**3.** **a)** 3 **b)** 14

**5.** 85

held on the same day. **3.** **a)** 3 **b)** 14 **5.** 85 **7.** Because there are four possible remainders when an integer is divided by 4, the pigeonhole principle implies that given five integers, at least two have the same remainder. **9.** Let $a, a+1, \dots,$

at least two have the same remainder. **9.** Let $a, a+1, \dots,$
$a+n-1$ be the integers in the sequence. The integers $(a+i)$ **mod** $n$, $i = 0, 1, 2, \dots, n-1$, are distinct, because $0 < (a+j)-(a+k) < n$ whenever $0 \le k < j \le n-1$. Because there are $n$ possible values for $(a+i)$ **mod** $n$ and there are $n$ different integers in the set, each of these values is taken on exactly once. It follows that there is exactly one integer in the sequence that is divisible by $n$. **11.** 4951

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**13.** The midpoint of the segment joining the points $(a, b, c)$ and $(d, e, f)$ is $((a+d)/2, (b+e)/2, (c+f)/2)$. It has integer coefficients if and only if $a$ and $d$ have the same parity, $b$ and $e$ have the same parity, and $c$ and $f$ have the same parity. Because there are eight possible triples of parity [such as (*even*, *odd*, *even*)], by the pigeonhole principle at least two of the nine points have the same triple of parities. The midpoint of the segment joining two such points has integer coefficients. **15.** **a)** Group the first eight posi-

has integer coefficients. **15.** **a)** Group the first eight posi-
tive integers into four subsets of two integers each so that the integers of each subset add up to 9: $\{1, 8\}$, $\{2, 7\}$, $\{3, 6\}$, and $\{4, 5\}$. If five integers are selected from the first eight positive integers, by the pigeonhole principle at least two of them come from the same subset. Two such integers have a sum of 9, as desired. **b)** No. Take $\{1, 2, 3, 4\}$, for example.

**17.** 4

**19.** 21,251