Machine Learning and Causal Inference

Mixtape Track

Allow me to introduce myself

- Economics professor at Brigham Young University in Utah

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- 4 kids, most of whom can now run and mountain bike faster than me

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- Economics professor at Brigham Young University in Utah
- 4 kids, most of whom can now run and mountain bike faster than me
- A big fan of causal inference in observational settings:
  - Quasi-experimental evaluations of the effects of unions (Frandsen 2016, 2017, 2021; Chen, Frandsen, Grabowski, Town, Sojourner 2015)
  - Distributional effects (Frandsen and Lefgren 2018, 2021; Frandsen, Froelich, Melly 2012)
- And of exploring machine learning in applied economics:
  - Teach Machine Learning for Economists at BYU
  - Research on the power of ML in empirical strategies (Angrist and Frandsen 2022)

Welcome to the Machine: Where we're going

Machine Learning + Causal Inference I

Day 1 (today)

- Prediction vs. Causality
- Conceptual and practical (python!) intro to supervised machine learning methods
  - Lasso
  - Ridge
  - Neural Networks
  - Random Forests
- Day 2: How modern prediction methods can be deployed in the service of causal inference
  - Post double selection lasso (PDS lasso)
  - Double/de-biased machine learning (DML)

Machine Learning + Causal Inference II (starts Nov 10)

- Predicting heterogeneous treatment effects
- Random Causal Forests

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

- male

Prediction vs. Causality

Imagine you are a life insurance underwriter. You receive an application for life insurance from someone with the following characteristics:

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- age 67

Prediction vs. Causality

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- high blood pressure

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- family history of heart disease

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- and . . .

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- family history of heart disease
- and . . .
- was admitted to the hospital yesterday

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Now imagine you are a loved one of someone with the following characteristics:

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- is having chest pains.

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- family history of heart disease
- and . . .
- is having chest pains.
- Should you take him to the hospital?

Prediction vs. Causality: Purpose

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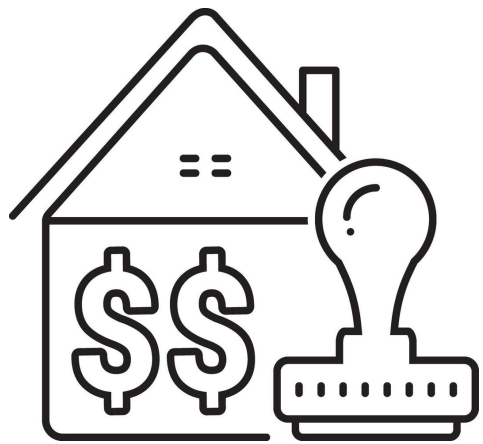
Prepare

- A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.



Prediction vs. Causality: Purpose

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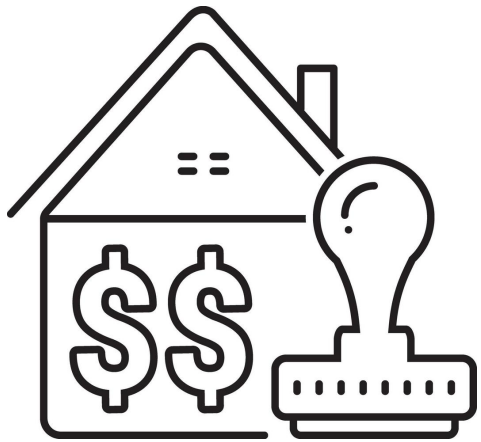
- A loan officer wants to know the likelihood of an individual repaying a loan based on income, employment, and other characteristics.

Influence

- A mortgage lender wants to know if direct debit will increase loan repayments



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Prediction vs. Causality: Purpose

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Influence

- A policy maker needs to know the effect of being released on bail (rather than detained) prior to trial on ultimate conviction

Prediction vs. Causality: Purpose

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- A home seller wants to know what price homes with the characteristics of his or her home typically sell for

Prediction vs. Causality: Purpose

Prepare

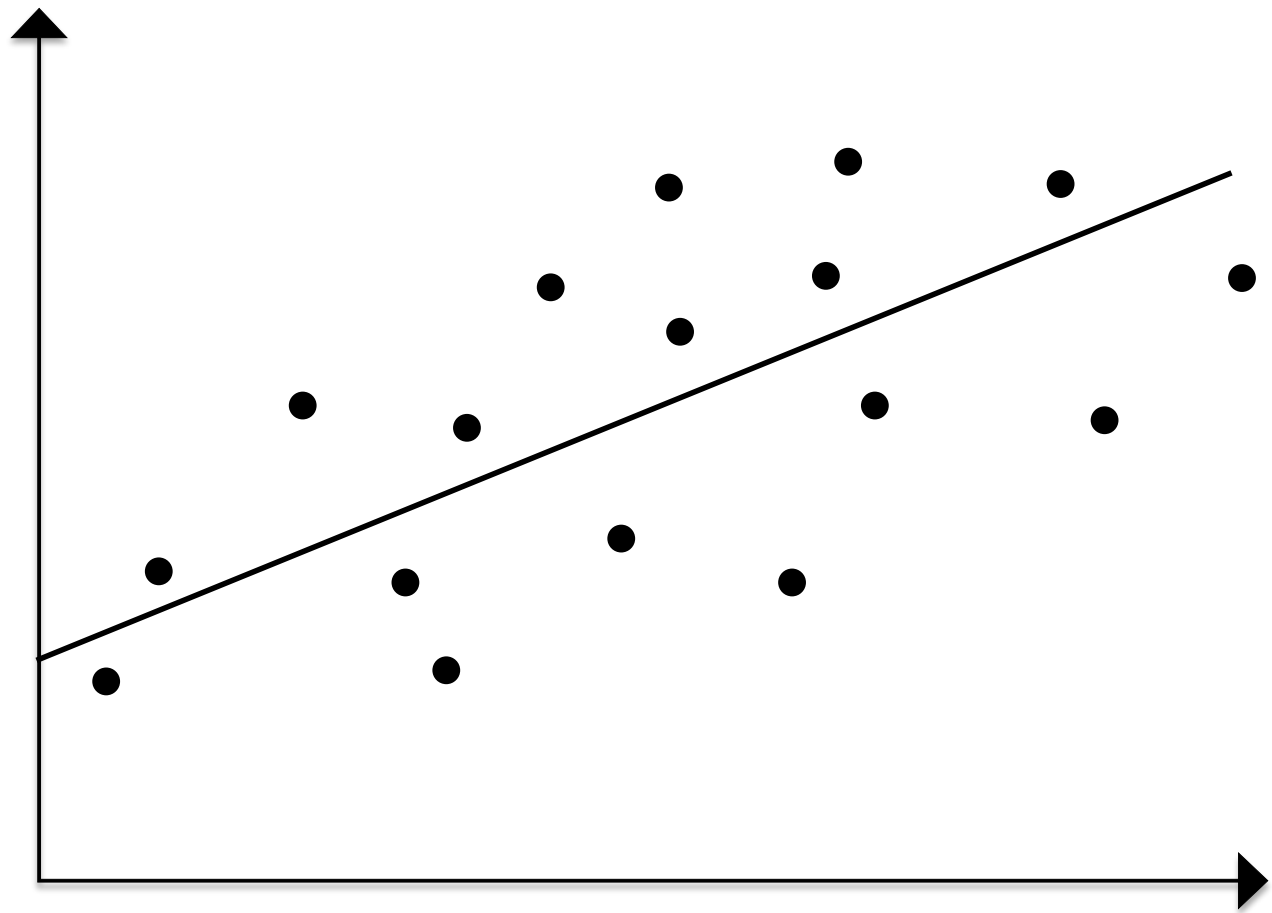
- A home seller wants to know what price homes with the characteristics of his or her home typically sell for

Influence

- A home seller wants to know by how much installing new windows will raise the value of his or her home

Prediction vs. Causality: Target

$$y_i = \alpha + \beta x_i + \varepsilon_i$$



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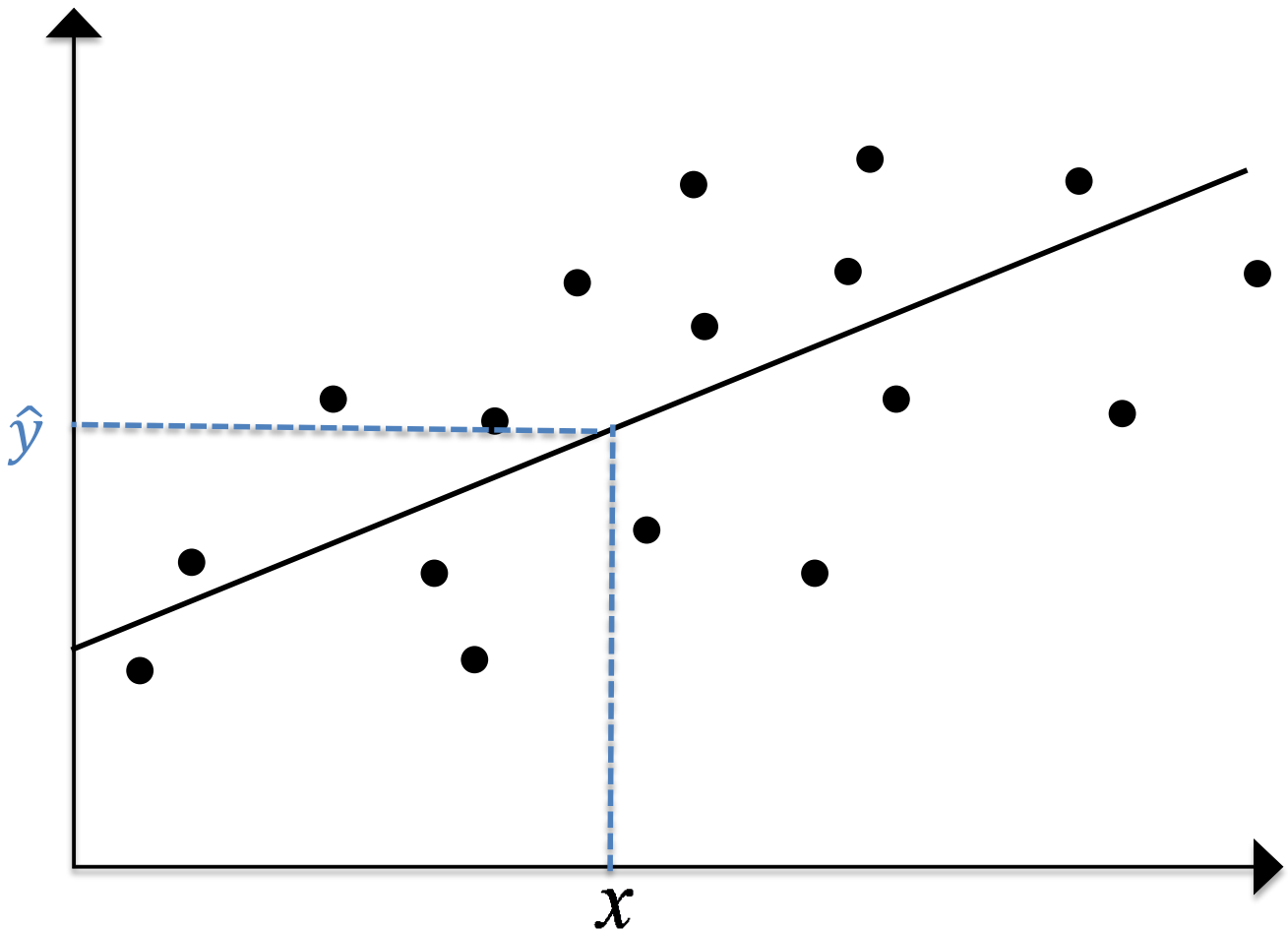

