Practice Exam 1
QUESTIONS PACKET
EECS 203
Fall 2023

Name (ALL CAPS): _____

Uniqname (ALL CAPS): _____

8-Digit UMID: _____

*****MAKE SURE YOU HAVE PROBLEMS 1 - 16 IN THIS BOOKLET.*****

General Instructions

You have 120 minutes to complete this exam. You should have two exam packets.

- **Questions Packet:** Contains ALL the questions for this exam, worth 90 points total:
  - 5 Single Answer Multiple Choice questions (4 points each),
  - 4 Multiple Answer Multiple Choice questions (4 points each),
  - 2 Short Answer questions (6 points each), and
  - 5 Free Response questions (8 or 9 points each)

  Questions Packet is for scratch work only. Work in this packet will not be graded.
- **Answers Packet:** Write all of your answers in the Answers Packet, including your answers to multiple choice questions.

  For free response questions, you must show your work! Answers alone will receive little or no credit.
- You may bring **one** 8.5" by 11" note sheet, front and back, created by you.
- You may **NOT** use any other sources of information, including but not limited to electronic devices (including calculators), textbooks, or notes.
- After you complete the exam, sign the Honor Code on the front of the Answers Packet.

- You must turn in both parts of this exam.
- **You are not to discuss the exam until the solutions are published.**

Part A1: Single Answer Multiple Choice

Problem 1. (4 points)

How many rows does the truth table for $(p \vee q) \wedge (r \to (\neg r \wedge q))$ have?

(a) 4

(b) 5

(c) 8

(d) 16

(e) 32

Problem 2. (4 points)

Consider the proposition

$$(s \vee \neg b) \to h$$

where:

- s is "Shubh overslept"
- b is "The busses are on time"
- h is "Shubh is late to office hours"

Which of the following is a correct translation of the given proposition?

(a) If Shubh overslept or the busses are not on time, then Shubh is late to office hours

(b) If the busses are not on time and Shubh overslept, then Shubh is late to office hours

(c) If Shubh is late to office hours, then either they overslept or the busses are not on time

(d) Shubh is late to office hours if and only if they overslept or the busses are not on time

(e) If Shubh overslept, then the busses are not on time and they are late to office hours

Problem 3. (4 points)

Which of the following is the correct assumption to start a proof by **contradiction** for the statement:

"If n^2 is even, n is also even."

"Seeking a contradiction, assume ..."

(a) n^2 is odd and n is even.

(b) n^2 is even and n is odd.

(c) if n is odd, then n^2 is odd.

(d) if n^2 is even, then n is odd.

(e) if n^2 is even, then n is even.

Problem 4. (4 points)

Let $P(x, y)$ mean "person x can work on day y". Which of the following statements has the same meaning as this sentence:

"No one can work every day."

(a) $\exists x \neg \exists y \, P(x, y)$

(b) $\neg \forall x \exists y \, P(x, y)$

(c) $\neg \exists x \forall y \, P(x, y)$

(d) $\forall x \forall y \, \neg P(x, y)$

(e) $\exists x \forall y \, \neg P(x, y)$

Problem 5. (4 points)

Define the follow predicates:

- $O(x, y)$ means person x ordered pizza y
- $D(x, y)$ means person x delivered pizza y

Which is an equivalent translation to the following: Some delivery-person delivered every pizza that they themselves ordered.

(a) $\exists d \forall p[O(d,p) \rightarrow D(d,p)]$

(b) $\exists d \forall p[O(d,p) \wedge D(d,p)]$

(c) $\forall p \exists d[O(d,p) \rightarrow D(d,p)]$

(d) $\forall p \exists d[O(d,p) \wedge D(d,p)]$

Part A2: Multiple Answer Multiple Choice

Problem 6. (4 points)

Let the domain of x and y be the **non-zero integers**. Which of the following are true?

