

EECS 203: Discrete Mathematics
Fall 2023
Homework 1

Due **Thursday, Sept. 7**, 10:00 pm

No late homework accepted past midnight.

Number of Problems: $8 + 2$

Total Points: $100 + 24$

- **Match your pages!** Your submission time is when you upload the file, so the time you take to match pages doesn't count against you.
- Submit this assignment (and any regrade requests later) on Gradescope.
- Justify your answers and show your work (unless a question says otherwise).
- By submitting this homework, you agree that you are in compliance with the Engineering Honor Code and the Course Policies for 203, and that you are submitting your own work.
- Check the syllabus for full details.

Individual Portion

1. Collaboration and Support [1.5 points]

- (a) Give the names and usernames of 2 of your EECS 203 classmates (these could be members of your homework group or other classmates).
- (b) When you have questions about the course content, where can you ask them? Where are *you* most likely to ask questions?
- (c) Name one self-care action you plan to do this semester to maintain your overall well-being.

Solution:

- (a) Any 2 classmates' information.
- (b) Options include: discussion, office hours, Piazza, lecture, and potentially the Admin Form (if it's a personal/individual question).
- (c) Some great ideas we've heard: you could block time on your calendar to relax (read, take a walk, chat with friends, Netflix, etc.); set a bedtime and stick to it (at least on weekdays); set aside time every week to connect with the people you are close with (e.g., close friends, parents, siblings, other relatives); get a massage/manicure/pedicure; and lots of other ideas!

Grading Guidelines [1.5 points]

For each part:

+0.5 adequate answer provided

2. Propositions? [12 points]

Which of the following are propositions? For those that are propositions, determine their truth value.

- (a) Do not worry.
- (b) Water cannot boil.
- (c) $e^x = -1$ has a solution, where the domain of x is all **real** numbers.
- (d) Excuse me!

(e) $9 - 4 = 5$.

(f) There is an **integer** k such that $4k + 3 = 10$.

Solution:

(a) No. This is a command.

(b) Yes. False.

(c) Yes. False: for any real number x , e^x will be positive.

(d) No. This is a request.

(e) Yes. True.

(f) Yes. False: the only real solution to this equation is $k = \frac{7}{4}$ which is not an integer.

Draft Grading Guidelines [12 points]

Parts a-f:

+1 correctly states yes/no

Parts b, c, e, and f:

+1.5 correctly states true/false (no justification required)

3. To Tell the Truth [16 points]

Construct a truth table for each of these compound propositions. Include a column for each proposition you evaluate along the way in order to reach the final column (these columns are worth points!).

(a) $(p \vee q) \wedge \neg r$

(b) $((p \rightarrow q) \rightarrow p) \rightarrow r$

Solution:

(a)

p	q	r	$p \vee q$	$\neg r$	$(p \vee q) \wedge \neg r$
F	F	F	F	T	F
F	F	T	F	F	F
F	T	F	T	T	T
F	T	T	T	F	F
T	F	F	T	T	T
T	F	T	T	F	F
T	T	F	T	T	T
T	T	T	T	F	F

(b)

p	q	r	$p \rightarrow q$	$(p \rightarrow q) \rightarrow p$	$((p \rightarrow q) \rightarrow p) \rightarrow r$
T	T	T	T	T	T
T	T	F	T	T	F
T	F	T	F	T	T
T	F	F	F	T	F
F	T	T	T	F	T
F	T	F	T	F	T
F	F	T	T	F	T
F	F	F	T	F	T

Draft Grading Guidelines [16 points]

Part a:

- +2 truth table contains 8 rows of outputs (correct setup)
- +2 $p \vee q$ column is correct
- +2 $\neg r$ column is correct
- +2 $(p \vee q) \wedge \neg r$ column is correct

Part b:

- +2 truth table contains 8 rows of outputs (correct setup)
- +2 $p \rightarrow q$ column is correct
- +2 $(p \rightarrow q) \rightarrow p$ column is correct
- +2 $((p \rightarrow q) \rightarrow p) \rightarrow r$ column is correct

4. Take Flight [12 points]

Let t , l , and d be the propositions:

- t : You get stuck in traffic.

- l : You are late for your flight.
- d : Your flight is delayed.

Write these propositions using t , l , and d and logical connectives (including negations, if necessary).

- You are late for your flight whenever you get stuck in traffic.
- You get stuck in traffic, but your flight is delayed, and you are not late for your flight.
- You will be late for your flight if and only if you get stuck in traffic or your flight is not delayed.

Solution:

- $t \rightarrow l$
- $t \wedge d \wedge \neg l$
- $l \leftrightarrow (t \vee \neg d)$

Draft Grading Guidelines [12 points]

For each part:

- +2 answer contains correct variables with the correct number of occurrences (not necessarily conjoined with the correct connectives)
- +2 correct answer

5. Negate and Celebrate! [16 points]

Write the negation of the following statements in plain English. **None of your answers should use the word “not.”** The domain is all things.

