Heuristic Analysis

1. Heuristic Function

I implemented three functions in my game playing against the improved heuristic function.

Custom Score

This is naive variation of improved score. It calculates the difference between the number of own player moves and opponent player moves. It wights the number of opponent player moves with the factor 2.

Custom Score2

This function is the improved center_score in sample_players. It calculates the difference between own player distance and opponent distance from center.

Custom Score3:

This function calculate the distance between own player and opponent player.

2. Agent Information

- Random: Agent that randomly choose a move each turn
- MM_Null: Agent using fixed-depth mini-max search and the null_score function
- MM_Open: Agent using fixed-depth mini-max search and the open_move_score function
- MM_Improved: Agent using fixed-depth mini-max search and the improved_score function
- AB_Null: Agent using fixed-depth alpha-beta search and the null_score function
- AB_Open: Agent using fixed-depth alpha-beta search and the open_move_score function
- AB_Improved: Agent using fixed-depth alpha-beta search and the improved_score function

3. Results

We could evaluate our heuristic functions using tournament.py as below.

```
                     **************************
                          Playing Matches
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 Match #   Opponent    AB_Improved   AB_Custom   AB_Custom_2  AB_Custom_3
                        Won | Lost   Won | Lost   Won | Lost   Won | Lost
    1       Random       10  |   0     6  |   4     9  |   1     10  |   0
    2       MM_Open       8  |   2     9  |   1     7  |   3      7  |   3
    3     MM_Center      10  |   0    10  |   0    10  |   0      9  |   1
    4    MM_Improved      8  |   2     9  |   1     9  |   1      8  |   2
    5       AB_Open       5  |   5     5  |   5     4  |   6      4  |   6
    6     AB_Center       5  |   5     7  |   3     6  |   4      5  |   5
    7    AB_Improved      5  |   5     8  |   2     4  |   6      5  |   5
------------------------------------------------------------------------
          Win Rate:      72.9%        77.1%        70.0%        68.6%
```

4. Heuristic function recommendation in my three functions

I recommend the custom_score function based on playing results following reasons.

1. It is relatively simple to implement.
2. If we would like to improve this function, we could tune using grid search for this function: A*own_moves - B*opp_moves
3. This function has the highest Win Rate.