Groupwork

1. Grade Groupwork 1

Using the solutions and Grading Guidelines, grade your Groupwork 1:

- Mark up your past groupwork and submit it with this one.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- Use the table below to calculate scores.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- What if my group changed?
  - If your current group submitted the same groupwork last time, grade it together.
  - If not, grade your version, which means submitting this groupwork assignment separately. You may discuss grading together.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 1	+4	+4	+2	+2	+2							/14
Problem 2	+5	+5										/10
Total:												24/24

Solution:

1. The Metaverse [14 points]

Suppose there is an island where there are exactly two types of people: truth-tellers and liars. A truth-teller always tells the truth, and a liar always lies.

Suppose a logician came across two inhabitants of this island, A and B. She asked A: "are you both truth-tellers?" A answered either yes or no. She stopped to think for a minute but could not determine what A and B were (truth-tellers or liars). She then asked "are you both of the same type?" (Same type means that they are either both truth-tellers or both liars.) A answered either yes or no, and then she knew what A and B were.

What are A and B?

Solution:

A	B	Q1	Q2
TT	TT	Yes	Yes
TT	L	No	No
L	TT	Yes	Yes
L	L	Yes	No

+8

We wrote it briefly and answered every point in a right way!

+6

From the table we draw we can see that, for the logician to be uncertain after Q1, A must have answered yes. Else, the logician would have deduced what A and B were if A answered no.For Q2, there are 3 possible options: 2 where A answers yes and 1 where A answers no. Since the logician knew what A and B after this question, A must have answered no. That makes A and B both liars.

2. Majority Rules [10 points]

Consider the ternary logical connective # where $\#PQR$ takes on the value that the majority of P, Q and R take on. That is $\#PQR$ is true if at least two of P, Q or R is true and is false otherwise. Express $\#PQR$ using **only** the symbols: $P, Q, R, \wedge, \vee$ and parenthesis.

Solution:

$((P \vee Q) \wedge (P \vee R) \wedge (Q \vee R))$

At least 2 of P, Q, R need to be T if and only if #PQR is T. Therefore we can take all the 2-combinations out of 3, and as long as one of them is T, the #PQR is T. This can be proven true by true value table below. And to express it in symbols, it is $(P \vee Q) \wedge (P \vee R) \wedge (Q \vee R)$.

+5

P	Q	R	$P \vee Q$	$P \vee R$	$Q \vee R$	$(P \vee Q) \wedge (P \vee R)$	$(P \vee Q) \wedge (P \vee R) \wedge (Q \vee R)$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	F	T	F
F	T	T	T	T	T	T	T
F	T	F	T	F	T	F	F
F	F	T	F	T	T	F	F
F	F	F	F	F	F	F	F

Solution:

P	Q	R	$\#PQR$
T	T	T	T
T	T	F	T
T	F	T	T
T	F	F	F
F	T	T	T
F	T	F	F
F	F	T	F
F	F	F	F

+5

We did a nice job!

2. Implication Inception [13 points]

Consider two propositions, A and B.

(a) Prove via a truth table that $A \equiv [(A \to B) \to A]$.

(b) Consider the compound proposition

$$\underbrace{((A \to B) \to A) \to B \dots}_{\text{203 letters}}$$

which has 203 total letters in it, alternating between A and B. Fill in the rest of the following truth table. You don't need to manually make all 203 columns of the truth table to solve this, so try to find a pattern and think of a shortcut.

A	B	$\underbrace{((A \to B) \to A) \to B \dots}_{\text{203 letters}}$
T	T	
T	F	
F	T	
F	F	

(c) Consider the following truth table. This is similar to the truth table in part (b), but it contains each iteration of the compound proposition from 1 to 203 letters.

A	B	$\underbrace{A}_{\text{1 letter}}$	$\underbrace{A \to B}_{\text{2 letters}}$	$\underbrace{(A \to B) \to A}_{\text{3 letters}}$	...	$\underbrace{((A \to B) \to A) \to B \dots}_{\text{203 letters}}$
T	T	T				
T	F	T				
F	T	F				
F	F	F				

What is the total number of cells that will be T among all 4 rows and 203 columns? (Note that the two initial A and B columns do *not* count as columns that should be counted. However, the $\underbrace{A}_{\text{1 letter}}$ column and all following columns do count.)

