

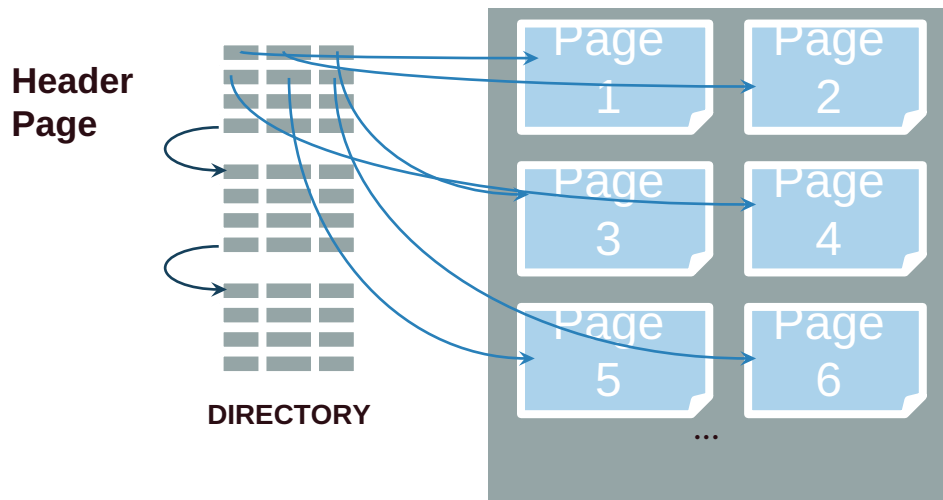
# Tree Indexes



R & G - Chapter 10

# Reminder on Heap Files

- Two access APIs:
  - fetch by recordId (pageId, slotId)
  - scan (starting from some page)

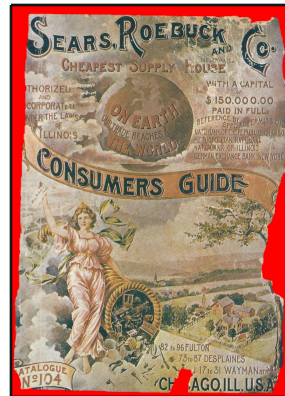


# Wouldn't it be nice...

- ...if we could look things up by value?
- Toward a Declarative access API
- But ... efficiency?

“If you don't find it in the index, look very carefully through the entire catalog. ”

—Sears, Roebuck, and Co., Consumers' Guide, 1897



# We've seen this before

- Data structures ... in RAM:
  - Search trees (Binary, AVL, Red-Black, ...)
  - Hash tables
- Needed: disk-based data structures
  - “paginated”: made up of disk pages!

# Index

An **index** is data structure that enables fast **lookup** and **modification** of **data entries** by **search key**

- **Lookup:** may support many different operations
  - **Equality**, 1-d range, 2-d region, ...
- **Search Key:** any subset of columns in the relation
  - Do not need to be unique
    - —e.g. (firstname) or (firstname, lastname)

# Index Part 2

An **index** is data structure that enables fast **lookup** and **modification** of **data entries** by **search key**

- **Data Entries:** items stored in the index
  - Assume for today: a pair (**k**, recordId) ...
    - Pointers to records in Heap Files!
    - Easy to generalize later
- **Modification:** want to support fast insert and delete

Many Types of indexes exist: B+-Tree, Hash, R-Tree, GiST, ...



# Simple Idea?

Input  
Heap  
File

3, 4, 5

1, 2, 7

8, 6, 9

10, \_, \_

- **Step 1:** Sort heap file & leave some space
  - Pages physically stored in logical order (sequential access)
  - Do we need “next” pointers to link pages?
    - No. Pages are physically sorted in logical order

1, 2, \_

3, 4, \_

5, 6, \_

7, 8, \_

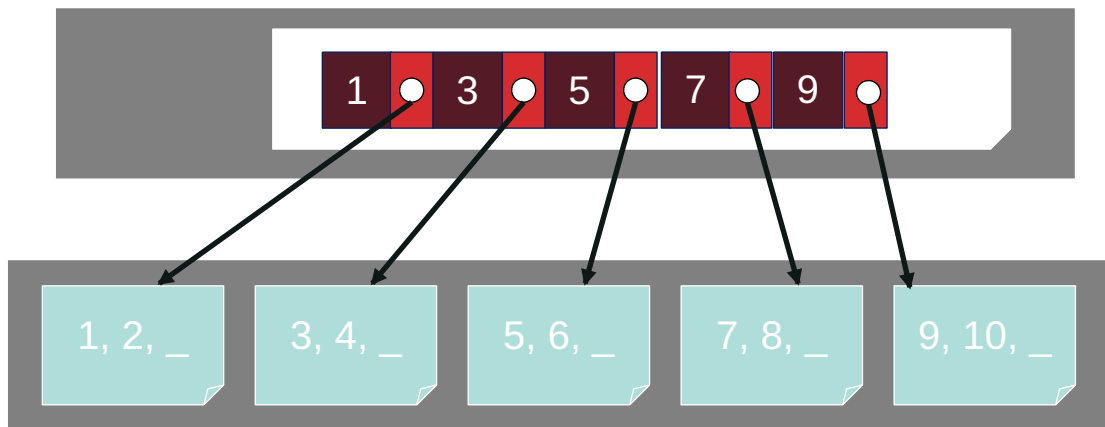
9, 10, \_

- **Step 2:** Build the index data structure over this...
  - Why not just use binary search in this heap file?
    - Fan-out of 2 → deep tree → lots of I/Os
    - Examine entire records just to read key during search



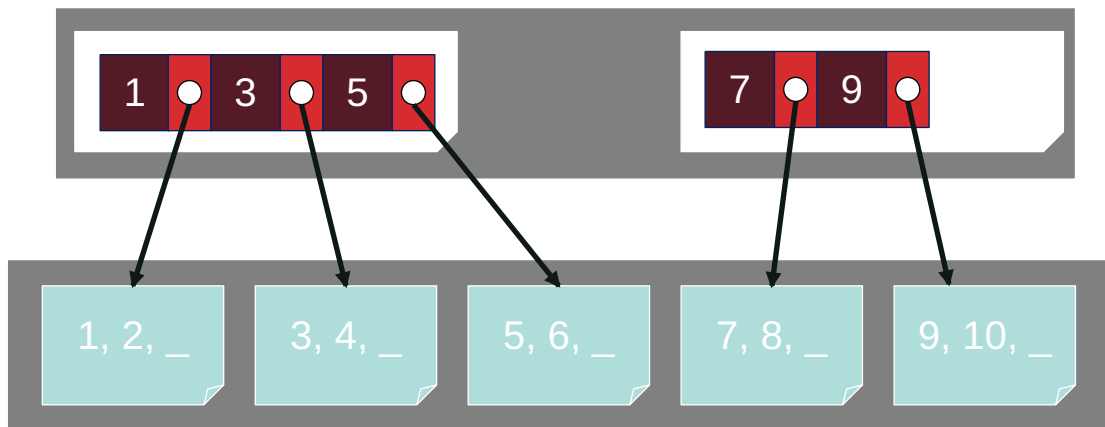
# Build a high fan-out search tree

- Start simple: *Sorted (key, page id) file*
  - No record data
  - Binary search in the key file. Better!
  - **Forgot:** Need to break across pages!



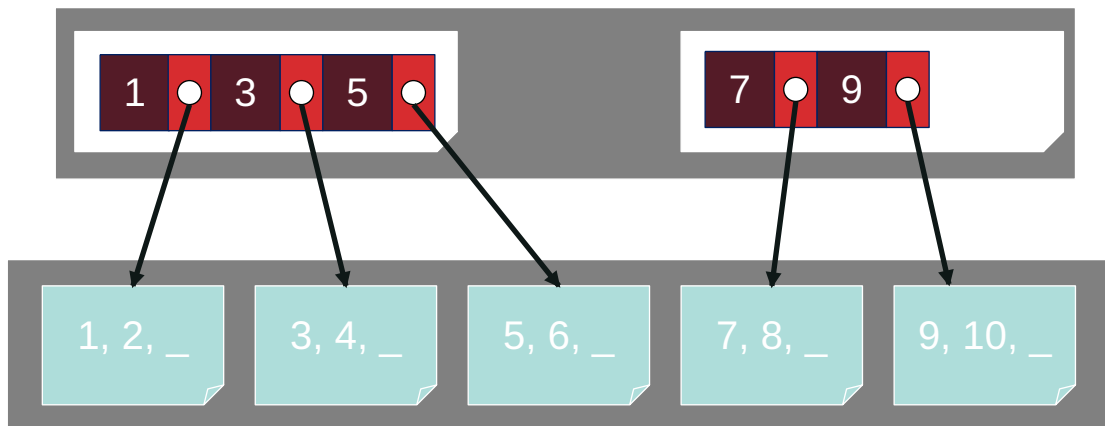
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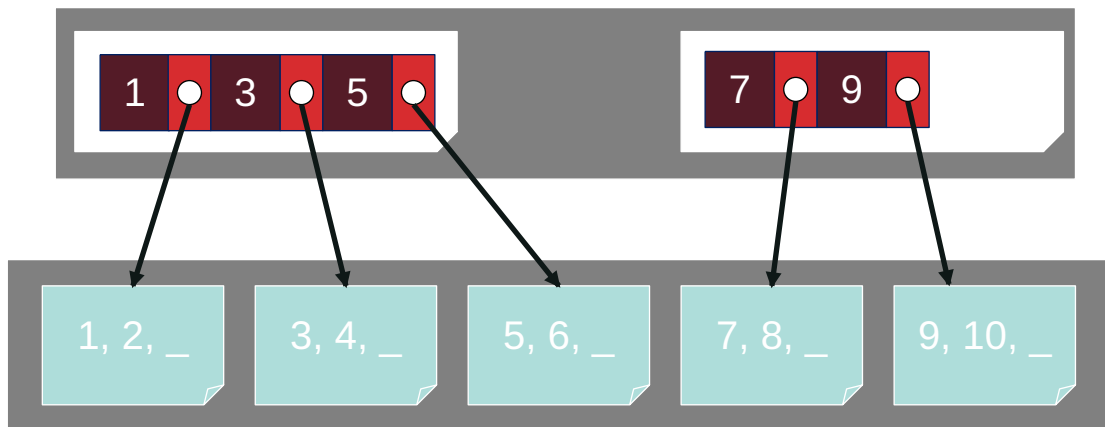
# Build a high fan-out search tree Part 2

- Start simple: *Sorted (key, page id) file*
  - No record data
  - Binary search in the key file. Better!
  - **Complexity?**



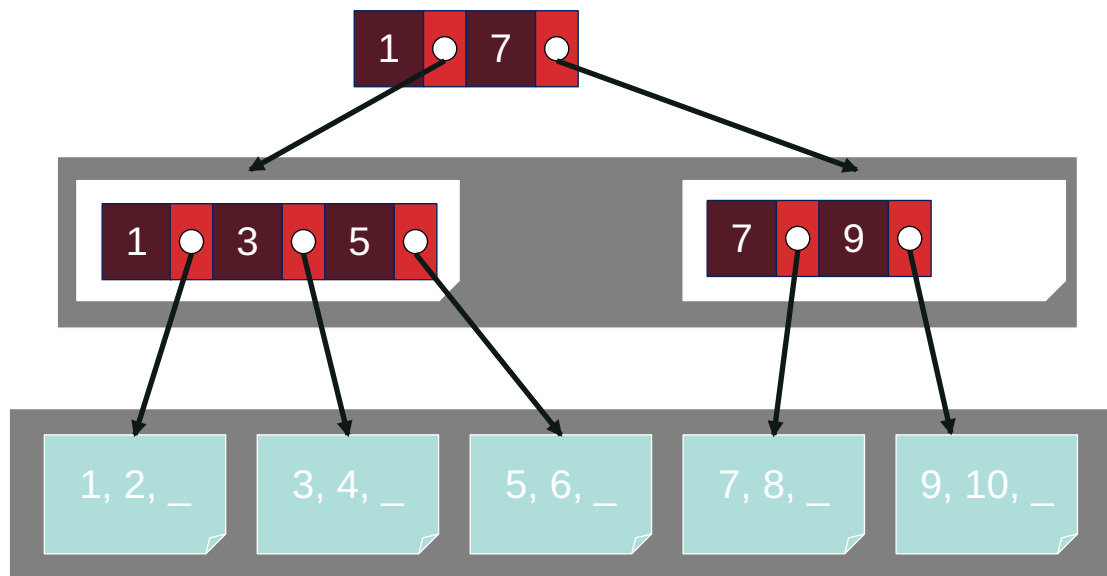
# Build a high fan-out search tree Part 3

