

1.  $p(T) = 1/4, p(H) = 3/4$

$p(6) = 1/16; p(2) = p(4) = 3/8$  5.  $9/49$  7. a)  $1/2$   
b)  $1/2$  c)  $1/3$  d)  $1/4$  e)  $1/4$  9. a)  $1/26!$  b)  $1/26$  c)  $1/2$

b)  $1/2$  c)  $1/3$  d)  $1/4$  e)  $1/4$  9. a)  $1/26!$  b)  $1/26$  c)  $1/2$   
d)  $1/26$  e)  $1/650$  f)  $1/15,600$  11. Clearly,  $p(E \cup F) \geq$

d)  $1/26$  e)  $1/650$  f)  $1/15,600$  11. Clearly,  $p(E \cup F) \geq$   
 $p(E) = 0.7$ . Also,  $p(E \cup F) \leq 1$ . If we apply Theorem 2 from  
Section 7.1, we can rewrite this as  $p(E) + p(F) - p(E \cap F) \leq 1$ ,  
or  $0.7 + 0.5 - p(E \cap F) \leq 1$ . Solving for  $p(E \cap F)$  gives  
 $p(E \cap F) \geq 0.2$ . 13. Because  $p(E \cup F) = p(E) + p(F) - p(E \cap F)$

$p(E \cap F) \geq 0.2$ . 13. Because  $p(E \cup F) = p(E) + p(F) - p(E \cap F)$   
and  $p(E \cup F) \leq 1$ , it follows that  $1 \geq p(E) + p(F) - p(E \cap F)$ .  
From this inequality we conclude that  $p(E) + p(F) \leq 1 +$   
 $p(E \cap F)$ . 15. We will use mathematical induction to prove

emathical induction. 17. Because  $E \cup \bar{E}$  is the entire sam-  
ple space  $S$ , the event  $F$  can be split into two disjoint events:  
 $F = S \cap F = (E \cup \bar{E}) \cap F = (E \cap F) \cup (\bar{E} \cap F)$ , using the distributive  
law. Therefore,  $p(F) = p((E \cap F) \cup (\bar{E} \cap F)) = p(E \cap F) + p(\bar{E} \cap F)$ ,  
because these two events are disjoint. Subtracting  $p(E \cap F)$   
from both sides, using the fact that  $p(E \cap F) = p(E) \cdot p(F)$   
(the hypothesis that  $E$  and  $F$  are independent), and factoring,  
we have  $p(F)[1 - p(E)] = p(\bar{E} \cap F)$ . Because  $1 - p(E) = p(\bar{E})$ ,  
this says that  $p(\bar{E} \cap F) = p(\bar{E}) \cdot p(F)$ , as desired. 19. a)  $1/12$

this says that  $p(\bar{E} \cap F) = p(\bar{E}) \cdot p(F)$ , as desired. 19. a)  $1/12$   
b)  $1 - \frac{11}{12} \cdot \frac{10}{12} \cdot \dots \cdot \frac{13-n}{12}$  c) 5 21. 614 23.  $1/4$  25.  $3/8$

23.  $1/4$

25.  $3/8$

27. a) Not independent

29.  $3/16$

35. a)  $p^n$  b)  $1 - p^n$  c)  $p^n + n \cdot p^{n-1} \cdot (1 - p)$   
d)  $1 - [p^n + n \cdot p^{n-1} \cdot (1 - p)]$  37.  $p(\bigcup_{i=1}^{\infty} E_i)$  is the sum of