

## Lower/Upper Formulae Used in `qe_plot.py`

In the current implementation, the kinematic edge is

$$Q(E, \theta) = \sqrt{\frac{E + 2E_f - 2\cos(2\theta)\sqrt{E_f(E + E_f)}}{\hbar^2/(2m_n)}}, \quad \frac{\hbar^2}{2m_n} = 2.072 \text{ meV } \text{\AA}^2. \quad (1)$$

Therefore, with scattering-angle bounds  $\theta_{\min}$  and  $\theta_{\max}$ ,

$$Q_{\text{lower}}(E) = Q(E, \theta_{\min}) = \sqrt{\frac{E + 2E_f - 2\cos(2\theta_{\min})\sqrt{E_f(E + E_f)}}{\hbar^2/(2m_n)}}, \quad (2)$$

$$Q_{\text{upper}}(E) = Q(E, \theta_{\max}) = \sqrt{\frac{E + 2E_f - 2\cos(2\theta_{\max})\sqrt{E_f(E + E_f)}}{\hbar^2/(2m_n)}}. \quad (3)$$

The dispersion used in the same script is

$$E_{\text{mag}}(\mathbf{q}) = S\sqrt{A(\mathbf{q})B(\mathbf{q})}, \quad (4)$$

$$A(\mathbf{q}) = J_1(3 + 2\gamma_1) + 2J_2(-1 + \gamma_2), \quad (5)$$

$$B(\mathbf{q}) = J_1(3 - \gamma_1) + 2J_2(-1 + \gamma_2), \quad (6)$$

$$\gamma_1 = \cos(2\pi q_x) + \cos(2\pi q_y) + \cos(2\pi(q_x + q_y)), \quad (7)$$

$$\gamma_2 = \cos(\pi q_z). \quad (8)$$

Equivalent coupling products are

$$J_1 S, \quad J_2 S, \quad (9)$$

because  $E_{\text{mag}}$  is proportional to  $S$ .

Path-coordinate transforms currently used are:

$$\text{(linear path)} \quad |x|_{\text{lower}} = \frac{Q_{\text{lower}}}{|Q_{\text{scale}}|}, \quad |x|_{\text{upper}} = \frac{Q_{\text{upper}}}{|Q_{\text{scale}}|}, \quad (10)$$

$$\text{(symmetric path)} \quad |x|_{\text{lower}} = \frac{\sqrt{Q_{\text{lower}}^2 - Q_{\text{center}}^2}}{|Q_{\text{scale}}|}, \quad |x|_{\text{upper}} = \frac{\sqrt{Q_{\text{upper}}^2 - Q_{\text{center}}^2}}{|Q_{\text{scale}}|}. \quad (11)$$

Main parameters used from `list.txt` are:

$$S, J_1, J_2, E_f, E_i^{\text{max}}, \theta_{\min}, \theta_{\max}, Q_{110}, Q_{1\bar{1}0}, Q_{001}. \quad (12)$$

Numerical values currently used (`code/list.txt`):

$$S = 1.0, \quad J_1 = 1.80 \text{ meV}, \quad J_2 = -4.30 \text{ meV}, \quad (13)$$

$$J_1 S = 1.80 \text{ meV}, \quad J_2 S = -4.30 \text{ meV}, \quad E_f = 4.0 \text{ meV}, \quad (14)$$

$$E_i^{\text{max}} = 18.0 \text{ meV}, \quad \theta_{\min} = 5.0^\circ, \quad \theta_{\max} = 115.0^\circ, \quad (15)$$

$$Q_{110} = 1.82016 \text{ \AA}^{-1}, \quad Q_{1\bar{1}0} = 1.05087 \text{ \AA}^{-1}, \quad Q_{001} = 0.49986 \text{ \AA}^{-1}. \quad (16)$$