- Start simple: *Sorted (key, page id) file*
  - No record data
  - Binary search in the key file. Better!
  - **Complexity:** Still binary search, just a constant factor smaller input



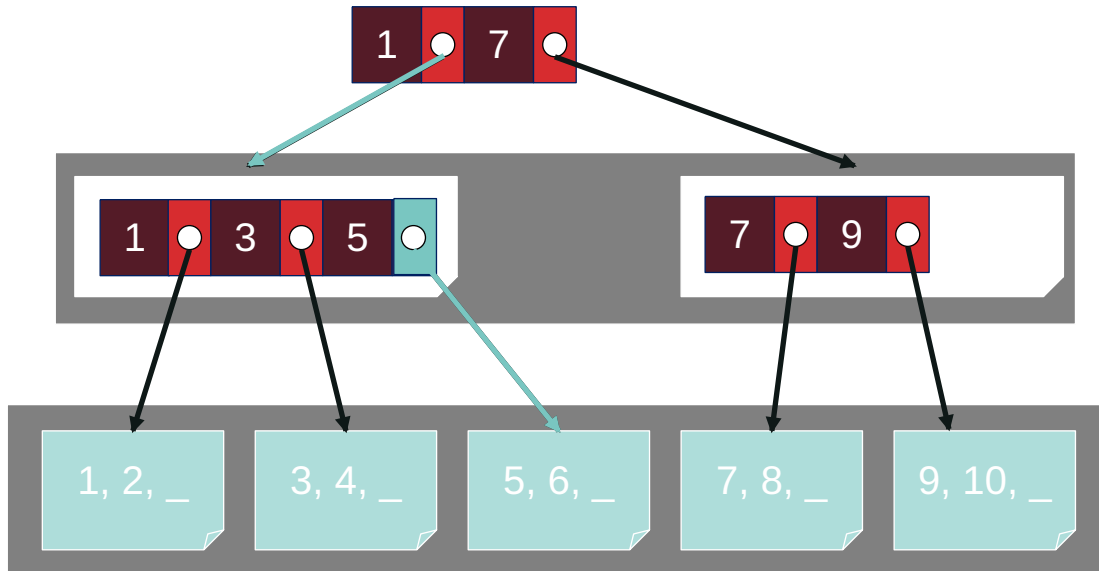
# Build a high fan-out search tree Part 4

- Recursively “index” key file
- **Key Invariant:**
  - Node  $[..., (K_L, P_L), (K_R, P_R), ...] \rightarrow$  All tuples in range  $K_L \leq K < K_R$  are in tree  $P_L$



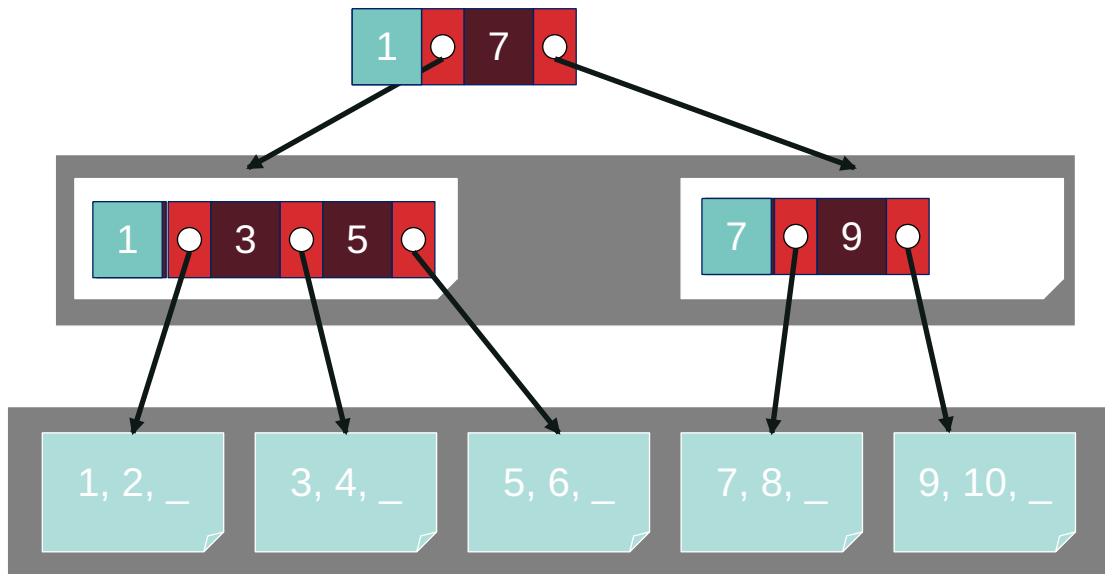
# Search a high fan-out search tree

- Searching for **5**?
  - Binary Search each node (page) starting at root
  - Follow pointers to next level of search tree
- Complexity?  $O(\log_F(\#\text{Pages}))$



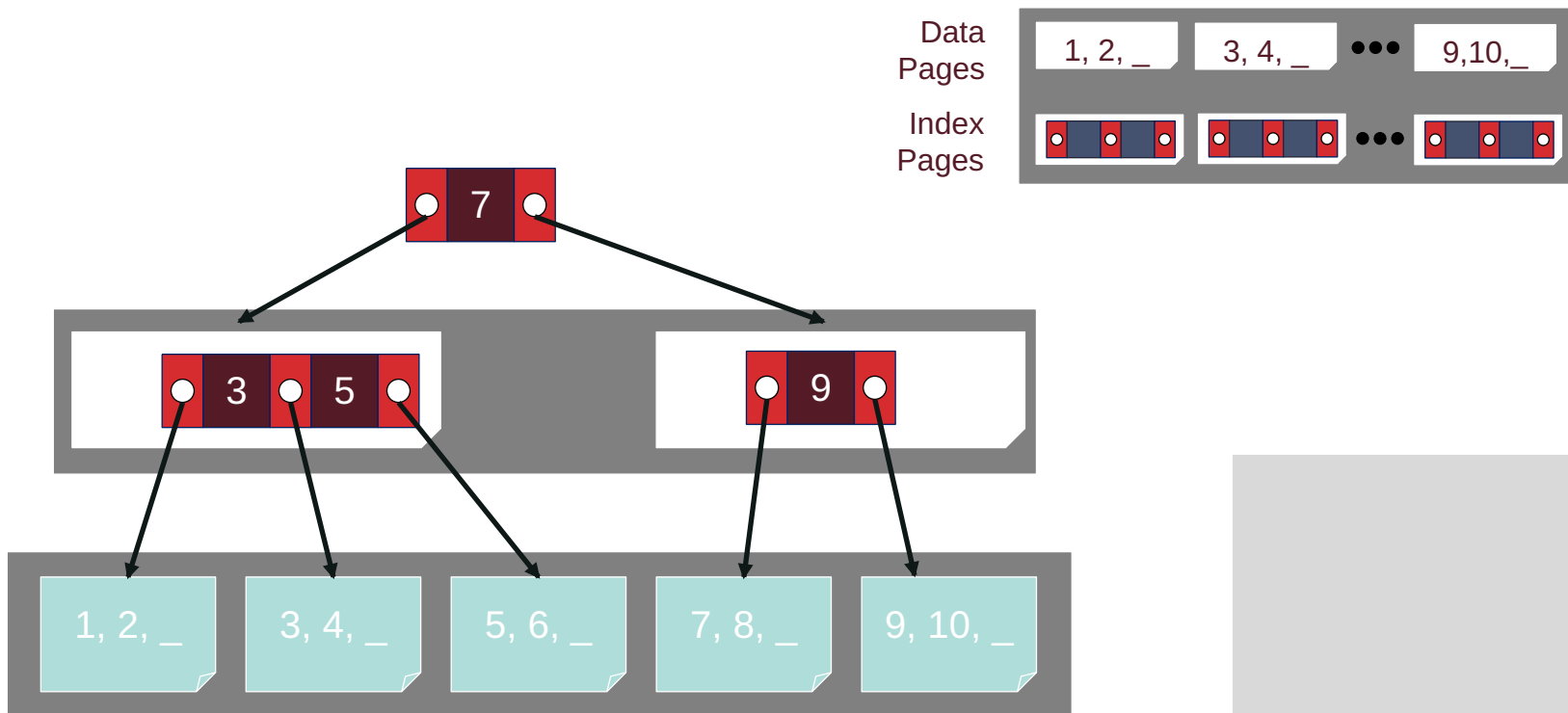
# Left Key Optimization?

- Optimization
  - Do we need the left most key?



# Build a high fan-out search tree

- Disk Layout? All in a single file, Data Pages first.





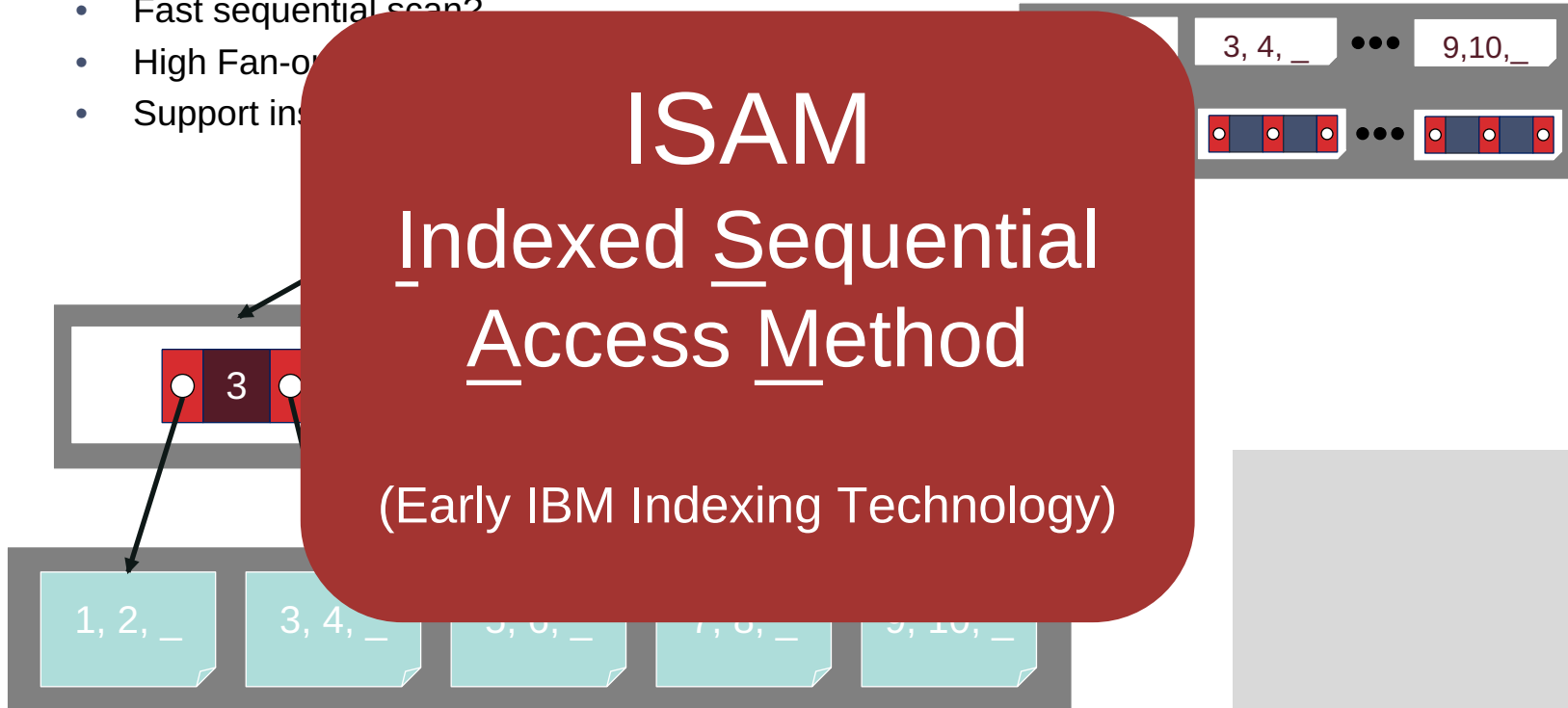
# Status Check

- Some design goals:
  - Fast sequential scan?
  - High Fan-out
  - Support insertions

