Problem set 0 - suggested solution

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1.1.3 Demand and supply in this market are given by:

$$\textbf{Demand: } \ln Q_t = \beta_1 - \beta_2 \ln P_t + \varepsilon_t^D$$

$$\textbf{Supply: } \ln P_t = \delta \ln Q_t + \ln \mu - \ln a_t$$

Using these equations, we can fully characterize the market equilibrium as a result of shocks and structural parameters representing preferences and technology:

$$\ln Q_t^* = q_t^* = \frac{\beta_1 - \beta_2(\ln \mu - \ln a_t) + \varepsilon_t^D}{1 + \delta\beta_2}$$

$$\ln P_t^* = p_t^* = \frac{\delta\beta_1 + \delta\varepsilon_t^D}{1 + \delta\beta_2} + \ln \mu - \ln a_t$$

The following graph represents the result of simulating the data for $T = 50$[1]

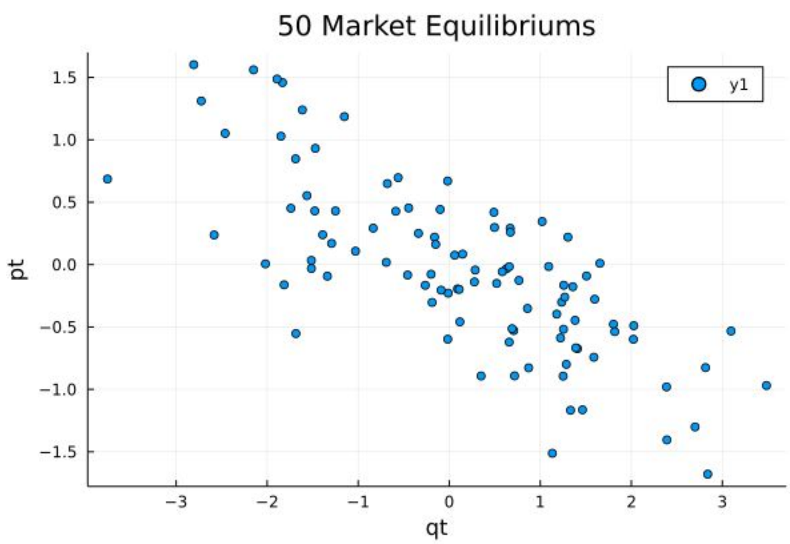

Figure 1: Market equilibrium

[1] Random.seed!(1)

1.2 We can estimate the demand elasticity as $\hat{\beta}_2^{OLS} = (p_t' p_t)^{-1} p_t' q_t$. However, this is not a consistent estimator because the price is a function of the demand shock and therefore $\mathbb{E}(p_t \varepsilon_t^D) = \mathbb{E}(p_t(\varepsilon_t^D)\varepsilon_t^D) \neq 0$. To prove this, we can re-center variables by subtracting their mean and then, using the fact that $\beta_1 = 0$, write the probability limit of the estimator as follows:

$$plim(\hat{\beta}_2^{OLS}) = \frac{plim \sum_i (p_i q_i)}{plim \sum_i (p_i^2)} = \frac{plim \sum_i (\beta_2 p_i^2 + p_i \varepsilon_i^D)}{plim \sum_i (p_i^2)} = \beta_2 + \underbrace{\frac{plim \sum_i (p_i \varepsilon_i^D)/n}{plim \sum_i (p_i^2)/n}}_{\neq 0} = \beta_2 + \underbrace{\frac{\mathbb{E}(p_t \varepsilon_t^D)}{\mathbb{E}(p_t^2)}}_{\neq 0}$$

As the title of the subsection suggests, you can use $\ln Z_t = z_t$ as an instrument for p_t, where you would need to prove that $\mathbb{E}(p_t z_t) \neq 0$. The intuition is that the supply shock z_t will shift only the supply curve, so it will make possible to identify the elasticity of demand β_2. Even though it is not said explicitly, the random variables $\ln Z_t$ and $\ln a_t^u$ are both independent. Notice that the variance of $\ln a_t$ is equal to the sum of the variance of its two components without any covariance term: $\sigma^S = \gamma^2 \sigma^z + \sigma^{\ln a_t^u}$.

How informative is the instrument will depend on the relative contributions of the variance components $\gamma^2 \sigma^z$ and $\sigma^{\ln a_t^u}$. In small samples, the more informative the first component the better the identification of β_2 will be. For IV, you can either run a single regression $\hat{\beta}_2^{IV} = (z_t' p_t)^{-1} z_t' q_t$ or do 2SLS estimating $\hat{p}_t$ using z_t in the first stage ($\hat{p}_t = \hat{\alpha}^{OLS} zt$ with $\alpha^{OLS} = (z_t' z_t)^{-1} z_t' p_t$) and then using $\hat{p}_t$ in the second stage ($\hat{\beta}_2^{IV} = (\hat{p}_t' \hat{p}_t)^{-1} \hat{p}_t' q_t$). The estimated parameters are $\hat{\beta}_2^{OLS} = 1.52$ and $\hat{\beta}_2^{IV} = 1.85$.