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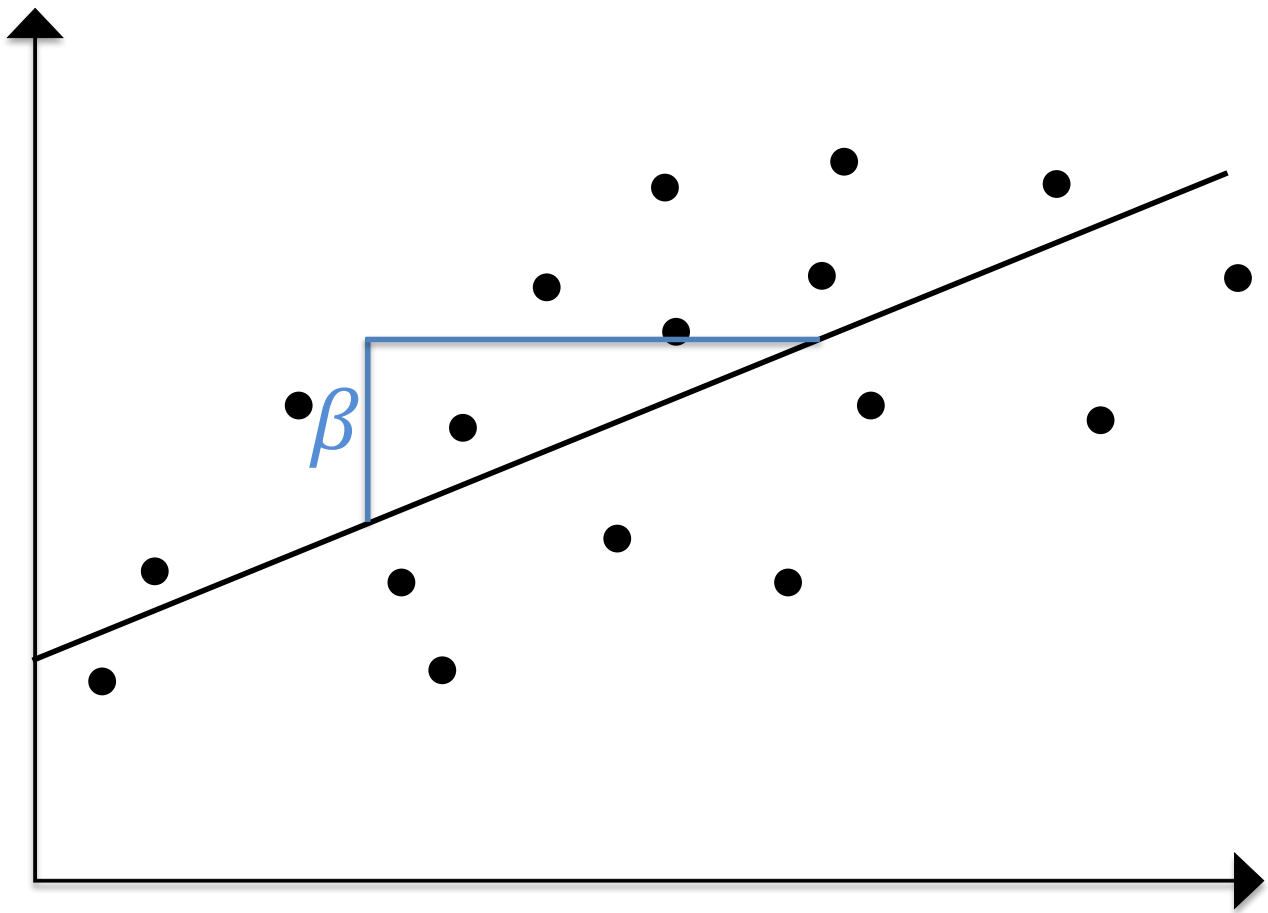

Prediction vs. Causality: Methods

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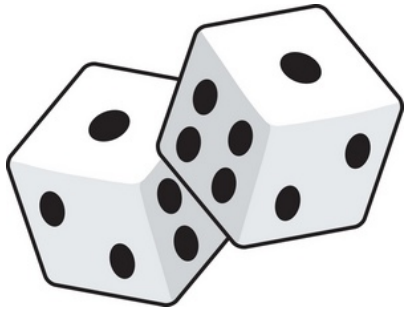

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Prediction

- Supervised machine learning algorithms

Prediction vs. Causality: Where shall the twain meet?

We've seen that prediction and causality

- answer different questions

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Different strokes for different folks, or complementary tools in an applied economist's toolkit?

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- Illustrate using the Oregon Health Insurance Experiment (go to python)

Where ML fits into causal inference

Traditional regression strategy:

1. Regress Y_i on X_i and compute the residuals,

$$\begin{aligned} \tilde{Y}_i &= Y_i - \hat{Y}_i^{OLS}, \\ \hat{Y}_i^{OLS} &= X_i' \left(X'X\right)^{-1} X'Y \end{aligned}$$

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When OLS might not be the right tool for the job:

- there are many variables in X_i
- the relationship between X_i and Y_i or D_i may not be linear

Where ML fits into causal inference

ML-augmented regression strategy:

1. Predict Y_i using X_i with ML and compute the residuals,

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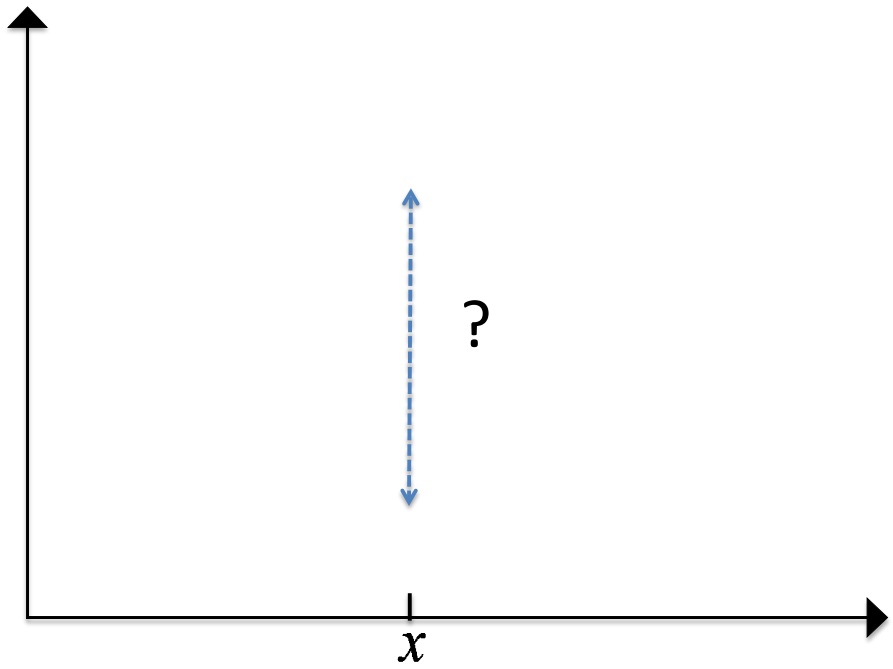
Most common ML methods in applied economics:

- Lasso
- Ridge
- Neural network
- Random forest

Getting serious about prediction

- **Goal:** Predict an out-of-sample outcome Y



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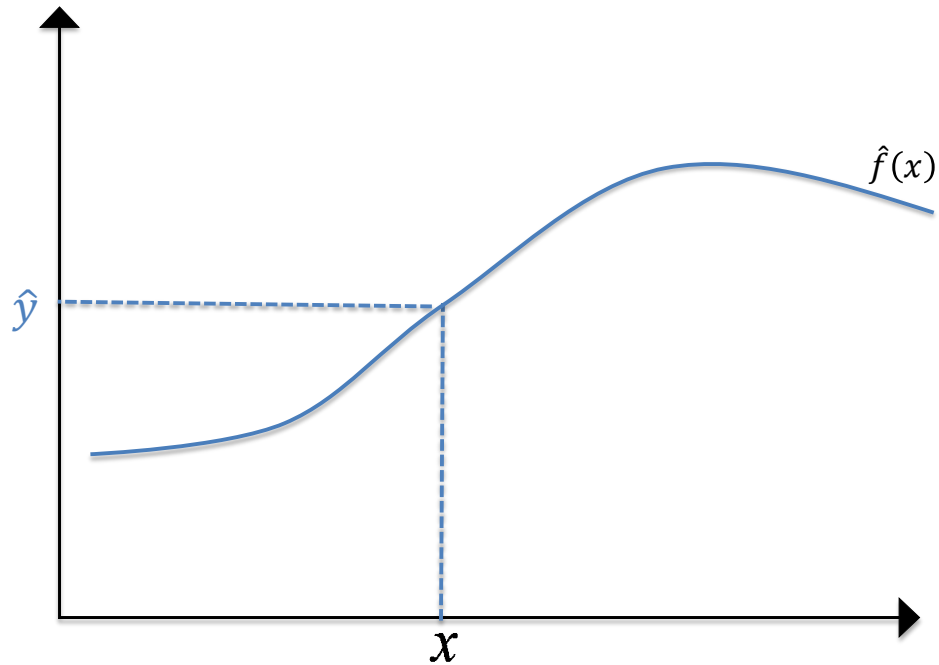