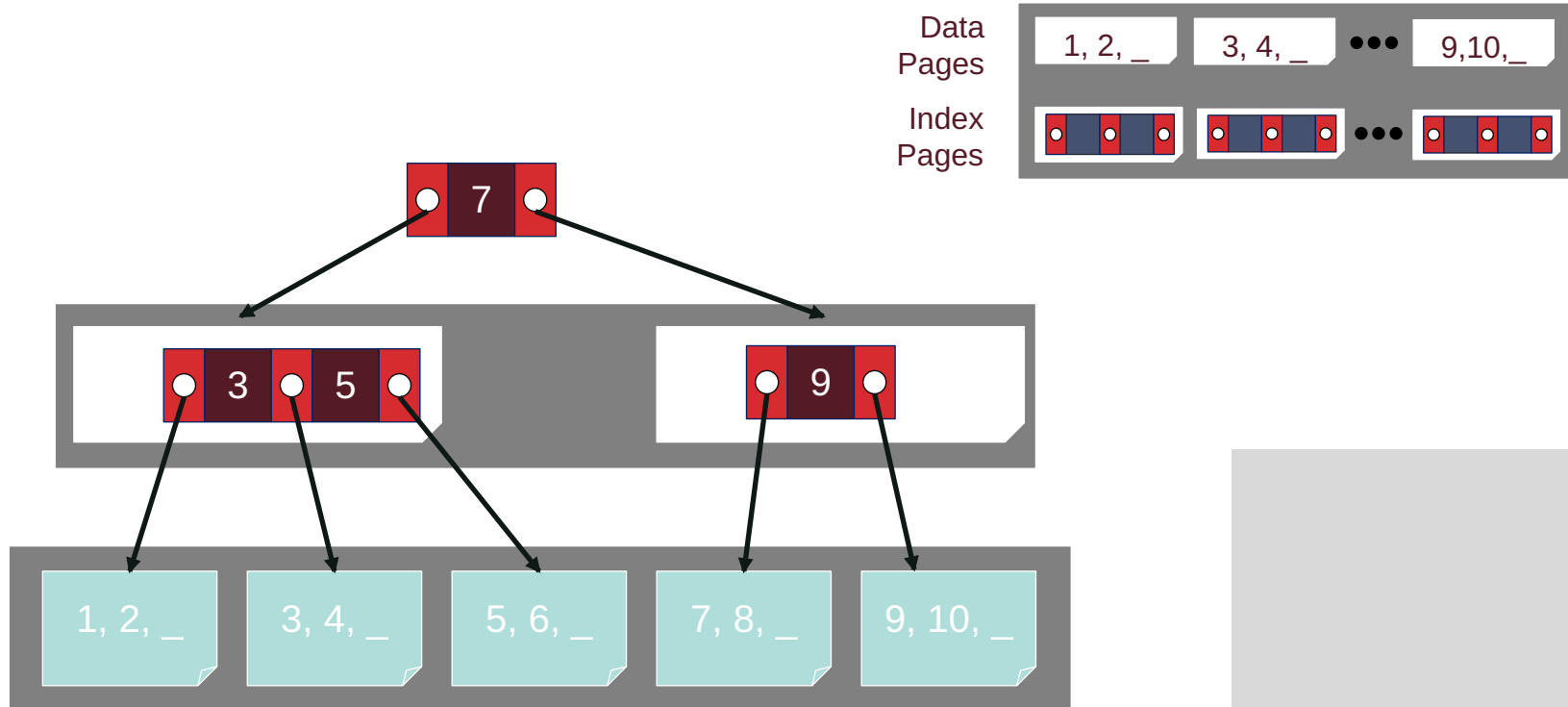
## ISAM Indexed Sequential Access Method

(Early IBM Indexing Technology)

Indexed File

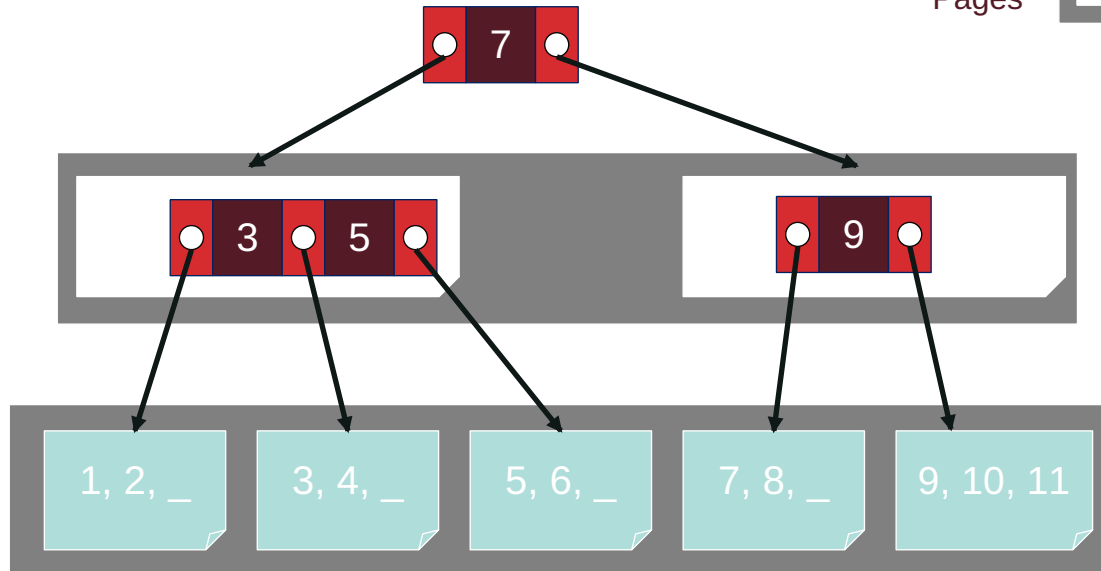


# Insert 11, Before



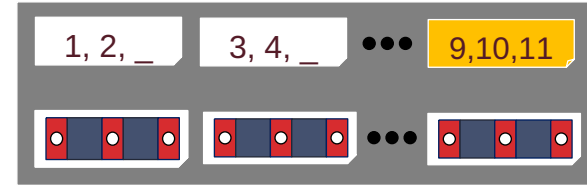
# Insert 11, After

- Find location
- Place in data page
  - Re-sort page ...



Data  
Pages

Index  
Pages



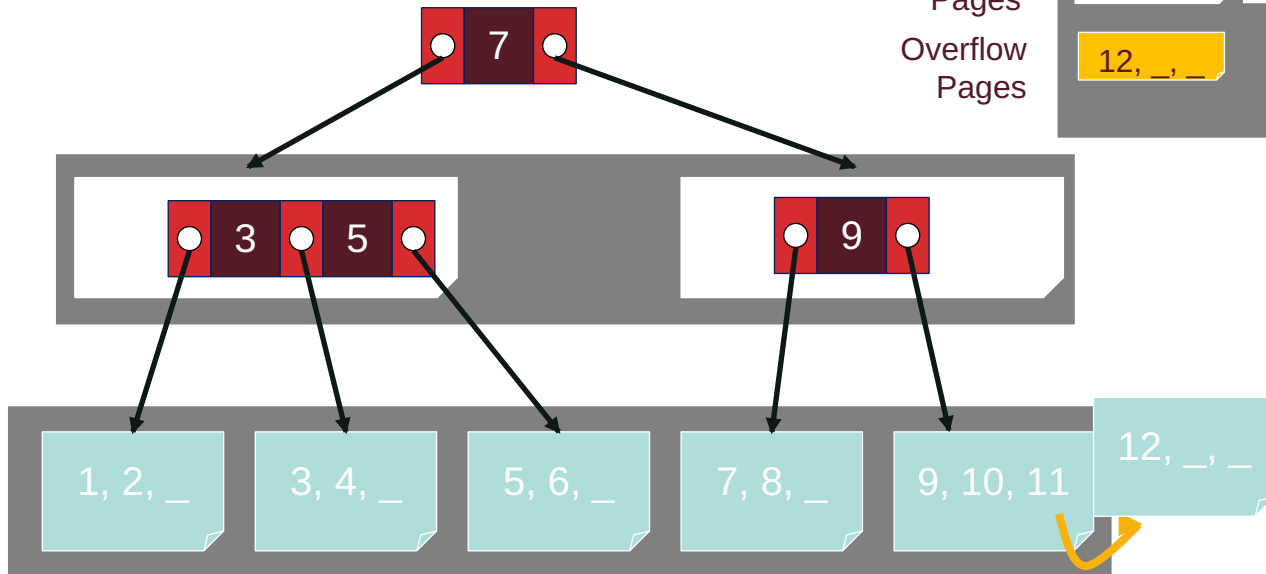
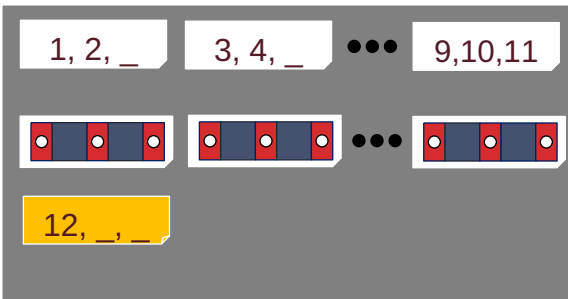
# Insert 12?

- Find location
- Place in data page
- Add overflow page if necessary ...

