Practice Exam 2
QUESTIONS PACKET
EECS 203
Fall 2023

Name (ALL CAPS): ______________________________

Uniqname (ALL CAPS): ______________________________

8-Digit UMID: ______________________________

*****MAKE SURE YOU HAVE PROBLEMS 1 - 19 IN THIS BOOKLET.*****

General Instructions

You have 120 minutes to complete this exam. You should have two exam packets.

- **Questions Packet:** Contains ALL of the questions for this exam, worth 90 points total. There are 9 Multiple Choice questions (4 points each), 4 Short Answer questions (5 or 6 points each), and 4 Free Response questions (8 points each). You may do scratch work on this part of the exam, but only work in the Answers Packet will be graded.
- **Answers Packet:** Write all of your answers in the Answers Packet, including your answers to multiple choice questions. **For free response questions, you must show your work! Answers alone will receive little or no credit.**
- You may bring **one** 8.5" by 11" note sheet, front and back, created by you.
- You may **NOT** use any other sources of information, including but not limited to electronic devices (including calculators), textbooks, or notes.
- After completing the exam, sign the Honor Code on the front of the Answers Packet.
- You must turn in both parts of this exam.
- **You are not to discuss the exam until the solutions are published.**

Part A: Multiple Answer Multiple Choice

For the following questions, select all the options that apply. **This could be all answers, no answers, or anything in between.**

Problem 1. (4 points)

Which of the following statements are logically equivalent to this statement:

"If it's Friday, Regan teaches discussion today."

(a) "If it's not Friday, Regan does not teach discussion today."

(b) "If Regan teaches discussion today, it's Friday."

(c) "If Regan does not teach discussion today, it's not Friday."

(d) "Regan does not teach discussion today or it's Friday."

(e) "It's not Friday or Regan teaches discussion today."

Problem 2. (4 points)

Which one of these proof methods would allow us to prove the following statement:
For an integer n, n^2 is odd if and only if n is odd.

(a) Consider the integer 3. 3 is odd and $3^2 = 9$ is also odd.

(b) List out all the possible odd numbers and show that all of their squares are odd as well.

(c) Let n be odd and show that n^2 is also odd. Let n^2 be even and show that n is even.

(d) Let n be even and show that n^2 is also even. Let n be odd and show that n^2 is also odd.

(e) Assume n^2 is odd and n is even and arrive at a contradiction.

Problem 3. (4 points)

Which of the following are equivalent to $\forall x \neg \exists y \forall z [P(x, y, z) \rightarrow Q(x, y, z)]$?

(a) $\neg \exists x \exists y \forall z \quad [P(x, y, z) \rightarrow Q(x, y, z)]$

(b) $\forall x \forall y \forall z \quad [P(x, y, z) \rightarrow Q(x, y, z)]$

(c) $\forall x \exists y \neg \exists z \; [P(x,y,z) \to Q(x,y,z)]$

(d) $\forall x \forall y \exists z \; \neg[\neg P(x,y,z) \vee Q(x,y,z)]$

(e) $\forall x \forall y \exists z \quad [P(x,y,z) \wedge \neg Q(x,y,z)]$

Problem 4. (4 points)

Let $P(x)$ and $Q(x)$ be predicates over the domain of integers. $a \oplus b$ is the XOR symbol, and $a \oplus b$ means a or b is true, but not both. Suppose we know that the following statement is true:

$$\forall x[P(x) \oplus Q(x)]$$

Which of the statements below **must** also be true?

(a) $\forall x[P(x) \vee Q(x)]$

(b) $\exists x[\neg Q(x) \wedge P(x)]$

(c) $\neg \exists x[P(x) \wedge Q(x)]$

(d) $\forall x[P(x)] \oplus \forall x[Q(x)]$

(e) $\exists x[P(x)] \vee \exists x[Q(x)]$

Problem 5. (4 points)

Which of the following are propositions?

(a) For all integers x, if x is odd, then x^2 is even

(b) $p \to q \equiv \neg p \vee q$, where p and q are propositions

(c) x is a power of 5

(d) Is mayonnaise an instrument?

(e) $\exists x[(x \in \mathbb{Z}) \wedge (x^3 = 27)]$

Problem 6. (4 points)

Which of the following proof outlines could be used to prove the given statement:

For all real numbers x, if x^3 is irrational, then x is irrational.

(a) **Direct Proof:**
Let x be an arbitrary real number.
Assume that x is irrational and show that x^3 must be irrational.

(b) **Proof by Contradiction:**
Let x be an arbitrary real number.
Assume that x^3 is irrational and assume that x is rational. Show that this leads to a contradiction.

(c) **Proof by Contrapositive:**
Let x be an arbitrary real number.
Assume that x^3 is rational and show that if x^3 is rational, then x must be rational.