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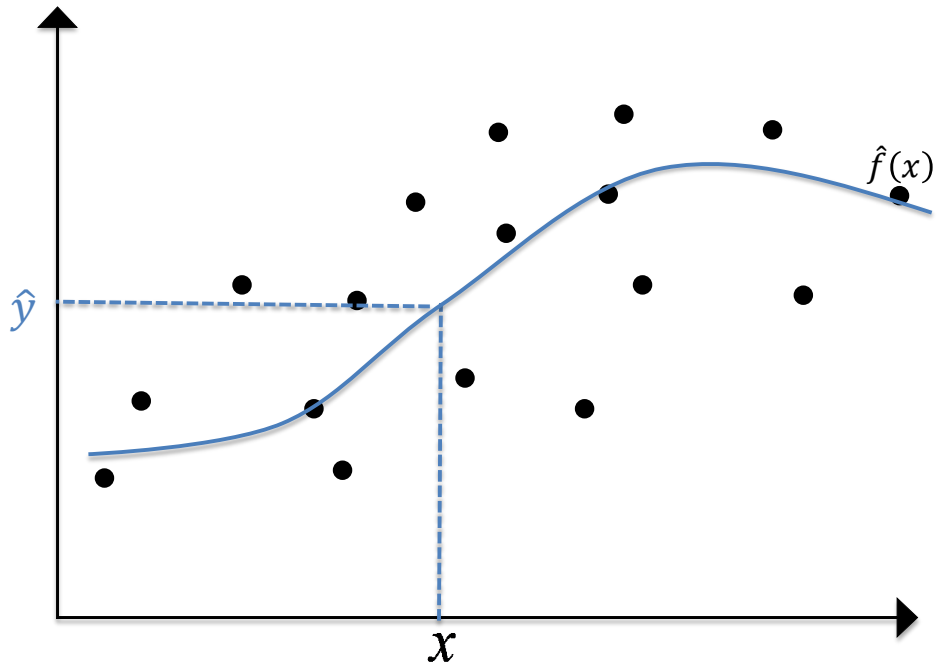
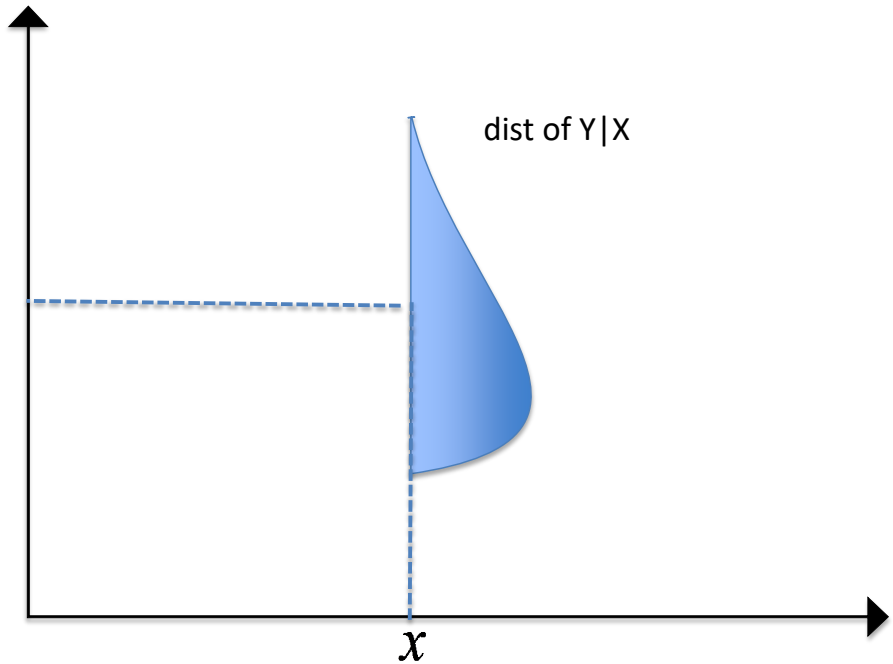
- Estimate the function $\hat{f}$ (aka "train the model") based on **training sample** $\{(Y_i, X_i); i = 1, ..., N\}$



Cutting our losses

- Want our prediction to be "close," i.e. minimize the expected **loss function**:

$$\min_{f(x)} E\left[L\left(f\left(x\right)-Y\right)|X=x\right]$$



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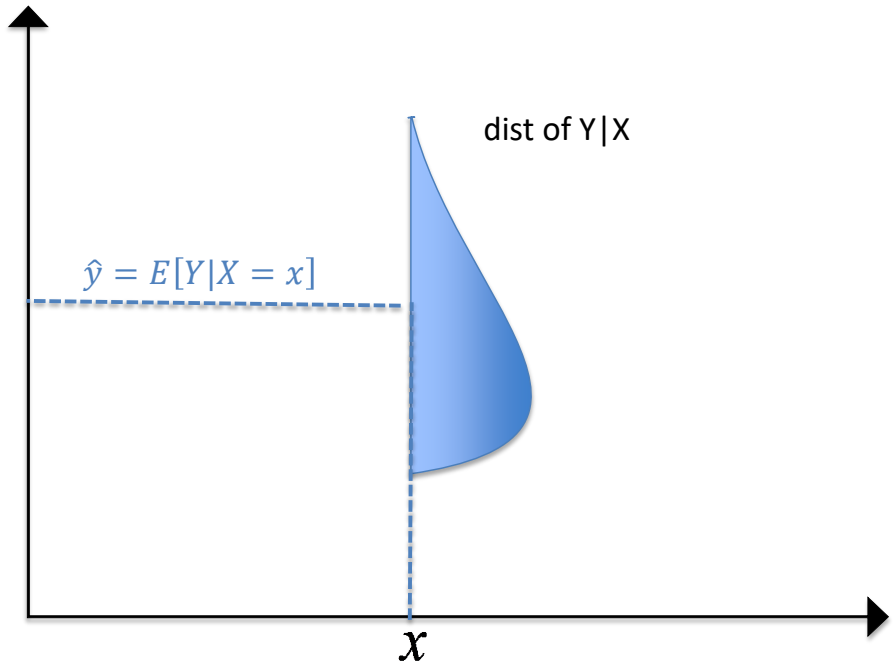

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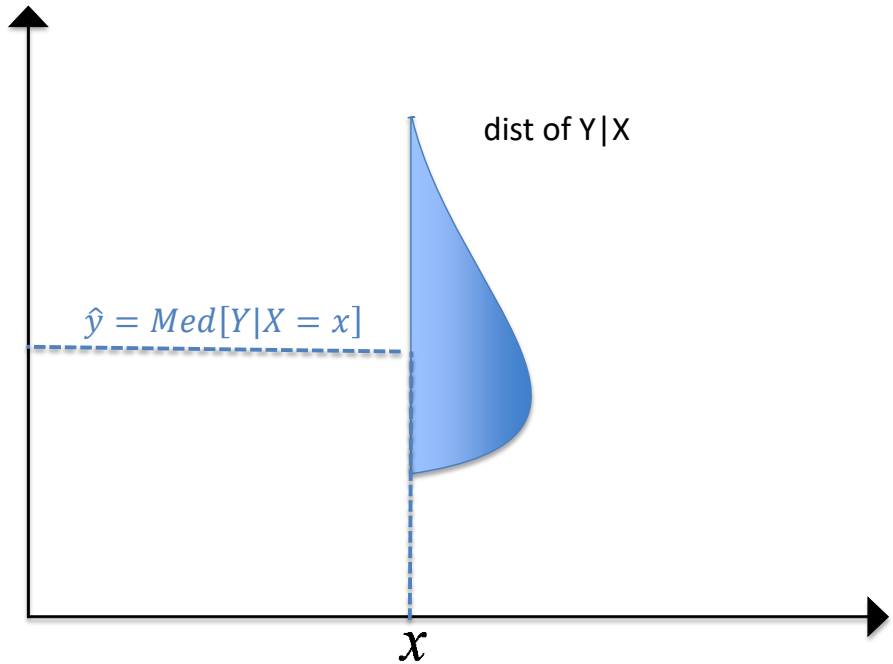

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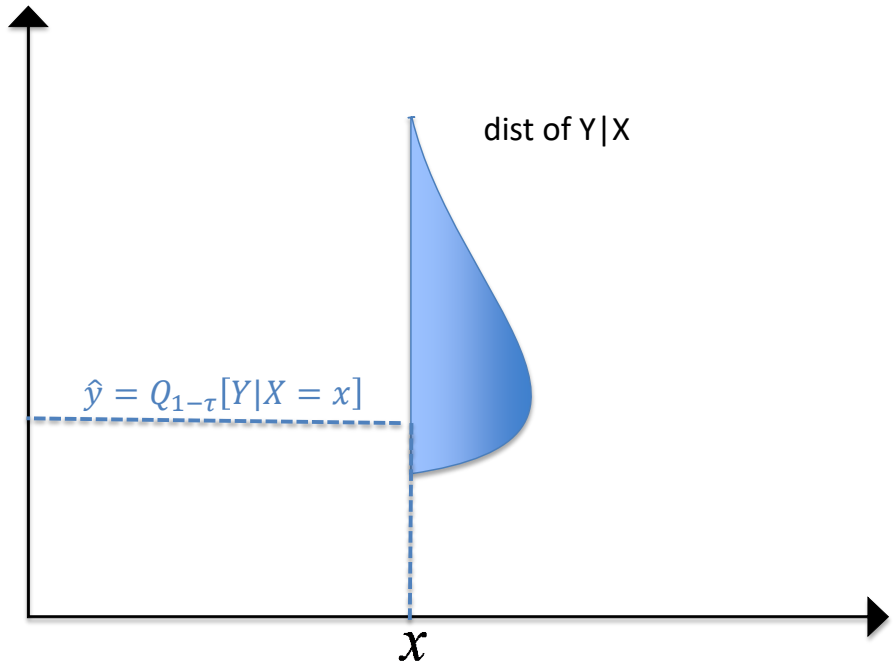
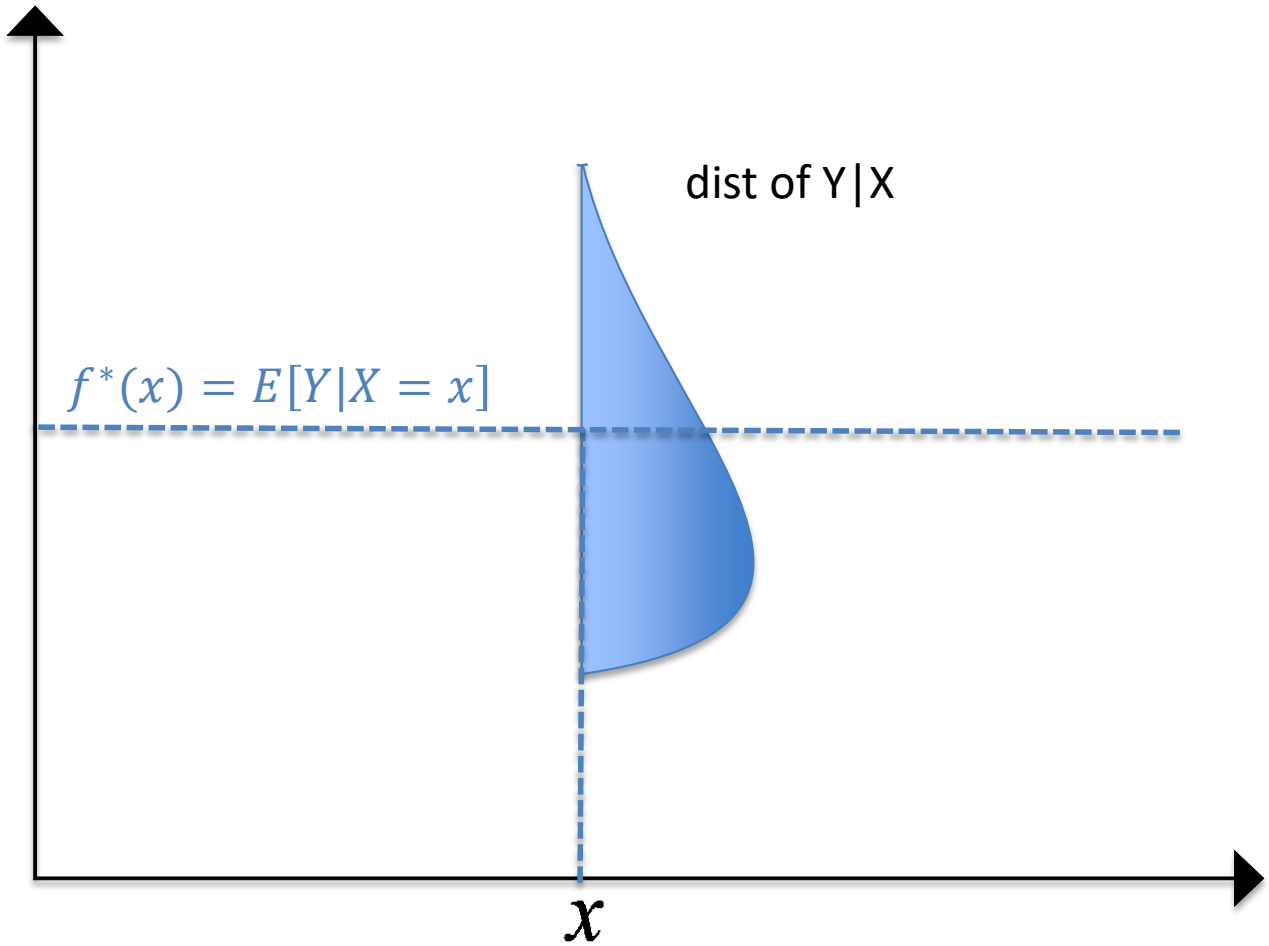
- **Asymmetric loss**:
  $L_\tau\left(d\right)=d\left(\tau-1\left(d<0\right)\right) \implies f^*(x)=Q_{1-\tau}\left[Y|X=x\right]$



