1. *Basis step:* We are told we can run one mile, so $P(1)$ is true. *Inductive step:* Assume the inductive hypothesis, that we can run any number of miles from 1 to k. We must show that we can run $k + 1$ miles. If $k = 1$, then we are already told that we can run two miles. If $k > 1$, then the inductive hypothesis tells us that we can run $k - 1$ miles, so we can run $(k - 1) + 2 = k + 1$ miles. **3. a)** $P(8)$ is true, because we can

$(k - 1) + 2 = k + 1$ miles. **3. a)** $P(8)$ is true, because we can
form 8 cents of postage with one 3-cent stamp and one 5-cent stamp. $P(9)$ is true, because we can form 9 cents of postage with three 3-cent stamps. $P(10)$ is true, because we can form 10 cents of postage with two 5-cent stamps. **b)** The statement that using just 3-cent and 5-cent stamps we can form j cents postage for all j with $8 \leq j \leq k$, where we assume that $k \geq 10$ **c)** Assuming the inductive hypothesis, we can form $k + 1$ cents postage using just 3-cent and 5-cent stamps **d)** Because $k \geq 10$, we know that $P(k - 2)$ is true, that is, that we can form $k - 2$ cents of postage. Put one more 3-cent stamp on the envelope, and we have formed $k + 1$ cents of postage. **e)** We have completed both the basis step and the inductive step, so by the principle of strong induction, the statement is true for every integer n greater than or equal to 8. **5. a)** 4,

true for every integer n greater than or equal to 8. **5. a)** 4, 8, 11, 12, 15, 16, 19, 20, 22, 23, 24, 26, 27, 28, and all values greater than or equal to 30 **b)** Let $P(n)$ be the statement that we can form n cents of postage using just 4-cent and 11-cent stamps. We want to prove that $P(n)$ is true for all $n \geq 30$. For the basis step, $30 = 11+11+4+4$. Assume that we can form k cents of postage (the inductive hypothesis); we will show how to form $k + 1$ cents of postage. If the k cents included an 11-cent stamp, then replace it by three 4-cent stamps. Otherwise, k cents was formed from just 4-cent stamps. Because $k \geq 30$, there must be at least eight 4-cent stamps involved. Replace eight 4-cent stamps by three 11-cent stamps, and we have formed $k+1$ cents in postage. **c)** $P(n)$ is the same as in part (b). To prove that $P(n)$ is true for all $n \geq 30$, we check for the basis step that $30 = 11+11+4+4$, $31 = 11+4+4+4+4+4$, $32 = 4+4+4+4+4+4+4+4$, and $33 = 11+11+11$. For the inductive step, assume the inductive hypothesis, that $P(j)$ is true for all j with $30 \leq j \leq k$, where k is an arbitrary integer greater than or equal to 33. We want to show that $P(k+1)$ is true. Because $k-3 \geq 30$, we know that $P(k-3)$ is true, that is, that we can form $k-3$ cents of postage. Put one more 4-cent stamp on the envelope, and we have formed $k + 1$ cents of postage. In this proof, our inductive hypothesis was that $P(j)$ was true for all values of j between 30 and k inclusive, rather than just that $P(30)$ was true. **7.** We can form all amounts except \$1 and

$P(30)$ was true. **7.** We can form all amounts except \$1 and
\$3. Let $P(n)$ be the statement that we can form n dollars using just 2-dollar and 5-dollar bills. We want to prove that $P(n)$ is true for all $n \geq 5$. (It is clear that \$1 and \$3 cannot be formed and that \$2 and \$4 can be formed.) For the basis step, note that $5 = 5$ and $6 = 2+2+2$. Assume the inductive hypothesis, that $P(j)$ is true for all j with $5 \leq j \leq k$, where k is an arbitrary integer greater than or equal to 6. We want to show that $P(k+1)$ is true. Because $k - 1 \geq 5$, we know that $P(k - 1)$ is true, that is, that we can form $k - 1$ dollars. Add another 2-dollar bill, and we have formed $k + 1$ dollars. **9.** Let $P(n)$ be the state-

in our game) can win. **13.** Let $P(n)$ be the statement that exactly $n - 1$ moves are required to assemble a puzzle with n pieces. Now $P(1)$ is trivially true. Assume that $P(j)$ is true for all $j \leq k$, and consider a puzzle with $k + 1$ pieces. The final move must be the joining of two blocks, of size j and $k+1-j$ for some integer j with $1 \leq j \leq k$. By the inductive hypothesis, it required $j - 1$ moves to construct the one block, and $k+1-j-1 = k-j$ moves to construct the other. Therefore, $1+(j-1)+(k-j) = k$ moves are required in all, so $P(k+1)$ is true. **15.** Let the Chomp board have n rows and n columns.

stronger statement $\forall n \geq 4\, T(n)$ in Exercise 17. **25. a)** The inductive step here allows us to conclude that $P(3)$, $P(5)$, ... are all true, but we can conclude nothing about $P(2)$, $P(4)$, ... **b)** $P(n)$ is true for all positive integers n, using strong induction. **c)** The inductive step here enables us to conclude that $P(2)$, $P(4)$, $P(8)$, $P(16)$, ... are all true, but we can conclude nothing about $P(n)$ when n is not a power of 2. **d)** This is mathematical induction; we can conclude that $P(n)$ is true for all positive integers n. **27.** Suppose, for a proof by contra-

29. The error is in going from the base case $n = 0$ to the next case, $n = 1$; we cannot write 1 as the sum of two smaller natural numbers. **31.** Assume that the well-ordering prop-