Data  
Pages

Index  
Pages

Overflow  
Pages



# Recap: ISAM

- Data entries in sorted heap file
- High fan-out static tree index
- Fast search + good locality
  - Assuming nothing changes
- Insert into overflow pages

# A Note of Caution

- ISAM is an old-fashioned idea
  - Introduced by IBM in 1960s
  - B+ trees are usually better, as we'll see
    - Though not always (← we'll come back to this)
- But, it's a good place to start
  - Simpler than B+ tree, many of the same ideas
- Upshot
  - Don't brag about ISAM on your resume
  - Do understand ISAM, and tradeoffs with B+ trees



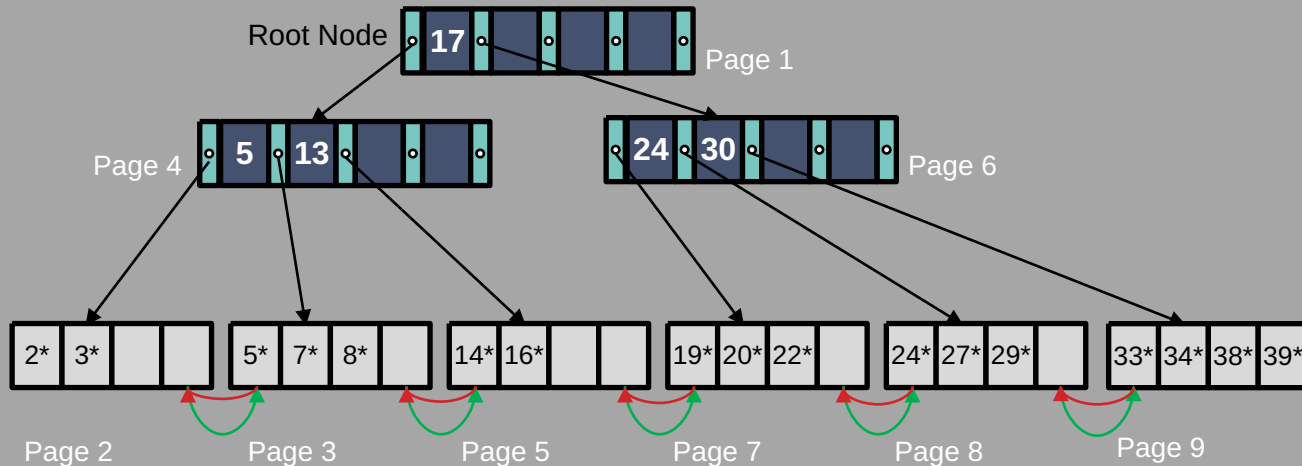
# B+-TREE



# Enter the B+ Tree

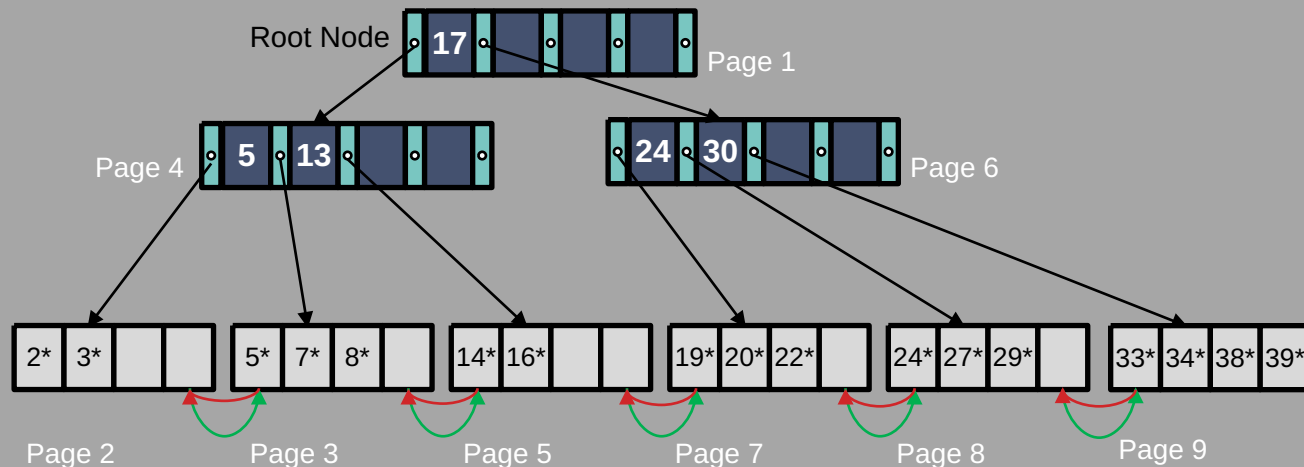
- Similar to ISAM
  - Same interior node structure
    - $\langle \text{Key}, \text{Page Ptr} \rangle$  pairs with same key invariant
  - Same search routine as before
- **Dynamic Tree Index**
  - Always Balanced
  - Support efficient insertion & deletion
    - Grows at root not leaves!
- “+”? B-tree that stores data entries in leaves only

# Example of a B+ Tree



- Occupancy Invariant
  - Each interior node is at least partially full:
    - $d \leq \text{\#entries} \leq 2d$
    - $d$ : order of the tree (max fan-out =  $2d + 1$ )
- Data pages at bottom need not be stored in logical order
  - Next and prev pointers

# Sanity Check



What is the value of d?

2

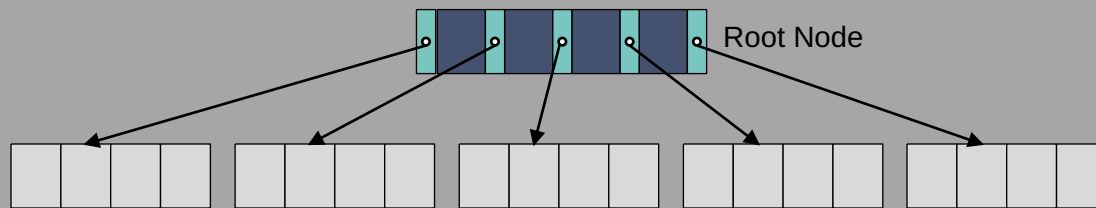
What about the root?

The root is special

Why not in sequential order?

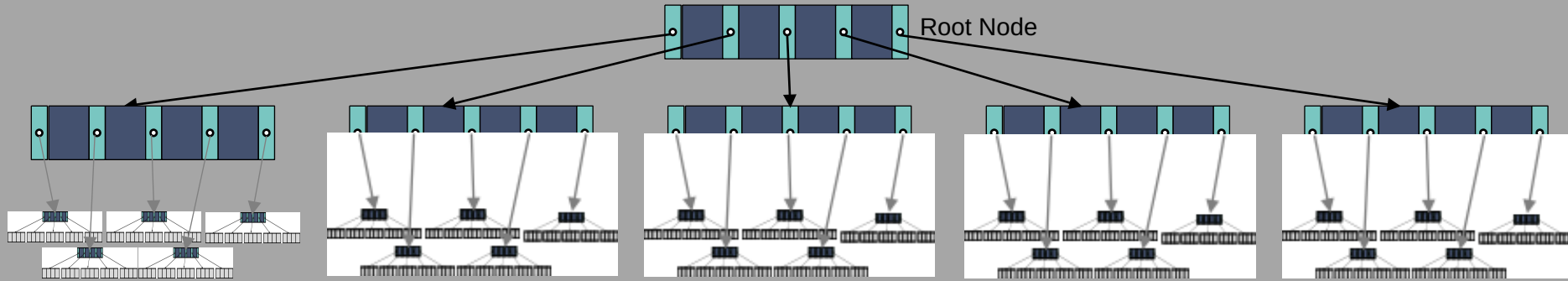
Data pages allocated dynamically

# B+ Trees and Scale



- How big is a height 1 B+ tree
  - $d = 2 \rightarrow$  Fan-out?
  - Fan-out =  $2d + 1 = 5$
  - **Height 1:**  $5 \times 4 = 20$  Records

# B+ Trees and Scale Part 2



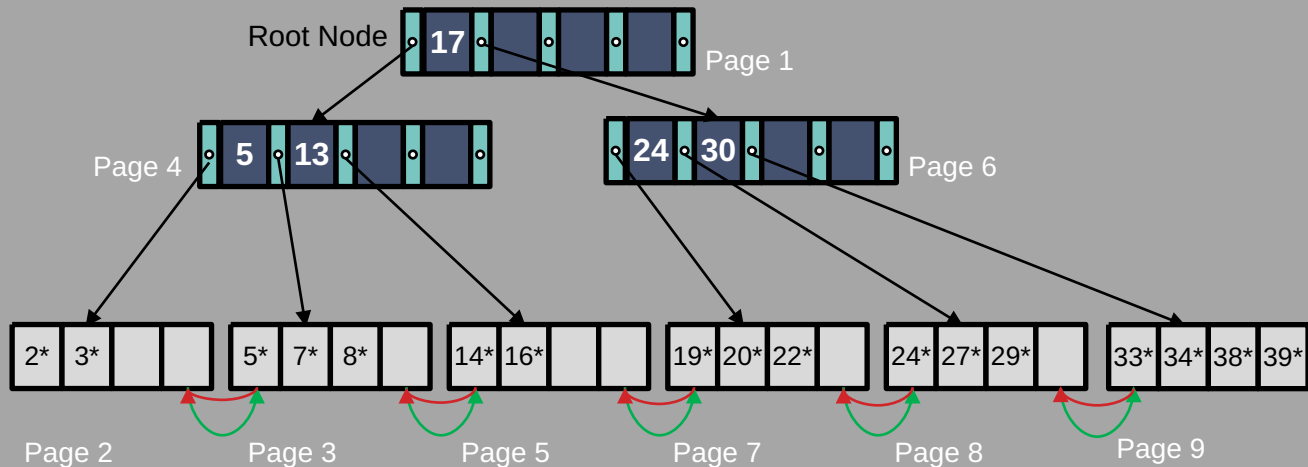
- How big is a height **3** B+ tree
  - $d = 2 \rightarrow$  Fan-out?
  - Fan-out =  $2d + 1 = 5$
  - **Height 3:**  $5^3 \times 4 = 500$  Records

# B+ Trees in Practice

- Typical order: 1600. Typical fill-factor: 67%.
  - average fan-out = 2144
  - (assuming 128 Kbytes pages at 40Bytes per record)
- At typical capacities
  - Height 1:  $2144^2 = \mathbf{4,596,736 \text{ records}}$
  - Height 2:  $2144^3 = \mathbf{9,855,401,984 \text{ records}}$



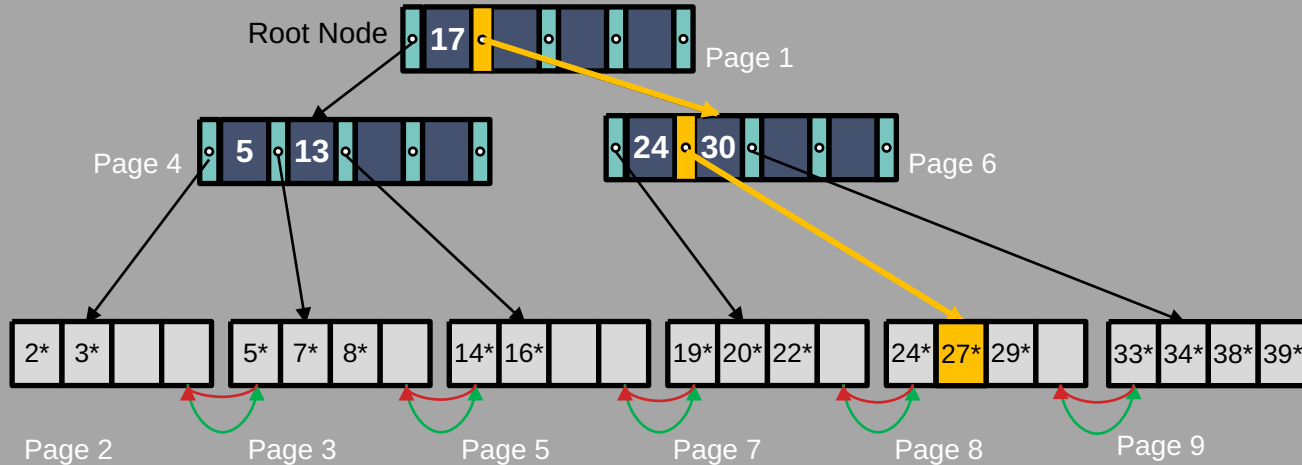
# Searching the B+ Tree



- Same as ISAM
- Find key = 27
  - Find split on each node (Binary Search)
  - Follow pointer to next node