Hint: Instead of saying “not in excellent condition,” you can say “in poor condition.”

- Something is in the incorrect place.
- My computer is both in the correct place and in excellent condition.
- Nothing is both in the correct place and in excellent condition.

Solution:

- (a) Everything is in the correct place. **Alternate:** Nothing is in the incorrect place.
- (b) My computer is in the incorrect place or in poor condition.
- (c) Something is both in the correct place and in excellent condition.

Draft Grading Guidelines [16 points]

For each part:

+4 correct negation of non-quantifier portion of the proposition

+2 *partial credit where relevant: applies De Morgan's Law but forgets to flip logical connective*

Part a and c:

+2 correct quantifier (everything/something)

Note to Graders:

Here is an example where the partial credit item **would** apply:

Student Answer: My computer is both in the incorrect place and in poor condition.

Explanation: The student negated both sides of the conjunction, but did not change the logical connective from \wedge to \vee .

6. Implication Station [12 points]

Rewrite each of these statements in the form “if p , then q ” in English.

Example: “You will enjoy visiting the Willis Tower if you’re not afraid of heights.” becomes “If you’re not afraid of heights, then you will enjoy visiting the Willis Tower.”

- (a) It is necessary to wash the boss’s car to get promoted.
- (b) Willy gets caught whenever he cheats.
- (c) You can access the website only if you pay a subscription fee.
- (d) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.

Solution:

- (a) We can determine that there will be a contradiction if you get promoted but did not wash the boss’s car. This corresponds to the statement “if you get promoted,

then you washed the boss's car."

- (b) There will be a contradiction if Willy cheats, but he does not get caught. This corresponds to the statement "If Willy cheats, then he gets caught."
- (c) There will be a contradiction if you can access the website, but you did not pay a subscription fee. This corresponds to the statement "If you can access the website, then you paid a subscription fee".
- (d) If you bought the computer less than a year ago, then the warranty is good.

Grading Guidelines [12 points]

For each part:

+3 correct re-writing of the sentence (the correct contrapositive should also be accepted)

7. Who's Who? [16 points]

In this problem, there are exactly two types of people: truth-tellers and liars. A truth-teller always tells the truth, and a liar always lies.

- (a) There are two people, A and B, each of whom is either a truth-teller or liar. A makes the following statement: "at least one of us is a liar." What are A and B?
- (b) There are three people, A, B, and C, each of whom, again, is either a truth-teller or liar. A and B make the following statements:
 - A: "All of us are liars."
 - B: "Exactly one of us is a truth-teller."

What are A, B, and C?

Solution:

- (a) We know that A is either a truth-teller or a liar, and likewise for B.

Suppose that A is a truth-teller. Then that means what A said is true, so since at least one of A and B is a liar, B must be the liar. So everything seems to work out.

Now we should check that A *cannot* be a liar. If A is a liar, then what they said is false, meaning that neither A nor B is a liar. But this is impossible if A is a liar! So A must be a truth-teller.

Thus A is a truth-teller and B is a liar.

- (b) First consider the statement that A makes: if it were true, then A would be a liar but telling the truth, which is not possible, so not everyone is a liar. Now knowing that A's statement is false, A must be a liar, since they are not telling the truth.

Now consider B's statement. If B were lying, then there must be 0, 2, or 3 truth-tellers among A, B, and C. We know there cannot be 3 since A is a liar, and there cannot be 0 since A's statement is false, meaning that B and C must both be truth-tellers. But then B made a false statement and is a truth-teller! So B must be telling the truth, meaning B is a truth-teller and C is a liar.

Overall: A is a liar, B is a truth-teller, and C is a liar.

Draft Grading Guidelines [16 points]

Part a:

- +3 considers whether or not A is a truth-teller
- +3 correct identification of A and B

Part b:

- +3 recognizes that A's statement is false
- +2 considers whether or not B is a truth-teller
- +2 determines that B cannot be lying
- +3 correct identification of A, B and C

8. The Discrete Life of Pets [14.5 points]

Harry, Emily, Grace, and Rohit each have a pet, and have picked a unique number from 1 to 4. Using the following clues, match each person with their pet and number.

- Emily owns the cat or the dog.
- Grace owns the Lizard.
- The bird owner picked the number 2 more than Rohit's number.
- The dog owner picked the number 3.

Solution:

Assume Emily owns the dog. Then they picked the number 3. This means that Rohit's number is not 1, otherwise the bird owner could not have picked 3. Moreover, Rohit cannot be the bird owner, otherwise point (c) creates a paradox. Therefore Harry owns the Bird. Moreover, Rohit's number must be 1 or 2 so that Harry can pick a valid number,

but since we know it's not 1, Rohit's number is 2, and Harry's number is 4. Then Grace's number, by process of elimination, is 1, and similarly by process of elimination Rohit has the cat.

