Exam 3
QUESTIONS PACKET
EECS 203
Practice Exam 1

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

8-Digit UMID: ______________________________

*****MAKE SURE YOU HAVE PROBLEMS 1 - 18 IN THIS BOOKLET.*****

General Instructions

You have 120 minutes to complete this exam. You should have two exam packets.

- **Questions Packet:** Contains ALL of the questions for this exam, worth 100 points total. There are 8 Single-Answer Multiple Choice questions (4 points each), 4 Multiple-Answer Multiple Choice questions (4 points each), and 6 Free Response questions (52 points total). You may do scratch work on this part of the exam, but only work in the Answers Packet will be graded.
- **Answers Packet:** Write all of your answers in the Answers Packet, including your answers to multiple choice questions. **For free response questions, you must show your work! Answers alone will receive little or no credit.**
- You may bring **one** 8.5” by 11” note sheet, front and back, created by you.
- You may **NOT** use any other sources of information, including but not limited to electronic devices (including calculators), textbooks, or notes.
- After you complete the exam, sign the Honor Code Pledge on the front of the Answers Packet.
- You must turn in both parts of this exam.
- **You are not to discuss the exam until the solutions are published.**

Part A1: Single Answer Multiple Choice

Problem 1. (4 points)

An EECS 203 student eats dinner at Bursley Hall. For each meal, they choose:

- one of 3 main dish options (pizza, hamburger, or salad)
- one of 3 drink options (coffee, juice, or water)
- one of 2 dessert options (cookie or ice cream)

Additionally, if they choose a hamburger for their main dish, they will **not** have coffee to drink.

How many different meal combinations are possible?

(a) 7

(b) 8

(c) 16

(d) 17

(e) 18

Solution: (c)
Since the student has to chose a main dish, a drink, and a dessert option, the student has to make all of these decisions, meaning that we use the product rule. If there were no restrictions, the student would have $3 \cdot 3 \cdot 2 = 18$ decisions. However, with the additional restriction, we need to remove the cases hamburger-coffee-cookie and hamburger-coffee-ice cream. Subtracting these 2 invalid cases leaves us with $18 - 2 = 16$.

Problem 2. (4 points)

Suppose you are given a compound proposition of the form

$$x_1 \vee x_2 \vee x_3 \vee \cdots \vee x_{n-1} \vee x_n.$$

The variables $x_1, \ldots, x_n$ are then independently and randomly set to either true or false with equal probability. What is the probability that the compound proposition is true?

(a) $\dfrac{1}{n}$

(b) $\dfrac{n-1}{n}$

(c) $\dfrac{1}{2^n}$

(d) $\dfrac{n}{2^n}$

(e) $\dfrac{2^n-1}{2^n}$

Solution: (e)
We first observe that if x_i is true for some $1 \leq i \leq n$, then the whole compound proposition must all be true as well. The only way for the compound proposition to be false is if every variable is false. There are 2^n assignments for the n variables, since each can either be true or false, and excluding the 1 case where all the variables are false gives us an event space with a cardinality of $2^n - 1$. Therefore, the overall probability is

$$\frac{2^n-1}{2^n}.$$

Problem 3. (4 points)

Consider a list of n answer choices for a multiple choice question, of which k are correct. How many ways could the list be rearranged such that the correct answers are the first k choices in the list?

For example, if the answer choices are {A, B, C, D, E} and A, C, and E are the correct answers, then (E, A, C, D, B) and (A, E, C, B, D) would be two such rearrangements.

(a) $n!$

(b) $n! - k!$

(c) $\binom{n}{k}$

(d) $k!$

(e) $k!(n-k)!$

Solution: (e)
We know that the k correct answers must occupy the first k spots in the list, so we only need to decide their order. This can be done $k!$ ways. Similarly, there are $n-k$ incorrect answers and they must occupy the $n-k$ last positions in the list. They can be ordered in $(n-k)!$ different ways. The order of the correct and incorrect answers can be chosen independently of one another, which gives $k!(n-k)!$ total possible arrangements of the list.

Problem 4. (4 points)

You have 10 cards numbered 1 through 10. After shuffling the cards, what is the probability that the top 3 cards are in ascending order? Note that $C(n,k) = \binom{n}{k}$.

(a) $\frac{1}{6}$

(b) $\frac{1}{3}$

(c) $\frac{P(10,3)}{C(10,3)}$

(d) $\frac{1}{C(10,3)}$

(e) $\frac{3}{10}$

Solution: (a)
We can take any 3 cards, and order them in exactly 1 way out of the $3! = 6$ possible orderings, so 1/6. Note that we can also find the number of ascending trios $\binom{10}{3}$, and divide by the total ways to order 3 objects out of 10 $P(10,3)$.

Problem 5. (4 points)

Professor Diaz rolls an unfair four-sided die and gets a number x, with probabilities

$$p(x=1) = \frac{1}{10}, \quad p(x=2) = \frac{2}{10}, \quad p(x=3) = \frac{3}{10}, \text{ and } p(x=4) = \frac{4}{10}.$$

- If x is odd, Professor Diaz will win x dollars (eg. rolling a 3 yields \$3).
- If x is even, she will win $2x$ dollars (eg. rolling a 4 yields \$8).

What is the expected value of Professor Diaz's winnings?

(a) 1.6

(b) 2.5

(c) 3

(d) 3.5

(e) 5

Solution: (e)
The expected value formula defines the expected value as the sum of the probabilities of each outcome times the value of that outcome.

$$\mathbb{E}[X] = \sum_{s \in S} p(s) \cdot X(s)$$

In this case, the random variable maps from the dice roll to the money earned from that roll. The outcomes are the dice rolls (1, 2, 3, or 4), and the value of these outcomes are defined by the value on the dice when the roll is odd and two times the value on the dice when the roll is even.

$$\begin{aligned}
\mathbb{E}[X] &= \sum_{s \in S} p(s) \cdot X(s) \\
&= p(1) \cdot X(1) + p(2) \cdot X(2) + p(3) \cdot X(3) + p(4) \cdot X(4) \\
&= \frac{1}{10} \cdot X(1) + \frac{2}{10} \cdot X(2) + \frac{3}{10} \cdot X(3) + \frac{4}{10} \cdot X(4) \\
&= \frac{1}{10} \cdot 1 + \frac{2}{10} \cdot 2 \cdot 2 + \frac{3}{10} \cdot 3 + \frac{4}{10} \cdot 4 \cdot 2 \\
&= \frac{1}{10} + \frac{8}{10} + \frac{9}{10} + \frac{32}{10} \\
&= \frac{50}{10} \\
&= 5
\end{aligned}$$

Thus, since $\mathbb{E}[X] = 5$, the correct answer is (e).

Problem 6. (4 points)

Regan flips a fair coin 10 times. What is the probability that at least 9 flips were heads?

