C + \log g(x) < 2 \log g(x)$ for sufficiently large x. Hence, $\log f(x)$ is $O(\log g(x))$. **43.** By