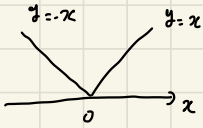
4-5

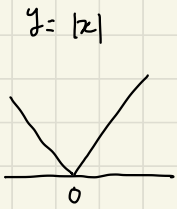
$$\mathbf{x}_i = \begin{pmatrix} 1 \\ x_{i1} \\ \vdots \\ x_{iD} \end{pmatrix}, \quad \mathbf{X} = \begin{pmatrix} \mathbf{x}_1^T \\ \mathbf{x}_2^T \\ \vdots \\ \mathbf{x}_N^T \end{pmatrix}, \quad \mathbf{t} = \begin{pmatrix} t_1 \\ t_2 \\ \vdots \\ t_N \end{pmatrix}, \quad \mathbf{w} = \begin{pmatrix} w_0 \\ w_1 \\ \vdots \\ w_D \end{pmatrix}$$

事例集合 $\{(\mathbf{x}_i, t_i)\}_{i=1}^N$

(1) $$E(\mathbf{w}) = \sum_{i=1}^{N} (t_i - \mathbf{w}^T \mathbf{x}_i)^2 + \lambda \sum_{i=1}^{D} |w_i|$$

(2) $y = |x|$





$$\partial|x| = \begin{cases} 1 & , x > 0 \\ [-1, 1] & , x = 0 \\ -1 & , x < 0 \end{cases}$$

(3) $w_k^{(t+1)} \leftarrow w_k^{(t)} - \eta \partial E(w_k^{(t)})$ であり、

$$w_k^{(t)} - \eta \partial E(w_k^{(t)}) = w_k^{(t)} - \eta \partial \left\{ \sum_{i=1}^{N} (t_i - w_k^{(t)} x_{ik})^2 + \lambda |w_k^{(t)}| \right\}$$

$$= w_k^{(t)} - \eta \partial \left[\left\{ \sum_{i=1}^{N} (t_i^2 - 2 t_i w_k^{(t)} + (w_k^{(t)} x_{ik}) \right\} + \lambda |w_k^{(t)}| \right]$$

$$= w_k^{(t)} - \eta \left[\left\{ 2 \sum_{i=1}^{N} x_{ik} (w_k^{(t)} x_{ik} - t_i) \right\} + \partial \lambda |w_k^{(t)}| \right] \cdots (*)$$

ここで

$$\partial |w_k| = \begin{cases} 1, & w_k > 0 \\ [-1, 1], & w_k = 0 \\ -1, & w_k < 0 \end{cases}$$ であるので、

$$(*) = w_k^{(t)} - 2\eta \sum_{i=1}^{N} x_{ik} (w_k^{(t)} x_{ik} - t_i) + \begin{cases} \eta\lambda, & w_k > 0 \\ [-\eta\lambda, \eta\lambda], & w_k = 0 \\ -\eta\lambda, & w_k < 0 \end{cases}$$

以上より、

$$w_k^{(t+1)} \leftarrow w_k^{(t)} - 2\eta \sum_{i=1}^{N} x_{ik} (w_k^{(t)} x_{ik} - t_i) + \begin{cases} \eta\lambda, & w_k > 0 \\ [-\eta\lambda, \eta\lambda], & w_k = 0 \\ -\eta\lambda, & w_k < 0 \end{cases}$$