Exam 3
QUESTIONS PACKET
EECS 203
Practice Exam 2

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

8-Digit UMID: ______________________________

*****MAKE SURE YOU HAVE PROBLEMS 1 - 19 IN THIS BOOKLET.*****

General Instructions

You have 120 minutes to complete this exam. You should have two exam packets.

- **Questions Packet:** Contains ALL the questions for this exam, worth 100 points total:
  - 10 Single Answer Multiple Choice questions (4 points each),
  - 2 Multiple Answer Multiple Choice questions (4 points each),
  - 5 Short Answer questions (6-8 points each), and
  - 2 Free Response questions (10 points each)

  Questions Packet is for scratch work only. Work in this packet will not be graded.

- **Answers Packet:** Write all of your answers in the Answers Packet, including your answers to multiple choice questions.

  For free response questions, you must show your work! Answers alone will receive little or no credit.

- You may bring **one** 8.5" by 11" note sheet, front and back, created by you.
- You may **NOT** use any other sources of information, including but not limited to electronic devices (including calculators), textbooks, or notes.
- After you complete the exam, sign the Honor Code on the front of the Answers Packet.

Part A1: Single Answer Multiple Choice

Problem 1. (4 points)

Suppose two fair six-sided dice are rolled, one yellow and the other blue. Given their sum is less than or equal to 5, what is the probability that the value of the blue die is 2?

(a) $\frac{3}{10}$

(b) $\frac{1}{3}$

(c) $\frac{4}{10}$

(d) $\frac{1}{2}$

(e) $\frac{2}{3}$

Problem 2. (4 points)

Nolan wears his lucky EECS 203 staff T-shirt to all of his job interviews. When Nolan wears the lucky shirt, he has a 70% chance of receiving the job offer. Suppose Nolan has 5 upcoming interviews this week (yikes!). What is the probability that he will receive exactly 3 job offers?

(a) $3!\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$

(b) $\binom{5}{3}\left(\frac{7}{10}\right)^2\left(\frac{3}{10}\right)^3$

(c) $\binom{5}{3}\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$

(d) $P\left(5,3\right)\left(\frac{7}{10}\right)^3\left(\frac{3}{10}\right)^2$

(e) $\frac{7^3 \cdot 3^2}{10^5}$

Problem 3. (4 points)

A bucket contains 20 indistinguishable Red Delicious apples, 20 indistinguishable Macintosh apples, and 20 indistinguishable Granny Smith apples. How many ways are there to choose 3 apples from the bucket?

(a) 9

(b) 10

(c) 11

(d) 12

(e) 13

Problem 4. (4 points)

What is the tightest big-O bound of the function: $f(n) = (n^2 \log n)(5n^3 + 4)(n^4 + 3^n)$?

(a) $O(n^5 \log n)$

(b) $O(3^n)$

(c) $O(3^n n^5 \log n)$

(d) $O(3^n n^3 \log n)$

(e) $O(n^5 \log n + 3^n)$

Problem 5. (4 points)

What is the runtime complexity of the following algorithm?

```
function GOOFYLOOP(n ∈ ℕ)
    for i ← 1 to 10 do
        GOOFYLOOP(⌊n/5⌋)
    end for
    for i ← 1 to n do
        for j ← 1 to 20 do
            print "I'm stuck in a loop"
        end for
    end for
    print "I'm freeee"
end function
```

(a) $\Theta(n)$

(b) $\Theta(n \log n)$

(c) $\Theta(n^{10})$

(d) $\Theta(n^{10} \log n)$

(e) $\Theta(n^{\log_5 10})$

Problem 6. (4 points)

Suppose 13 cards from a standard deck of 52 cards are dealt evenly across 4 players. How many ways are there for each player to receive exactly one card of each rank?

Reminder: A standard deck of cards has 4 suits and 13 ranks (A, 2, 3,..., 10, J, Q, K).

(a) $4!$

(b) $(4!)^{13}$

(c) $P(13, 4)^{13}$

(d) $\binom{52}{13}\binom{39}{13}\binom{26}{13}\binom{13}{13}$

(e) $(4^{13})^4$

Problem 7. (4 points)

What is the Big-Theta runtime of the following recurrence?

$$T(n) = 8T\left(\frac{n}{2}\right) + 203n^3$$

(a) $\Theta(n^3)$

(b) $\Theta(8n^3)$

(c) $\Theta(n^4)$

(d) $\Theta(n^3 \log n)$

(e) $\Theta(n^{\log_2 3})$

Problem 8. (4 points)

Yili repeatedly rolls a pair of dice: one **unfair** 4-sided die and one **fair** 6-sided die. The 4-sided die is weighted such that it has a $\frac{2}{5}$ probability of rolling a 1. She keeps rolling the two dice until **both** of the following conditions are met in the same turn:

- The 4-sided die rolls a 1.
- The 6-sided die rolls a 3, 4, 5, or 6.

What is the expected number of turns it would take for Yili to stop, including the final turn?

(a) $\dfrac{3}{2}$

(b) 2

(c) $\dfrac{15}{4}$

(d) 6

(e) 12

Problem 9. (4 points)

How many strings of 8 upper-case English letters begin with GO or end with BLUE?

(a) 26^8

(b) $26^6 + 26^4$

(c) $26^6 \cdot 26^4$

(d) $26^6 + 26^4 - 26^2$

(e) $26^6 + 26^2 - 26^4$

Part A2: Multiple Answer Multiple Choice

Problem 10. (4 points)

Trent rolls two fair, 6-sided dice. One die is red and the other die is blue. Consider the following events:

- R: The red die is a 1.
- B: The blue die is a 6.
- S: The sum of the dice is 8.

