3. Let $Q(x, y)$ be the statement "x has sent an e-mail message to y," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

a) $\exists x \exists y Q(x, y)$
b) $\exists x \forall y Q(x, y)$
c) $\forall x \exists y Q(x, y)$
d) $\exists y \forall x Q(x, y)$
e) $\forall y \exists x Q(x, y)$
f) $\forall x \forall y Q(x, y)$

9. Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a) Everybody loves Jerry.
b) Everybody loves somebody.
c) There is somebody whom everybody loves.
d) Nobody loves everybody.
e) There is somebody whom Lydia does not love.
f) There is somebody whom no one loves.
g) There is exactly one person whom everybody loves.
h) There are exactly two people whom Lynn loves.
i) Everyone loves himself or herself.
j) There is someone who loves no one besides himself or herself.

11. Let $S(x)$ be the predicate "x is a student," $F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate "x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.

a) Lois has asked Professor Michaels a question.
b) Every student has asked Professor Gross a question.
c) Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
d) Some student has not asked any faculty member a question.
e) There is a faculty member who has never been asked a question by a student.
f) Some student has asked every faculty member a question.
g) There is a faculty member who has asked every other faculty member a question.
h) Some student has never been asked a question by a faculty member.

25. Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a) $\exists x \forall y(xy = y)$

b) $\forall x \forall y(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$

c) $\exists x \exists y((x^2 > y) \wedge (x < y))$

d) $\forall x \forall y \exists z(x + y = z)$

27. Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a) $\forall n \exists m(n^2 < m)$

b) $\exists n \forall m(n < m^2)$

c) $\forall n \exists m(n + m = 0)$

d) $\exists n \forall m(nm = m)$

e) $\exists n \exists m(n^2 + m^2 = 5)$

f) $\exists n \exists m(n^2 + m^2 = 6)$

g) $\exists n \exists m(n + m = 4 \wedge n - m = 1)$

h) $\exists n \exists m(n + m = 4 \wedge n - m = 2)$

i) $\forall n \forall m \exists p(p = (m + n)/2)$

31. Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a) $\forall x \exists y \forall z T(x, y, z)$

b) $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$

c) $\forall x \exists y(P(x, y) \wedge \exists z R(x, y, z))$

d) $\forall x \exists y(P(x, y) \rightarrow Q(x, y))$

39. Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a) $\forall x \forall y(x^2 = y^2 \rightarrow x = y)$

b) $\forall x \exists y(y^2 = x)$

c) $\forall x \forall y(xy \geq x)$