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Exam 1 Alternate
EECS 203, Fall 2023

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Uniqname (ALL CAPS): RYNNEFAN

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Instructions

- When you receive this packet, fill in your name, Uniqname, and UMID above.
- Once the exam begins, make sure you have problems 1-18 in this booklet.
- Write your UMID in the blank at the top of every other page.
- No one may leave within the last 10 minutes of the exam.
- After you complete the exam, sign the Honor Code below. If you finish when time is called, your proctor will give you time to sign the Honor Code.
- Do not detach the scratch paper at the end of the packet.
- Do not discuss the exam until solutions have been released!

Materials

- No electronics allowed, including calculators.
- You may use one 8.5″ by 11″ note sheet, front and back, created by you.
- You may not use any other sources of information.

Honor Code

This exam is administered under the College of Engineering Honor Code. Your signature endorses the pledge below. We will not grade your exam without your signature.

I have neither given nor received unauthorized aid on this examination, nor have I concealed any violations of the Honor Code. I further agree not to discuss any aspect of this examination in any way, shape, or form until the solutions have been published.

Signature: Qiulin Fan

Part A: Single Answer Multiple Choice

Instructions

- There are 4 questions in this section.
- Shade the bubble you believe to be correct.
- There is only one correct answer option for each question in this section. If you shade more than one bubble, your answer will be marked as incorrect.

Problem 1. (4 points) ⓐ ⓑ ● ⓓ ⓔ

Consider the proposition:

Someone arrives early to lecture and sits in the front row.

Which of these is its negation? C

(a) Everyone arrives early to lecture and sits in the front row.

(b) No one arrives early to lecture or sits in the front row.

(c) Everyone arrives late to lecture or does not sit in the front row. ✓

(d) Someone arrives late to lecture and does not sit in the front row.

(e) No one arrives late to lecture or does not sit in the front row.

Problem 2. (4 points) ⓐ ⓑ ● ⓓ ⓔ

How many rows would a truth table for the following compound proposition need? C

$$(p \to q) \land (q \to r) \land (r \to p)$$

(a) 4

(b) 6

(c) 8

(d) 16

(e) 32

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Problem 3. (4 points)

ⓐ ⓑ ⓒ ● ⓔ

You are shipwrecked on an island with truth-tellers and liars. Truth-tellers always tell the truth and liars always lie. You meet two islanders - Alice and Bob. d

- Alice says "Exactly one of us is a truth-teller."
- Bob says "Alice is a liar."

A TT ⇒ B liar ✓ no TT X

A L ⇒ B liar TT X

What are Alice and Bob?

(a) Alice and Bob are both truth-tellers.

(b) Alice and Bob are both liars.

(c) Alice is a liar and Bob is a truth-teller.

(d) Alice is a truth-teller and Bob is a liar.

(e) There are multiple possible identities for Alice and Bob.

Problem 4. (4 points)

ⓐ ● ⓒ ⓓ ⓔ

Joseph wants to use proof by contradiction to prove the following statement:

For any integer n, if n^2 is even then n is even as well. b

What should he assume in order to derive a contradiction?

(a) "For some integer n, n and n^2 are not even."

(b) "For some integer n, n^2 is even but n is not even."

(c) "For any integer n, n and n^2 are even."

(d) "For any integer n, n is even but n^2 is not even."

(e) "For some integer n, n is even but n^2 is not even."

Part B: Multiple Answer Multiple Choice

Instructions

- There are 7 questions in this section.
- Shade whichever boxes you believe are correct. **This could be all answers, no answers, or anything in between.**
- If there are no correct answers, leave all the boxes blank.

Problem 5. (4 points)

a

Which of the following are **propositions**?

BCDE

(a) $x^2 - 2x + 1 = 0$ ✓

(b) $(1)^2 - 2(1) + 1 = 0$ ✓

(c) There exists an integer x such that $x^2 - 2x + 1 = 0$

(d) For all integers x, $x^2 - 2x + 1 = 0$

(e) Prof. Bodwin's favorite animal is the giraffe.

Problem 6. (4 points)

c e

Given that the following proposition is true: $\neg P(x) \wedge Q(x) = \neg(P(x) \vee \neg Q(x))$

$$\exists x[\neg P(x) \wedge Q(x)]$$

ABD

Which of the propositions below must also be true?