Solution:

(a) Truth Table:

A	B	$A \to B$	$(A \to B) \to A$
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	F

and so we have proved that $((A \to B) \to A) \equiv A$

(b) We have proved that $((A \to B) \to A) \equiv A$
Therefore $(((A \to B) \to A) \to B) \equiv (A \to B)$
and $((((A \to B) \to A) \to B) \to A) \equiv ((A \to B) \to A) \equiv A$.
We can see that it returns to the original proposition and thus forms a cycle because the next letter is always A and then B, continuing.
If we replace components from the inside out to be its logical equivalence and recursively perform this operation, we can find replace all the propositions by either A or $(A \to B)$. And by doing this, we can derive that propositions ended with A is equivalent to A, and those ended with B is equivalent to $(A \to B)$.
And since propositions ended with A have odd letters, and propositions ended with B have even letters (vice versa), the 203-letter-proposition is ended with A and has the same truth value with the A column.
$\therefore$

A	B	$\underbrace{((A \to B) \to A) \to B \dots}_{\text{203 letters}}$
T	T	T
T	F	T
F	T	F
F	F	F

(c) In question(b) we have justified that propositions ended with A are logically equivalent to A and have odd letters, while propositions ended with B are logically equivalent to $A \to B$ and have even letters (vice versa), the 203-letter-proposition is ended with A and has the same truth value with the A column.

$\therefore$ In the Table, odd number columns have the truth value of $TTFF$, the same as that of A; even number columns have the truth value of $TFTT$, the same as $(A \to B)$.
$\therefore$ There are 102 $TTFF$s and 101 $TFTT$s in the 203 columns.
$\therefore$ $102 \times 2 + 101 \times 3 = 505$ Ts.

A	B	$\underbrace{A}_{\text{1 letter}}$	$\underbrace{A \to B}_{\text{2 letters}}$	$\underbrace{(A \to B) \to A}_{\text{3 letters}}$	...	$\underbrace{((A \to B) \to A) \to B \ldots}_{\text{203 letters}}$
T	T	T	T	T		T
T	F	T	F	T		T
F	T	F	T	F		F
F	F	F	T	F		F

3. Functionally Complete [12 points]

A logical operator (or a set of logical operators) is considered to be *functionally complete* if it can be used to make any truth table.

(a) One set of functionally complete logical operators is $\{\vee, \neg\}$. In other words, we can use the $\vee$ and $\neg$ operators to make any truth table. Let's test this out with an example! Consider two propositions p and q. Write a compound proposition that is logically equivalent to $p \wedge q$ by only using p, q, $\vee$, $\neg$, and parentheses.

(b) Now, let's consider a new logical operator: NAND. The symbol for NAND is $\overline{\wedge}$. Below is the truth table for NAND. *(If you take EECS 370, you will get to use NAND even more!)*

p	q	$p \,\overline{\wedge}\, q$
T	T	F
T	F	T
F	T	T
F	F	T

Let's start trying to figure out whether $\overline{\wedge}$ is functionally complete. Is it possible to write a proposition that is logically equivalent to $\neg p$ by only using p and $\overline{\wedge}$? If so, write the proposition. If not, explain why it is impossible to do so.

(c) Is it possible to write a compound proposition that is logically equivalent to $p \vee q$ by only using p, q, $\overline{\wedge}$, and parentheses? If so, write the compound proposition. If not, explain why it is impossible to do so.

(d) Based on parts (a), (b), and (c), is $\overline{\wedge}$ functionally complete? Why or why not?

Solution:

(a) $\neg(\neg p \vee \neg q) \equiv (p \wedge q)$

p	q	$p \wedge q$	$\neg p$	$\neg q$	$\neg p \vee \neg q$	$\neg(\neg p \vee \neg q)$
T	T	T	F	F	F	T
T	F	F	F	T	T	F
F	T	F	T	F	T	F
F	F	F	T	T	T	F

(b) $(p \barwedge p) \equiv \neg p$

p	$\neg q$	$p \barwedge p$
T	F	F
F	T	T

(c) $(p \vee q) \equiv [(p \barwedge p) \barwedge (q \barwedge q)]$

p	q	$p \vee q$	$p \barwedge p$	$q \barwedge q$	$(p \barwedge p) \barwedge (q \barwedge q)$
T	T	T	F	F	T
T	F	T	F	T	T
F	T	T	T	F	T
F	F	F	T	T	F

(d) Yes. We are given that $\{\vee, \neg\}$ is functionally complete, and through (b) and (c) we have found equivalences of the two operators by using $\barwedge$. Therefore by substitution, we can say that $\barwedge$ is functionally complete.