Outline: 1. Predicates
(1) Recap of Quantifiers: For all (∀), There exist (∃)
(2) Nested Quantifier (嵌套)
(3) Translation Practice
2. DeMorgan's for Quantifiers
3. Domain Restrictions
4. Quantifier Scoping

9/6/23

Lec 4 Handout: Predicates & Quantifiers

(1) A **predicate** is a statement with some variables unspecified

Ex: $P(x)$ = "x likes Bubble Tea", $Q(x)$ = "$2x = 7$"

Quantifiers:

∀ means "for all" (like a big chain of ANDS)

∃ means "there exists" (like a big chain of ORS)

	True iff …	**False** iff …
$\forall x\, P(x)$	P(x) is T for ∀x in the domain	P(x) is ____ for ____ in the domain
$\exists x\, P(x)$	P(x) is ____ for ____ in the domain	P(x) is ____ for ____ in the domain

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Nested (嵌套的) Quantifiers – additional exercises

Let **P(x,y) = "4x – y = 0"**, domain = integers

Write each proposition using quantifiers, then determine whether it is true or false:

Quantifiers

Quantifiers		
∃x∃y	1. There exists **x** such that there exists **y** such that **P(x, y).**	True / False
∃x∀y	2. There exists **x** such that for all **y**, **P(x, y).**	True / False
∀y∃x	3. For all **y**, there exists **x** such that **P(x, y).**	True / False
∀x∃y	4. For all **x**, there exists **y** such that **P(x, y).**	True / False
∃y∀x	5. There exists **y** such that for all **x**, **P(x, y).**	True / False
∀x∀y	6. For all **x**, for all **y**, **P(x, y).**	True / False

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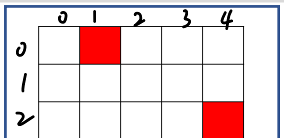
Nested Quantifiers – additional exercises

P(x,y) = "the square at row **x**, column **y** is colored"



- *Note*: we'll start our indexing at 0, so P(0, 0) refers to the square at the top left corner of the grid

(a) P(0, 0) is false

P(0, 1) is true

P(1, 0) is false

P(2, 4) is true

(b) What is the **domain of x**? {0, 1, 2}

What is the **domain of y**? {0, 1, 2, 3, 4}

(c) If $\exists x \exists y\, P(x, y)$ is **true**, the minimum number of shaded squares is 1, and the maximum number is 15 (all)

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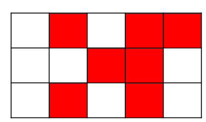
Nested Quantifiers – additional exercises

P(x,y) = "the square at row **x**, column **y** is colored"

Select the logical expression that **matches the English statement.**

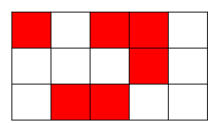
(d) *English*: **The grid has an entire column that is shaded.** D.

Logic: ∃y ∀x P(x,y)



(e) *English*: **Every row has at least one shaded square.** C.

Logic: ∀x ∃y P(x,y)



Answer options:

A. $\exists x\, \forall y\ \ P(x, y)$

B. $\forall y\, \exists x\ \ P(x, y)$

C. $\forall x\, \exists y\ \ P(x, y)$

D. $\exists y\, \forall x\ \ P(x, y)$

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Translating Logic to English #1

Handout

Domain of:
m: all movies
x, y: people in this room

$V(x, m)$: "person x has seen movie m"

a) $\exists x\, \exists y\, [x \neq y\ \wedge\ \exists m\, (V(x,m) \wedge V(y,m))]$

There are (at least) two different people in this room who have seen (at least) one same movie.

b) $\exists x\, \exists y\, [x \neq y\ \wedge \forall m\, (V(x,m) \leftrightarrow V(y,m))]$

There are (at least) two different people in this room who have seen the exact same set of movies

Symbol	"Main" Words
¬p	"not p"
p∧q	"p and q"
p∨q	"p or q"
p→q	"if p, then q"
∀x P(x)	"for all x, P(x)"
∃x P(x)	"there exists x such that P(x)"

DeMorgan's Laws for Quantifiers

$\neg\exists x\, P(x) \equiv \underline{\forall x \neg P(x)}$

$\neg\forall x\, P(x) \equiv \underline{\exists x \neg P(x)}$

Exercise: Simplify each statement (so that negation appears only directly before a predicate):

a) $\neg\, \exists x\, \forall y\, \exists z\, [\neg P(x, y, z) \vee \neg Q(x, y, z)]$

$\equiv \forall x \neg \forall y \exists z\, [\neg P(x,y,z) \vee \neg Q(x,y,z)]$

$\equiv \forall x \exists y \neg \exists z\, [\neg P(x,y,z) \vee \neg Q(x,y,z)]$

$\equiv \forall x \exists y \forall z \neg [\neg P(x,y,z) \vee \neg Q(x,y,z)]$

$\equiv \forall x \exists y \forall z\, [P(x,y,z) \wedge Q(x,y,z)]$

b) $\neg\, \exists x\, [P(x) \rightarrow \neg Q(x)]$

$\equiv \forall x \neg [P(x) \rightarrow \neg Q(x)]$

$\equiv \forall x\ \neg [\neg P(x) \vee \neg Q(x)]$

$\equiv \forall x\, [P(x) \wedge Q(x)]$

How would you translate this?

"There is a person in this class all of whose friends in this class will get As"

C(x) : "x is in this class" F(x,y) : "x and y are friends"

A(x) : "x will get an A"

Select all correct translations.

C, D

$\exists x [C(x) \wedge \forall y [[C(y) \wedge F(x,y)] \rightarrow A(y)]]$

(A) ∃x [C(x) → ∀y (F(x,y) ∧ C(y) ∧ A(y))] ✗

(B) ∃x∀y [(C(x) ∧ F(x,y)) → (C(y) ∧ A(y))] ✗

(C) ∃x [C(x) ∧ ∀y [(F(x,y) ∧ C(y)) → A(y)]] ✓

(D) ∃x [C(x) ∧ ∀y [F(x,y) → (C(y) → A(y))]] ✓

(E) ∃x [C(x) ∧ ∀y [F(x,y) ∧ (C(y) → A(y))]] ✗

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Scoping

Translate the following logical statements into English.

Do the two statements have the same meaning? No

B(x, y) = "x buys a y"

Logic	English
∀x [B(x, umbrella) ∨ B(x, raincoat)]	Everyone buys an umbrella or a raincoat
[∀x B(x, umbrella)] ∨ [∀x B(x, raincoat)]	Everyone buys an umbrella or everyone buys a raincoat

everyone chose either

everyone buys the same!

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Example 1. negation

a) $\exists x\,(-4 < x \leq 1)$

Answer: $\forall x, ((x > 1) \vee (x \leq -4))$

b) $\forall z\, \exists x\, \exists y\, (x^3 + y^3 = z^3)$

Answer: $\exists z\, \forall x\, \forall y\, (x^3 + y^3 \neq z^3)$

Example 2. Find counterexamples (domain: all integers for all variables)

a) $\forall x \exists y \left(x = \frac{1}{y}\right)$ CE: $x = 0$, $\forall y \left(x \neq \frac{1}{y}\right)$

b) $\forall x \exists y \left(y^2 - x < 100\right)$. CE: $x = -100$, $\forall y \left(y^2 < 0\right)$

c) $\forall x \forall y \left(x^2 \neq y^3\right)$ CE: $x = 1$, $y = 1$