(a) $\exists x[\neg P(x)] \wedge \exists x[Q(x)]$ $\neg\forall x\,[\neg$

(b) $\exists x[P(x)] \vee \exists x[Q(x)]$

(c) $\neg\forall x[\neg P(x) \wedge Q(x)]$

(d) $\exists x[\neg P(x) \vee Q(x)]$

(e) $\forall x[\neg R(x) \vee R(x)]$ X

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Problem 7. (4 points)

a b c d e

Which of the following propositions are logically equivalent to $(p \wedge \neg q) \to r$?

(a) $\neg(\neg p \vee q) \to r$ ✓ $p \wedge \neg q$

(b) $\neg p \vee q \vee r$ ✓

(c) $\neg(p \vee \neg q) \to r$ ✗

(d) $\neg p \wedge q \wedge r$ ✗

(e) $\neg(p \to q) \to r$

abc

$\neg(p \wedge \neg q) \vee r$

$(\neg p \vee q) \vee r$ $\quad (\neg(p \to \neg q)) \to r$

Problem 8. (4 points)

a b c d e

Which of the following statements are tautologies?

(a) $p \wedge T$ ✗

(b) $p \wedge \neg p$ ✗

(c) $(p \wedge q) \vee (\neg p \wedge \neg q)$ ✗

(d) $(p \to q) \vee p$ ✓

(e) $(p \to q) \vee (\neg q \to \neg p)$

d

Problem 9. (4 points)

a b c d e

Which of the following are logically equivalent to $\neg \forall x \forall y \exists z\, P(x, y, z)$?

(a) $\neg \exists x \exists y \forall z\, P(x, y, z)$

(b) $\exists x \neg \forall y \exists z\, P(x, y, z)$ ✓

(c) $\exists x \exists y \forall z\, \neg P(x, y, z)$ ✓

(d) $\forall x \forall y \exists z\, \neg P(x, y, z)$

(e) $\exists x \exists y \forall z\, P(x, y, z)$

bc

Problem 10. (4 points)

a b c

Which of the following are valid ways to prove that the product of any two even integers must be an even integer? de

(a) Find a counterexample. The product of 3 (an odd integer) and 5 (an odd integer) equals 15 (an odd integer). X

(b) Find an example. The product of 2 (an even integer) and 4 (an even integer) equals 8 (an even integer). ✓

(c) Let m, n be arbitrary odd integers, and show that mn is an odd integer.

(d) Let m, n be arbitrary even integers, and show that mn is an even integer. ✓

(e) Assume that there are two even integers m, n where mn is odd, and then find a contradiction. ✓

Problem 11. (4 points)

b c

Which of the following are true, where the domain of x and y are **positive integers** (integers greater than or equal to 1)?

(a) $\forall x \forall y \ (xy \geq 1)$ ✓

(b) $\exists x \exists y \ (x + y = 0)$ X

(c) $\forall x \exists y \ (\sqrt{x} = y)$ X $\sqrt{2}$

(d) $\forall y \exists x \ (x - y \leq 0)$ ✓ $x \leq y$ $y \geq x$

(e) $\exists x \forall y \ (x - y \leq 0)$ $x=1$ ✓

Part C: Short Answer

Instructions

- There are 3 questions in this section
- Write your solution in the space provided below the question
- Don't simplify your answers
- Show your work and include justification

Problem 12: Divides Proof (8 points)

Prove that for all integers n, if n is odd, then $4 \mid (n^2 - 1)$.

Reminder: $a \mid b$ means "a divides b", i.e., "there exists an integer q such that $b = a \cdot q$."

Final Answer:

Proof Style: direct proof.

~~Let x be an arbitrary odd number~~

Then for some integer k, $x = 2k+1$,

then $n^2 - 1 = (2k+1)^2 - 1 = 4k^2 + 4k + 1 - 1 = 4k^2 + 4k = 4(k^2+4k)$

since k is an integer, (k^2+4k) is an integer

then $4 \mid 4(k^2+4k)$

then $4 \mid (n^2-1)$

Therefore we have proved that for all integer n, if n is odd, then $4 \mid (n^2-1)$

Problem 13: Translating English to Logic (5 points)

Let:

- $R(x, y)$ mean person x runs on day y
- $T(x, y)$ mean person x plays tennis on day y

$\neg\exists x\ (P(x,y) \land T(x,y))$

Translate the following into propositional logic:

"Nobody who goes for a run will play tennis on the same day."