(a) $\frac{9}{10}$

(b) $\left(\frac{1}{2}\right)^9$

(c) $11 \cdot \left(\frac{1}{2}\right)^{10}$

(d) $10 \cdot \left(\frac{1}{2}\right)^9$

(e) $\left(\frac{1}{2}\right)^9 + \left(\frac{1}{2}\right)^{10}$

Solution: (c)
We can split this into two cases: Regan flipped exactly 9 heads, or she flipped exactly 10 heads. In the first case, there are $\binom{10}{9}$ places for the heads to occur among the 10 flips. The probability of any one of these arrangements is $\left(\frac{1}{2}\right)^{10}$. In the second case there is only one way to flip exactly 10 heads, and this has a probability of $\left(\frac{1}{2}\right)^{10}$. So the overall probability is $\binom{10}{9} \cdot \left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10} = 10 \cdot \left(\frac{1}{2}\right)^{10} + \left(\frac{1}{2}\right)^{10} = 11 \cdot \left(\frac{1}{2}\right)^{10}$.

Problem 7. (4 points)

What is the Big-Θ bound on the runtime of this algorithm?

```
procedure loopy(n):
    if n ≤ 1: return
    for i := 1 to 4:
        loopy(n/4)
    for i := 1 to n:
        for j := 1 to n:
            print 'I am feeling silly!'
    for i := 1 to 4:
        loopy(n/4)
```

(a) $\Theta(\log_4 n)$

(b) $\Theta(n)$

(c) $\Theta(n^{1.5})$

(d) $\Theta(n^2)$

(e) $\Theta(n^2 \log n)$

Solution: d
This calls itself 8 times (4 at the start and 4 at the end), each time with 1/4 the size. It additionally has 2 nested loops from 1 to n, making $O(n^2)$ additional work.
This gives a recurrence of $T(n) = 8T(n/4) + n^2$. The master theorem makes cases based on how $a/b^d = 8/4^2 = 8/16 = 1/2$ compares to 1. Because this is smaller than 1, most of the work is done right at the top, with n^2 work in the original function call.

Problem 8. (4 points)

Reminders: A standard deck has 52 cards with 4 suits ($\heartsuit, \diamondsuit, \clubsuit, \spadesuit$) and 13 ranks (2, 3, ..., 10, Jack (J), Queen (Q), King (K), Ace (A)).

How many unique five-card hands have exactly 2 Queens and exactly 1 Heart? (The heart **is allowed** to be one of the queens.)

(a) $\binom{4}{2}\binom{13}{1}\binom{36}{2}$

(b) $\binom{4}{2}\binom{13}{1}\binom{36}{2} + \binom{4}{1}\binom{13}{2}\binom{36}{2}$

(c) $\binom{4}{2}\binom{36}{3} + \binom{3}{2}\binom{13}{1}\binom{36}{2}$

(d) $\binom{3}{1}\binom{36}{3} + \binom{3}{2}\binom{12}{1}\binom{36}{2}$

Solution: (d)
If we have the $Q\heartsuit$ as our one heart then we need to choose one other queen and 3 other non-heart and non-queen cards, so $\binom{3}{1}\binom{36}{3}$.

If don't have the $Q\heartsuit$ then we need two queens of non-heart suits, one heart that is not a queen, and 2 other cards, so $\binom{3}{2}\binom{12}{1}\binom{36}{2}$.

We have to add these two distinct cases, so our final answer is $\binom{3}{1}\binom{36}{3} + \binom{3}{2}\binom{12}{1}\binom{36}{2}$.

Part A2: Multiple Answer Multiple Choice

Problem 9. (4 points)

A simple undirected graph G has 5 vertices and 5 edges. Which of the following are possible?

(a) G contains a spanning tree

(b) G contains two or more cycles

(c) G is a complete graph

(d) G is a tree

(e) G is bipartite

Solution: (a),(b),(e)

(a) Possible. If we drew the graph as a pentagon, a spanning tree would be just one of the edges removed. So, it is possible for G to contain a spanning tree.

(b) Possible. If we have an isolated vertex (degree 0), then we can bound the remaining four vertices as a square using four edges, then create a diagonal with the remaining edge to split the square into two cycles.

(c) Not possible. A complete graph with n vertices always has $(n(n-1))/2$ edges, which is not the case here.

(d) Not possible. A tree with n vertices always has $n-1$ edges, which is not the case here.

(e) Possible. G could be a cycle of 4, with one additional vertex connected to a vertex of the cycle, which is bipartite.

Problem 10. (4 points)

Which of the following are scenarios are counted with $\binom{10}{5}$?

(a) Number of binary strings of 10 bits, 5 of which are 1's.

(b) Number of ways to give 1st through 5th awards to 10 racers

(c) Number of ways to choose a five digit PIN number

(d) Number of 5 digit numbers

(e) Number of ways to put 10 distinct balls into two identical boxes

Solution: (a)

(a) we pick which 4 of the 10 bits are 1s. Picking the 4th bit and then the 5th bit is the same as picking the 5th bit and then the 4th bit, so we don't care about the order of these 5 choices.

(b) Here, while we are picking 5 of 10 things, the order of our choices matters, as medals for 1st and 2nd place are different.

(c) This can be counted with 10^5 rather than $\binom{10}{5}$

(d) Similar to the previous, but even more complicated because a 5 digit number can't begin with 0.

(e) This allows us to put the balls in unevenly if we wanted. For example, we might put 7 of them in the first box and the other 3 balls in the second box. If we required that the boxes each have 5 balls in them, it would still be off by a factor of 2 because the boxes are identical.

Problem 11. (4 points)

Let f, g be functions where

- $f(n) = 5n^3 + 2n + \log n + 1$
- $g(n) = n! + n^4$

Which of the following are true?

(a) $f(n)$ is $O(g(n))$

(b) $f(n)$ is $\Theta(g(n))$

(c) $f(n)$ is $\Omega(g(n))$

(d) $\dfrac{g(n)}{f(n)}$ is $O(n!)$

(e) $g(n) \cdot f(n)$ is $O(n!)$

Solution: a, d
a) is true since we know that $n!$ will grow at a faster rate than polynomial terms (e.g., n^3), which $f(n)$ is dominated by
b) is not true since $g(n)$ does not provide a lower bound for $f(n)$ and does not represent the exact performance value of $f(n)$
c) is not true for the same reason as b)
d) is true since when we have $\frac{g(n)}{f(n)}$, the fastest growing term will be $\frac{n!}{n^3}$, which is $O(n!)$
e) is not true since the fastest growing term in $g(n) \cdot f(n)$ will be $n! \cdot 5n^3$, which grows faster than $n!$

Problem 12. (4 points)

Suppose that there are independent events E and F in a sample space S. Both events have non-zero probability. Which of the following **must** be true?

(a) $p(E \cap F) = 0$

(b) $p(E|\overline{F}) = p(E)$

(c) $p(\overline{F}) = 1 - p(E \cap F)$

(d) $p(E|F) = p(F|E)$

(e) $p(E \cap F) = p(E|F) \cdot p(F|E)$

Solution: b, e

(a) Not necessarily true. Two events with non-zero probability and being independent in the same sample space does not imply they cannot both happen. In fact, we can be certain that $P(E \cap F) \neq 0$. Because E and F are independent, we know $P(E \cap F) = P(E) \cdot P(F)$, with both terms being positive.