Person	Pet	Number
Harry	Bird	4
Emily	Dog	3
Grace	Lizard	1
Rohit	Cat	2

Since we made the assumption that Emily owns the dog, we have to check that our solution really is valid, since Emily might not actually own the dog. Our solution works with the first clue, since Emily does own the dog or cat. In particular, they own the dog. It works with the second clue, since Grace owns the lizard. It works with the third clue, since the bird owner's number (Harry's number 4) is 2 more than Rohit's number (2). It also works with the fourth clue, since the dog owner (Emily) picked the number 3. So, our solution is valid.

Alternate Solution:

Rohit cannot own the Lizard or the Bird. Moreover, if Rohit owned the Dog, the bird owner must pick the number 5, which is impossible. So Rohit must own the cat, Emily owns the dog, Grace the lizard, and Harry the remaining animal - the bird. Therefore, Emily picked number 3. The bird owner (Harry) must have a number 2 larger than Rohit's (leaving 4 as the only remaining possibility). Finally, then Rohit must have 2 and Grace has 1.

Grading Guidelines [14.5 points]

- +4 assigns at least two people the correct pet and number
- +4 correctly matches all people to their pet and number
- +6.5 correct justification for matching
- +3 *partial credit: justification is partially correct*

Groupwork

1. The Metaverse [14 points]

Suppose there is an island where there are exactly two types of people: truth-tellers and liars. A truth-teller always tells the truth, and a liar always lies.

Suppose a logician came across two inhabitants of this island, A and B. She asked A: “are you both truth-tellers?” A answered either yes or no. She stopped to think for a minute but could not determine what A and B were (truth-tellers or liars). She then asked “are you both of the same type?” (Same type means that they are either both truth-tellers or both liars.) A answered either yes or no, and then she knew what A and B were.

What are A and B?

Solution:

First let’s try to figure out how A answered the first question. If they answered no, then we end up with the same situation as individual homework question 7(a), meaning the logician could have determined what A and B were. So since she could not figure it out after A’s response, A must have answered yes.

Now let’s try to figure out A’s answer to the second question.

Suppose A answered no. If A told the truth, then one of A or B is a truth-teller and one of A or B is a liar. Since A’s answer to the first question is a lie in this scenario, A must be the liar, but that contradicts the fact that they answered the second question truthfully. So if A is a liar, then both A and B are of the same type. They cannot both be truth-tellers otherwise A would have lied when answering the second question, so they must both be liars.

We should also check that A could not have answered yes. If A answered yes and told the truth, then both A and B would have been truth-tellers. On the other hand, if A lied then A would have been a liar and B a truth-teller. Since there’s more than one possible scenario in this case, it’s not possible that the logician determined what A and B were. So A could not have answered yes.

So overall, A and B are both liars.

Alternate Solution (Table):

Another way to approach this is to make a table with every combination of inhabitant types and how they would answer the questions.

Let Q_1 be the answer that A gives to the first question: “are you both truth-tellers?” Let Q_2 be the answer that A gives to the next question: “are you both of the same type?”

For clarity, I will italicize answers that are lies (spoken by a liar) and not italicize answers that are true (spoken by a truth-teller).

A	B	Q ₁	Q ₂
TT	TT	yes	yes
TT	L	no	
L	TT	<i>yes</i>	<i>yes</i>
L	L	<i>yes</i>	<i>no</i>

We are told that after the logician heard the answer to her first question, she still could not determine what type A and B were. If we look at the Q₁ column, we can see that there is only 1 row with “no”, so that answer would reveal what type they are. We therefore know that the answer to the first question was “yes”, and can eliminate row 2. We are told that after the logician heard the answer to her second question, she now knew what types A and B were. Of the remaining 3 possible scenarios, two of them answer “yes” and one of them answers “no.” If the answer were “yes,” the logician would still not be able to deduce A and B’s types, so the answer must be “no,” and we know both inhabitants are liars.

Grading Guidelines [14 points]

- (i) +4 determines that A answered yes to the first question
- (ii) +4 rules-out that A answered yes to the second question
- (iii) +2 determines that A cannot be a truth-teller
- (iv) +2 determines that A and B cannot both be truth-tellers
- (v) +2 determines that A and B are both liars

2. Majority Rules [10 points]

Consider the ternary logical connective $\#$ where $\#PQR$ takes on the value that the majority of P, Q and R take on. That is $\#PQR$ is true if at least two of P, Q or R are true and is false otherwise. Express $\#PQR$ using **only** the symbols: P, Q, R, \wedge, \vee , and parentheses.

Solution:

Notice that $\#PQR$ is true exactly when at least two of the propositions P, Q , or R are true. In other words, if any two of P, Q , or R are true, then so is $\#PQR$; otherwise, it is false. Therefore,

$$\#PQR \equiv (P \wedge Q) \vee (P \wedge R) \vee (Q \wedge R).$$

Grading Guidelines [10 points]

- (i) +5 writes $\#PQR$ in terms of the logical connectives discussed in class
- (ii) +5 correctly writes expression in terms of the allowed symbols