Reminder: Two events are *mutually exclusive* when they cannot happen at the same time.

Which of the following statements are true?

(a) R and B are mutually exclusive.

(b) R and B are independent.

(c) R and S are mutually exclusive.

(d) R and S are independent.

(e) B and S are mutually exclusive.

Problem 11. (4 points)

Which of the following graphs **must** be bipartite?

(a) A tree

(b) K_5

(c) C_8

(d) A simple graph with four vertices with degrees $2, 2, 3$, and 3

(e) W_4

Problem 12. (4 points)

Let f, g be functions where

- $f(n) = 6n^4 + 4n + \log n + 4$
- $g(n) = n! + n^5$

Which of the following are true?

(a) $f(n)$ is $O(g(n))$

(b) $f(n)$ is $\Theta(g(n))$

(c) $f(n)$ is $\Omega(g(n))$

(d) $\dfrac{g(n)}{f(n)}$ is $O(n!)$

(e) $g(n) \cdot f(n)$ is $O(n!)$

Part B: Short Answer

Questions 13-16: You DO NOT need to simplify your answer.
Question 17: You DO need to fully simplify your answer.

Problem 13. (6 points)

How many distinct permutations of the 10-letter string BOOKKEEPER are there?

Problem 14. (6 points)

Jasmine lives on an island in the archipelago Discretopia. Discretopia is a ring of n islands, with n bridges built all the way around the ring, making a cycle. Recently, there was an earthquake that damaged the bridges. Each bridge independently has a 10% chance of collapsing. What is the probability that Jasmine can still reach all of the islands in Discretopia?

Note: Your answer should be left in terms of n.

Problem 15. (6 points)

You are selecting an All Star Team from a set of 20 finalists, 7 of whom are wearing green shirts. If you select 9 people uniformly at random for your All Star Team, what is the probability that you end up with **at least** 1 green-shirted person on the team?

Problem 16. (6 points)

A binary string is **palindromic** if it reads the same backwards as forwards. For example,

- 1001 is a 4-bit palindromic binary string
- 01110 is a 5-bit palindromic binary string.

What is the probability that a randomly selected 11-bit binary string is palindromic?

Problem 17. (8 points)

Abby and Nihar both plan to volunteer this weekend. They will each volunteer one shift at one organization. They each select which organization to volunteer at with the probabilities outlined below.

Organization	Length of Volunteer Shift	Probability of Abby selecting this	Probability of Nihar selecting this
Humane Society	3 hours	1/3	1/6
Food Bank	6 hours	5/12	1/3
Big Brothers Big Sisters	10 hours	1/4	1/2

Let X be a random variable representing the total hours volunteered by Abby and Nihar this weekend. Find $E(X)$.

Express your answer as a single, **fully simplified** number.

Part C: Free Response

Problem 18. (10 points)

Consider a non-standard 104-card deck with 8 suits ($\heartsuit$, $\diamondsuit$, $\clubsuit$, $\spadesuit$, $\bigcirc$, $\triangle$, $\bigstar$, $\blacksquare$) and the usual 13 ranks per suit (A, 2, 3, ..., 10, J, Q, K). The IAs play a game where a hand contains 6 cards each.

We define a *full mansion* as a 6-card hand that can be split into two sets of three cards each, where each set of three are of the same rank. The ranks of the two sets do **not** need to be different. (In other words, 6 cards of the same rank does qualify as a *full mansion*.)

For example, both of these hands are *full mansions*:

$$\{2\bigcirc, 2\clubsuit, 2\diamondsuit, \mathrm{K}\diamondsuit, \mathrm{K}\spadesuit, \mathrm{K}\heartsuit\}$$

$$\{7\spadesuit, 7\bigstar, 7\clubsuit, 7\heartsuit, 7\triangle, 7\blacksquare\}$$

How many hands of 6 cards are a *full mansion*?

You do **not** need to simplify your answer.

Problem 19. (10 points)

Suppose that Alicia selects one of three books. Each book has a different number of pages and a different probability that she selects it.

- Book 1 has 100 pages, and is selected with probability 2/7.
- Book 2 has 200 pages, and is selected with probability 4/7.
- Book 3 has 300 pages, and is selected with probability 1/7.

Alicia flips to a random page of the selected book, such that each page in the book is flipped to with equal probability.

Answer each question below, and **simplify your answer to a single, fully-simplified number.**

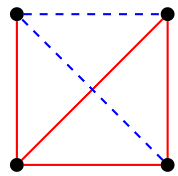
(a) What is the probability that Alicia flips to a page number that is **greater than** 150?

(b) Given that Alicia flips to a page number greater than 150, what is the probability that she has selected Book 2?

Problem 20. (8 points)

We define a spanning tree of a graph G to be a connected subgraph that still has every vertex and contains no cycles (hence a tree). Then, let G be a graph in which each **edge** has been colored red or blue. G contains a **"monochromatic spanning tree"** if it contains a spanning tree in which all edges are the same color (i.e., all edges are red or all edges are blue.)

For example, the following red-blue coloring of K_4 *contains* a solid red monochromatic spanning tree:



Let $P(n)$ be the predicate

"Any possible red-blue coloring of K_n contains a monochromatic spanning tree."

Using induction, prove $\forall n \geq 2, P(n)$.