Navigating the Bias-Variance Tradeoff

- Prediction problem solved if we knew $f^*(x) = E[Y|X = x]$



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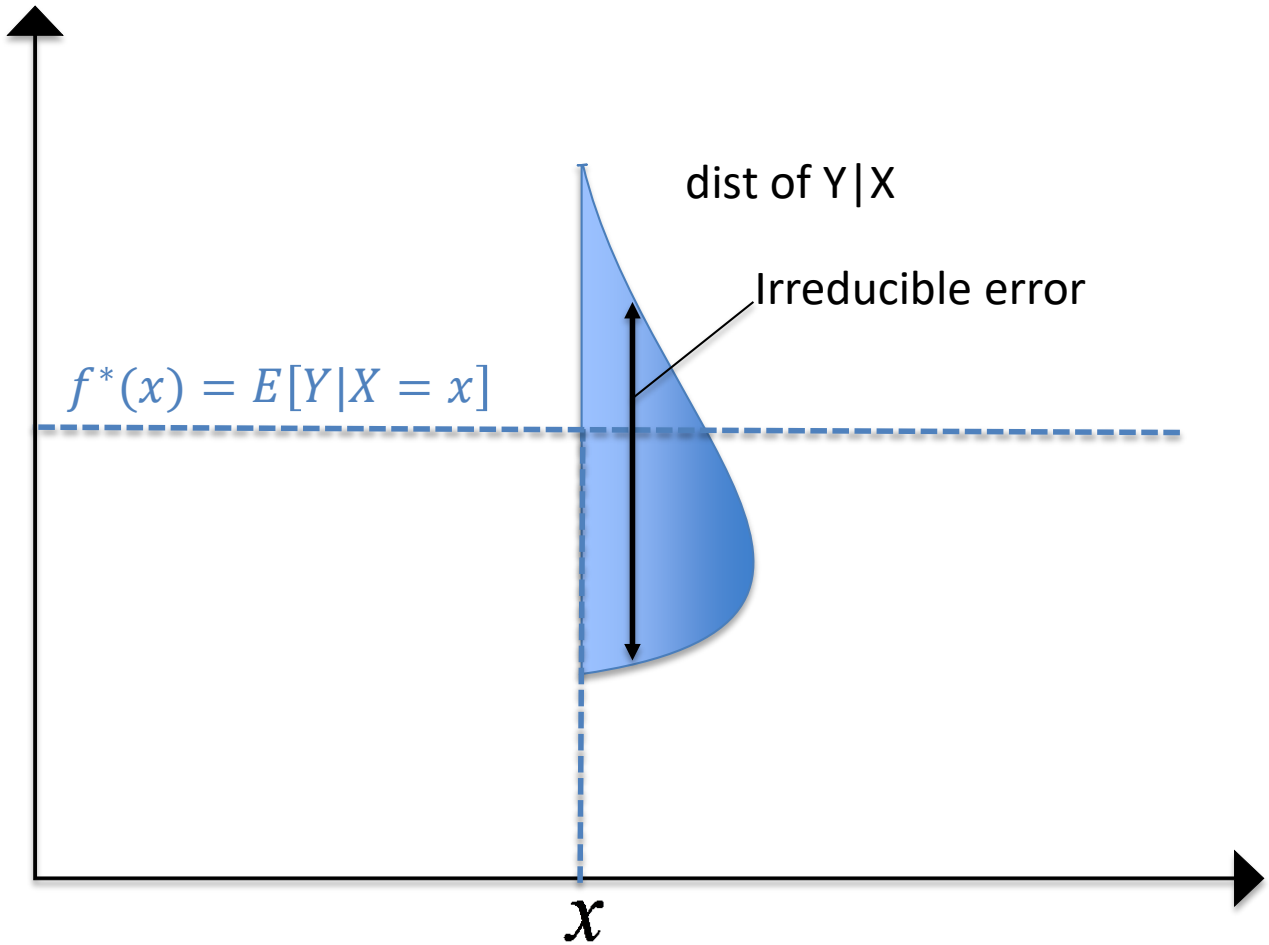

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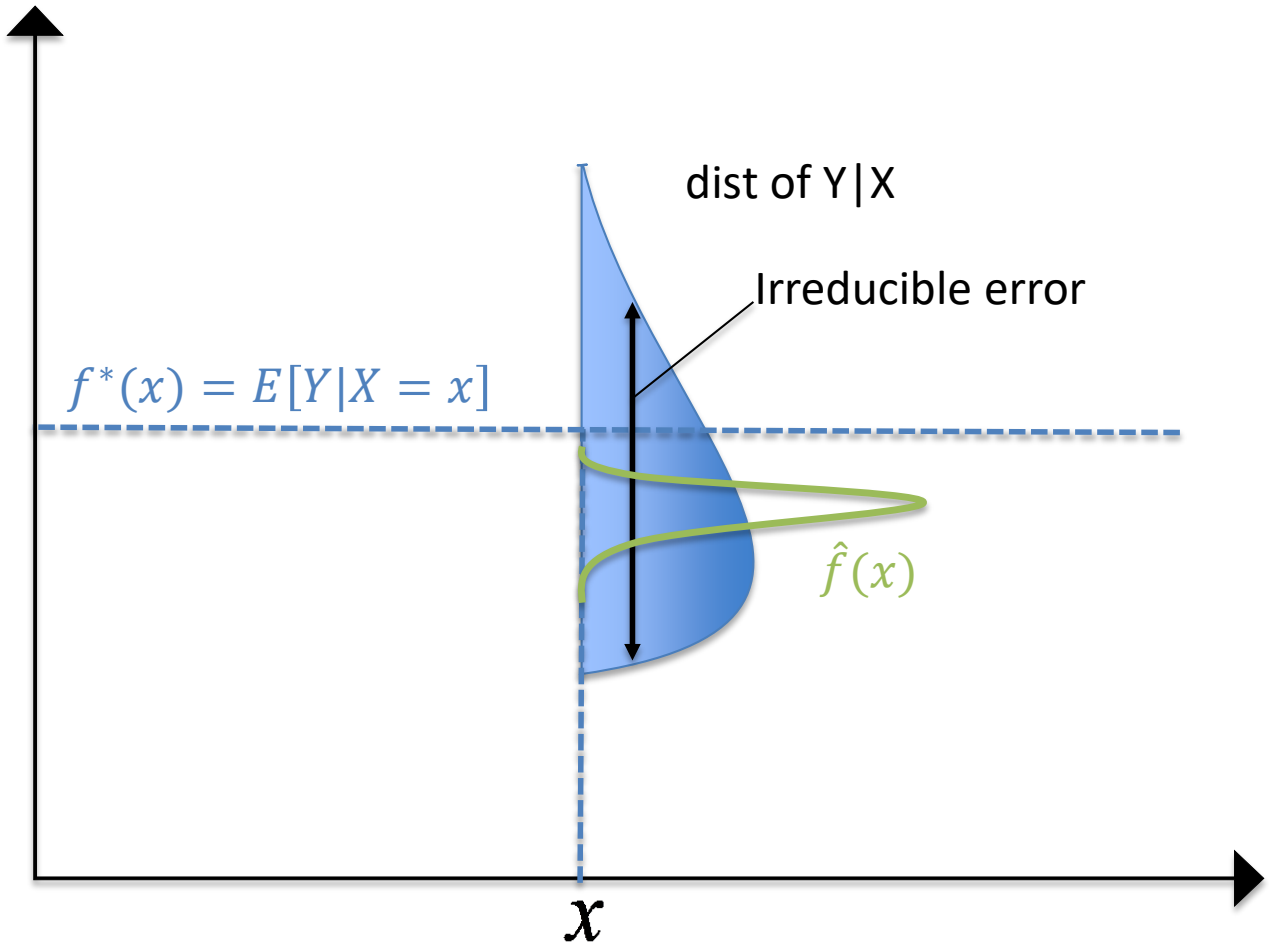

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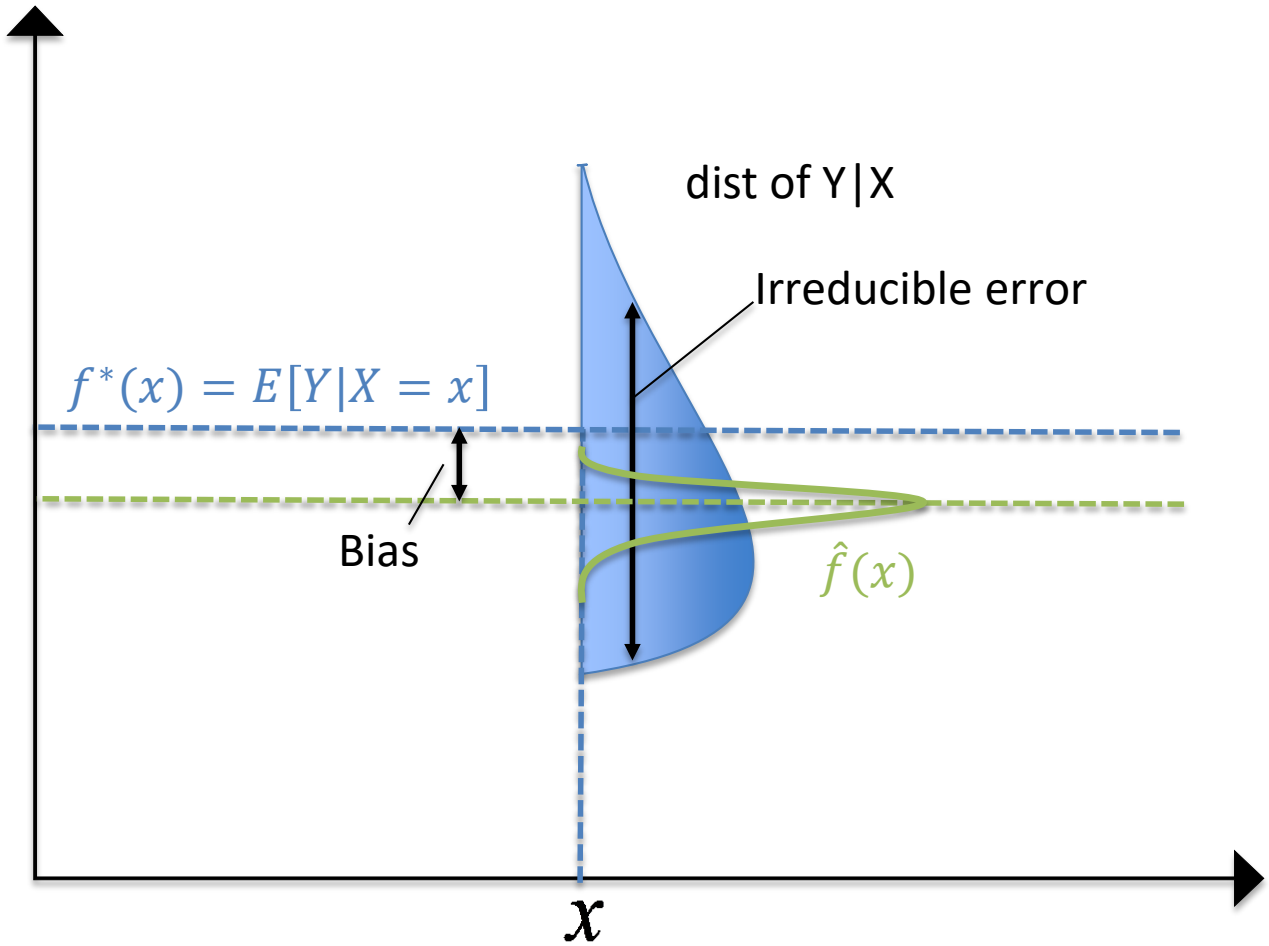