(b) Necessarily true. Since E and F are independent, whether F happens or not does not affect E

(c) Not necessarily true. The only scenario where this would be true is if $p(E \cap F) = p(F)$, but we cannot make that assumption

(d) Not necessarily true. If we know that E and F are independent, that means that $P(E|F) = P(E)$ and also that $P(F|E) = P(F)$. This means that statement (d) is equivalent to $P(E) = P(F)$, which we cannot guarantee.

(e) Necessarily true. We know that $p(E \cap F) = p(E|F) \cdot p(F)$. Since E and F are independent, $p(F) = p(F|E)$ so $p(E \cap F) = p(E|F) \cdot p(F|E)$

Problem 13. (4 points)

Ashu wants to name his child using only the letters from his name. More specifically, he wants to create a name with 3 A's, 2 S's, 2 H's and 3 U's. Which of the following represent the number of names that consist of exactly 3 A's, 2 S's, and 2 H's and 3 U's? Select all that apply.

(a) 4^{10}

(b) $\binom{10}{3} \cdot \binom{10}{2} \cdot \binom{10}{2} \cdot \binom{10}{3}$

(c) $\frac{10!}{3! \cdot 2! \cdot 2! \cdot 3!}$

(d) $\binom{10}{3} \cdot \binom{7}{2} \cdot \binom{5}{2} \cdot \binom{3}{3}$

(e) $3! \cdot 2! \cdot 2! \cdot 3!$

Solution: (c), (d)
(a) This answer does not account for the fixed number of each letter
(b) This answer does not account for the fact that you cannot pick any spot twice
(c) Using multinomial theorem this is the number of permutations of $N = 10$ total letters, $n_1 = 3, n_2 = 2, n_3 = 2$ and $n_4 = 3$.
(d) First pick the spots for the A's $\binom{10}{3}$, then S's $\binom{7}{2}$, then H's $\binom{5}{2}$and finally U's $\binom{3}{3}$
(e) This is the number of ways 10! counts any single string, the factor by which it overcounts. That is not the number of different strings there are.

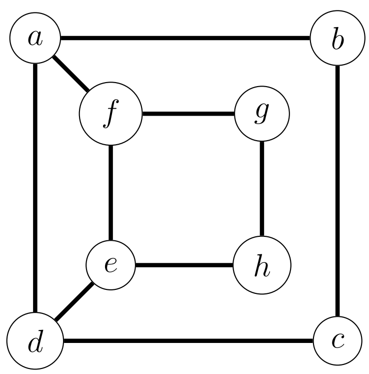
Part B: Short Answer

Problem 14. (6 points)

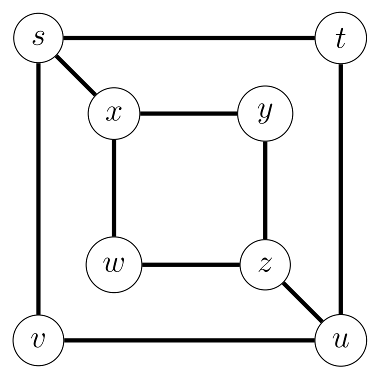
Are these graphs G and H isomorphic?

- If so, describe an isomorphism.
- If not, describe an invariant that one graph has that the other does not.

You do **not** have to prove that a named isomorphism is correct, or that G and H do/don't have the invariant that you name.



G



H

Solution: Not isomorphic. There are many options for invariants (but note that the degree sequences are identical, so pure degree-counting won't work). For example:

- G has three C_4 subgraphs, while H has two.
- G has two C_6 subgraph, while H has four.
- G has a Hamiltonian cycle (C_8 subgraph), while H has none.
- G contains a pair of adjacent vertices with degree 2, while H does not. Or equivalently, all degree 2 nodes in H are only adjacent to degree 3 nodes.
- G contains a C_4 subgraph where all vertices have degree 3, while H doesn't
- G contains a degree 3 node adjacent to two vertices of degree 3, while H doesn't.
- G contains a vertex of degree 2 that is adjacent to one vertex of degree 3 and one vertex of degree 2. H does not have any vertex with such description.

Grading Guidelines:

- +2: Claims "not isomorphic"
- +4: Valid broken invariant shown as a counterexample

Common mistakes:

- Using inner vs outer loop. This is not an invariant. we could have switched the inner and outer loop in G and it will be isomorphic to G.
- Miscounting the number of subgraphs. e.g. claiming that there are 2 C_4 subgraphs in G and 1 C_4 subgraph in H.
- Specifying a mapping of G to H, even if partial mapping. e.g. Let node a map to s in H. We want to show that no possible mapping is possible. This is a forall statement, so it cannot be proven with providing an example of a mapping and proving that that specific mapping is not possible.
- Stating that H does not have two degree 3 nodes adjacent to each other. This is false as s and x have degree 3, and are adjacent.

Problem 15. (7 points)

Is there a directed graph $G = (\{r, s, t, u, v\}, E)$ with the following properties? If so, draw the graph. If not, explain why no such graph can exist.

Note: for a node x in a directed graph, an

- "outgoing edge" is an edge that has arrow pointing away from x (x as its first node)
- "incoming edge" is an edge that has arrow pointing toward x (x as its second node)

(a) Properties of G :

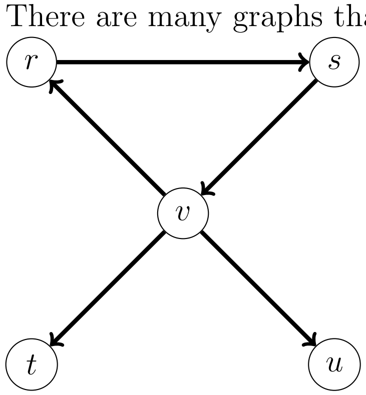
- r, s have 1 incoming edge and 1 outgoing edge.
- t, u have 1 incoming edge and 0 outgoing edges.
- v has 1 incoming edge and 3 outgoing edges.

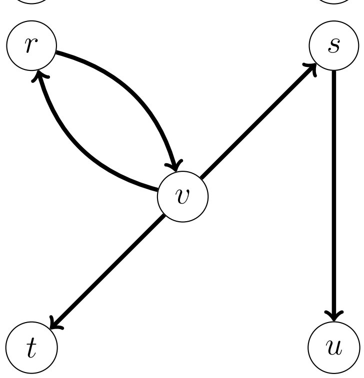
(b) Properties of G :

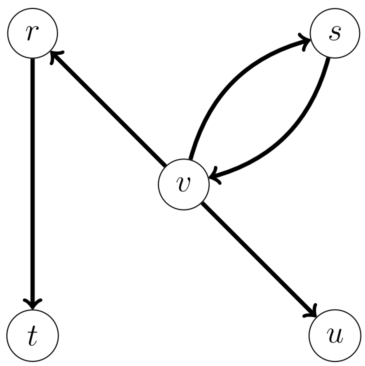
- r, s, t have 1 incoming edge and 1 outgoing edge.
- u, v have 1 incoming edge and 2 outgoing edges.