(a) $\exists x \exists y \left[x^2 + y^2 = 3\right]$

(b) $\forall x \exists y \left[(y < 0) \rightarrow (y < x^2)\right]$

(c) $\forall x \forall y \left[(y < 0) \rightarrow (y < x^2)\right]$

(d) $\forall y \exists x \left[\frac{x}{y} = 1\right]$

(e) $\exists x \forall y \left[\frac{x}{y} = 1\right]$

Problem 7. (4 points)

Which of the following statements are equivalent to the **negation** of the proposition below?

$$\exists x \forall y \left[P(x, y) \rightarrow (x \neq y)\right]$$

(a) $\forall x \exists y \left[P(x, y) \wedge (x = y)\right]$

(b) $\forall x \exists y \left[\neg P(x, y) \rightarrow (x = y)\right]$

(c) $\forall x \neg \forall y \left[P(x, y) \rightarrow (x \neq y)\right]$

(d) $\exists x \forall y \left[P(x, y) \wedge (x = y)\right]$

(e) $\exists x \forall y \left[\neg P(x, y) \wedge (x \neq y)\right]$

Problem 8. (4 points)

Which of the following are tautologies?

(a) $p \vee q$

(b) $p \wedge \neg p$

(c) $p \vee \neg p$

(d) $(p \to q) \vee (p \wedge \neg q)$

(e) $p \vee (p \to q)$

Problem 9. (4 points)

Which of these are true over the domain of discourse $\mathbb{R}$? For this, we will define $\frac{0}{0} = 0$, while anything else divided by 0 is undefined, and therefore not equal to 0.

(a) $\forall x \exists y (\frac{x}{y} = 0)$

(b) $\exists x \forall y (\frac{x}{y} = 0)$

(c) $\exists x \exists y (\frac{x}{y} = 0)$

(d) $\exists y \forall x (\frac{x}{y} = 0)$

(e) $\forall y \exists x (\frac{x}{y} = 0)$

Part B: Short Answer

Problem 10. (6 points)

Prove or disprove the following statement:

$$\forall x \exists y \, [y > x]$$

Note: The domain for x and y is **integers.**

Problem 11. (6 points)

Using a proof by contradiction, prove that if $3n^2 + 3$ is even, then n is odd.
Note: You cannot use the lemmas "even + odd = odd", "even $\cdot$ even= even", etc. without proving it.

Part C: Free Response

Problem 12. (8 points)

p	q	r	s	t	w
T	T	T	T	T	F
T	T	F	T	F	F
T	F	T	T	T	F
T	F	F	T	F	F
F	T	T	T	T	F
F	T	F	F	T	T
F	F	T	T	T	T
F	F	F	T	T	T

Use the truth table for the compound propositions s, t, and w given above to answer the following questions.

(a) Is $(s \wedge w) \vee t$ a tautology? Briefly explain your answer.

(b) For each unknown proposition, s, t, and w:

- Find an expression for the proposition as a compound proposition using p, q, and/or r.
- You may use **only** $\wedge$, $\vee$, $\neg$, and parentheses in each expression.
- You may use p, q, and r **at most once** in each expression.

Problem 13. (9 points)

Prove or disprove each of the following statements.
Note: If you use a specific irrational number in your proof/disproof, you do **not** need to prove that it is irrational. You can simply state that it is.

(a) Prove or disprove: For all rational numbers x and irrational numbers y, their sum $x+y$ is irrational.

(b) Prove or disprove: For all irrational numbers x and y, their difference $x-y$ is irrational.

Problem 14. (9 points)

Use the definitions of "even" and "odd" to prove the following:

$$\text{If either } x \text{ is odd or } y \text{ is even, then } y^2(x-1) \text{ is even.}$$

Note: You cannot use the lemmas "even + odd = odd", "even $\cdot$ even= even", etc. without proving it.

Problem 15. (9 points)

Prove the following statement, and state which proof method you are using. The domain for a and b is the set of all **integers**.

"If $a + b > -1$ and $ab > 0$, then a and b are both positive."

Problem 16. (9 points)

Prove or disprove that for all integers x and y:

If $x - 2xy$ is even, then one of the variables is even and the other is odd.

Hint: You may find it easier to consider the contrapositive of the given statement.

If you choose to prove, you may find it helpful to use the following 6 properties about odd and even numbers. For this question only, you may use them without proving them.

- Odd + Odd = Even
- Odd + Even = Odd
- Even + Even = Even
- Odd $\times$ Odd = Odd
- Odd $\times$ Even = Even
- Even $\times$ Even = Even

Scratch paper. **Nothing written on this page will be graded.**

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