1. What probability should be assigned to the outcome of heads when a biased coin is tossed, if heads is three times as likely to come up as tails? What probability should be assigned to the outcome of tails?

7. What is the probability of these events when we randomly select a permutation of $\{1, 2, 3, 4\}$?

a) 1 precedes 4.
b) 4 precedes 1.
c) 4 precedes 1 and 4 precedes 2.
d) 4 precedes 1, 4 precedes 2, and 4 precedes 3.
e) 4 precedes 3 and 2 precedes 1.

9. What is the probability of these events when we randomly select a permutation of the 26 lowercase letters of the English alphabet?

a) The permutation consists of the letters in reverse alphabetic order.
b) z is the first letter of the permutation.
c) z precedes a in the permutation.
d) a immediately precedes z in the permutation.
e) a immediately precedes m, which immediately precedes z in the permutation.
f) m, n, and o are in their original places in the permutation.

11. Suppose that E and F are events such that $p(E) = 0.7$ and $p(F) = 0.5$. Show that $p(E \cup F) \geq 0.7$ and $p(E \cap F) \geq 0.2$.

13. Show that if E and F are events, then $p(E \cap F) \geq p(E) + p(F) - 1$. This is known as **Bonferroni's inequality**.

17. If E and F are independent events, prove or disprove that $\overline{E}$ and F are necessarily independent events.

In Exercises 18, 20, and 21 assume that the year has 366 days and all birthdays are equally likely. In Exercise 19 assume it is equally likely that a person is born in any given month of the year.

19. **a)** What is the probability that two people chosen at random were born during the same month of the year?

b) What is the probability that in a group of n people chosen at random, there are at least two born in the same month of the year?

c) How many people chosen at random are needed to make the probability greater than $1/2$ that there are at least two people born in the same month of the year?

23. What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

25. What is the conditional probability that a randomly generated bit string of length four contains at least two consecutive 0s, given that the first bit is a 1? (Assume the probabilities of a 0 and a 1 are the same.)

27. Let E and F be the events that a family of n children has children of both sexes and has at most one boy, respectively. Are E and F independent if

a) $n = 2$? **b)** $n = 4$? **c)** $n = 5$?

29. A group of six people play the game of "odd person out" to determine who will buy refreshments. Each person flips a fair coin. If there is a person whose outcome is not the same as that of any other member of the group, this person has to buy the refreshments. What is the probability that there is an odd person out after the coins are flipped once?

35. Find each of the following probabilities when n independent Bernoulli trials are carried out with probability of success p.

a) the probability of no failures

b) the probability of at least one failure

c) the probability of at most one failure

d) the probability of at least two failures