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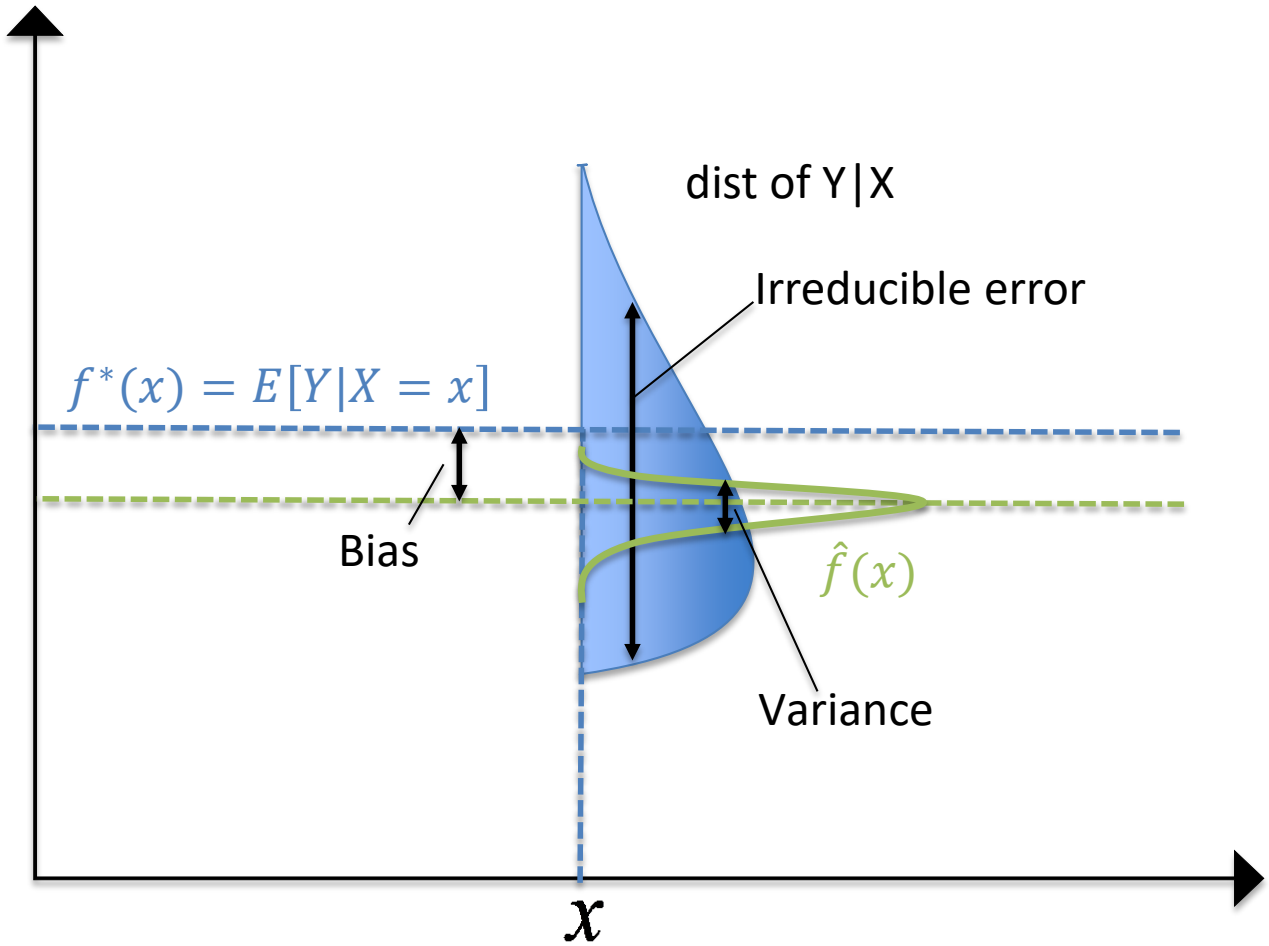

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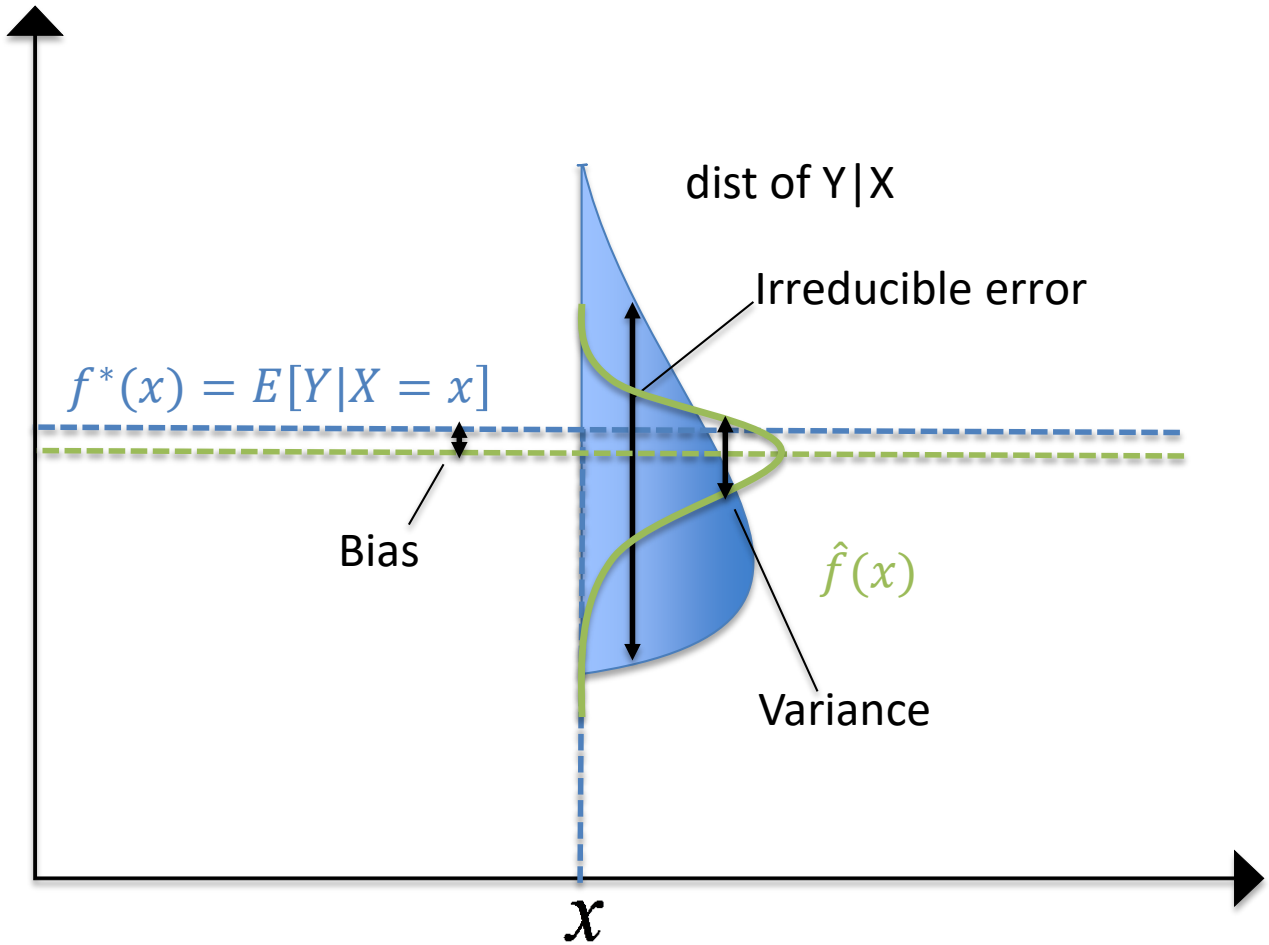

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$$
\begin{aligned}
&\left(E\left[\hat{f}(x) - f^*(x)\right]\right)^2 && \text{prediction bias squared} \\
+\ &E\left[\left(\hat{f}(x) - E\left[\hat{f}(x)\right]\right)^2\right] && \text{prediction variance} \\
+\ &E[(Y - f^*(x))^2 | X = x] && \text{irreducible error.}
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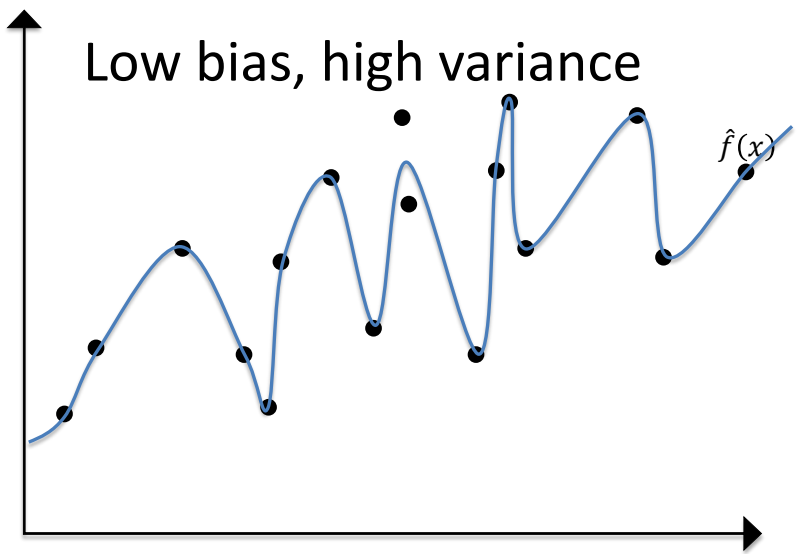