(d) **Proof by Contrapositive:**
Let x be an arbitrary real number.
Assume that x is rational, and show that if x is rational, then x^3 must be rational.

(e) **Proof by Example:**
Find a specific value of x where x^3 is irrational and x is irrational.

Problem 7. (4 points)

For $x, y, z \in \mathbb{R}$, which of the following are true?

(a) $\exists x \forall y (xy = 0)$

(b) $\exists x \forall y (xy = 1)$

(c) $\forall x \exists y (y^2 = x)$

(d) $\forall x \exists y (x^2 = y)$

(e) $\forall z \exists x \exists y (\frac{x}{y} = z)$

Problem 8. (4 points)

Let the domain of x and y be the **integers**. Which of the following are true?

(a) $\exists x \exists y \; [x^2 + y^2 = 0]$

(b) $\exists x \forall y \; [y = 3x]$

(c) $\forall y \exists x \; [y = 3x]$

(d) $\exists x \forall y \; [x + y^2 = 7]$

(e) $\forall y \exists x \; [x + y^2 = 7]$

Problem 9. (4 points)

Emily X, Emily Y, and Emily Z walk into a room. One is a Professor, one is a GSI, and one is an IA.

- Emily X says: "Neither Y nor Z are professors."
- Emily Y says: "I'm not an IA."
- Emily Z says: "I've never heard of EECS 203."

It turns out all the Emilys are lying. Which Emily is the professor and which is the GSI?

(a) Professor: X, GSI: Y

(b) Professor: Y, GSI: X

(c) Professor: X, GSI: Z

(d) Professor: Z, GSI: X

(e) Professor: Z, GSI: Y

Part B: Short Answer

For the following questions, keep the answer brief. If there are intermediate steps involved, you would need to show work and justification to get full credit.

Problem 10. (6 points)

Prove that if $5n^2 - 2$ is odd, then n is odd.
Note: You cannot use the lemmas "even + odd = odd", "even · even= even", etc. without proving it.

Problem 11. (5 points)

Let $P(x, y)$ be the predicate "x is taller than y", defined on the domain of all people.
What is the truth value of the following proposition? Briefly explain your answer by explaining the meaning of the proposition in English.

$$\exists x \exists y \exists z \, [P(x, y) \wedge P(y, z) \wedge P(z, x)]$$

Problem 12. (6 points)

Let the domain of x be all people. Let $N(x)$ be the statement, "x is a nice person". Let $T(x)$ be the statement, "x drinks tea".

Translate each of the following English statements into logical statements.

(a) All nice people drink tea.

(b) Everyone who drinks tea is nice.

(c) There is a nice person who drinks tea.

(d) Not everybody is nice, but everybody drinks tea.

Problem 13. (5 points)

Claim: "The sum of two non-negative integers is always non-negative."

Suppose we want to prove the above claim using proof by contradiction. Complete the following sentence that would begin such a proof.

"Seeking contradiction, assume that ..."

Notes:

- Your answer *can* use the phrase "non-negative", but should **not** contain the word "not" or any other negation.
- Complete the given sentence. Do **not** complete the full proof.

Part C: Free Response

Problem 14. (8 points)

p	q	r	a	b	c
T	T	T	F	T	T
T	T	F	F	T	T
T	F	T	F	F	F
T	F	F	F	T	F
F	T	T	T	T	F
F	T	F	T	T	F
F	F	T	F	T	T
F	F	F	F	T	T

Use the truth table for the compound propositions a, b, and c given above to answer the following question.

For each unknown proposition, a, b, and c, find an expression for the proposition as a compound proposition using p, q, and/or r. Note the following requirements:

- You may use **only** $\wedge$, $\vee$, $\neg$, $\rightarrow$, $\leftrightarrow$, and parentheses in each expression.
- You may use p, q, and r **at most once** in each expression.

Problem 15. (8 points)

Let a, b, c be consecutive integers with $a < b < c$. Prove that the sum $a + b + c$ is a multiple of 3.

Note: For example, 3,4,5 are consecutive integers, but 3,4,6 are not. An integer x is a multiple of an integer y if and only if there is an integer k with $x = ky$.

Problem 16. (8 points)

Let x and y be integers. Prove that if $x^2 + y^2 - 3x^2y$ is even, then x and y are both even.

For this question only, you may use the following 8 properties about odd and even numbers without proof.

- Odd + Odd = Even
- Odd + Even = Odd
- Even + Even = Even
- Odd $\times$ Odd = Odd
- Odd $\times$ Even = Even
- Even $\times$ Even = Even
- Odd^2 is Odd
- Even^2 is Even

Problem 17. (8 points)

Prove or disprove the following claim:

"For all positive integers a, b, and d, if $d|a$ or $d|b$, then $d|a^2b$".

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