Final Answer:

$\neg\exists x\ [P(x,y) \land T(x,y)]$

$\equiv \forall x \neg [P(x,y) \land T(x,y)]$

$\equiv \forall x\ [\neg P(x,y) \lor \neg T(x,y)]$

Problem 14: Even Odd Proof 1 (8 points)

Use proof by **CONTRADICTION** to prove the following statement:

For all integers x, if $11x + 7$ is even, then x is odd.

Note: You may **not** use lemmas such as even + even = even, etc. without proof.

Proof by Contradiction:

Assume its negation: "There exist an integer x such that $11x+7$ is even and x is even.", seeking a contradiction

$(\neg\forall x\ [P(x) \to Q(x)] \equiv \exists x \neg [P(x) \to Q(x)] \equiv \exists x\ [P(x) \land \neg Q(x)])$

Let x be an arbitrary even integer,

then for some integer k, $x = 2k$

then $11x+7 = 22k+7 = 22k+6+1 = 2(11k+3)+1$

Since k is an integer, $(11k+3)$ is an integer

Therefore $[2(11k+3)+1]$ is odd, then $(11x+7)$ is odd, which causes a contradiction with the condition that $(11x+7)$ is even

Therefore we have proved the statement that for all integer x, if $11x+7$ is even, then x is odd by disproving its negation by contradiction.

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Part D: Free Response

Instructions

- There are 4 questions in this section
- Write your solution in the space provided
- Write down your answer with care: **answers that are unreadable (such as too faint or too messy) will not be graded**
- If you have multiple answers, you must indicate which one you want graded. Otherwise, we will grade your least favorable answer.
- Show your work and include justification

Problem 15: Prove or Disprove (8 points)

Prove or disprove the following proposition where the domain is all **real numbers**.

$$\exists x \forall y [9x = y^2]$$

Reminder: Your proof/disproof must be mathematically rigorous.

Bubble prove or disprove, then continue your work below. **Show your work**.

○ Prove ● Disprove

Proof by contradiction:
We can disprove it by contradiction
Assume there exists a real number x such that for all y, $9x = y^2$
Consider $y=1$, then $9x = 1$, $x = \frac{1}{9}$
Then consider $y=2$, then $9x=4$, $x = \frac{4}{9}$
Since x cannot be $\frac{1}{9}$ and $\frac{4}{9}$ at the same time, this causes a contradiction
Therefore we have disproved the proposition. $\exists x \forall y [9x = y^2]$ by contradiction.

Problem 16: Rational/Irrational Proof (7 points)

Prove or disprove the following statements. Note: If your proof/disproof uses a specific irrational number, you do not need to prove that the number you choose is irrational.

(a) For all real numbers x, y, if $xy + y^2$ is irrational, then x is irrational or y is irrational.

● Prove ○ Disprove

Proof by contrapositive:

Prove its contrapositive. "If x is rational and y is rational, then $xy+y^2$ is rational".

Let x, y be arbitrary rational numbers.

Then for some integers p, q, m, n, $x = \frac{p}{q}$ and $y = \frac{m}{n}$

then $xy + y^2 = \frac{pm}{qn} + \frac{m^2}{n^2} = \frac{pmn + qm^2}{qn^2}$

$\because$ m, n, p, q are integers, qn^2, pmn, qm^2 are integers, then $(pmn + qn^2)$ is an integer.

then $\frac{pmn+qm^2}{qn^2}$ is rational, $xy+y^2$ is rational

Then we have proved the original proposition that for all real numbers x,y, if $xy+y^2$ is irrational, then x is irrational or y is irrational by proving its contrapositive.

(b) For all real numbers x, y, if $xy + y^2$ is rational, then x is rational or y is rational.

○ Prove ● Disprove

Disprove by a counterexample:

Consider $x = y = \sqrt{2}$

then $xy + y^2 = 2 + 2 = 4$, is rational

but x and y are both irrational

Then we have disproved the original statement that for all real number x, y, if $xy+y^2$ is rational, then x is rational or y is rational by a counterexample.

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Problem 17: Even Odd Proof 2 (8 points)

Prove that for all integers x and y, if $4xy + x + 2y$ and $2xy + 3x + 3y + 7$ are both odd, then x and y are both odd. You **may** use the following 6 axioms about odd and even numbers without proof.