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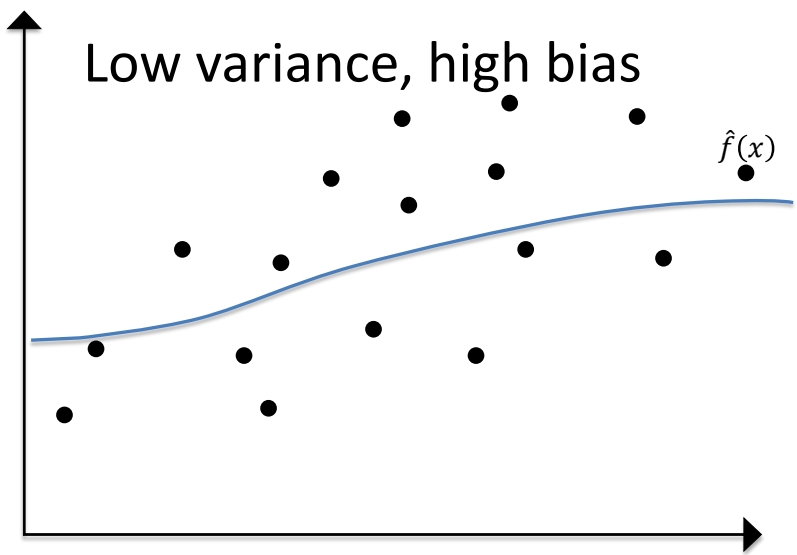


Python example: predicting earnings in the NLSY

Penalized Regression: Lasso

- When is it the right tool for the job:
  - When you have a large number of potential regressors (including powers or other transformations), maybe even more than the sample size!
  - Out of these, only a relatively few (but you don't know which) really matter (what do we mean by "matter?"). We call this **approximate sparsity**
- Theoretical definition:

$$\arg\min_{b} \sum_{i=1}^{n} \left(y_i - x_i' b\right)^2 + \lambda \sum_{j=1}^{k} |b_j|$$

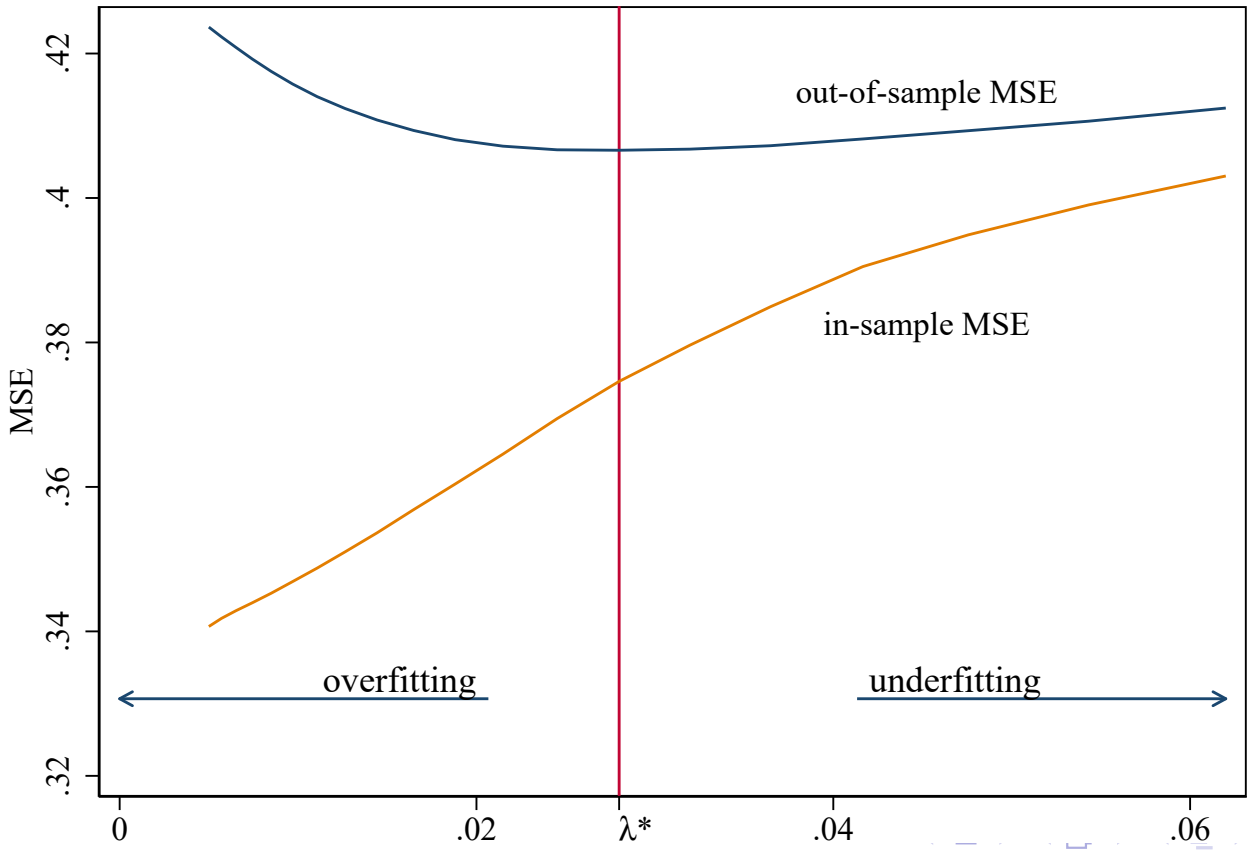
  What does λ do and how do we choose it?
- Caveats and considerations:
  - Important to standardize regressors pre-lasso
  - Can give unexpected results with dummy variables
  - Resist the temptation to interpret coefficients or the included variables as the "true model!"
- Let's give it a go in python!

Choosing Tuning Parameters: Cross-Validation

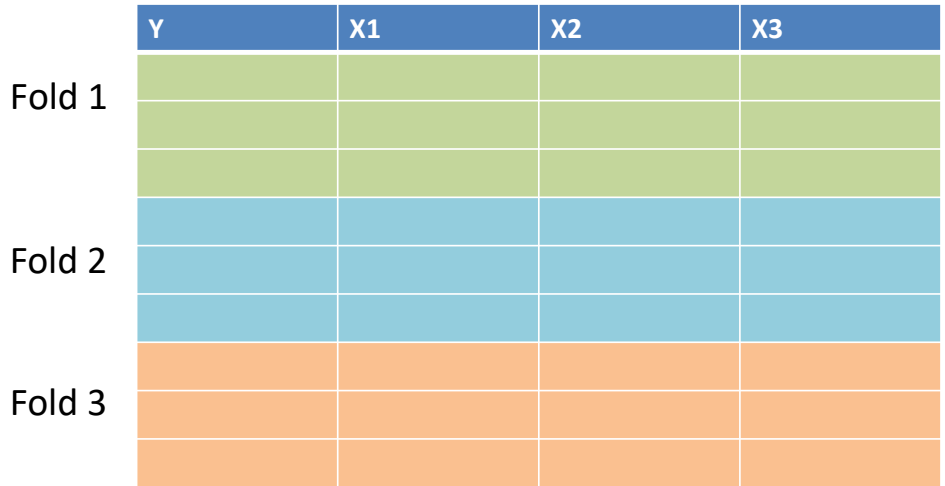
All supervised ML methods have tuning parameters:

- Lasso, Ridge: λ
- Neural networks: number of hidden layers, nodes per layer
- Random forests: tree depth, etc.

Tuning parameters are the rudder by which we navigate the bias-variance tradeoff.



Choosing Tuning Parameters: Cross-Validation



Cross-validation procedure: Divide sample in K folds

- Choose some value of the tuning parameter, λ
- For each fold $k = 1, \ldots, K$
  1. Train model leaving out fold k
  2. Generate predictions in fold k
  3. Compute MSE for fold k: $MSE_k = \frac{1}{n_k} \sum_{i \in k} \left(Y_i - \hat{Y}_i \right)^2$
- Compute overall MSE correponding to the current choice of λ: $MSE\left(\lambda\right) = \frac{1}{K} \sum_{k=1}^{K} MSE_k$

Repeat the above for many values of λ, and choose the value λ^* with the lowest cross-validated MSE—time for python!

Penalized Regression: Ridge

- When is it the right tool for the job:
  - When you have a large number of regressors including highly collinear ones
- Theoretical definition:

$$\arg\min_{b} \sum_{i=1}^{n} \left(y_i - x_i' b\right)^2 + \alpha \sum_{j=1}^{k} b_j^2$$

$$= \left(X'X + \alpha I\right)^{-1} X'Y$$

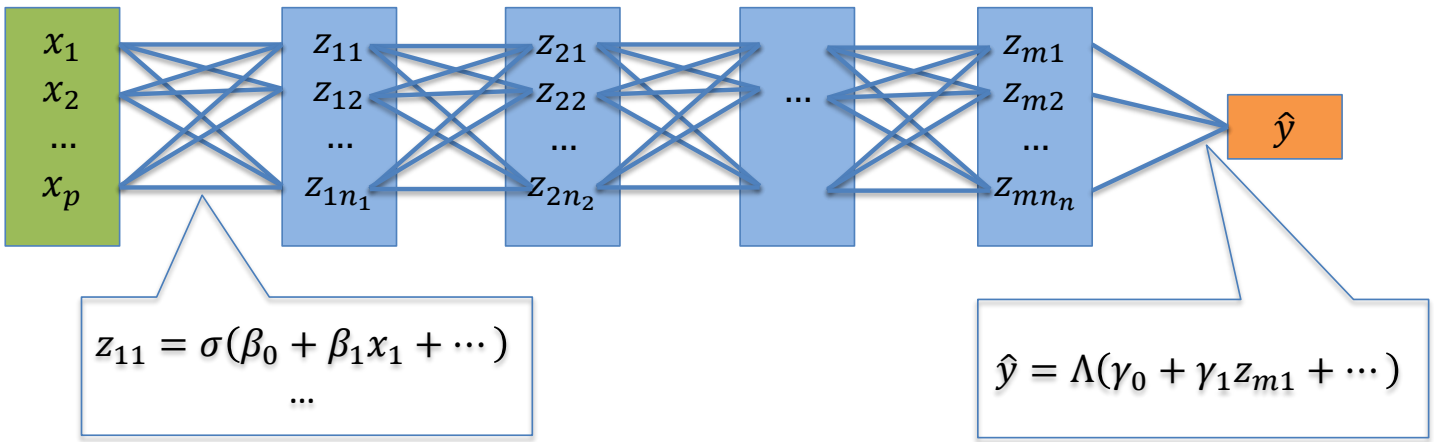
- Caveats and considerations:
  - Important to standardize regressors pre-ridge
  - Shrinks (biases) coefficients towards zero, but not all the way (unlike lasso)
- Let's give it a go in python!

