EECS 203: Discrete Mathematics
Fall 2023
Discussion 1 Notes

1 Definitions

- **Proposition**:

- **Proof**:

- **Negation**:

- **Disproof/Refutation**:

- **Even**:

- **Odd**:

Solution:

- **Proposition**: A statement about the world that has a truth value (either true or false) or false, but not both.

- **Proof**: A proof shows that a proposition is true by a step-by-step explanation. Each step should be easy enough to follow, even if the overall statement is not obvious.

- **Negation**: The negation of a proposition is the statement with the opposite truth value. In English, we often just add or remove the word "not" to get the negation.

- **Disproof/Refutation**: When you prove that a proposition is false, this is called a disproof or a refutation. You have disproved or refuted the statement. This generally involves proving that the negation is true.

- **Even**: An integer x is even if there exists an integer k with $x = 2k$.

- **Odd**: An integer x is odd if there exists an integer k with $x = 2k + 1$

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2   Symbolic Logic Review

- $p \land q$:

- $p \lor q$:

- $p \to q$:

- $\neg p$:

Solution:

- $p \land q$: p and q

- $p \lor q$: p or q

- $p \to q$: if p then q

- $\neg p$: not p (this is a negation of p)

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3   Exercises

1. Meet your Classmates!

Say hello to your classmates. Your peers are an amazing resource, so this exercise is to help you meet some of them.

Tip: One of the questions in Homework 1 is to give some classmates' names and uniqnames, so this is your head start on that!

In a group of 3 or more:

a) Introduce yourself

b) Write down each others' names and uniqnames

c) Find one thing you all have in common

Solution: Answers will vary. :-)

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2. Propositions

Which of these are propositions? What are the truth values of those that are propositions?

a) Do not pass go.

b) What time is it?

c) There are no black flies in Maine.

d) The moon is made of green cheese.

e) $2^n \geq 100$

f) Your discussion leader's favorite number is 2

g) $4 + x = 5$

Solution:

a) This is **not** a proposition; it's a command and does not have a truth value.

b) This is **not** a proposition; it's a question.

c) This **is** a proposition that is false, as anyone who has been to Maine knows.

d) This **is** a proposition that is false.

e) This is **not** a proposition; its truth value depends on the value of n. This is a predicate, which you will hear more of later.

f) This **is** a proposition, even if you don't know whether it is true or false.

g) This is **not** a proposition; its truth value depends on the value of x. This is also a predicate.

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3. Beginning to Think Logically $\star$

Find the negations of each of the following statements. Discuss with a neighbor what that negation means and if there are any other ways to express that statement. (*We will formally define some rules for logic later on.*)

a) I am a student.

b) My sister and my mom like ice cream.

c) I will wear a dress or skirt today.

d) We all have phones.

e) Nobody knows everything.

f) Someone in this class is a math major.

Solution:

a) I am not a student.

b) My sister does not like ice cream or my mom does not like ice cream.

c) I will not wear a dress nor a skirt today.
   Equivalently, I will not wear a dress and I will not wear a skirt today.

d) We don't all have phones.
   In other words, at least one of us does not have a phone.

e) Somebody knows everything

f) No one in this class is a math major.

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4. Truth Tables

Fill in the following truth table.
*Reminder: $\wedge$ denotes "and", $\vee$ denotes "or",   and $\rightarrow$ denotes "implies"/"if…then".

p	q	r	$p \rightarrow q$	$q \rightarrow r$	$(p \rightarrow q) \wedge (q \rightarrow r)$	$p \vee r$	$[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \vee r)$
T	T	T					
T	T	F					
T	F	T					
T	F	F					
F	T	T					
F	T	F					
F	F	T					
F	F	F					

Solution:

p	q	r	$p \to q$	$q \to r$	$(p \to q) \land (q \to r)$	$p \lor r$	$[(p \to q) \land (q \to r)] \to (p \lor r)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	F	T	F	T	T
T	F	F	F	T	F	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	T	F	F

5. Logic Puzzle

Knights always tell the truth and Knaves always lie. Figure out what A and B are in each scenario!

a) A says, "Either I am a Knave, or B is a Knight."

b) A says, "I am a Knave, but B isn't."

Solution:

a) A is a Knight and B is a Knight. A cannot be a Knave, because then their claim would be a truth. Because A must be a Knight and can only tell the truth, B must be a Knight.

b) A is a Knave and B is a Knave. If A were a Knight, A would not be able to claim to be a Knave. In order to make A's statement a lie, B must be a Knave.