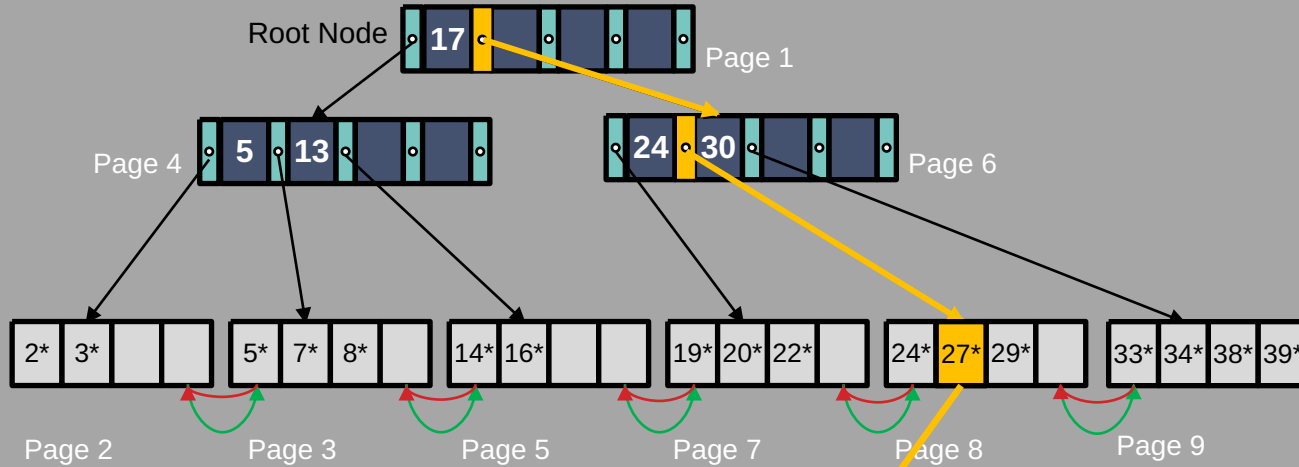


# Searching the B+ Tree: Find 27



- Same as ISAM
- Find key = 27
  - Find split on each node (Binary Search)
  - Follow pointer to next node

# Searching the B+ Tree: Fetch Data



Page 1

(20, Tim)	(7, Dan)
-----------	----------

Page 2

(5, Kay)	(3, Jim)
----------	----------

Page 3

(27, Joe)	(34, Kit)
-----------	-----------

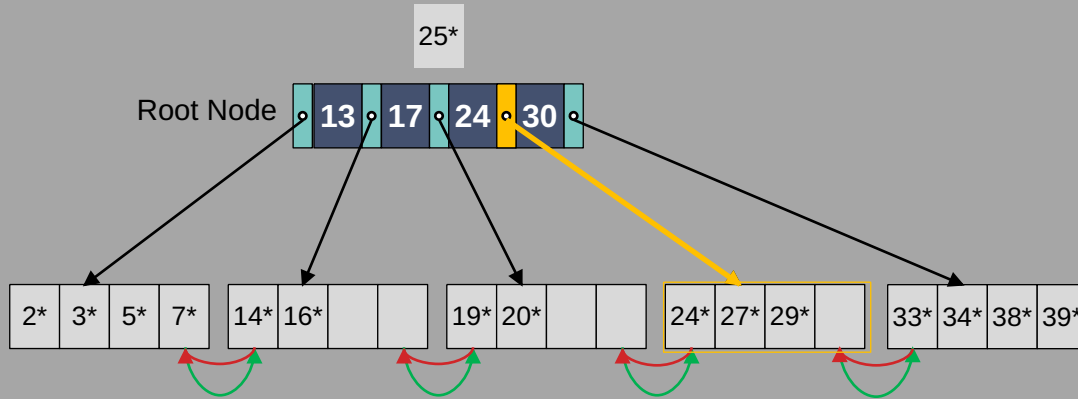
Page 4

(1, Kim)	(42, Hal)
----------	-----------

Page Id,  
Record Id

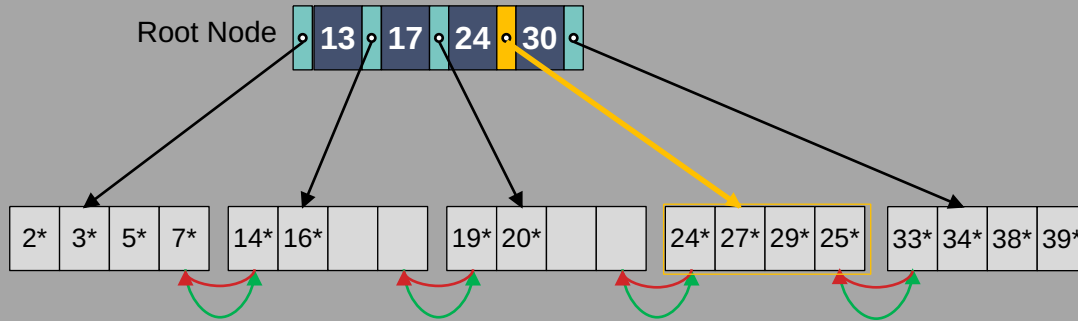


# Inserting 25\* into a B+ Tree Part 1



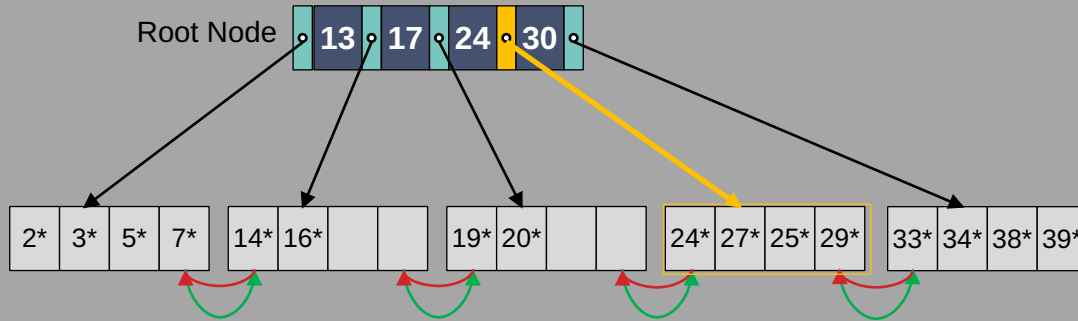
- Find the correct leaf

# Inserting 25\* into a B+ Tree Part 2



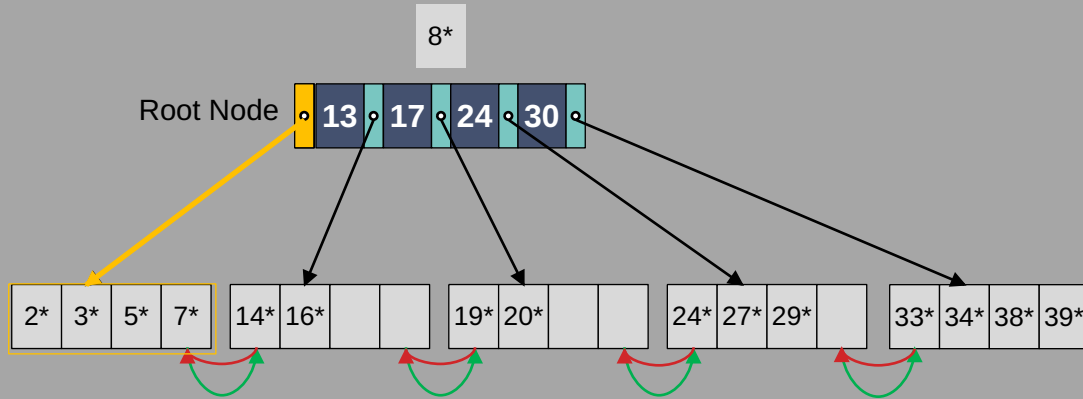
- Find the correct leaf
- If there is room in the leaf just add the entry

# Inserting 25\* into a B+ Tree Part 3



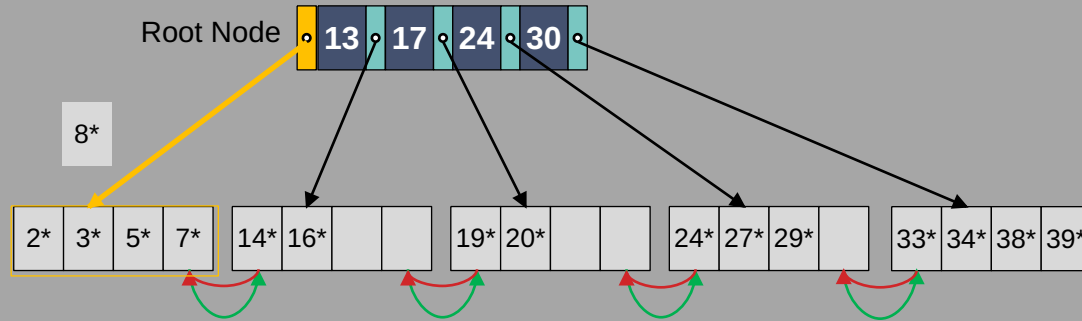
- Find the correct leaf
- If there is room in the leaf just add the entry
  - Sort the leaf page by key

# Inserting 8\* into a B+ Tree: Find Leaf



- Find the correct leaf

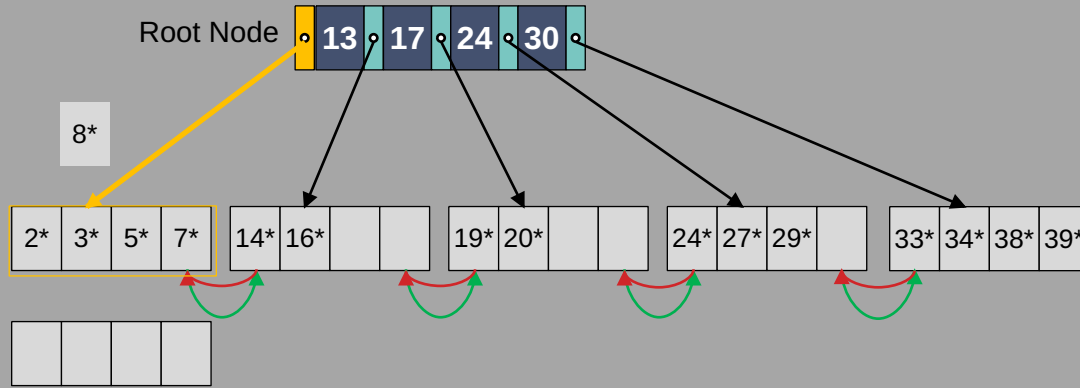
# Inserting 8\* into a B+ Tree: Insert



- Find the correct leaf
  - Split leaf if there is not enough room

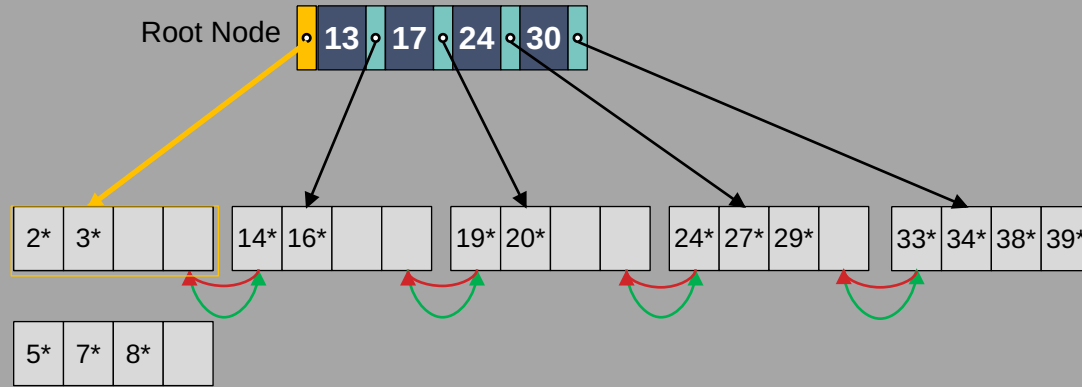


# Inserting 8\* into a B+ Tree: Split Leaf



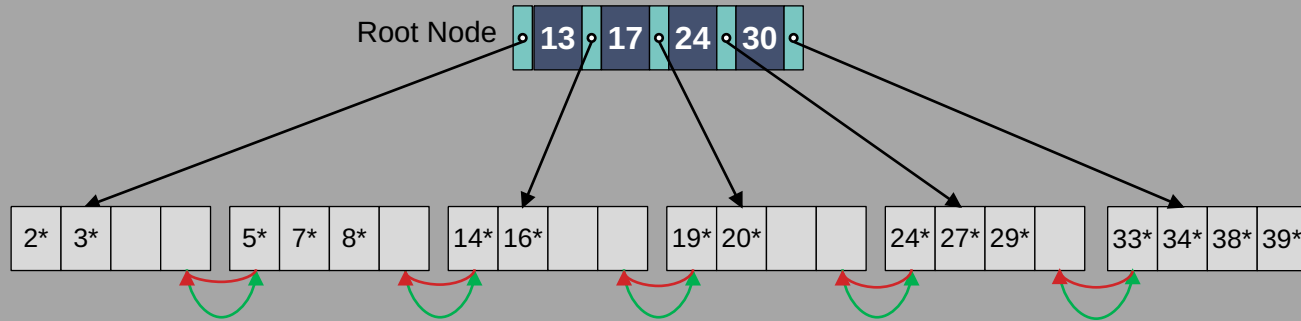
- Find the correct leaf
  - Split leaf if there is not enough room
  - Redistribute entries evenly

