

レーザー物理学 レポート No.6

82311971 佐々木良輔

問 11

(2.30) 式の左辺は

$$\begin{aligned}\frac{d\rho}{dt} &= \frac{d}{dt} (|\psi\rangle\langle\psi|) \\ &= \left(\frac{d}{dt} |\psi\rangle \right) \langle\psi| + |\psi\rangle \frac{d}{dt} \langle\psi| \end{aligned} \quad (11.1)$$

ここで Schrödinger 方程式から

$$\frac{d}{dt} |\psi\rangle = -\frac{i}{\hbar} H |\psi\rangle \quad (11.2)$$

$$\frac{d}{dt} \langle\psi| = \frac{i}{\hbar} \langle\psi| H \quad (11.3)$$

を用いて (11.1) は

$$\begin{aligned}\frac{d\rho}{dt} &= -\frac{i}{\hbar} H |\psi\rangle\langle\psi| + \frac{i}{\hbar} |\psi\rangle\langle\psi| H \\ &= -\frac{i}{\hbar} H \rho + \frac{i}{\hbar} \rho H \\ &= \frac{i}{\hbar} [\rho, H] \end{aligned} \quad (11.4)$$

となる.

問 13

$[\rho, H]$ を計算する.

$$\begin{aligned} [\rho, H] &= \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} - \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{21} & \rho_{22} \end{pmatrix} \\ &= \begin{pmatrix} \rho_{12}H_{21} - \rho_{21}H_{12} & \rho_{11}H_{12} + \rho_{12}H_{22} - H_{11}\rho_{12} - H_{12}\rho_{22} \\ \rho_{21}H_{11} + \rho_{22}H_{21} - H_{21}\rho_{11} - H_{22}\rho_{21} & \rho_{21}H_{12} - H_{21}\rho_{12} \end{pmatrix} \end{aligned} \quad (13.1)$$

したがって Schrödinger 方程式は

$$\begin{pmatrix} \dot{\rho}_{11} & \dot{\rho}_{12} \\ \dot{\rho}_{21} & \dot{\rho}_{22} \end{pmatrix} = \frac{i}{\hbar} \begin{pmatrix} \rho_{12}H_{21} - \rho_{21}H_{12} & \rho_{11}H_{12} + \rho_{12}H_{22} - H_{11}\rho_{12} - H_{12}\rho_{22} \\ \rho_{21}H_{11} + \rho_{22}H_{21} - H_{21}\rho_{11} - H_{22}\rho_{21} & \rho_{21}H_{12} - H_{21}\rho_{12} \end{pmatrix} \quad (13.2)$$

この行列の 22 成分から 11 成分を引くと

$$\rho_{22} - \rho_{11} = \frac{i}{\hbar} (2\rho_{21}H_{12} - 2H_{21}\rho_{12}) \quad (13.3)$$

ここで左辺は $\rho_{22} - \rho_{11} = \dot{r}_3$ である. また

$$\begin{aligned} (\mathbf{F} \times \mathbf{r})_3 &= F_1r_2 - F_2r_1 \\ &= \frac{i}{\hbar} (H_{12} + H_{21})(\rho_{21} - \rho_{12}) + \frac{i}{\hbar} (H_{12} - H_{21})(\rho_{12} + \rho_{21}) \\ &= \frac{i}{\hbar} (2H_{12}\rho_{21} - 2H_{21}\rho_{12}) \end{aligned} \quad (13.4)$$

である. (13.3), (13.4) から

$$\dot{r}_3 = (\mathbf{F} \times \mathbf{r})_3 \quad (13.5)$$

である. 次に (13.2) の 12 成分と 21 成分を足すと

$$\begin{aligned} \rho_{12} + \rho_{21} &= \frac{i}{\hbar} (\rho_{11}H_{12} - \rho_{11}H_{21} + \rho_{12}H_{22} - \rho_{12}H_{11} \\ &\quad + \rho_{21}H_{11} - \rho_{21}H_{22} + \rho_{22}H_{21} - \rho_{22}H_{12}) \\ &= \frac{i}{\hbar} (H_{21} - H_{12})(\rho_{22} - \rho_{11}) + \frac{i}{\hbar} (H_{22} - H_{11})(\rho_{12} - \rho_{21}) \\ &= (F_2r_3 - F_3r_2) = (\mathbf{F} \times \mathbf{r})_1 \end{aligned} \quad (13.6)$$

また $\rho_{12} + \rho_{21} = \dot{r}_1$ より

$$\dot{r}_1 = (\mathbf{F} \times \mathbf{r})_1 \quad (13.7)$$

である. 次に (13.2) の 12 成分と 21 成分を引くと

$$\begin{aligned}
 \rho_{12} - \rho_{21} &= \frac{i}{\hbar} (\rho_{11}H_{12} + \rho_{11}H_{21} + \rho_{12}H_{22} - \rho_{12}H_{11} \\
 &\quad - \rho_{21}H_{11} + \rho_{21}H_{22} - \rho_{22}H_{21} - \rho_{22}H_{12}) \\
 &= \frac{i}{\hbar} (H_{22} - H_{11})(\rho_{12} + \rho_{21}) - \frac{i}{\hbar} (H_{21} + H_{12})(\rho_{22} - \rho_{11}) \\
 &= i(F_3r_1 - F_1r_3) = i(\mathbf{F} \times \mathbf{r})_2
 \end{aligned} \tag{13.8}$$

また $(\rho_{12} - \rho_{21})/i = r_2$ より

$$r_2 = (\mathbf{F} \times \mathbf{r})_2 \tag{13.9}$$

以上から

$$\frac{d}{dt}\mathbf{r} = \mathbf{F} \times \mathbf{r} \tag{13.10}$$

が示された.