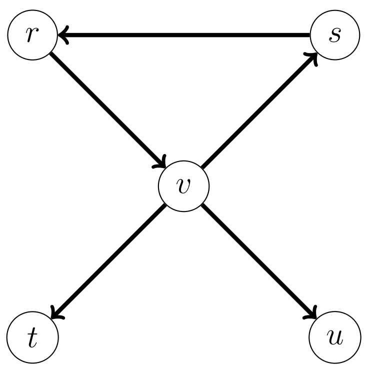
Solution:

a) There are many graphs that work. Here are provided four common ones.









b) Impossible. Let x be the sum of incoming edges over all nodes, and let y be the sum of outgoing edges over all nodes. We count $x = 5$ and $y = 7$ for the degrees described here. But, every directed edge must 1 to x and 1 to y, which means we must have $x = y$ in any graph – this forms a contradiction because $5 \neq 7$.

NOTE: A less formal explanation than this would still get full credit, as long as it points at the right idea.
NOTE: Other similar answer types (e.g. there are 7 outgoing edges, but there are only 5 vertices where each vertex only accepts 1 incoming edge) were treated as equivalent to this.

Alternate Solution: Many students attempted some sort of case reasoning. The following is an example of how that could be done successfully. It doesn't come out nice looking, but if you successfully rule out all possible cases, you could earn full credit:

We first consider the case that vertex r's outgoing edge connects to either s or t – without loss of generality, let r connect to s. This means that the 4 total outgoing edges from u and v may only point to vertices r, t, u, and v, because s has its 1 incoming edge fulfilled. Since all remaining vertices only need 1 incoming edge (4 total), u and v must connect to all 4 vertices from that set. Now, every vertex has its incoming-edge requirement fulfilled, but s and t each have one remaining outgoing edge. Every possible connection from s or t violates the incoming-edge requirement of the connected node, and thus it is impossible to construct a graph *in this case.*

Next, we must consider the case that vertex r's outgoing edge connects to either u or v – without loss of generality, let r connect to u. With similar reasoning as before, the 4 total outgoing edges from u and v may only point to vertices r, s, t, and v, because u has its 1 incoming edge fulfilled. Since all remaining vertices only need 1 incoming edge (4 total), u and v connect to all 4 remaining vertices, resulting in every vertex fulfilling its incoming-edge requirement. Once again, s and t each have one remaining outgoing edge, and every possible connection from these vertices violates the incoming-edge requirement of the connected node. Therefore, it is also impossible to construct a graph in this case.

The case of r connecting to either s or t and the case of r connecting to either u or v cover all possible outcomes. Because it is impossible to construct a valid graph in both cases, it is impossible to construct a valid graph with the given specifications.

NOTE: Many students used similar reasoning, but had some assumption that barred their explanation from being general enough to account for all possible graphs (e.g. if r, s, t all connect to each other as a cycle, then it's impossible to satisfy u, v's conditions).

Grading Guidelines:

(a) [+3.5] Correct
The remaining Part A rubric items are for partially correct solutions. Note that if a solution has "No" circled, it gets a score of 0.0/3.5, regardless of whether a graph was drawn.
[+1.5] Correctly selected "Yes"
[+0.4] Had vertex r correct
[+0.4] Had vertex s correct
[+0.4] Had vertex t correct
[+0.4] Had vertex u correct
[+0.4] Had vertex v correct

(b) [+3.5] Correct
[+2.5] Correctly selected "No" with partially correct justification
[+1.5] Correctly selected "No" without correct justification
Partial credit was awarded to solutions that used the correct approach but miscomputed something and to solutions that tried to note that there were too many outgoing edges without having a completely correct justification.

Common Mistakes:

Part A:

- Incorrectly using the Handshake Theorem for undirected graphs
- Incorrectly stating that no such graph can exist because of a mismatch between the amounts of incoming and outgoing edges

Part B:

- Attempting to use an explanation of why a particular attempt at drawing the graph fails as a general justification for why drawing the graph is impossible (case-by-case proofs are very difficult to execute correctly due to normally not being general enough, but are possible)
- Misinterpreting the statement of edges (e.g. saying there are 2 incoming edges and 3 outgoing edges)
- Misinterpreting incoming/outgoing edges (e.g. saying there are 12 edges total)
- Asserting there are 5 or 7 edges in the graph – there is no concrete number of edges, which is part of why this graph is impossible
- Citing Handshake theorem without explaining why it is useful
- Incorrectly using the Handshake Theorem for undirected graphs

Part C: Free Response

Problem 16. (8 points)

Suzanne just started working for a new company. At this company, each employee receives one free gift per month. Each month there is a 40% chance of receiving a pen, a 35% chance of receiving socks, and a 25% chance of receiving a mug.

(a) Over the course of a year (12 months), what is the probabiltiy that Suzanne receives exactly 2 mugs?

(b) What is the expected number of months that Suzanne must work for the company to receive a mug?

(c) What is the expected number of mugs Suzanne recieves over the course of a 3 years?

Solution:

(a) If Suzanne receives **exactly** 2 mugs, that means that she did NOT receive a mug for the other 10 months which we must take into account. The probability of Suzanne receiving a mug is 0.25 and the probability of her NOT receiving a mug is 0.75. Thus, just like (a), we find the probabilty of one of these permutations and multiply by the number of permutations (12 months choose 2 to be mugs or 10 to not be mugs). Both solutions are correct as they are equivalent to each other.

$$\binom{12}{2} \cdot (0.25)^2 \cdot (0.75)^{10}$$

$$= \binom{12}{10} \cdot (0.25)^2 \cdot (0.75)^{10}$$

(b) This is a geometric distribution with success probability $p = 0.25$. We can find the expected value as follows: $E(C) = 1/p = 1/0.25 = 4$

(c) This is a binomial distribution with success probability $p = 0.25$ and $n = 36$ trials. We can find the expected value as follows: $E(D) = np = 36 \cdot 0.25 = 9$

Grading Guidelines [8 Points]:

Part a:

- +0.4: Multiplies the probability of Suzanne receiving a mug by itself: $(0.25)^2$
- +0.6: Multiplies the probability of Suzanne NOT receiving a mug by itself 10 times: $(0.75)^{10}$

- +0.5: Multiplies those two things together: $(0.25)^2 \cdot (0.75)^{10}$
- +0.5: Multiplies by number of arrangements: $\binom{12}{2}$

Part b:

- +1: Correct justification
- +1: 1/(25%)=4 months

Part c:

- +1.0 Correct justification, by identifying the binomial distribution and using $E(X) = n * p$
- +1.0 Correct calculation, derived from correct information: $36 * (25\%) = 9$ mugs

Common Mistakes:
Part a:

- Some solutions forgot to multiply by the binomial coefficient. The probability, $(0.25)^2 \cdot (0.75)^{10}$, is only the probability of one arrangement (for instance, mug, mug, not mug, not mug,...). The probability of Suzanne does not imply an order and thus you must account for all the different orders that could've led to this event.

Part b:

- Quite a few students approached this problem as though it were asking How many months does it take such that the expected number of mugs received is 1. This would be a binomial distribution, and they set up the equation $np = 1$ and solved for n getting $n = 1/p$, which you may notice gives the same result. We did not give credit to students who solved the problem like this, as that does not demonstrate an understanding of what the problem is asking. The problem wants an average across the number of months needed to wait, not an average across the number of mugs received.