# Inserting 8\* into a B+ Tree: Split Leaf, cont



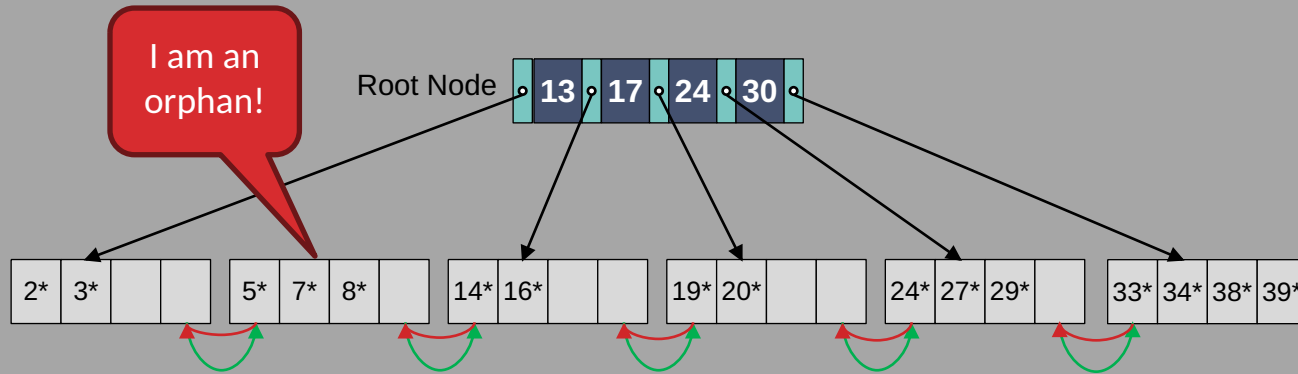
- Find the correct leaf
  - Split leaf if there is not enough room
  - Redistribute entries evenly
  - Fix next/prev pointers

# Inserting 8\* into a B+ Tree: Fix Pointers



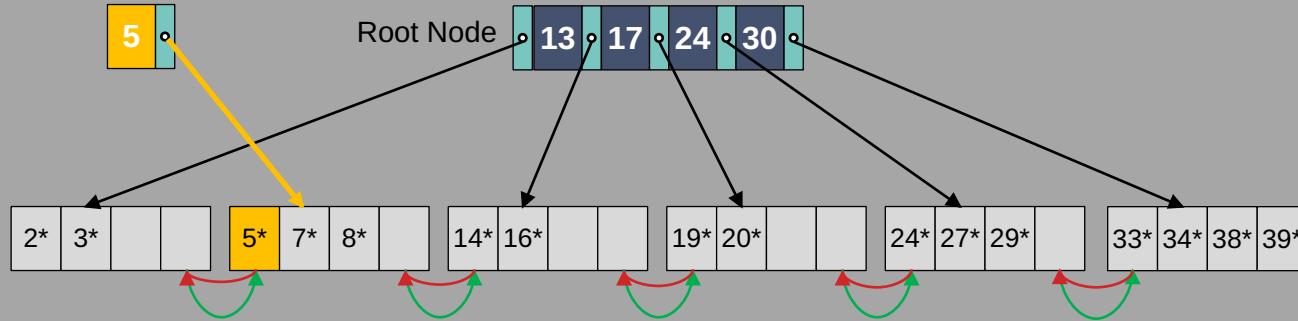
- Find the correct leaf
  - Split leaf if there is not enough room
  - Redistribute entries evenly
  - Fix next/prev pointers

# Inserting 8\* into a B+ Tree: Mid-Flight



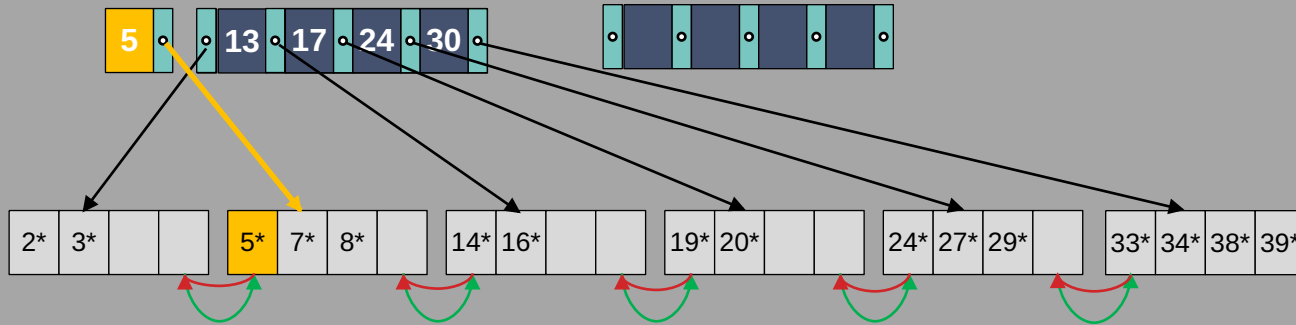
- Something is still wrong!

# Inserting 8\* into a B+ Tree: Copy Middle Key



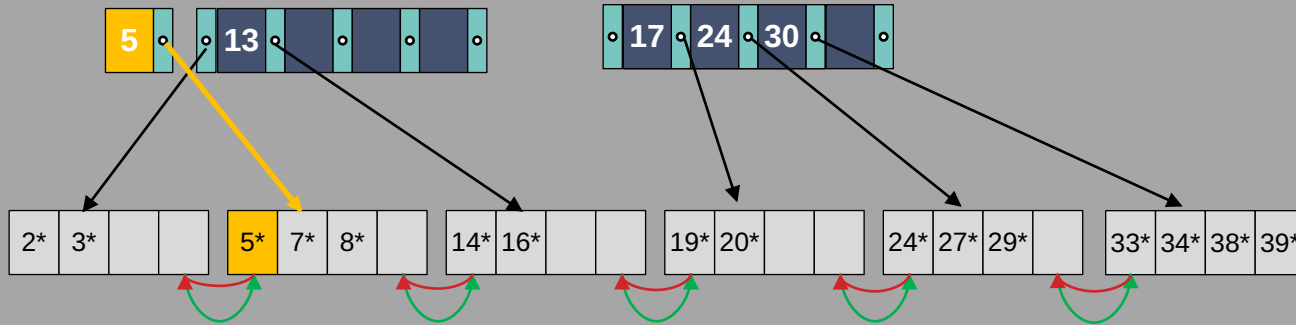
- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes

# Inserting 8\* into a B+ Tree: Split Parent, Part 1



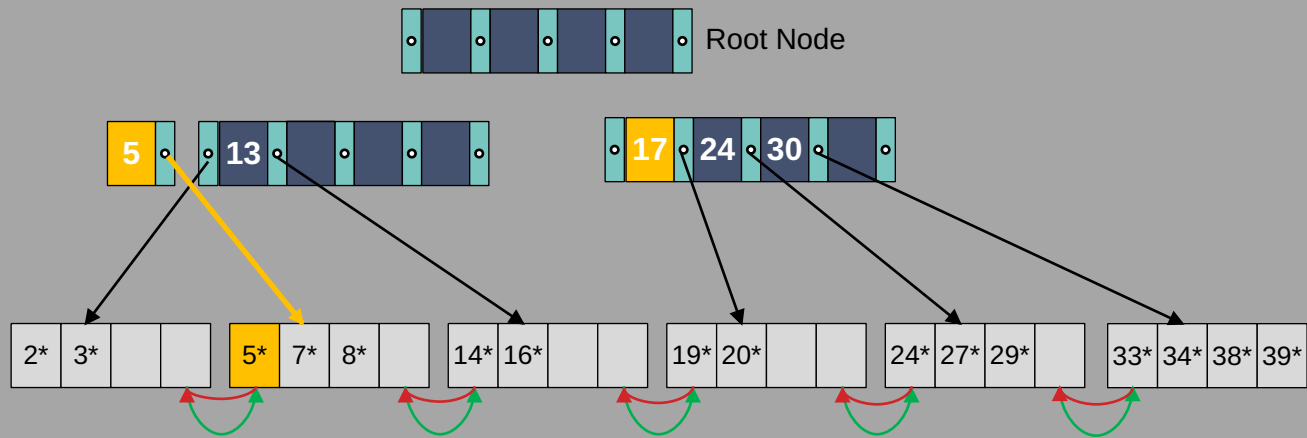
- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes
  - Redistribute the rightmost  $d$  keys

# Inserting 8\* into a B+ Tree: Split Parent, Part 2



- **Copy up from leaf** the middle key
- No room in parent? Recursively split index nodes
  - Redistribute the rightmost  $d$  keys

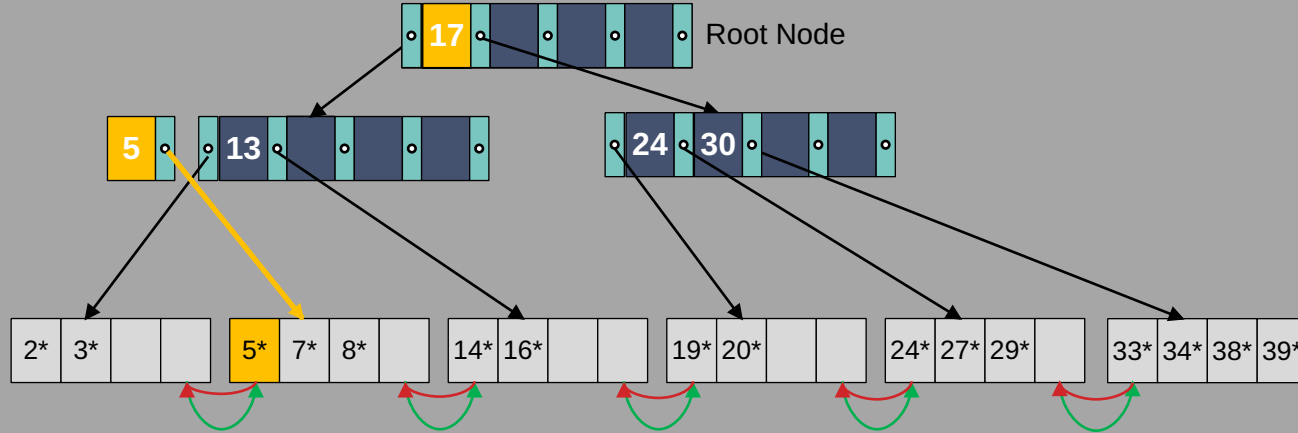
# Inserting 8\* into a B+ Tree: Root Grows Up



