Lec 4 Handout: Predicates & Quantifiers

A **predicate** is a statement with _____

Ex: $P(x)$ = "x likes Bubble Tea", $Q(x)$ = "$2x = 7$"

Quantifiers:

∀ means _____ (like a big chain of _____)

∃ means _____ (like a big chain of _____)

	True iff …	**False** iff …
$\forall x\, P(x)$	P(x) is _____ for _____ in the domain	P(x) is _____ for _____ in the domain
$\exists x\, P(x)$	P(x) is _____ for _____ in the domain	P(x) is _____ for _____ in the domain

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Nested Quantifiers – additional exercises

Let **P(x,y) = "4x – y = 0",** domain = integers

Write each proposition using quantifiers, then determine whether it is true or false:

Quantifiers

_____ 1. There exists **x** such that there exists **y** such that **P(x, y).** True / False

_____ 2. There exists **x** such that for all **y**, **P(x, y).** True / False

_____ 3. For all **y**, there exists **x** such that **P(x, y).** True / False

_____ 4. For all **x**, there exists **y** such that **P(x, y).** True / False

_____ 5. There exists **y** such that for all **x**, **P(x, y).** True / False

_____ 6. For all **x**, for all **y**, **P(x, y)**. True / False

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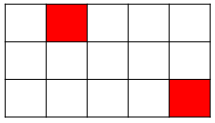
Nested Quantifiers – additional exercises

P(x,y) = "the square at row **x**, column **y** is colored"



(a) P(0, 0) is false

P(0, 1) is true

P(1, 0) is _____

P(2, 4) is _____

- *Note*: we'll start our indexing at 0, so P(0, 0) refers to the square at the top left corner of the grid

(b) What is the **domain of x**? _____

What is the **domain of y**? _____

(c) If $\exists x \exists y\, P(x, y)$ is **true**, the minimum number of shaded squares is _____, and the maximum number is _____.

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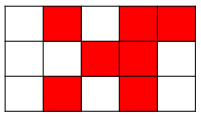
Nested Quantifiers – additional exercises

P(x,y) = "the square at row **x**, column **y** is colored"

Select the logical expression that **matches the English statement**.

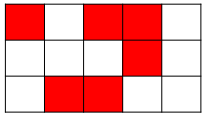
(d) *English*: **The grid has an entire column that is shaded.**

Logic: _____



(e) *English*: **Every row has at least one shaded square.**

Logic: _____



Answer options:

A. $\exists x \forall y\ P(x, y)$

B. $\forall y \exists x\ P(x, y)$

C. $\forall x \exists y\ P(x, y)$

D. $\exists y \forall x\ P(x, y)$

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Translating Logic to English #1

Handout

Domain of:
m: all movies
x, y: people in this room

$V(x, m)$: "person x has seen movie m"

a) $\exists x \, \exists y \, [x \neq y \, \wedge \, \exists m \, (V(x, m) \wedge V(y, m))]$

b) $\exists x \, \exists y \, [x \neq y \, \wedge \forall m \, (V(x, m) \leftrightarrow V(y, m))]$

Symbol	"Main" Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q"
p→q	"if p, then q"
∀x P(x)	"for all x, P(x)"
∃x P(x)	"there exists x such that P(x)"

DeMorgan's Laws for Quantifiers

$\neg \exists x \, P(x) \equiv$ _____

$\neg \forall x \, P(x) \equiv$ _____

Exercise: Simplify each statement (so that negation appears only directly before a predicate):

a) ¬ ∃x ∀y ∃z [¬P(x, y, z) ∨ ¬Q(x, y, z)]

b) ¬ ∃x [P(x) → ¬Q(x)]

How would you translate this?

"There is a person in this class all of whose friends in this class will get As"

C(x) : "x is in this class" F(x,y) : "x and y are friends"

A(x) : "x will get an A

Select all correct translations.

(A) ∃x [C(x) → ∀y (F(x,y) ∧ C(y) ∧ A(y))]

(B) ∃x∀y [(C(x) ∧ F(x,y)) → (C(y) ∧ A(y))]

(C) ∃x [C(x) ∧ ∀y [(F(x,y) ∧ C(y)) → A(y)]]

(D) ∃x [C(x) ∧ ∀y [F(x,y) → (C(y) → A(y))]]

(E) ∃x [C(x) ∧ ∀y [F(x,y) ∧ (C(y) → A(y))]]

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Scoping

Translate the following logical statements into English.

Do the two statements have the same meaning? _____

B(x, y) = "x buys a y"

Logic

∀x [B(x, umbrella) ∨ B(x, raincoat)]

English

[∀x B(x, umbrella)] ∨ [∀x B(x, raincoat)]

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