Neural Networks

Input layer: Original features

Hidden layers: Nonlinear transformations of input layer and previous hidden layers

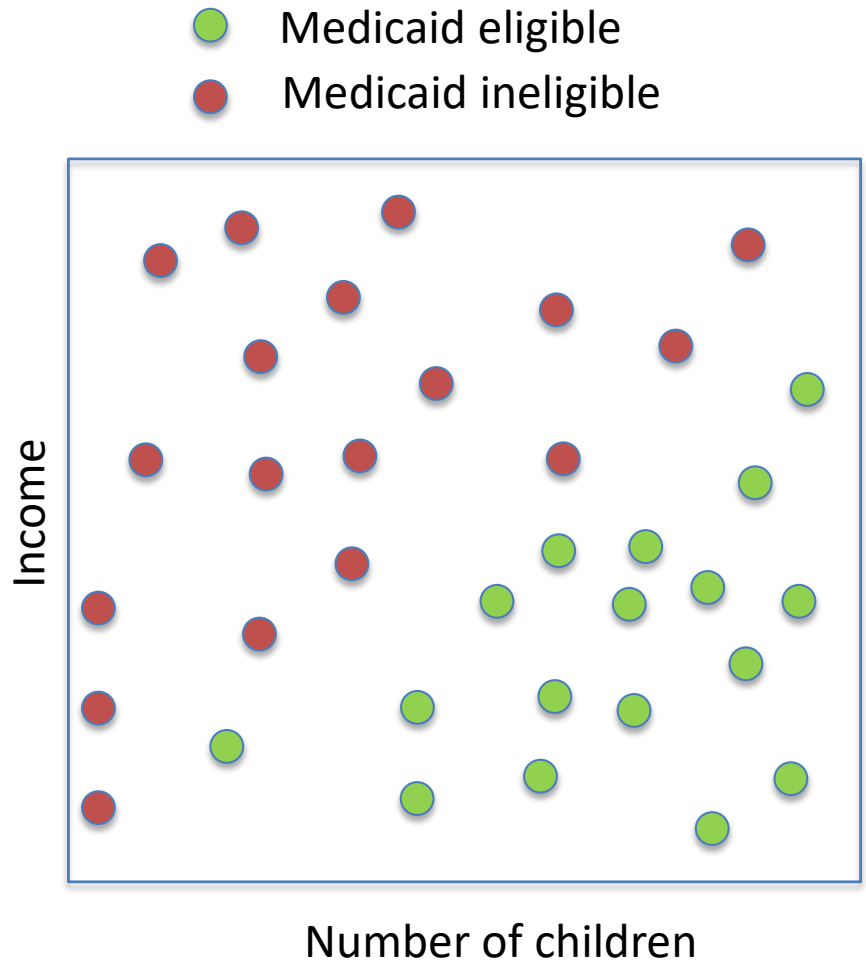
Output layer: Final prediction



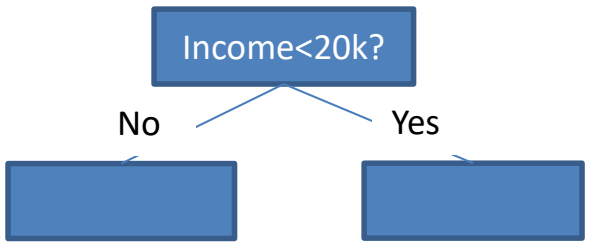
- Tuning parameters
  - Number of hidden layers, m, and number of nodes per layer, $n_1, \ldots, n_m$
  - **Activation functions**, $\sigma(), \Lambda()$ examples
- Let's give it a go in python!