Part c:

- Using the result from part (c) to solve part (d). (c) asked for expected number of months through a geometric distribution, and (d) asked for expected number of mugs through a binomial distribution. While the numbers actually ended up working out, this is NOT the correct approach for this problem, and received zero credit since it showed no understanding of the binomial distribution. Of note, if you solved (c) incorrectly, treating it as a binomial distribution as described in the above common mistake, and then used that value when solving (d), you should have gotten credit for (d), as given that mistake for (c), you solved (d) correctly.

- Realizing that 3 mugs are expected per year, and then using this to find 9 mugs for 3 years. This conclusion could be reached in any number of correct ways, and therefore received credit for the calculation. However, points for justification were not given, since the binomial distribution formula was not used or addressed in any way. Also, oftentimes no explanation was offered, just '3 mugs per year' as a statement of fact.
- Needed to identify the distribution and/or the formula in order to get credit for justification. Just writing the number is only half the battle.

Problem 17. (8 points)

Preeti has to teach discussion in a tiny classroom next semester. Her classroom has 8 different rows with 2 chairs in each row. She has 7 Right Handed People (RHP) and 3 Left Handed People (LHP) in her class. They must sit such that:

- Every RHP sits in one of the 8 chairs on the right
- Every LHP sits in one of the 8 chairs on the left
- Every row has at least one student.

How many ways can the students be seated?

Solution:

$$P(8,7) \cdot 3 \cdot P(7,2)$$

First, we seat the RHP. We pick 7 of the 8 seats for them. People are always distinguishable, so picking the same 7 seats in a different order would put different people in the seats, so we want $P(8,7)$, not a combination.

Next, in order to have every row have at least one student, the 8th row needs to have an LHP in it. We select which of the 3 to put there (3 options).

Finally, we need to seat the remaining 2 LHP in the remaining 7 left-handed seats. Similarly to the first step, there are $P(7,2)$ options.

If you are curious, the number comes out to 5080320

As is the case with many counting problems, there are many different and equally valid ways to approach the problem and arrive at the correct answer. We include a few common alternate solutions here.

Alternate Solution 1 - Seating LHP First

$$P(8,3) \cdot P(7,5) \cdot P(3,2)$$

This is very similar to the above solution except it seats the LHP before the RHP. There are $P(8,3)$ to choose 3 of the 8 rows for the LHP and then permute them. There are 5 rows left open that need to have an RHP sit in them, so we select 5 RHP from among the pool of 7 to sit there (in seat order, so we want a permutation), $P(7,5)$. Finally, we pick which 2 of the 3 rows that already have LHP to put the remaining 2 RHP, $P(3,2)$.

Alternate Solution 2 - Subtraction Rule

$$P(8,7)P(8,3) - C(8,1)P(8,7)P(8,3)$$

First count the number of ways to seat the students if we were allowed to have empty rows. There are $P(8,7)$ ways to choose 7 of the 8 rows for the RHP and then permute them. There are $P(8,3)$ to choose 3 of the 8 rows for the LHP and then permute them. So $P(8,7)P(8,3)$ ways overall.
Now we find the number of ways to seat the students that result in empty rows so that we can subtract it from our total. Since there are 7 RHP and 8 chairs on the right, it is impossible to have more than one empty row. There are $C(8,1)$ ways to choose which of the 8 rows will be empty. There are $P(7,7)$ ways to choose 7 of the 7 non-empty rows for the RHP and then permute them. There are $P(7,3)$ to choose 3 of the 7 non-empty rows for the LHP and then permute them. So $C(8,1)P(7,7)P(7,3)$ ways overall.
By subtraction, we get $P(8,7)P(8,3) - C(8,1)P(7,7)P(7,3)$ ways to seat the students.

Alternate Solution 3 - Choosing Which Rows are Full First

$$\binom{8}{2}\binom{3}{2}\binom{7}{2} \cdot 2! \cdot 2! \cdot 6!$$

Choose the 2 rows which will be full. The choose the 2 LHP from total 3 and choose the 2 RHP from the total 7. There are 2! ways to seat the 2 RHP and 2! ways to seat the 2 LHP. There are 6 rows left and 6 students, so 6! ways to seat them.
The 6! term can alternatively be reached by choosing where the remaining LHP goes, and then permuting the remaining RHP in the remaining seats. Since we already placed students in the full rows, there are 6 rows empty left, so $C(6,1)$ ways to choose the seat for the remaining LHP. There are now 5 empty rows left and 5 RHP remaining to place, so $P(5,5)$ ways. Note that $C(6,1)P(5,5) = 6!$

Grading Guidelines: Note that there are many many slightly different ways you can write these. For a very simple example, $P(8,7)$ would earn you both of the first two rubric items.

Answers without subtraction for overcounting:

- +1.5 for factor of $C(8,7)$, which picks rows RHP sit in (note that this equals 8)
- +1 for factor of 7! which assigns RHP to seats
- +1.5 for factor of 3, to choose which LHP sits in empty row
- +1.5 for factor of $C(7,2)$ to choose seats of remaining LHP
- +1 for factor of 2! to assign remaining LHP to seats
- +1.5 for sufficient justification
- -2 if a factor is added rather than multiplied

Answers with subtraction for overcounting (answers that count the total without considering the constraint were also graded on this rubric) :

- +1.5 for factor of $P(8,7)$, which picks seats for RHP
- +1.5 for factor of $P(8,3)$ which picks seats for LHP
- +0.5 for recognizing that this is overcounting and subtracting something
- +1 for having subtraction factor of $C(8,1)$ which picks the empty seats.
- +1 for subtraction factor of $P(7,7)$ to assign RHP to remaining seats
- +1 for subtraction factor of $P(7,3)$ to assign LHP to seats
- +1.5 for sufficient justification
- -2 if a factor is added rather than multiplied

Common Mistakes

- Taking students to be indistinguishable (Note that people are in general distinguishable unless otherwise stated) $\binom{8}{7}\binom{7}{2}$. With sufficient explanation, this could earn 4.5/8 points.
- Not taking into account the empty row. This answer correctly counts the total number of ways for students to sit, but does not subtract out the ways where 1 row is empty.

$$P(8,7) \cdot P(8,3)$$

- Taking students to be indistinguishable and not taking into account the empty row:

$$\binom{8}{7}\binom{8}{3}$$

- Forgetting to choose LHP to sit in empty row (drops factor of 3 in correct answer).

$$P(8,7) \cdot P(7,3)$$

Problem 18. (9 points)

Brian has a knapsack with 2 red, 3 yellow, and 4 green balls. Brian draws a random ball from the knapsack 50 times in a row, with replacement.

Consider the random variable:

$$D_i = \begin{cases} 1 & \text{if Brian draws a red ball in round } i \text{ and a yellow ball in round } i+1 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Compute $E[D_1]$.

(b) Are $D_1 = 1$ and $D_2 = 1$ independent events? Justify your answer.

