EECS 203 Exam 1 Review

Day 2

Today’s Agenda

- Review Questions
- Test Taking Strategies
- Open Q&A

Problem 1. (4 points) Determine if the following propositions are a contradiction, satisfiable, or tautology.

(a) $(p \to q) \wedge (\neg p \vee \neg q)$

(b) $(p \vee \neg p) \wedge (q \vee \neg q)$

(c) $(p \wedge q) \wedge (\neg p \vee \neg q)$

Solution:

(a) This is satisfiable. Take when p is false and q is true. However, this is not a tautology, since the statement is false when p is true and q is true.

(b) this is a tautology. $p \vee \neg p$ is always true, and similarly, $q \vee \neg q$ is always true, meaning this statement will always be true.

(c) This is a contradiction. $p \wedge q$ and $\neg p \vee \neg q$ are negations of each other, meaning, both can never be true at the same time, and giving us a contradiction

Problem 2. (4 points)

Which of the following are statements are true? Assume the domain is all positive integers.

(a) $\forall x \forall y \exists z (\frac{x}{y} = z)$

(b) $\forall x \exists y (x \geq y)$

(c) $\exists x \exists y (xy < 0)$

(d) $\exists x \forall y (x \neq y \rightarrow x < y)$

(e) $\forall x \exists y \exists z (\frac{y}{z} = x)$

Solution:

(a) This is false, take $x = 3$ and $y = 2$ for example.

(b) This is true, you can just let $y = x$.

(c) This is false. This statement is saying that the product of x and y is negative, and since no two positives can yield a negative, this is false.

(d) This is true. Take $x = 1$. Since we only need to consider cases in which $x \neq y$, all other possible values are greater than x when $x = 1$.

(e) This is true. All integers can be represented by two other integers in fractional form. For example let $y = 2x$ and $z = 2$.

Problem 3. (4 points)

Translate the below statements to logic given the following predicates. Assume the domain is all UMich students.

- $L(x, y)$: Person x is roommates with person y.
- $D(x, y)$: Person x is eating dinner with person y.
- $H(x)$: Person x is hungry.

(a) Everyone has a roommate

(b) Someone only ever eats alone or with their roommates.

(c) No one will eat dinner with their roommates when they are hungry

Solution:

(a) $\forall x \exists y L(x, y)$

(b) $\exists x \forall y[(D(x, y) \wedge x \neq y) \to L(x, y)]$

(c) $\forall x \forall y[H(x) \to \neg(D(x, y) \wedge L(x, y))]$
or equivalently, $\neg \exists x \exists y[H(x) \wedge (D(x, y) \wedge L(x, y))]$

Problem 4. (4 points)

Prove the following statement for all integers:

If n is even and m is odd, then $nm + n + m$ is odd.

Solution:

Let n be an arbitrary even element and m be an arbitrary odd element. This means that $n = 2k$ where k is some integer and $m = 2j + 1$ where j is some integer. Then we can evaluate the following:

$$\begin{aligned} nm + n + m &= 2k \cdot (2j + 1) + 2k + 2j + 1 \\ &= 4jk + 4k + 2j + 1 \\ &= 2(2jk + 2k + j) + 1 \end{aligned}$$

Since j and k are both integers, $2jk+2k+j$ is also an integer, meaning that the previous statement is odd, since we have the form of $2m + 1$ where m is some integer. Thus, If n is even and m is odd, then $nm + n + m$ is odd.

Problem 5. (4 points)

Prove the following statement:

If $4x + 3y$ is even or $5x + 7y$ is odd, then x is even or y is even

We will prove the contrapositive: If x is odd and y is odd, then $4x + 3y$ is odd and $5x + 7y$ is even.

Let x be an arbitrary odd integer and y be an arbitrary odd integer. This means that $x = 2k + 1$ and $y = 2j + 1$ where k and j are integers. Then, we can evaluate the following:

Solution:

$$\begin{aligned}
4x + 3y &= 4(2k + 1) + 3(2j + 1) \\
&= 8k + 4 + 6j + 3 \\
&= 8k + 6j + 6 + 1 \\
&= 2(4k + 3j + 3) + 1
\end{aligned}$$

Since we can represent the statement as $2m + 1$ where m is some integer, the statement is odd.

$$\begin{aligned}
5x + 7y &= 5(2k + 1) + 7(2j + 1) \\
&= 10k + 5 + 14j + 7 \\
&= 10k + 14j + 12 \\
&= 2(5k + 7j + 6)
\end{aligned}$$

Since we can represent the statement as $2m$ where m is some integer, the statement is even.
Thus, We have proven the contrapositive, proving the original statment.

Suppose that it snows for at least 1 in 3 days in the month of November.
Prove that for any 25 days chosen in November, it snows for at least 5 of them.

(As a reminder, there are 30 days in November.)

Solution:

Seeking contradiction, assume that for any 25 days chosen in November, it snows for less than 5 of them. This means that it does not snow for more than out 4 of any 25 days in November. This would mean that there would be at least 21 days on which it did not snow. This contradicts our statement, however, that it snows for at least 1 in 3 days in the month of November. This leaves a possible minimum of 10 snow days and a possible maximum of 21 non-snow days. Therefore, it must snow for at least five days out of any 25 days in November.

