

EECS 203: Discrete Mathematics  
Fall 2023  
Homework 9

Due **Thursday, November 16**, 10:00 pm

No late homework accepted past midnight.

Number of Problems:  $9 + 2$

Total Points:  $100 + 30$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

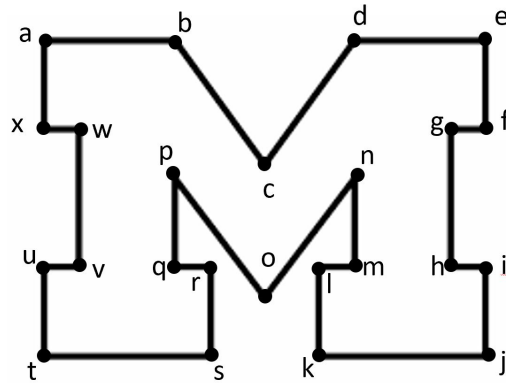
# Individual Portion

## 1. Shortest Paths [12 points]

Provide the shortest path distances between the following point pairings in their respective graphs. Justify your answer by providing a shortest path, or by stating that there is no such path.

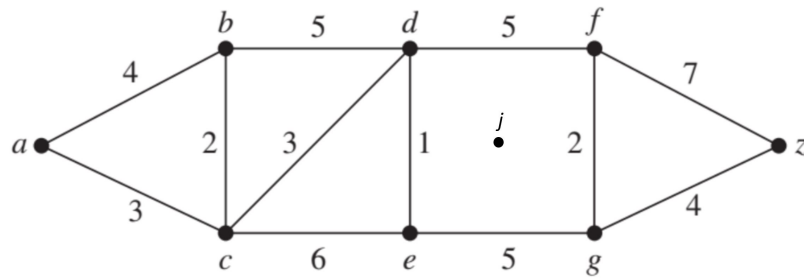
(a) (i)  $(x, p)$

(ii)  $(c, o)$

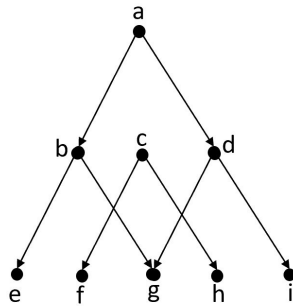


(b) (i)  $(a, z)$

(ii)  $(b, j)$



- (c) (i)  $(a, g)$   
(ii)  $(i, b)$



**Solution:**

## 2. Alexander Hamiltonian [12 points]

For each of the following parts, state whether a graph with  $n \geq 3$  vertices and the given properties **always**, **sometimes**, or **never** contains a Hamiltonian cycle. Justify your response for each part.

- (a) The complete graph  $K_n$ .  
(b) A tree.  
(c) A bipartite graph.  
(d) A graph that contains a vertex  $v$  where  $\deg(v) = 1$ .

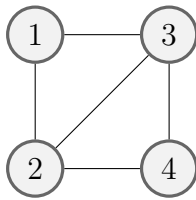
**Solution:**

## 3. Bipartite? [12 points]

For each of the following parts, determine if the graph is bipartite and justify your answer.

- (a) The cycle  $C_5$   
(b) The hypercube  $Q_2$   
(c) The complete graph  $K_5$

(d)



**Solution:**

#### 4. Melman the Graph [12 points]

In a simple, undirected graph with 5 vertices, what are all possible values for the number of vertices with odd degree? Justify your answer.

For each possible value  $k$ , construct a graph with 5 vertices that has  $k$  vertices of odd degree.

**Solution:**

#### 5. Keeping things Merry! [12 points]

Mary has three dogs, Sarge, Duke, and Brady. However, when left alone, Duke will pick a fight with either of the other two dogs. Mary wants to walk all 3 dogs from her house across the road to the dog park, can only walk 1 dog at a time, and wants to avoid a fight between any of the dogs. She can make multiple trips across the road, and can leave some of her dogs on one side of the road when walking another across.

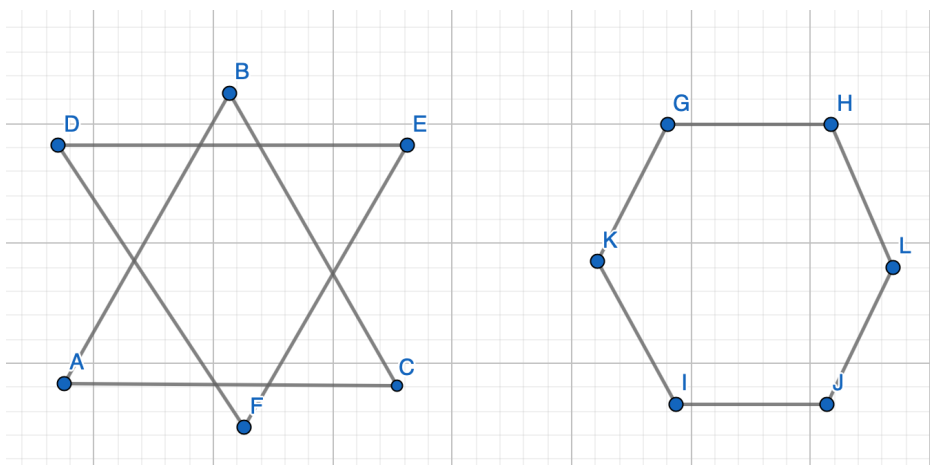
- Draw a graph where the nodes are the legal configurations of the puzzle and the edges represent possible transitions from one state to another. Each node should contain M, S, D, B, and || representing Mary, Sarge, Duke, Brady, and the road. For instance SB||MD would represent the Sarge and Brady being the the left side of the road at home with Mary and Duke on the other side at the park. Start with MSDB|| as your initial state.
- Identify the nodes that represent the start and end configurations of the puzzle. Is the puzzle solvable? Explain in terms of your graph.

