

3. Use a proof by cases to show that 100 is not the cube of a positive integer. [*Hint*: Consider two cases: (i) $1 \leq x \leq 4$, (ii) $x \geq 5$.]
5. Prove that if x and y are real numbers, then $\max(x, y) + \min(x, y) = x + y$. [*Hint*: Use a proof by cases, with the two cases corresponding to $x \geq y$ and $x < y$, respectively.]
19. Suppose that a and b are odd integers with $a \neq b$. Show there is a unique integer c such that $|a - c| = |b - c|$.
23. Prove that given a real number x there exist unique numbers n and ϵ such that $x = n - \epsilon$, n is an integer, and $0 \leq \epsilon < 1$.
31. Prove that there is no positive integer n such that $n^2 + n^3 = 100$.