磁性物理学 レポート No.4

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(2)

副格子 A, B の磁化はそれぞれ

$$\begin{aligned} M_A &= \frac{C}{T}(H_{\mathrm{ext}} + H_w^A) \\ &= \frac{C}{T}\left(H_{\mathrm{ext}} + \gamma_{AA}M_A - \gamma_{AB}M_B\right) \end{aligned} \tag{1}$$

$$\begin{aligned} M_B &= \frac{C}{T}(H_{\mathrm{ext}} + H_w^B) \\ &= \frac{C}{T}\left(H_{\mathrm{ext}} + \gamma_{BB}M_B - \gamma_{AB}M_A\right) \end{aligned} \tag{2}$$

であった. ここで簡単のため $M_A = x$, $M_B = y$, $C\gamma_{AA}/T = a$, $C\gamma_{BB}/T = b$, $C\gamma_{AB}/T = c$, $CH_{\mathrm{ext}}/T = h$ と置くと

$$\begin{cases} x = h + ax - cy \\ y = h + by - cx \end{cases} \tag{3}$$

$$\iff \begin{cases} x = \dfrac{h - cy}{1 - a} \\ y = \dfrac{h - cx}{1 - b} \end{cases} \tag{4}$$

(16) から

$$\begin{aligned} & y = \frac{h}{1-b} - \frac{c}{1-b}\frac{h - cy}{1-a} = \frac{h(1-a-c) + c^2 y}{(1-a)(1-b)} \\ \iff \quad & y\left(1 - \frac{c^2}{(1-a)(1-b)}\right) = \frac{h(1-a-c)}{(1-a)(1-b)} \\ \iff \quad & y = \frac{h(1-a-c)}{1-a-b+ab-c^2} \end{aligned} \tag{5}$$

x については a と b を入れ替えれば

$$x = \frac{h(1-b-c)}{1-a-b+ab-c^2} \tag{6}$$

である. 正味の磁化は $M = M_A + M_B = x + y$ なので

$$
\begin{aligned}
&& x+y = \tfrac{h(2-a-b-2c)}{1-a-b+ab-c^2} \\
\Longleftrightarrow && \frac{h}{x+y} = \frac{1-a-b+ab-c^2}{2-a-b-2c} \\
\Longleftrightarrow && \frac{\frac{C}{T}H_{\mathrm{ext}}}{M} = \frac{1-\frac{C}{T}\gamma_{AA}-\frac{C}{T}\gamma_{BB}+\left(\frac{C}{T}\right)^2\gamma_{AA}\gamma_{BB}-\left(\frac{C}{T}\right)^2{\gamma_{AB}}^2}{2-\frac{C}{T}\gamma_{AA}-\frac{C}{T}\gamma_{BB}-2\frac{C}{T}\gamma_{AB}} \\
\Longleftrightarrow && \frac{H_{\mathrm{ext}}}{M} = \frac{\frac{T}{C}-(\gamma_{AA}+\gamma_{BB})+\frac{C}{T}\left(\gamma_{AA}\gamma_{BB}-{\gamma_{AB}}^2\right)}{2-\frac{C}{T}(\gamma_{AA}-\gamma_{BB}-2\gamma_{AB})}
\end{aligned}
\tag{7}
$$

を得る. また $1/\chi_D = (2\gamma_{AB}-\gamma_{AA}-\gamma_{BB})/4$, $b = C(\gamma_{AA}-\gamma_{BB})^2/8$, $\theta = C(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})/2$ とおいたとき

$$
\begin{aligned}
\frac{T}{2C}+\frac{1}{\chi_D}-\frac{b}{T-\theta} &= \frac{T}{2C}+\frac{2\gamma_{AB}-\gamma_{AA}-\gamma_{BB}}{4}-\frac{\frac{C}{4T}(\gamma_{AA}-\gamma_{BB})^2}{2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})} \\
&= \frac{1}{2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})}\Bigg(-\frac{C(\gamma_{AA}-\gamma_{BB})^2}{4T} \\
&\quad +\frac{2\gamma_{AB}-\gamma_{AA}-\gamma_{BB}}{4}\left(2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})\right) \\
&\quad +\frac{T}{2C}\left(2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})\right)\Bigg) \\
&= \frac{1}{2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})}\Bigg(-\frac{C}{4T}\gamma_{AA}^2+\frac{C}{2T}\gamma_{AA}\gamma_{BB}-\frac{C}{4T}\gamma_{BB}^2 \\
&\quad +\gamma_{AB}-\frac{\gamma_{AA}}{2}-\frac{\gamma_{BB}}{2}-\frac{C}{4T}\left(4\gamma_{AB}^2-(\gamma_{AA}+\gamma_{BB})^2\right) \\
&\quad +\frac{T}{C}-\frac{1}{2}\left(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB}\right)\Bigg) \\
&= \frac{1}{2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})}\Bigg(-\frac{C}{4T}\gamma_{AA}^2+\frac{C}{2T}\gamma_{AA}\gamma_{BB}-\frac{C}{4T}\gamma_{BB}^2 \\
&\quad +\gamma_{AB}-\frac{\gamma_{AA}}{2}-\frac{\gamma_{BB}}{2}-\frac{C}{T}\gamma_{AB}^2+\frac{C}{4T}\left(\gamma_{AA}^2+2\gamma_{AA}\gamma_{BB}+\gamma_{BB}^2\right) \\
&\quad +\frac{T}{C}-\frac{1}{2}\left(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB}\right)\Bigg) \\
&= \frac{1}{2-\frac{C}{T}(\gamma_{AA}+\gamma_{BB}+2\gamma_{AB})}\Bigg(0+\frac{C}{T}\gamma_{AA}\gamma_{BB}-0 \\
&\quad +0-\gamma_{AA}-\gamma_{BB}-\frac{C}{T}\gamma_{AB}^2+\frac{T}{C}\Bigg) \\
&= \frac{\frac{T}{C}-(\gamma_{AA}+\gamma_{BB})+\frac{C}{T}\left(\gamma_{AA}\gamma_{BB}-{\gamma_{AB}}^2\right)}{2-\frac{C}{T}(\gamma_{AA}-\gamma_{BB}-2\gamma_{AB})}
\end{aligned}
\tag{8}
$$

以上から

$$
\frac{H_{\mathrm{ext}}}{M} = \frac{T}{2C}+\frac{1}{\chi_D}-\frac{b}{T-\theta} \tag{9}
$$

である.