1.2.2 The empirical moments $\mathbb{E}[g_1(W,\theta)] = \mathbb{E}[qz]$ and $\mathbb{E}[g_2(W,\theta)] = \mathbb{E}[pz]$ can be calculated after substituting p, q and z for their components. In this case we have

$$\mathbb{E}[g_1(W,\theta)] = \mathbb{E}\left[z_t \frac{\beta_1 - \beta_2(\ln\mu - \ln a_t) + \varepsilon_t^D}{1 + \delta\beta_2}\right] = \frac{\gamma\beta_2\sigma^z}{1 + \delta\beta_2} \tag{1}$$

$$\mathbb{E}[g_2(W,\theta)] = \mathbb{E}\left[z_t \left(\frac{\delta\beta_1 + \delta\varepsilon_t^D}{1 + \delta\beta_2} + \ln\mu - \ln a_t\right)\right] = -\gamma\sigma^z \tag{2}$$

For the simulation I picked $\gamma = 2$ and $\sigma^z = 0.2$. The point estimates for $\theta = \beta_2$ were $\beta_2^{GMM} = 2.16$ and $\beta_2^{GMM-s} = 1.62$ using the theoretical and the simulated moments, respectively. Furthermore, in general, the choice of the norm matters. However, you can check that in this case it doesn't, because once you have γ and σ^z only the first moment is needed to identify β_2. In fact, you can try by estimating β_2^{GMM} with different norms and without the second moment, but the point estimate won't be affected. As an additional test, you can plot the functions you used to estimate β_2 by comparing the data moments with both the simulated and the theoretical moments.

Finally, there is no demand shifter available to recover the supply side parameter δ. If we were doing macro, we probably would be able to assume that some technology shocks follow an auto-regressive process and therefore use quantity in previous periods to recover the supply curve.

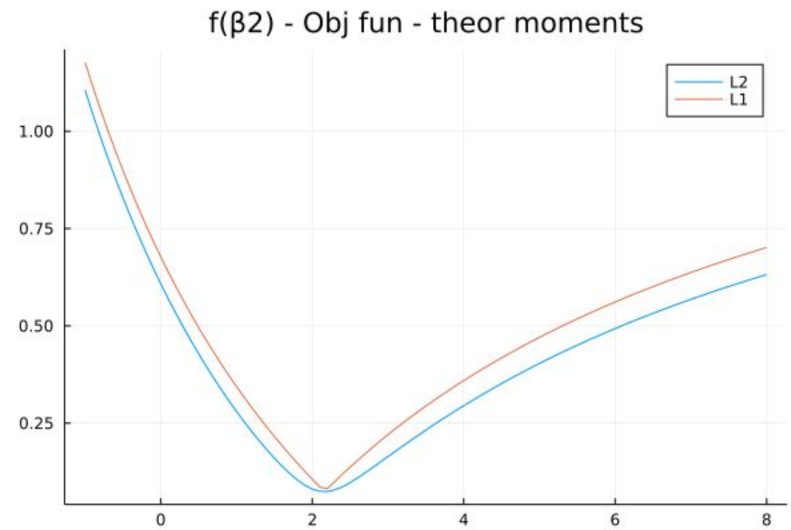

Figure 2: Objective function using theoretical moments

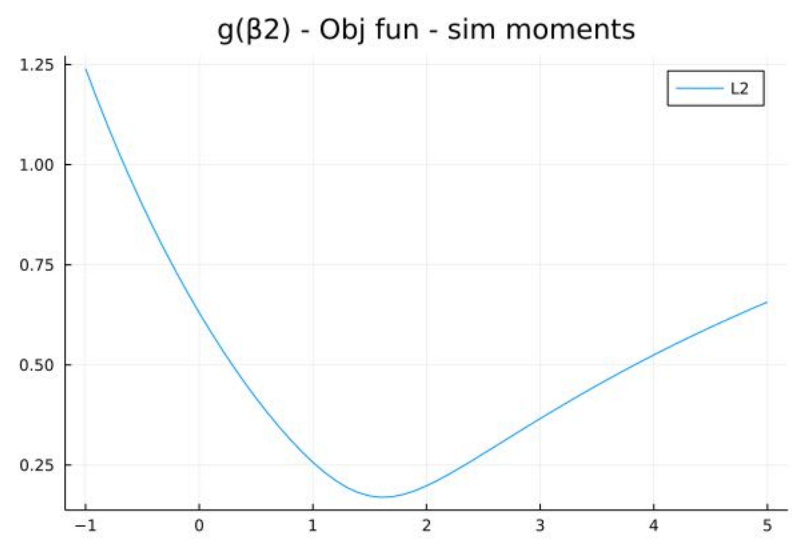

Figure 3: Objective function using simulated moments