- **Push up from interior node** the middle key
  - Now the last key on left
- No room in parent? Recursively split index nodes
  - Redistribute the rightmost  $d$  keys

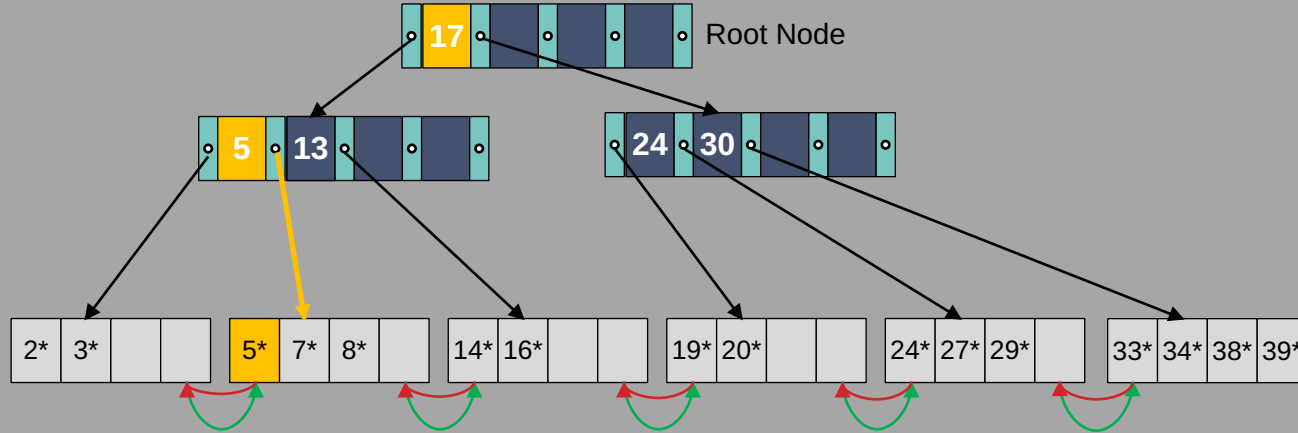


# Inserting 8\* into a B+ Tree: Root Grows Up, Pt 2



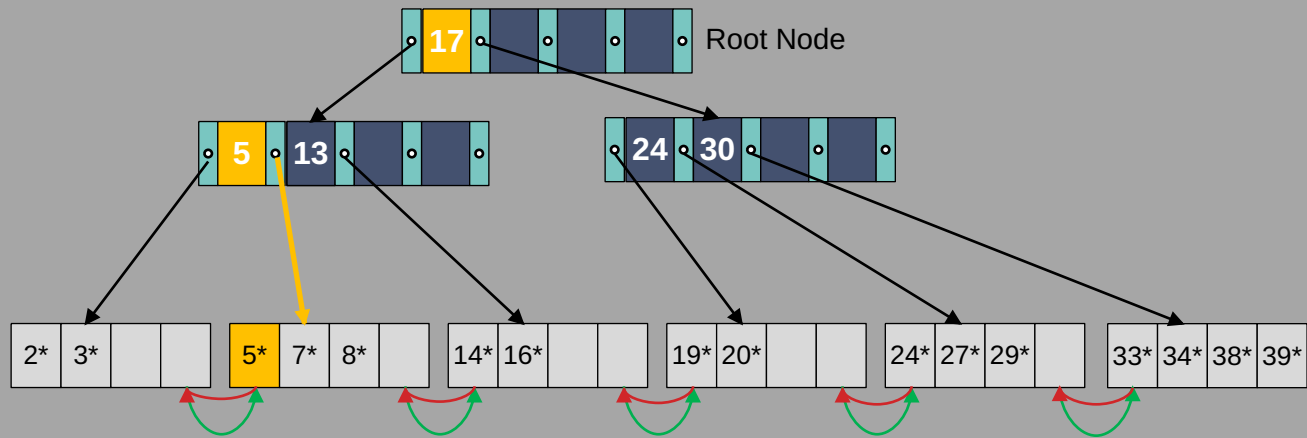
- Recursively split index nodes
  - Redistribute right d keys
  - **Push** up middle key

# Inserting 8\* into a B+ Tree: Root Grows Up, Pt 3



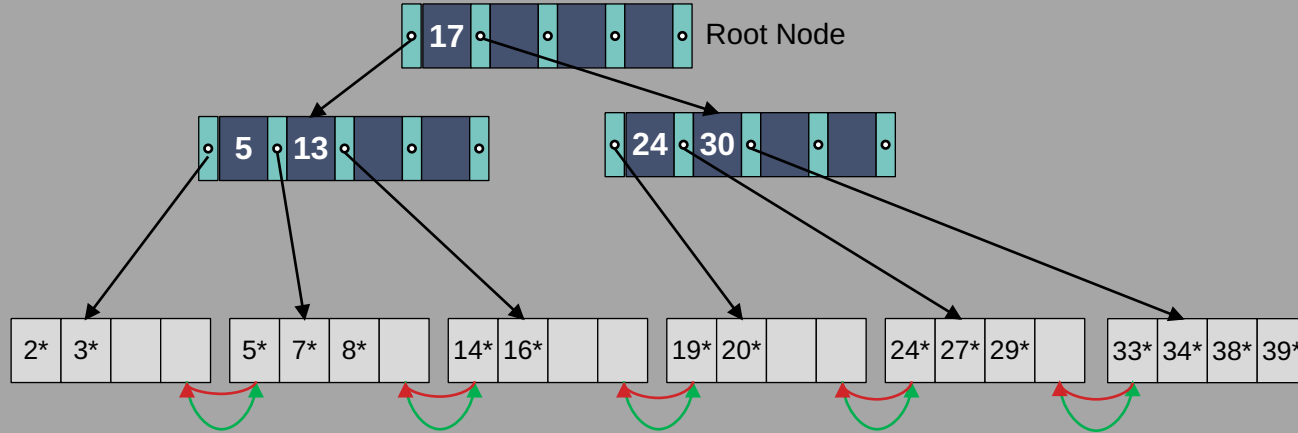
- Recursively split index nodes
  - Redistribute right d keys
  - **Push** up middle key

# Copy up vs Push up!



- Notice:
  - The **leaf** entry (5) was **copied** up
  - The **index** entry (17) was **pushed** up

# Inserting 8\* into a B+ Tree: Final



- Check invariants
- **Key Invariant:**
  - $\text{Node}[\dots, (K_L, P_L), \dots] \rightarrow K_L \leq K$  for all  $K$  in  $P_L$  Sub-tree
- **Occupancy Invariant:**
  - $d \leq \# \text{ entries} \leq 2d$



# B+ Tree Insert: Algorithm Sketch

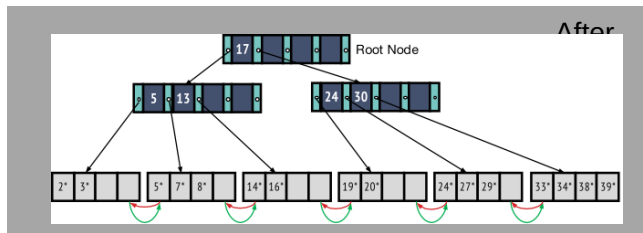
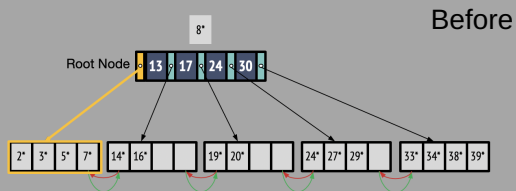
1. Find the correct leaf L.
2. Put data entry onto L.
  - If L has enough space, done!
  - Else, must split L (into L and a new node L2)
    - Redistribute entries evenly, copy up middle key
    - Insert index entry pointing to L2 into parent of L.

# B+ Tree Insert: Algorithm Sketch Part 2

- Step 2 can happen recursively
  - To split index node, redistribute entries evenly, but push up middle key. (Contrast with leaf splits)
- Splits “grow” tree; root split increases height.
  - Tree growth: gets wider or one level taller at top.

# Before and After Observations

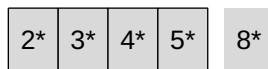
- Notice that the root was split to increase the height
  - Grow from the root not the leaves
  - All paths from root to leaves are equal lengths
- Does the occupancy invariant hold?
  - Yes! All nodes (except root) are at least half full
  - Proof?



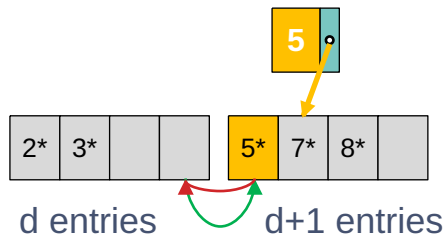


# Splitting a Leaf

- Start with full leaf ( $2d$ ) entries (let  $d = 2$ )
  - Add a  $2d + 1$  entry ( $8^*$ )



- Split into leaves with  $(d, d+1)$  entries
  - Copy key up to parent
- Why copy key and not push key up to parent?



# Splitting an Inner Node

- Start with full interior node ( $2d$ ) entries: (let  $d = 2$ )
  - Add a  $2d + 1$  entry



- Split into nodes with  $(d, d+1)$  entries
  - Push** key up to parent



$d$  entries



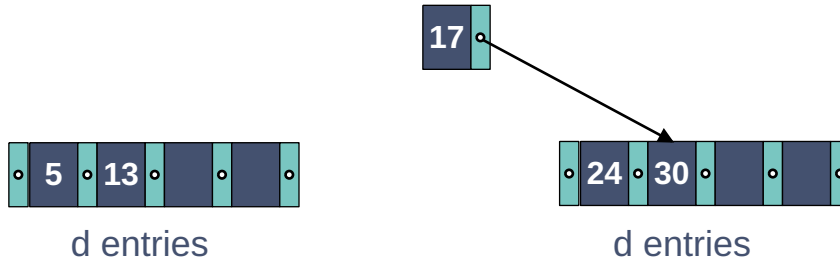
$d+1$  entries

# Splitting an Inner Node Pt 2

- Start with full interior node ( $2d$ ) entries: (let  $d = 2$ )
  - Add a  $2d + 1$  entry

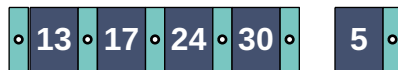


- Split into nodes with  $(d, d)$  entries
  - Push** key up to parent

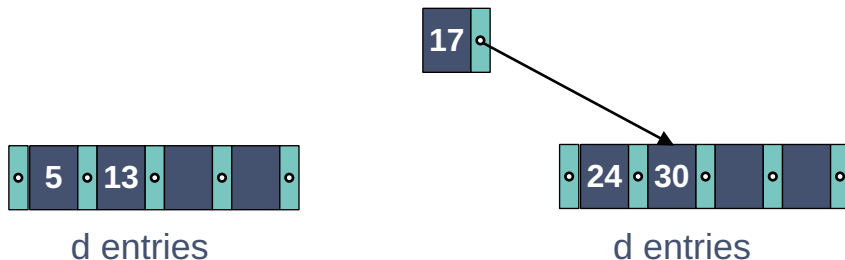


# Splitting an Inner Node Pt 3

- Start with full interior node ( $2d$ ) entries: (let  $d = 2$ )
  - Add a  $2d + 1$  entry



- Split into nodes with  $(d, d)$  entries
  - Push** key up to parent



Why push not copy?

