6.3

$$\sigma(a) = \frac{1}{1+\exp(-a)}$$

1. $\sigma(-a) = 1 - \sigma(a)$ を示す.

$$1-\sigma(a) = 1 - \frac{1}{1+\exp(-a)}$$

$$= \frac{\exp(-a)}{1+\exp(-a)}$$

$$= \frac{1}{\exp(a)+1} = \sigma(-a)$$

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2. $\frac{d}{da}\sigma(a) = \sigma(a)(1-\sigma(a))$ を示す.

$$\frac{d}{da}\sigma(a) = \frac{d}{da}\frac{1}{1+\exp(-a)}$$

$$= \frac{\exp(-a)}{(1+\exp(-a))^2}$$

$$= \frac{1}{1+\exp(-a)} \times \frac{\exp(-a)}{1+\exp(-a)}$$

$$= \frac{1}{1+\exp(-a)} \times \left\{1 - \left(\frac{1}{1+\exp(-a)}\right)\right\}$$

$$= \sigma(a)\,(1-\sigma(a))$$

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3. $a \in \mathbb{R}$のとき、$\sigma(a) \in [0,1]$を示す

$$\frac{1}{1+e^{-a}}$$

前問より、

$$\frac{d}{da}\sigma(a) = \sigma(a)(1-\sigma(a))$$

$$= \frac{1}{1+\exp(-a)} \times \frac{1}{\exp(a)+1}$$

$$= \frac{1}{1+e^{-a}} \times \frac{1}{e^{a}+1} > 0$$

よって $\sigma(a)(1-\sigma(a)) > 0$ であるので、

$$0 < \sigma(a) < 1$$

したがって $\sigma(a) \in [0,1]$

4. $\log\left(\frac{P(t=1|x)}{1-P(t=1|x)}\right) = w^T x$ は $P(t=1|x) = \sigma(w^T x)$ と等価であることを示す

$$\log\left(\frac{1-P(t=1|x)}{P(t=1|x)}\right) = -w^T x$$

$P(t=1|x) \neq 0$

$$\frac{1-P(t=1|x)}{P(t=1|x)} = \exp(-w^T x)$$

$$\frac{1}{P(t=1|x)} - 1 = \exp(-w^T x)$$

$$\frac{1}{P(t=1|x)} = \exp(-w^T x) + 1$$

$$P(t=1|x) = \frac{1}{\exp(-w^T x)+1} = \sigma(w^T x)$$

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