Dhj: 1. Technical Vocab: logical equivalence; contrapositive i tau indigy; contradiction; of 2. Key Logical equivalence mes: DeMorgan's, distributive law, implication breakon

9/5/23 1 Double Negation law: 7(7p) = p

3. the contrapositive of an if-then statement
4. Negate an if-then statement

5. A compound proposition is a toutohy, a contradiction, or neither

6. Logical symbols for "there exists" and for all " (3, 4)

## Lecture 3 Handout: Logical Equivalence

Translate each English statement to logic, then complete the truth table for each.

1. If I pet my cat then she is happy.

2. If my cat is unhappy then I didn't pet her.  $\underline{\neg h} \rightarrow \underline{\neg} P$ 

3. *I di* 

lidn't pet my	 7P \	<u>/_/</u>				
I. I	1			. 1	٠.	

p	h	-, ρ	つん	p→h	7h->7p	7PVh
Т	Т	F	F	T	au	T
Т	F	۴	1	F	F	F
F	Т	Τ	F	7	Ť	Т
F	F	7	1	T	T	T

## Logical Equivalence Two compound propositions A, Gare logically equivalent if they have the same truth value for any instantiation of their input truth values (i.e., if they have the same, but table. Some notations for logical equivalence: • "p, q are logically equivalent" (p = q) f p if and only if q" ( "p is necessary and sufficient for q" • "p iff q"

## DeMorgans Laws in Action (Words)

Negate each of the following:

Proposition 1: I will go to the store or I will go to the park.

Negation: 14's not true that I will go to S or I will go to P

Simplify: I will not go to S and L will not go to P.

Proposition 2:
Negation: It's not true that I is and I was Quad.
Simplify: I am met 18 and I don't live h wa.

(ADMORGAN'S Law #2: CZCY/W)=77 V7W

(negating an and statement)

Useful Logical Equivalence Rules

 $p \vee F \equiv P$  $p \vee T \equiv T$ 

 $p \wedge F \equiv F$ 

 $s \vee p$ 

 $v \wedge w$ 

Distributive Laws: 6 Distributive Law  $p \lor (q \land r) \equiv (p \lor q) \land (q \lor r)$ 

 $p \wedge (q \vee r) \equiv (P \wedge q) \vee (P \wedge q)$ DeMorgan's Laws:

 $\neg (p \lor q) \equiv \neg p \land \neg D$ 

 $\neg(p \land q) \equiv \neg p \lor \neg q,$ 

"Implication breakout" rule:  $p \rightarrow q \equiv$ 

Contrapositive:

 $p \rightarrow q \equiv 19.77$ 

Negating an "implies"

 $\neg(p \rightarrow q) \equiv$ 

Which of the following ALWAYS has the same truth value as  $p \rightarrow q$ ?

A) Converse:

 $q \rightarrow p$ 

B) Inverse:  $\neg p \rightarrow \neg q$ 

C) Contrapositive:  $\neg q \rightarrow \neg p$ 

(P/2)/r = p/g/r) (P/2)/r = p/(g/r)

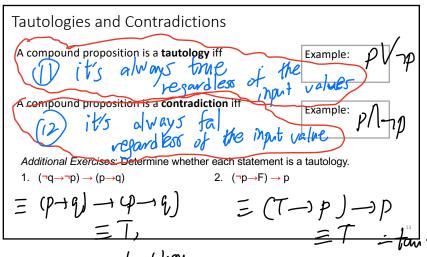
Contrapositives

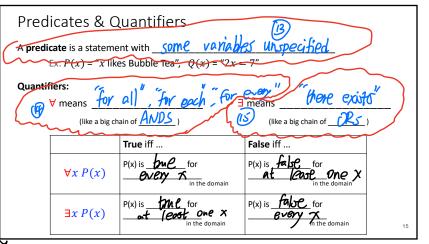
Statement: "If p, then q"  $p \rightarrow q$ Contrapositive of statement: "If not q, then not p"  $q \rightarrow q$   $p \rightarrow q \equiv q \rightarrow q$ Find the contrapositive of each statement:

• If it's Tuesday, then we have EECS 203 class. (PQSY)

• If you don't live in Michigan, then you don't live in Ann Arbor.

• If  $not \ p$ , then q. (Here p, q can stand for any propositions.)





P  $T \rightarrow P$   $(T \rightarrow P) \rightarrow P$   $Y \times P(X) : Every one likes <math>\sim$   $T \times P(X) : Someone likes <math>\sim$   $T \times P(X)$ 

总结:

3. Implication Breakart
$$p \rightarrow q = 7p \vee q$$

5 Associative Laws
$$p N(q Nr) = I p N q N r$$

$$p V(q Vr) = (p V q) V r$$

2. Identity Laws
$$pNT \equiv P$$

$$pVF \equiv P$$

4 Commutative Canu-  

$$P \land Q = Q \land P$$
  
 $P \lor Q = Q \lor P$ 

b Distributive Laws
$$pV(q \Lambda r) \equiv (pVq) \Lambda(pVr)$$

$$p\Lambda(q Vr) \equiv (p\Lambda q)V(p\Lambda r)$$

Eg1: Is this a tautology? EP 1 (p-19)] -> -9 SI. (Truth table) Sol 2. There is only one oftenton where A ->B = F, that is, A = T, B=F. Agrume 79, EF, then 9, ET then no matter p = T or P = F, (p-19)=T then if pet, TPEF, then [TP 1 (p-19)] = F.  $(7P \wedge 9) \rightarrow 79 = (F \rightarrow F) = 7$ de if p = F, \np = T, then Inp/ cp-19 1]= T, then [7p / (p-)2]) -> 9, = (T-)F] Eg2: Is it a tautology? i. not tambology b [PM(p-19)]-19  $= \frac{p \wedge (\neg p \vee 2)}{= (P \wedge \neg p) \vee (p \wedge 2) - 1}$ FV(pla)-9,

 $= (PN2) \rightarrow 2$  = 7, = 7, = 1 tautology

135  $Q(x) = \pi + 1 > 2x,$ a) Q(-1) = Tb) Qu) = Fc)  $3 \times Q(x) = T$ d)  $\forall x \hat{U}(x) \equiv T$ e)  $\exists x \land U(x) \in T$ 

FI HAT QCX) EF

domain: all interest