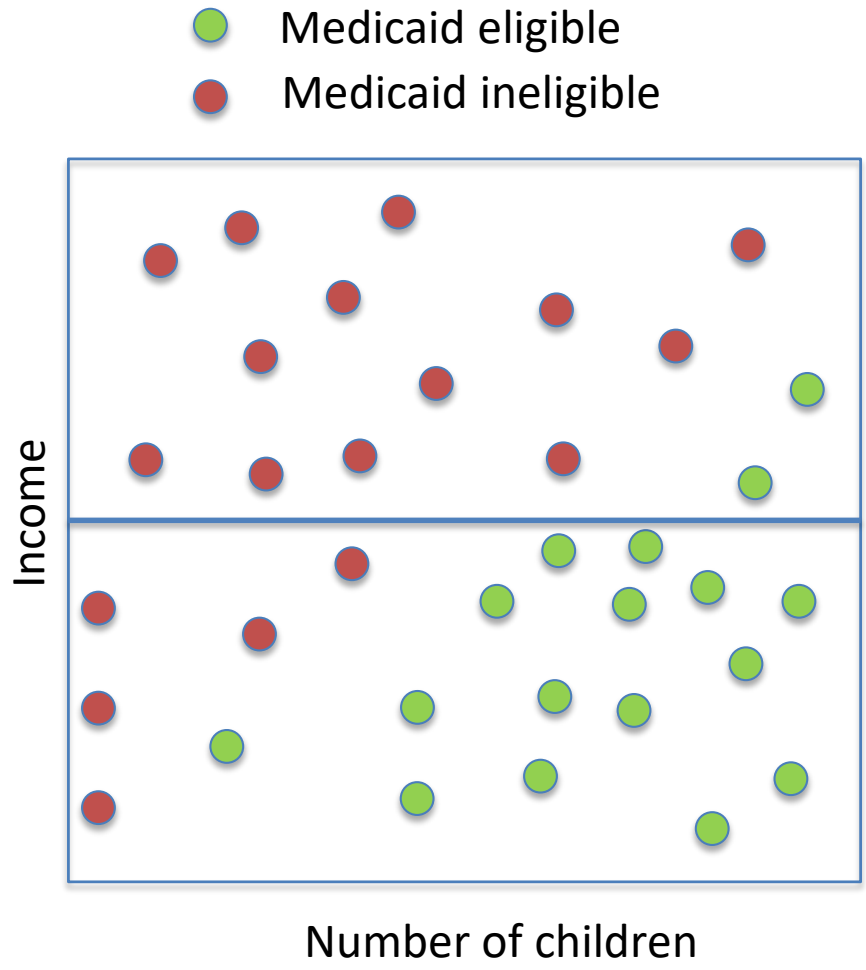
Decision Trees

Initial node

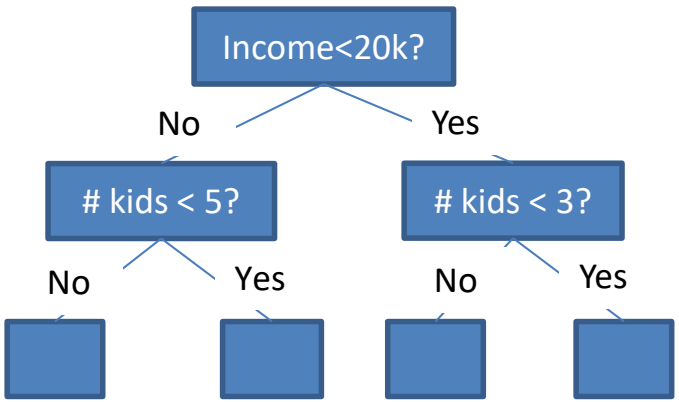


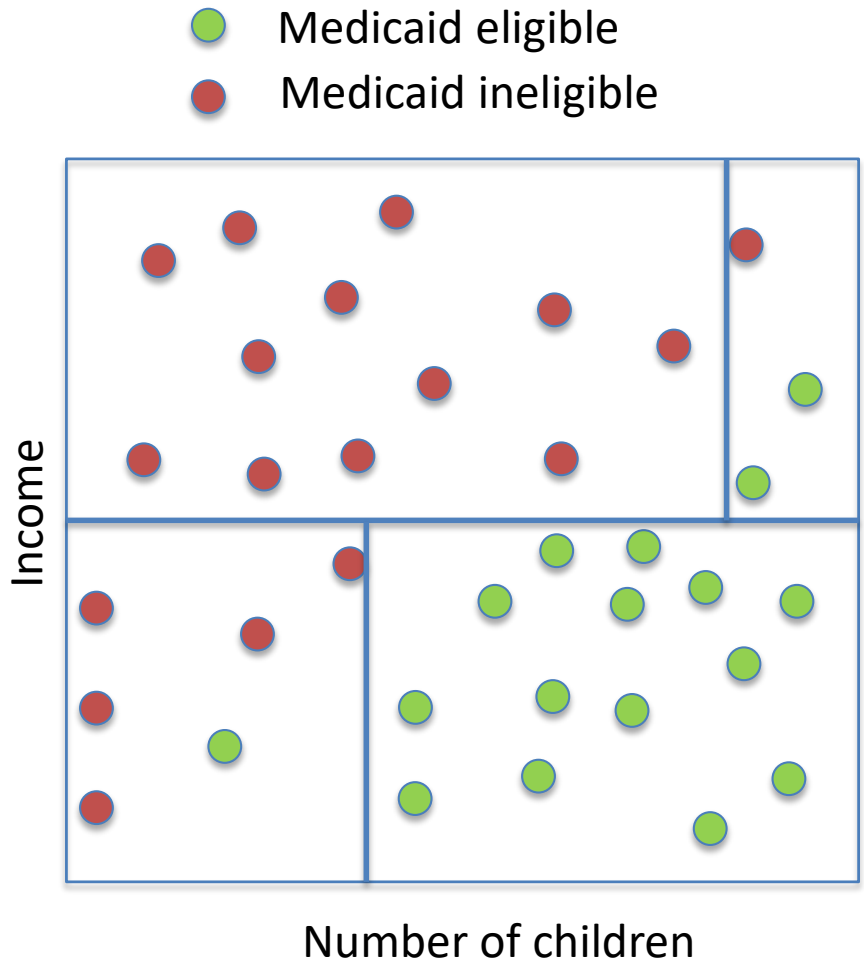
Decision Trees



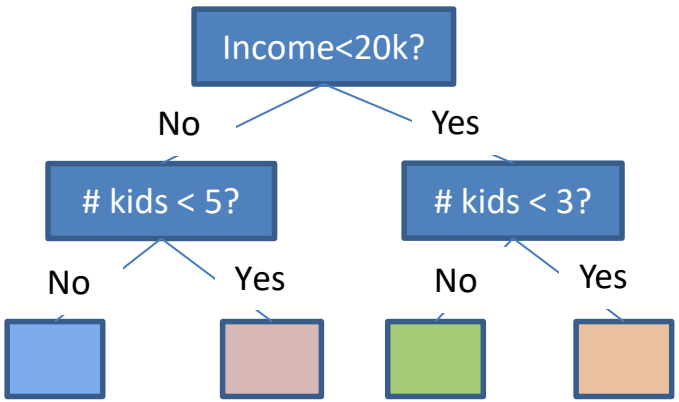


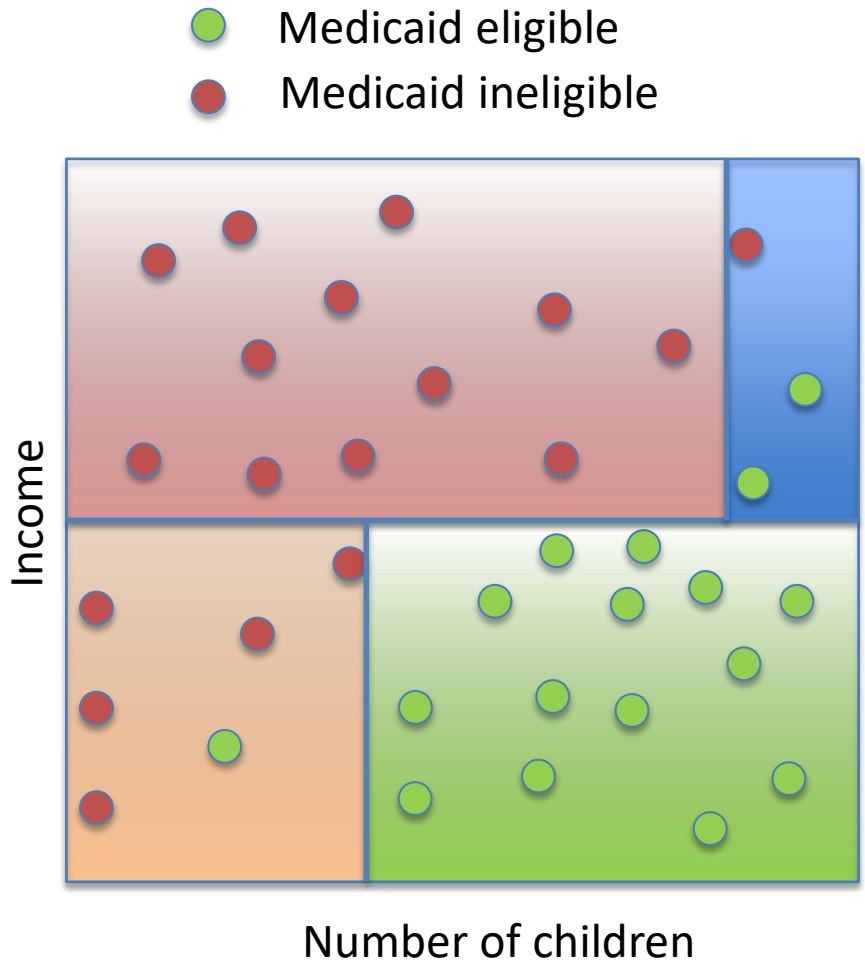
Decision Trees



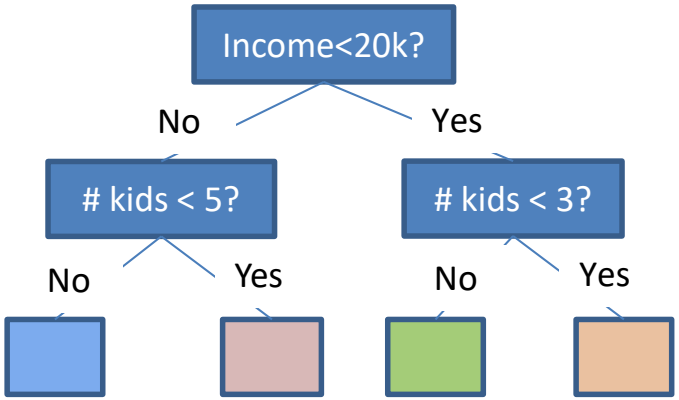


Decision Trees



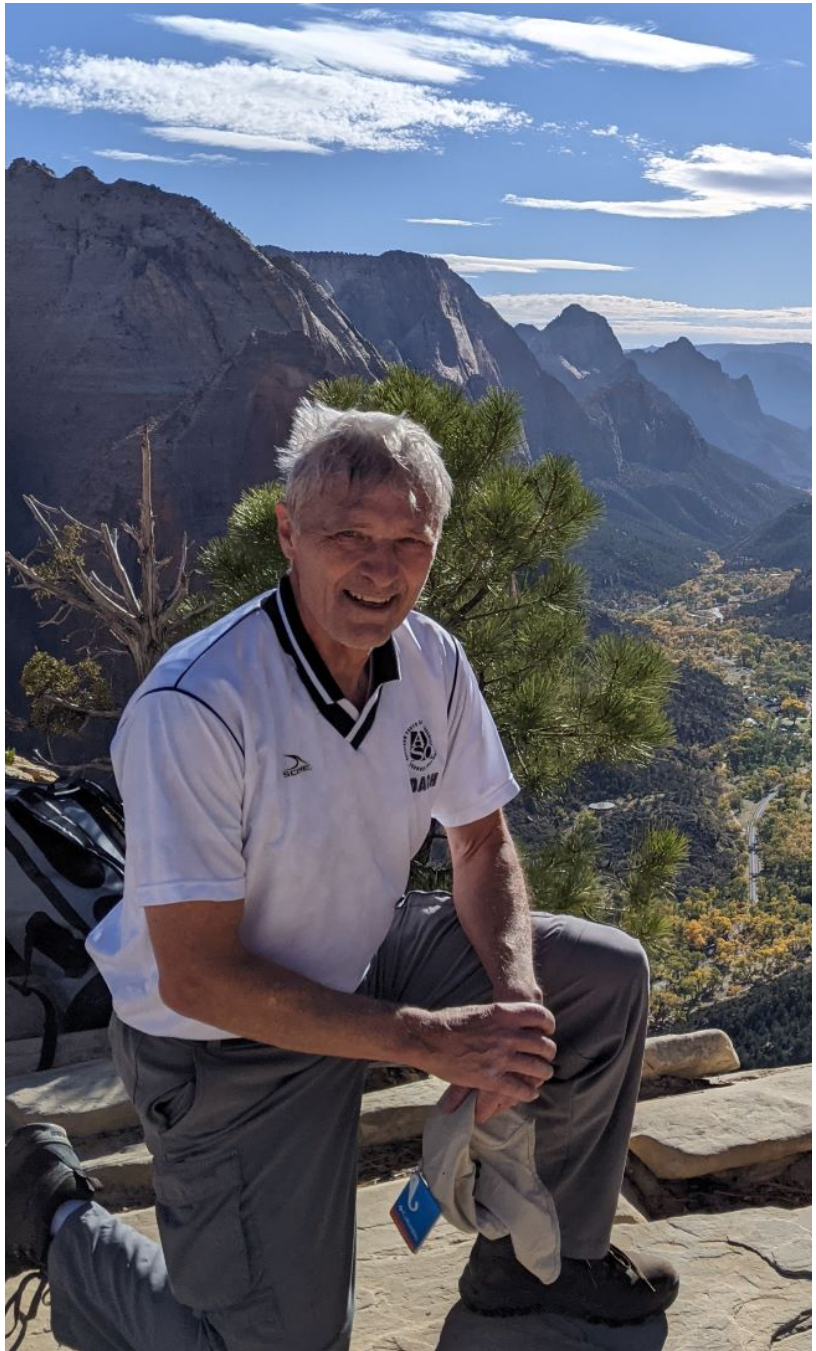


Decision Trees

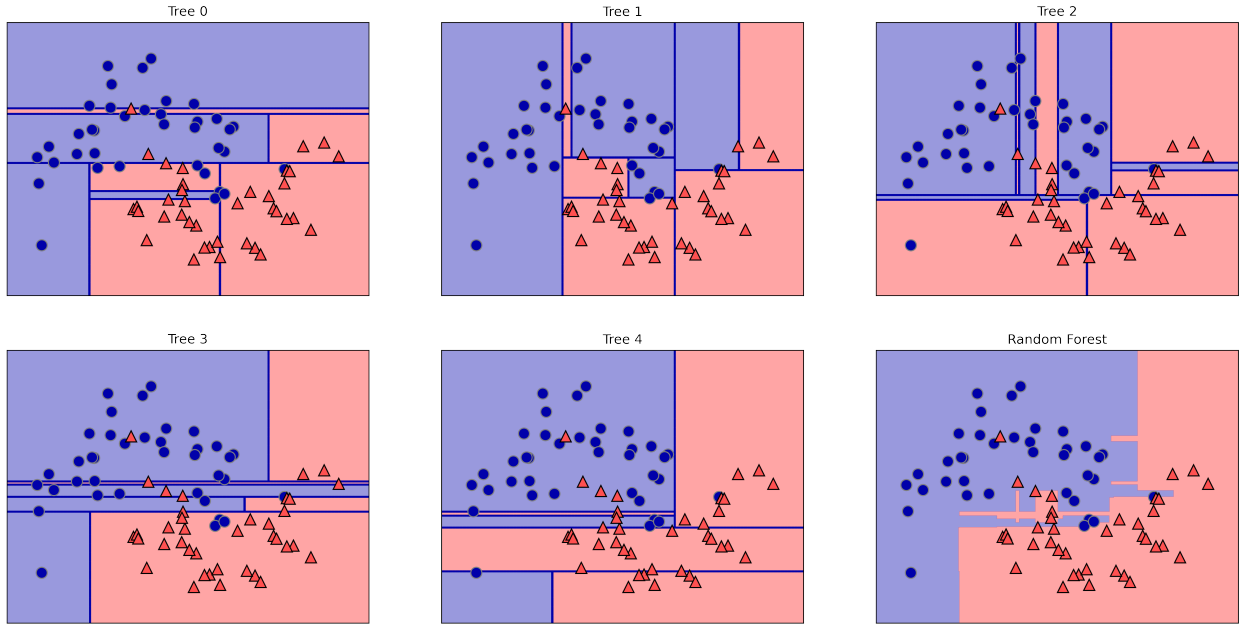


- Where to split:
  Choose the feature from $\{x_1, \ldots, x_p\}$ and the value of that feature to minimize MSE in the resulting child nodes
- Tuning parameters
  - Max depth
  - Min training obs per leaf
  - Min improvement in fit in order to go ahead with a split
- Let's try it in python!

Wisdom of the crowd: predict my father's age!



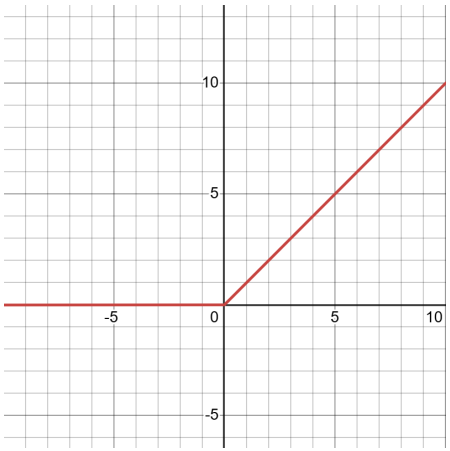
Forest for the Trees



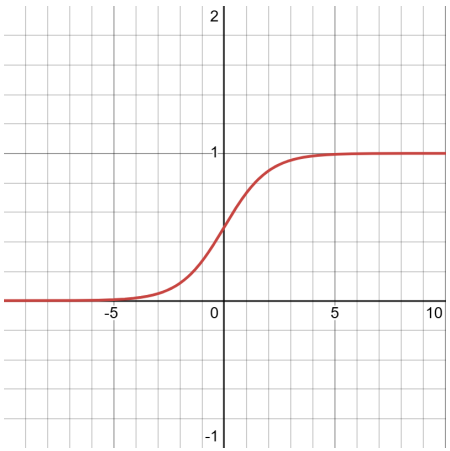
- Value proposition: reduce variance by averaging together multiple predictions
- The catch: individual trees need to be **de-correlated**
- Algorithm:
  - Grow B trees, each on a different bootstrapped sample
  - At each split, consider only a random subset of features
  - Average together the individual predictions
- Let's grow some trees in python!

Neural Network Activation Functions back

- Relu: $\sigma(x) = \max\{0, x\}$



- Logistic: $\sigma(x) = \frac{\exp(x)}{1+\exp(x)}$ (also for output layer for continuous outcomes)



- For output layer, if continuous outcome: $\Lambda(x) = x$