**Solution:**

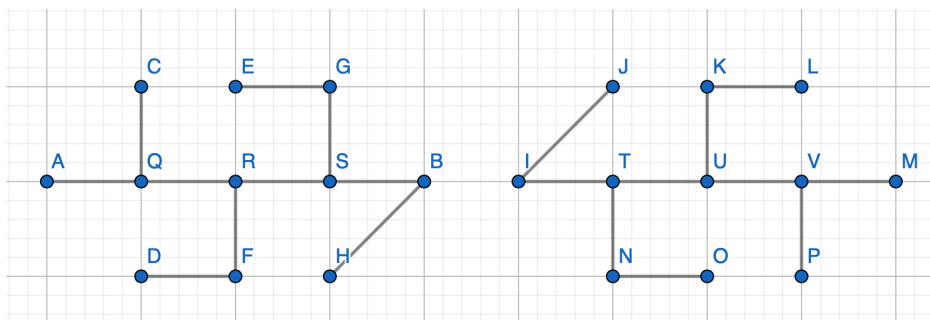
### 6. It's Iso-Morphin' Time [12 points]

Determine whether or not each of the following pairs of graphs are isomorphic. If yes, provide an isomorphism. If not, explain why.

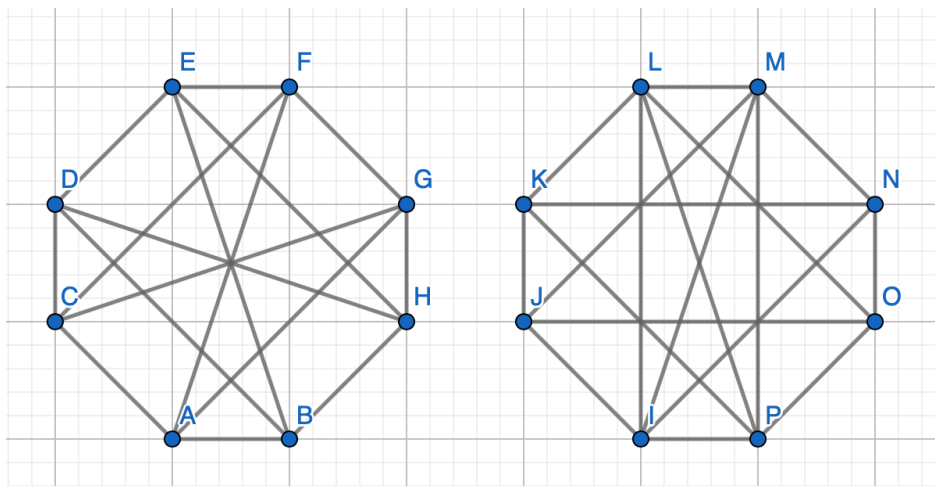
(a)



(b)



(c)



**Solution:**

## 7. Reduce, Reuse, Recycle [12 points]

For which values of  $n \geq 4$  do these graphs have an Euler cycle?

- (a) The complete graph  $K_n$
- (b) The cycle  $C_n$
- (c) The wheel  $W_n$
- (d) The hypercube  $Q_n$

**Solution:**

## 8. Counting [8 points]

How many positive integers less than 1000

- (a) are divisible by 7 but not by 11?
- (b) have distinct digits?
- (c) have distinct digits and are even?

You do **not** need to simplify your answer.

**Solution:**

**9. (Not So) Round and Round [8 points]**

Suppose we have a square-shaped table which seats 3 people on each side. How many ways are there to seat 12 people at the table, where seatings are considered the same if everyone is in the same group of 3 on a side?

**Solution:**

# Groupwork

## 1. Grade Groupwork 8

Using the solutions and Grading Guidelines, grade your Groupwork 8:

- Mark up your past groupwork and submit it with this one.
- Write whether your submission achieved each rubric item. If it didn't achieve one, say why not.
- Use the table below to calculate scores.
- For extra credit, write positive comment(s) about your work.
- You don't have to redo problems correctly, but it is recommended!
- What if my group changed?
  - If your current group submitted the same groupwork last time, grade it together.
  - If not, grade your version, which means submitting this groupwork assignment separately. You may discuss grading together.

	(i)	(ii)	(iii)	(iv)	(v)	(vi)	(vii)	(viii)	(ix)	(x)	(xi)	Total:
Problem 2												/12
Problem 3												/30
Total:												/42

## 2. Square the Cycle [15 points]

Prove that every  $n$ -node graph ( $n \geq 3$ ) in which all nodes have degree at least  $\lceil \sqrt{n} \rceil$  has a 3-cycle subgraph or a 4-cycle subgraph.

**Hint:** One useful concept is the neighborhood of a vertex; the neighborhood of  $v \in V$  is the set  $N(v) = \{u \in V : u \text{ is adjacent to } v\}$ . We can also define the neighborhood of a set  $A \subseteq V$ :

$$N(A) = \{u \in V : u \text{ is adjacent to some } v \in A\}.$$

We recommend using a proof by contradiction, although this can also be done with a clever direct proof. Suppose a graph satisfying the above condition does not have a 3-cycle or 4-cycle. Fix a vertex  $v \in V$ . What can we say about the size of  $N(v)$ ? What about  $N(N(v))$ ?



**Solution:**

### **3. The Office Allocation [15 points]**

Consider a new office building with  $n$  floors and  $k$  offices per floor in which you must assign  $2nk$  people to work, each sharing an office with exactly one other person. Find a closed form solution for the number of ways there are to assign offices if from floor to floor the offices are distinguishable, but any two offices on a given floor are not.

**Solution:**