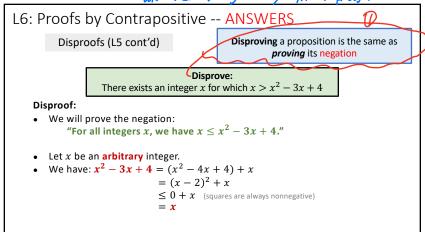
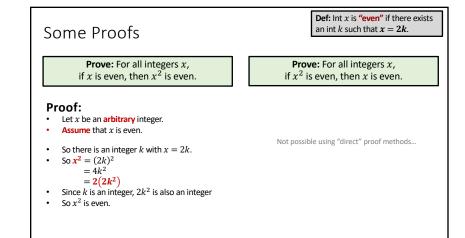
Technical Vocab: O proof by contrapasitive

D'untract loss of generality

2. Recognize propositions for which proof by contrapositive
might be helpful

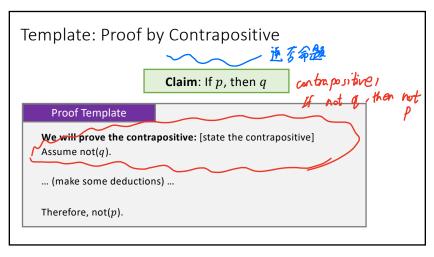
3. Understand when it is and issist valid to use
"with loss of generality" in a proof.

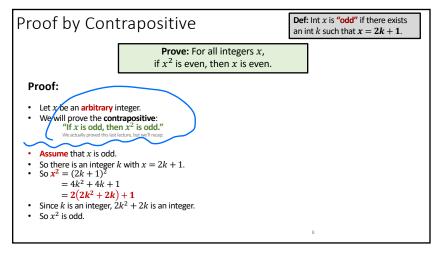




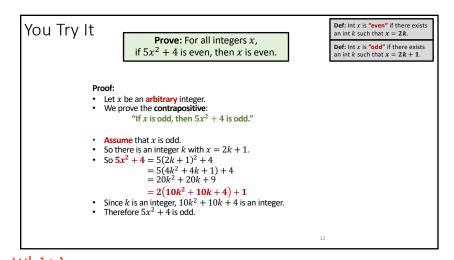
Proofs by contrapositive:

Modify the proposition using logical equivalences to make it easier to prove it





## **Def:** Int x is **"even"** if there exists **Another Contrapositive** an int k such that x = 2k. **Def:** Int x is "odd" if there exists an int k such that x = 2k + 1. **Prove:** For all integers x, if $x^2 + 6x + 5$ is even, then x is odd. Proof: • Let x be an arbitrary integer. We will prove the contrapositive: "If x is even, then $x^2 + 6x + 5$ is odd." Assume that x is even • So there is an integer k with x = 2k• So $x^2 + 6x + 5 = (2k)^2 + 6(2k) + 5$ $=4k^2+12k+5$ $=2(2k^2+6k+2)+1$ • Since k is an integer, $2k^2 + 6k + 2$ is an integer • So $x^2 + 6x + 5$ is odd.



Without Last of generality: (Assume WLOG)
we can we it when : (1) There are several possibilities about
the state of the world But these possibilities, are completely **Def:** Int x is "even" if there exists You Try It an int k such that x = 2k. **Prove:** For all integers x, y, Def: Int r is "odd" if there exists if xy is even, then x is even or y is even. an int k such that x = 2k + 1. Proof: • Let x, y be arbitrary integers. • We prove the contrapositive: "If x is odd and y is odd, then xy is odd." • Assume that x is odd and y is odd. So there are integers j, k with x = 2j + 1 and y = 2k + 1. • So xy = (2j + 1)(2k + 1)= 4jk + 2j + 2k + 1=2(2jk+j+k)+1• Since j, k are integers, 2jk + j + k is an integer. Therefore xy is odd.

A Fork in the Road Def: Int x is "even" if there exists Def: Int x is "odd" if there exists an int k such that x = 2k. an int k such that x = 2k + 1. **Prove:** For all integers x, y, if x + y is even, then x, y have the same parity (meaning both are even or both are odd) Let x, y be arbitrary integers. · We will prove the contrapositive: "If x, y have different parities (one is even and the other is odd), then x + y is odd." Assume that x, y have different parities. Assume without loss of generality (WLOG) that x is even and y is odd. So there are integers j, k with x = 2j and y = 2k + 1. So x + y = 2i + 2k + 1=2(j+k)+1• Since j, k are integers, j + k is an integer • So x + y is odd.