- Routing key not needed in child

Occupancy invariant holds after split

# Nice Animation Online

- [Great animation online of B+ Trees](#)
- One small difference to note
  - Upon deletion of leftmost value in a node, it updates the parent index entry
  - Incurs unnecessary extra writes

# B+-TREE DELETION

# We will skip deletion

- In practice, occupancy invariant often not enforced
- Just delete leaf entries and leave space
- If new inserts come, great
  - This is common
- If page becomes completely empty, can delete
  - Parent may become underfull
  - That's OK too
- Guarantees still attractive:  $\log_F(\text{max size of tree})$

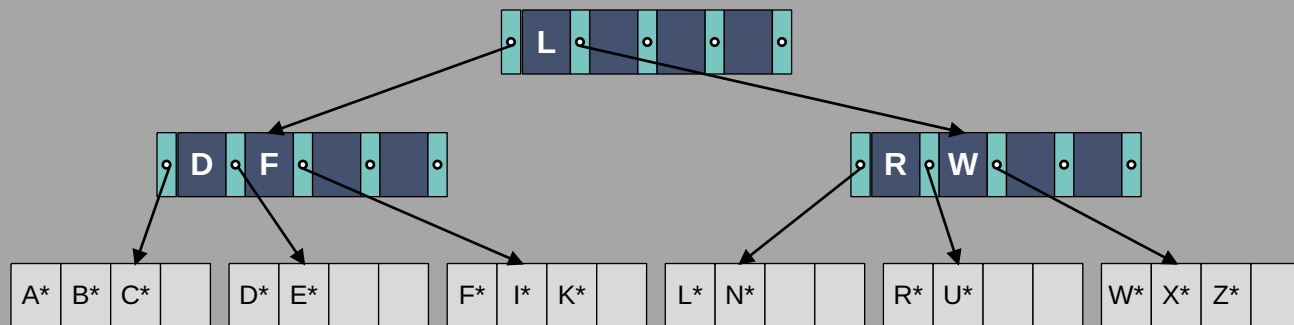
# **BULK LOADING B+-TREES**



# Bulk Loading of B+ Tree Part 1

- Suppose we want to build an index on a large table
- Would it be efficient to just call insert repeatedly
  - No ... Why not?
  - Random Order: CLZARNDXEKFWIUB. Order 2.
  - Try it: [Interactive demo](#)

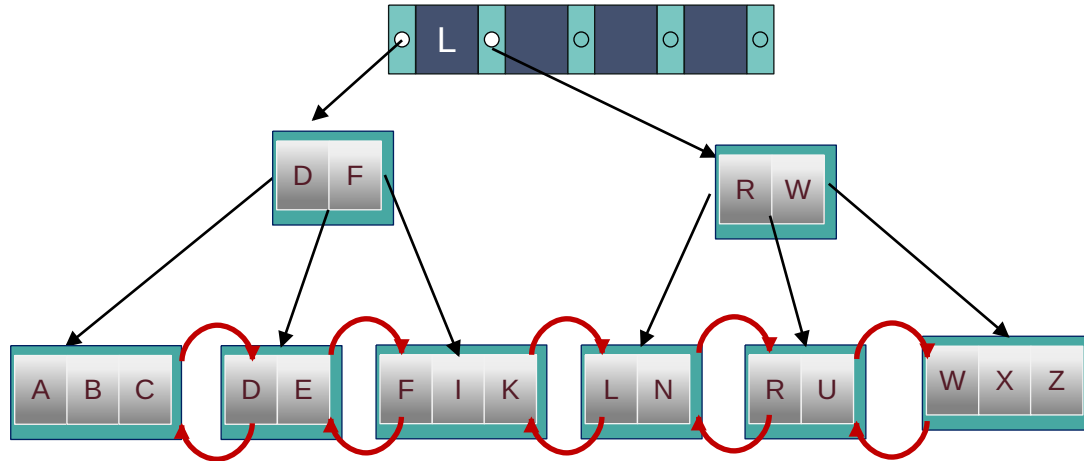
# Bulk Loading of B+ Tree Part 2



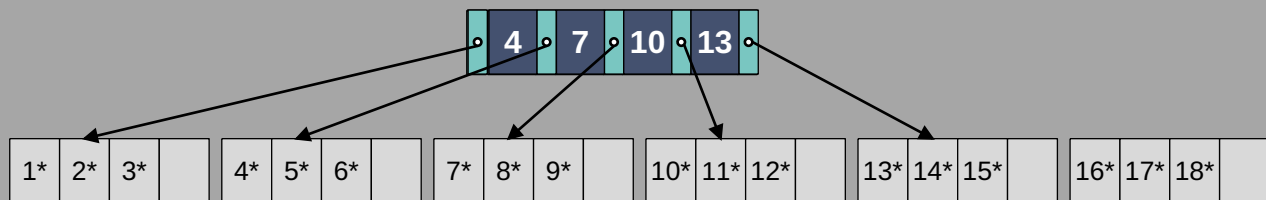
- Constantly need to search from root
- Leaves and internal nodes mostly half-empty
- **Modifying random pages:  
poor cache efficiency**

# Bulk Loading of B+ Tree Part 2

- Constantly need to search from leaf
- Leaves and nodes are mostly half full
- **Modifying random pages -> poor cache efficiency**

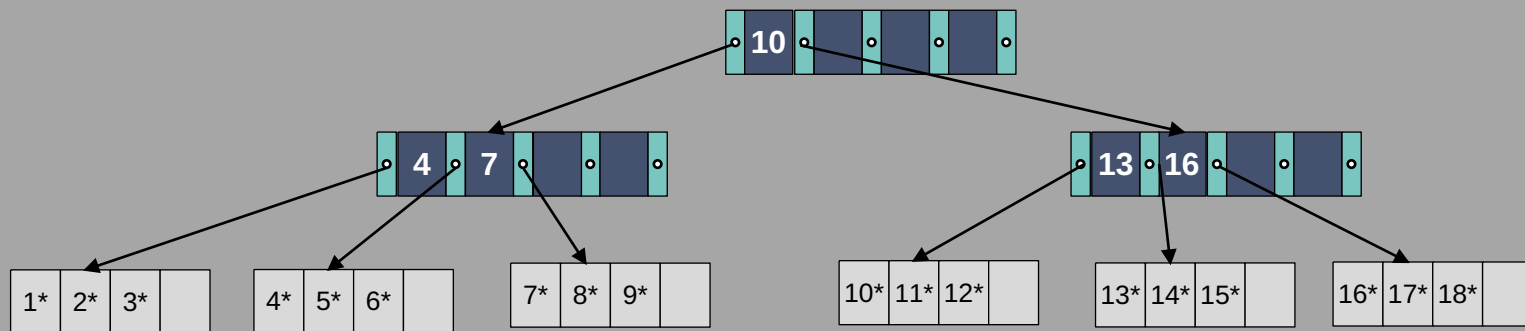


# Smarter Bulk Loading a B+ Tree



- Sort the input records by key:
  - 1\*, 2\*, 3\*, 4\*, ...
  - We'll learn a good disk-based sort algorithm soon!
- Fill leaf pages to some fill factor (e.g.  $\frac{3}{4}$ )
  - Updating parent until full

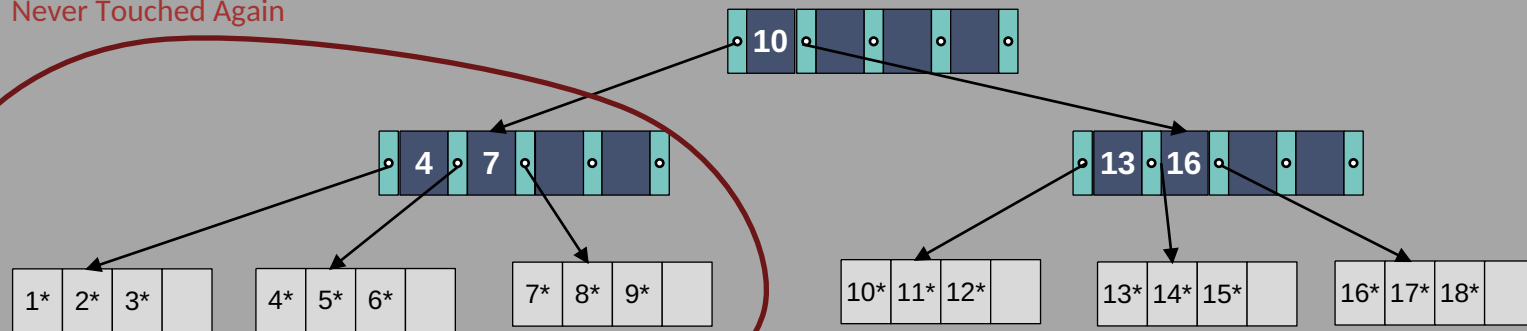
# Smarter Bulk Loading a B+ Tree Part 2



- Sort the input records by key:
  - 1\*, 2\*, 3\*, 4\*, ...
- Fill leaf pages to some fill factor (e.g.  $\frac{3}{4}$ )
  - Update parent until full
  - Then split parent (50/50) and copy to sibling

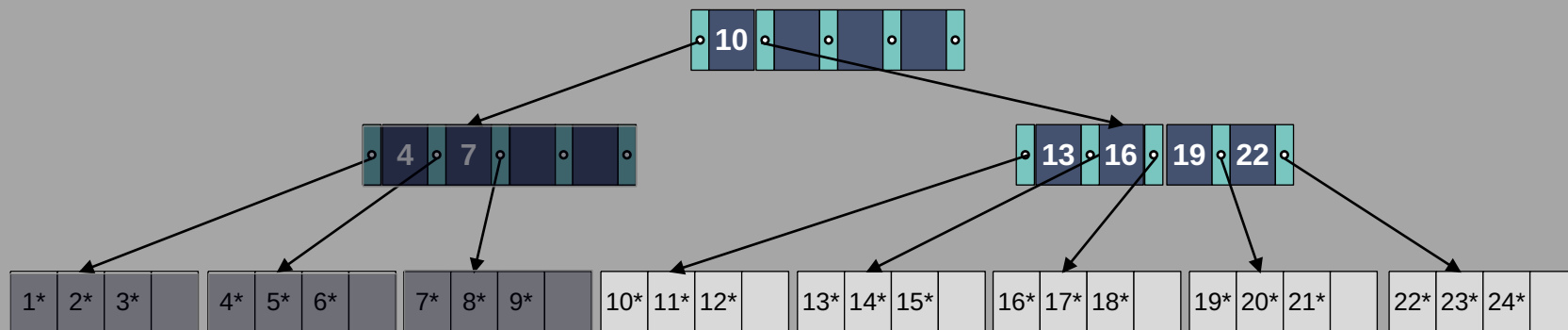
# Smarter Bulk Loading a B+ Tree Part 3

Never Touched Again



- Lower left part of the tree is never touched again
- Occupancy invariant maintained

# Smarter Bulk Loading a B+ Tree Part 4



- Sort the input records by key:
  - 1\*, 2\*, 3\*, 4\*, ...
- Fill leaf pages to some fill factor (e.g.  $\frac{3}{4}$ )
  - Update parent until full
  - Then split parent

# Summary of Bulk Loading

- Option 1: Multiple inserts
  - **Slow**
  - Does not give sequential storage of leaves
- Option 2: Bulk Loading
  - Fewer I/Os during build. (Why?)
  - Leaves will be stored sequentially (and linked, of course)
  - Can control “fill factor” on pages.





# Summary

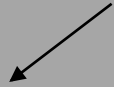
- ISAM is a static structure
  - **Only leaf pages modified**; overflow pages needed
  - Overflow **chains can degrade performance** unless size of data set and data distribution stay constant
- **B+ Tree is a dynamic structure**
  - Inserts/deletes leave tree height-balanced;  $\log_F N$  cost
  - High fanout (F) means depth rarely more than 3 or 4.
  - Almost always better than maintaining a sorted file.
  - Typically, 67% occupancy on average
  - Usually preferable to ISAM; adjusts to growth gracefully.

# Summary Cont.

- Bulk loading can be much faster than repeated inserts for creating a B+ tree on a large data set.
- B+ tree widely used because of its versatility
  - One of the most optimized components of a DBMS.
  - Concurrent Updates
  - In-memory efficiency



# Graphic Components



Root Node