(c) Let X be the total number of times Brian draws a red ball followed immediately by a yellow ball over all 50 draws. Find $E[X]$.

Solution:

(a) $\frac{2}{27}$

$$\begin{aligned} \mathbb{E}[D_1] &= 1 \cdot P[D_1 = 1] + 0 \cdot P[D_1 = 0] \\ &= P[D_1 = 1] \\ &= P[\text{Brian draws a red ball in round 1 and a yellow ball in round 2}] \\ &= P[\text{Red in round 1}] \cdot P[\text{Yellow in round 2} \mid \text{Red in round 1}] \\ &= \frac{2}{2+3+4} \cdot \frac{3}{2+3+4} = \frac{2 \cdot 3}{9 \cdot 9} = \frac{2}{27} \end{aligned}$$

Note that each draw itself is independent of each other draw, so we use that $P[\text{Yellow in round 2} \mid \text{Red in round 1}] = P[\text{Yellow in round 2}] = 3/9$

Alternatively, make each ball distinguishable. Then there are $2 \cdot 3 = 6$ ways to draw a red and yellow ball in order, by the product rule. There are also $9 \cdot 9$ total outcomes. So the probability of success is $P[D_1 = 1] = 6/81 = 2/27$.

(b) They are not independent.

- Main Solution: We have $P[D_1 = 1] = \frac{2}{27}$ from part (a). By the same reasoning, $P[D_2 = 1] = \frac{2}{27}$. However, $P[(D_1 = 1) \cap (D_2 = 1)] = 0$, since if $D_1 = 1$ then the second draw is yellow, and if $D_2 = 1$ then the second draw is red, which cannot happen at the same time. Thus we have

$$P[D_1 = 1] \cdot P[D_2 = 1] = \left(\frac{2}{27}\right)^2 \neq 0 = P[(D_1 = 1) \cap (D_2 = 1)]$$

- Alternate solution: We have $P[D_1 = 1] = \frac{2}{27}$ from part (a). However, we have $P[D_1 = 1 \mid D_2 = 1] = 0$, since if $D_2 = 1$ then the second draw is red, which implies that $D_1 = 0$. Since $P[D_1 = 1] \neq P[D_1 = 1 \mid D_2 = 1]$, these events are not independent.

(c) $49 \cdot \dfrac{2}{27} = \dfrac{98}{27}$

Observe that

$$X = \sum_{i=1}^{49} D_i$$

(note that the sum is over 49 variables, not 50. This is because D_i talks about draws i and $i+1$, so D_{49} talks about draw 49 and 50, the last draw). By linearity of expectation, we have

$$\mathbb{E}[X] = \sum_{i=1}^{49} \mathbb{E}[D_i].$$

From part a, we have $\mathbb{E}[D_1] = 2/27$, and by a similar argument (or symmetry of the situation), we have $\mathbb{E}[D_i] = 2/27$. We therefore have

$$\mathbb{E}[X] = 49 \cdot 2/27 = 98/27.$$

Grading Guidelines:

(a) 3 points total
[+0.5] Correct $P[\text{Red 1st}] = \frac{2}{9}$
[+0.5] Correct $P[\text{Yellow 2nd}] = \frac{3}{9} = \frac{1}{3}$
[+2] Correct $\mathbb{E}[D_1] = P[\text{Red 1st}] \times P[\text{Yellow 2nd}]$

(b) 3 points total
[+1] Circle "not independent"
[+2] Sufficient explanation that the second draw cannot be both red and yellow

(c) 3 points total
[+1] Correct $X = D_1 + D_2 + ... + D_{49}$
[+1] Correct use of linearity of expectations, so $\mathbb{E}[X] = \mathbb{E}[D_1] + \mathbb{E}[D_2] + ... + \mathbb{E}[D_{49}]$
[+1] Correct $\mathbb{E}[D_i] = \mathbb{E}[D_1] \quad \forall i \in \mathbb{N} \cap [1, 49]$

Common Mistakes:
Part (a)

1. [-2] Several students wrote that $\mathbb{E}[D_1] = P[\text{Red in round 1}] + P[\text{Yellow in round 2}]$. Recall that the product rule should be applied here, not the sum rule, because the choices occur in sequence.

2. [−0] Many students multiplied fractions incorrectly. $\frac{2}{9} \times \frac{3}{9} = \frac{6}{81}$, not $\frac{1}{9}$, $\frac{6}{9}$, $\frac{6}{18}$, $\frac{5}{18}$, 6, etc.

Part (b)

3. [−3] Incorrect solutions almost uniformly claimed that the events were independent because draws occur with replacement. Note that this makes the outcome of each *draw* independent of the others, but the events defined in the problem each span multiple draws.

4. [−0] Many students incorrectly referred to the random variables as events or outcomes, for example $P[D_1]$, "D_1 is true," "D_1 is yellow," etc. The correct formulation would be $P[D_1 = 1]$ or "$D_1 = 1$."

Part (c)

5. [−1] Several students wrote that $X = \sum_{i=1}^{50} D_i$ and $\mathbb{E}[D_i] = \mathbb{E}[D_1] \quad \forall i \in \mathbb{N} \cap [1, 50]$. Since the i-th indicator references the $(i+1)$-th draw, D_{50} should either be treated as undefined and excluded from the sum, or equal to zero with probability 1 (meaning its expectation is zero), so that it doesn't contribute to the final result.

6. [−2] Many solutions were influenced by the conclusion in part (b) which states that the indicators are not independent. These responses typically summed over alternating terms in the sequence, for example $\mathbb{E}[X] = \mathbb{E}[D_1 + D_3 + ... + D_{47} + D_{49}]$, to conclude that $\mathbb{E}[X] = 25 \cdot \mathbb{E}[D_1]$. Recall that linearity of expectations works *regardless* of whether the terms are independent. By skipping terms such as D_2, an arrangement that has RY happen in draws 2 and 3 (e.g. RRYYYYYYYYY...Y) would not have the RY counted.

7. [−0.5] Many students lost points for not fully showing their work. These solutions correctly arrived at $49 \cdot \mathbb{E}[D_1]$ but skipped explaining the relationship between X and the indicators D_i. Specifically, students needed to explicitly state that $X = \sum_{i=1}^{49} D_i$, cite linearity of expectations, and claim equality of all values $\mathbb{E}[D_i]$ as part of their justification.

8. [−0] Many students expressed the *expectation* of X as a random value, writing things like $\mathbb{E}[X] = \sum_{i=1}^{49} D_i = 49D_1$. A marginal expectation is an actual number, and should not contain random variables on the right-hand side! The correct formulation has expectations on *both* sides of the equality.

Problem 19. (9 points)

Xinhao wants to assign 9 IAs to ride 3 identical Blue Buses. Each Blue Bus should have at least 2 IAs assigned. How many different ways can he make this assignment?

Solution:
Given that each bus has to have at least 2 IA's assigned to it, there are 3 possible distributions for how many IAs are on each bus. The 3 cases are:

1. (2, 2, 5) case: From 9 total IAs, select which 2 will be in the first bus, $\binom{9}{2}$ ways, then which 2 will be in the second bus, $\binom{7}{2}$ ways, and the remaining 5 will be in the final bus: $\binom{5}{5} = 1$ way. Using the product rule to combine this sequence of choices, our initial count for this case is $\binom{9}{2}\binom{7}{2}\binom{5}{5}$.