- Odd + Odd = Even
- Odd + Even = Odd
- Even + Even = Even
- Odd × Odd = Odd
- Odd × Even = Even
- Even × Even = Even

You can use any number of these axioms in one step. For example, it is okay to say "if a is even and b is odd, then $a^2 + b^2 + ab + b$ is even."

Proof by contrapositive (and then by cases).

We prove it by its contrapositive. If x is even or y is even, then $4xy+x+2y$ is even or $2xy+3x+3y+7$ is even.

Let x, y be arbitrary integers s.t. at least one of them is even. Then it falls into 3 cases

Case 1: x is even and y is odd,
then $x = 2k$ for some integer k,
then $4xy + x + 2y = 8ky + 2k + 2y = 2(4ky + k + y)$
Since k, y are integers, $4ky$ is an integer, $(4ky+k+y)$ is an integer
Therefore $\underbrace{2(4ky+k+y)}_{4xy+x+2y\ =}$ is even.
The proposition is True in this case.

Case 2: x is odd and y is even
then $y = 2k$ for some integer k, $x = 2m+1$ for some integer m.
then $2xy + 3x + 3y + 7 = 2xy + 3(2m+1) + 7 + 6k$
$= 2xy + 6m + 10 + 6k$
$= 2(xy + 3m + 5 + 3k)$
Since x, y, m, k are integers, $xy, 3m, 3k$ are integers, $(xy+3m+5+3k)$ is an integer
Therefore $2 \mid 2(xy+3m+5+3k)$, $2xy+3x+3y+7$ is even
The proposition is True in this case.

Case 3: x and y are both even
then $x = 2k$ for some integer k
then $4xy + x + 2y = 2(4ky + k + y)$, the same as Case 1.
Since k, y are integers, $4ky$ is an integer, $(4ky + k + y)$ is an integer
Therefore $2 \mid 2(4ky+k+y)$, $4xy+x+2y$ is even. The proposition is true in this case

Since the proposition is true in every case, we have proved that for all integers x and y, if $4xy+x+2y$ and $2xy+3x+3y+7$ are both odd, then x and y are both odd by proving its contrapositive.

Problem 18: Even Schmevens (10 points)

Define a real number x to be "schmeven" if there is an integer k for which $x = \frac{3}{2}k$.

Prove or disprove the following propositions:

(a) For all real numbers x, if x is schmeven, then x^2 is schmeven.

○ Prove ● Disprove

Disproof by counterexample:

Consider $x = \frac{3}{2}$. For $k=1$, $x = \frac{3}{2}k$, therefore it is schmeven.

$x^2 = \frac{9}{4}$. $\frac{3}{2} \times 1 = \frac{3}{2}$, $\frac{3}{2} \times 2 = 3$.

$\because x^2 = \frac{9}{4} \in (\frac{3}{2}, 3)$. Assume there is an integer m for which $x^2 = \frac{3}{2}m$, then m must be between 1 and 2.

Since there is no more integer between 1 and 2, x^2 is not schmeven.

(b) For all real numbers x and y, if x is schmeven and y is **not** schmeven, then $x + y$ is **not** schmeven.

● Prove ○ Disprove

Proof by contradiction.

Assume its negation: "There exist real number x and y such that x is schmeven and y is not schmeven and $x+y$ is schmeven." $(\neg \forall x,y\,[P(x,y) \to Q(x,y)] \equiv \exists x,y\, \neg[P(x,y) \to Q(x,y)]$
Seeking a contradiction $\equiv \exists x,y\,[P(x,y) \land \neg Q(x,y)])$

Since x is schmeven and $x+y$ is schmeven, for some integer p, q, $x = \frac{3}{2}p$, $x+y = \frac{3}{2}q$

Then $y = \frac{3}{2}q - \frac{3}{2}p = \frac{3}{2}(q-p)$

Since p, q are integers, $(q-p)$ is an integer, $y = \frac{3}{2}(q-p)$ is schmeven, which causes a contradiction with the condition that y is not schmeven

Then we have proved the original statement: for all real numbers x and y, if x is schmeven and y is not schmeven, then $x+y$ is schmeven by contradicting its negation

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This is a space for scratch work. DO NOT DETACH THIS PAPER FROM YOUR EXAM.

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