Prove the following:

$5x^3 - 5x^2 + 3x - 19$ is negative whenever x is negative.

Solution:

Seeking contradiction, assume that x is negative and $5x^3 - 5x^2 + 3x - 19$ is positive. $5x^3$ is negative because x is negative. Subtracting $5x^2$, a positive value, increases this quantity in the negative direction. Adding $3x$, a negative value, and subtracting 19 again increases this value in the negative direction. Therefore, the polynomial itself is negative, resulting in a contradiction.

Problem 6. (4 points)

Prove the following statement:

If x is even or y is even, then $3xy + 3x$ is even or $3xy + 3y$ is even.

Solution:

We will prove by cases:

Case 1: x is even

Let x be an arbitrary even integer. Thus, $x = 2k$ where k is some integer. We will prove the first statement of the or.

$$\begin{aligned} 3xy + 3x &= 3(2k)y + 3(2k) \\ &= 2(3ky + 3k) \end{aligned}$$

Thus, the statement is even.

Case 2: y is even Let y be an arbitrary even integer. Thus, $y = 2j$ where j is some integer. We will prove the second statement of the or.

$$\begin{aligned} 3xy + 3y &= 3x(2j) + 3(2j) \\ &= 2(3xj + 3j) \end{aligned}$$

Thus, the statement is even. Since all cases have been exhausted, the original statement is true.

Problem 7

Prove/Disprove: “For all real numbers x, there exists some y s.t. $x^2 + y^2 = 1$.”

Consider $x = 8$. The only solution for this is

$$64 + y^2 = 1$$

$$y^2 = -63.$$

The only solution would be $(-63)^{1/2}$, which would be an imaginary number, thus disproving the statement.

How does take 203?? exam

Components of Exam

1. Single Answer Multiple Choice
2. Multi Answer Multiple Choice
3. Short Answer
4. Free Response (long answer)

Part I: Single Answer Multiple Choice

- Exactly one correct option
- Strategies:
  - Process of elimination
  - Scratch work (will not be graded)

Part II: Multi Answer Multiple Choice

- Any number of options could be correct
  - Could be none of them
  - Could be all of them!
- Strategies
  - Scratch work (will not be graded)
  - Find one correct answer and then find answers equivalent to it
    - Namely for “which of the following are equivalent…” problems

Part III: Short Answer

- Designed to be like “mini-FRQs”
  - Closer to homework/discussion diffculty
- Strategies
  - Remember: it’s *short* answer
    - Answers do not need to have a page’s worth of text in the given box
  - Brainstorm an answer/explanation *before* writing it out
    - Use scratch paper if it helps
  - Put pen(cil) to paper
    - Make sure it is formal and thorough
    - Re-read after writing and make sure it flows well

Part IV: Free Response

- Designed for longer response questions
  - Between homework/groupwork difficulty
- Strategies
  1. Read the question thoroughly (re-read if it helps)
     a. If it seems like a lot to handle at the moment, come back to it later
     b. Understand what the question is asking
  2. Brainstorm an answer/explanation *before* writing it out
     a. Use scratch work and write/sketch an outline, keeping it all in your head under pressure may lead to forgetting some details
     b. Develop a conceptual answer to the question
  3. Put pen(cil) to paper
     a. Make sure it is formal and thorough
     b. Re-read after writing and make sure it flows well

Problem Type Strategies: Logic

- What strategy to use when evaluating a statement?
  - Truth tables
    - Brute force solution
    - Complete picture of everything
    - Becomes very expensive to write out as there are more propositions involved\
  - Logical Equivalences
    - Helps simplify things to more understandable forms
    - If you have to do a lot of them it may be exhausting
  - Intuition/Rationale
    - Allows you to “fill in the gaps”
    - Sort of combines the best of both worlds
- Remember the difference b/w (for all, there exists) and (there exists, for all)

Problem Type Strategies: Proofs

- "What proof method should I use?"
  - Ask yourself: "What will make my life easier?"
  - If the original problem has you dealing with something annoying to define (i.e. irrational, prime)...
    - Consider negating them with contrapositive or contradiction
      - Irrational -> Rational
      - Prime -> Composite
  - If the problem is asking you to split something apart
    - Try putting it together instead
    - i.e. (if xy is even, then x is even or y is even)
- "What can/can't I assume?"
  - Unless we say otherwise…
  - If we give you a definition, you can't use it without proof
  - i.e. rational*rational = rational vs integer*integer = integer

General Test Taking Strategies

- Read/graze over the entire exam first
  - Decide where to allocate your time before committing to where you will begin
- *Always* know how much time you have left
  - Clock in room
  - Staff assigned to room will periodically update the time left (1 hr, 30 min, etc.)
  - If a question is taking a lot of time/is stumping you, come back to it later
    - Give your mind a break
    - Be sure to write down/keep track of skipped questions
- If you have time, check your answers
- The night before/day of:
  - Get enough sleep
  - Fuel up
  - Do your best to keep your nerves in check

General Studying Strategies

- **Take the practice exams**
  - Closest thing you will have to the real deal
  - Allows best to identify what your strengths and weaknesses are
  - After reviewing, take another practice exam and see how you improve
- Review homework/groupwork/discussion
- Review problem roulette
- Come to us!
  - Lots of OH leading up to the exam
  - Will help you with literally anything

Open Q&A :D