   Next, we need to adjust for overcounting. The two buses with 2 IAs on them are interchangeable, so our initial count represents an overcount (for example, selecting IAs {A,B} for the first bus and {C,D}for the second bus yields the same outcome as selecting {C,D} for the first bus and {A,B}for the second bus). To adjust for this overcounting, we need to divide by the number of ways to rearrange the first two buses, which is $2! = 2$.

   Final count for the (2,2,5) case is: $\dfrac{\binom{9}{2}\binom{7}{2}\binom{5}{5}}{2!}$

2. (2, 3, 4): From 9 total IAs, select which 2 will be in the first bus, $\binom{9}{2}$ ways, then which 3 will be in the second bus, $\binom{7}{3}$ ways, and the remaining 4 will be in the final bus: $\binom{4}{4} = 1$ way. Using the product rule to combine this sequence of choices, our initial count for this case is $\binom{9}{2}\binom{7}{3}\binom{4}{4}$.

   No overcounting exists here, as every bus has a different number of IAs.

   Final count for the (2,3,4) case is: $\binom{9}{2}\binom{7}{3}\binom{4}{4}$

3. (3, 3, 3): From 9 total IAs, select which 3 will be in the first bus, $\binom{9}{3}$ ways, then which 3 will be in the second bus, $\binom{6}{3}$ ways, and the remaining 3 will be in the final bus: $\binom{3}{3} = 1$ way. Using the product rule to combine this sequence of choices, our initial count for this case is $\binom{9}{3}\binom{6}{3}\binom{3}{3}$.

   Next, we need to adjust for overcounting. Since all three buses have the same number of IAs, our initial count represents an overcount (for example, selecting IAs {A,B,C} for Bus 1, {D,E,F}for the Bus 2, and {G,H,I} for Bus 3 yields the same outcome as selecting {C,D,E} for Bus 1, and {G,H,I}for Bus 2, and {A,B,C} for Bus 3). To adjust for this overcounting, we need to divide by the number of ways to rearrange the three buses, which is 3!.

Final count for the (3,3,3) case is: $\dfrac{\binom{9}{3}\binom{6}{3}\binom{3}{3}}{3!}$

Using the sum rule to combine the counts from the three cases, we get a total of:

$$\frac{\binom{9}{2}\binom{7}{2}\binom{5}{5}}{2!} + \binom{9}{2}\binom{7}{3}\binom{4}{4} + \frac{\binom{9}{3}\binom{6}{3}\binom{3}{3}}{3!}$$

Another correct way to express this result is using factorials:

$$\frac{9!}{2!2!5!} \cdot \frac{1}{2!} + \frac{9!}{2!3!4!} + \frac{9!}{3!3!3!} \cdot \frac{1}{3!}$$

Note for checking alternate solutions: the correct numerical answer is 1918.

Common Mistakes

1. The number one most common mistake was to try to assign 2 IAs to each bus first – solutions using the approach were awarded partial credit for good attempts at the solution using this strategy (see grading guidelines for details). This approach involves enough inherent overcounting that it is not a viable route to a correct solution. A "best attempt" at this incorrect approach generally looked as follows.
   - First, assign 2 IAs to each bus: $\binom{9}{2}\binom{7}{2}\binom{5}{2}$ ways to do this.
   - Assign the remaining 3 IAs. Since the buses now have different people on them, they are now distinguishable. Each of the 3 remaining IAs can go to any of the 3 buses, so there are $3 \cdot 3 \cdot 3 = 3^3$ ways to do this.
   - Attempt to adjust for overcounting. This usually took the form of dividing the initial 3 terms (distributing 2 IAs to each bus) by the number of ways to arrange the 3 buses, which is 3!
   - This gives a final answer of $\dfrac{\binom{9}{2}\binom{7}{2}\binom{5}{2} \cdot 3^3}{3!}$.

   The arrangements were counted more times than 3!, but not a consistent number of times. For example, if we put people A and B in the first bus and later put C, D, and E with them, we could also have put C and D in the first bus and later put A, B, and E with them and that would give the same scenario.

2. Not adjusting for overcounting in two of the three cases, i.e. picking two groups of two in 2! ways and picking three groups of three in 3! ways. Not accounting for overcounting at all gives a final answer of $\binom{9}{2}\binom{7}{2}\binom{5}{5} + \binom{9}{2}\binom{7}{3}\binom{4}{4} + \binom{9}{3}\binom{6}{3}\binom{3}{3}$.

3. Adjusting for overcounting in the 2-3-4 case, where it is unnecessary since the buses are actually distinguishable at this point, due to the number of people in them. If we switched the people on the busses, that would not have been something we counted because the number of people don't match up.
4. Adjusting for overcounting by dividing all cases by 3!. Each case overcounts by a different amount, so dividing the final answer by 3! does not accurately correct for that.
5. Adjusting for overcounting by considering the order of IAs within buses rather than the order that buses with the same number of IAs are selected.
6. Overlooking the requirement that each bus has at least 2 IAs, and therefore considering too many cases.
7. Counting the number of cases (e.g. saying the answer is 3) instead of the number of arrangements per case. This treats the IAs as indistinguishable (i.e., any (2,2,5) arrangement is the same as any other (2,2,5) arrangement), but IAs are people too, and thus distinguishable.
8. Mixing up when to use sum/product rules; multiple sequential choices for one case should be multiplied together, counted arrangements for different cases should be added together.
9. Treating IAs as indistinguishable and/or buses as distinguishable.
10. Allowing IAs to be chosen for more than one bus. For example, computing the 2-3-4 case as $\binom{9}{2}\binom{9}{3}\binom{9}{4}$.

Grading Guidelines [9 points]
Primary approach: cases (and only correct approach we found)
+1 for recognizing need to use cases
+1 Correct case breakdown
+1.25 {2,2,5} case: correct initial count (includes overcounting)
+1.25 {2,3,4} case: correct count
+1.25 {3,3,3} case: correct initial count (includes overcounting)
+1 {2,2,5} case: correctly adjust for overcounting
+1 {3,3,3} case: correctly adjust for overcounting
+1 adding the three cases together (sum rule)

Incorrect approach of assigning 2 IAs to each bus first [max of 6 points]
+1 Attempts to assign 2 IAs to each bus first
+1.5 Correctly assigns 2 IAs to each bus

+1 Attempts to assign remaining 3 IAs
+1.5 Correctly assigns remaining 3 IAs
+1 Attempts to adjust for overcounting inherent in this approach (e.g., divides by 3!)

Problem 20. (9 points)

Shubh's keyboard is broken such that when they press a key, the letter appears as normal with probability $\frac{3}{5}$, and otherwise nothing happens. They choose a string from the set $\{$`an, can, tan`$\}$ at random and try to type that string.

- Let A be the event that Shubh tried to type `an`, with $p(A) = 1/2$
- Let C be the event that Shubh tried to type `can`, with $p(C) = 1/4$
- Let T be the event that Shubh tried to type `tan`, with $p(T) = 1/4$
- Let R be the event that the string `an` appears (with no other letter)

(a) Compute $p(R \mid C)$ and $p(R \cap C)$.

(b) If the string `an` appears, what is the probability that Shubh tried to type `can`?

Solution:

(a) $p(R|C)$ is the probability that Shubh tries to type "`can`", but only "`an`" shows up. This means they tried to type `c` which fails, `a` which succeeds, and `n` which succeeds. the probability of this happening is $(1-\frac{3}{5})(\frac{3}{5})(\frac{3}{5}) = \frac{18}{125}$. From the conditional probability formula, we have that $p(R \cap C) = p(R|C)P(C) = \frac{18}{125} \cdot \frac{1}{4} = \frac{9}{250}$

(b) This is asking us to compute $p(C|R)$. In order to do this, we'll need $p(R \cap C), p(R \cap A)$ and $p(R \cap T)$. similar to above, we compute $p(R \cap A)$ as $\frac{3}{5} \cdot \frac{3}{5} \cdot \frac{1}{2} = \frac{9}{50}$ and we notice that $p(R \cap T) = p(R \cap C) = \frac{9}{250}$. Then using Bayes' Theorem, we get

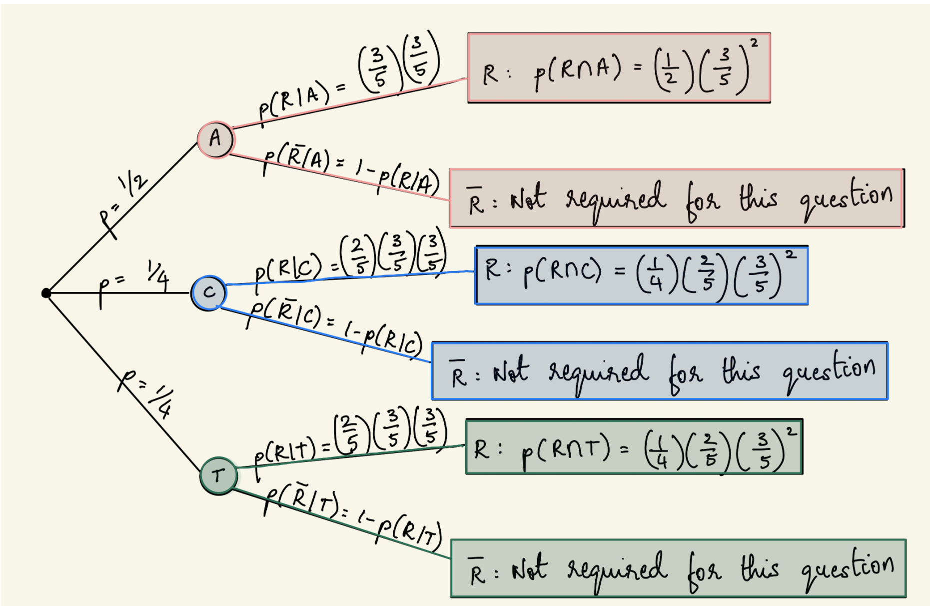
$$P(C|R) = \frac{p(R \cap C)}{p(R \cap A) + p(R \cap C) + p(R \cap T)} = \frac{\frac{9}{250}}{\frac{9}{50} + \frac{9}{250} + \frac{9}{250}} = \frac{9}{45+9+9} = \frac{1}{7}$$

Understanding the problem using the Box method:

	A	C	T
R	$p(A)p(R\|A) = (\frac{1}{2})(\frac{3}{5})(\frac{3}{5})$	$p(C)p(R\|C) = (\frac{1}{4})(\frac{2}{5})(\frac{3}{5})(\frac{3}{5})$	$p(T)p(R\|T) = (\frac{1}{4})(\frac{2}{5})(\frac{3}{5})(\frac{3}{5})$

Our answer is the middle box divided by the sum of the boxes.

Understanding the problem using the Tree method:



Note that summing over all the R nodes gives you, $p(R)$. Our answer is $p(C \cap R)$ over $p(R)$.

Grading Guidelines:

part (a) 3.5 points:
+1: correct terms in expression for $p(R|C)$
+1: correctly uses independence to combine terms for $p(R|C)$
+1.5: correct expression for $p(R \cap C)$ (in terms of $p(R|C)$)

part (b) 5.5 points:
+0.5: attempts to compute $p(C|R)$
+1.5: uses bayes' theorem, including correct numerator
+1.5: correct expression for denominator using law of total proability
+1: correct expression for $p(R \cap A)$
+1: correct expression fro $p(R \cap T)$

Common Mistakes:

- General confusion surrounding what the term $\frac{2}{5} \cdot \frac{3}{5} \cdot \frac{3}{5}$ represents. This is $p(R|C)$ because we need to assume that Shubh tried to type `can` in order for the probabilities to make sense. The most common mistakes were:
  - Claiming that this expression is $p(R \cap C)$. students would often then divide by $p(C)$ to get $p(R|C)$, obtaining an answer of $\frac{72}{125}$
  - Claiming that this expression is $p(C|R)$. students would then often use Bayes' to try to get our desired probabilities
- Expanding $p(R)$ as $p(R|C)p(C) + p(R|\bar{C})p(\bar{C})$. This is a correct statement, but it would frequently lead to errors in expanding $p(R|\bar{C})$. In particular:
  - Students would often claim $p(R|\bar{C}) = p(R|A) + p(R|T)$. Attempting to add together conditional probabilities with different given spaces will almost always give an answer without meaning. We need to multiply by $p(A)$ and $p(T)$ to get probabilities from the same sample space
  - Other students would claim $p(R|\bar{C}) = p(R|A)p(A) + p(R|T)p(T)$. This is closer to the truth, but still isn't quite right. This expression actually gives us $p(R \cap A) + p(R \cap T) = p(R \cap \bar{C})$. We would then need to divide by $p(\bar{C})$ to get a correct expression for $p(R|\bar{C})$
  - Some students claimed $p(R|\bar{C}) = 1 - p(R|C)$. This isn't correct; the true statement is $p(R|\bar{C}) = 1 - p(\bar{R}|\bar{C})$, which is not particularly easy to compute
- attempting to compute $p(R \cap C)$ using $p(R) \cdot p(C)$. This only holds when R and C are independent, which they aren't
- Attempting to compute $p(R \cap C)$ using the inclusion-exclusion principle. That is, using the fact that $p(R \cap C) = p(R) + p(C) + p(R \cup C)$. This is a correct statement, but typically leads you down the wrong path, since computing $p(R)$ is more complex than these students typically assume and $p(R \cup C)$ does not have a very intuitive meaning in terms of the problem.
- Claiming $p(R|A) = 1$. Even though we know that the string `an` appears, it is not guaranteed whenever Shubh types `an`
- Not labeling terms in solutions. We would often see fractions with no explanation of where they came from, and we have a hard time giving partial credit to these solutions. It's better to keep your answer in symbols until the very end.