

$$z \in \mathbb{R}^n, c \in \mathbb{R}$$

demostrar  $\text{softmax}(z) = \text{softmax}(z + c \cdot \mathbf{1})$

$$\text{softmax}(z)_i = \frac{e^{z_i}}{\sum_{j=1}^n e^{z_j}}, \text{ así que } \text{softmax}(z + c \cdot \mathbf{1})_i = \frac{e^{z_i + c}}{\sum_{j=1}^n e^{z_j + c}}$$

Utilizando  $e^{a+b} = e^a \cdot e^b$  tenemos que:

$$\begin{aligned} \text{softmax}(z + c \cdot \mathbf{1})_i &= \frac{e^{z_i + c}}{\sum_{j=1}^n e^{z_j + c}} = \frac{e^{z_i} \cdot e^c}{\sum_{j=1}^n e^{z_j} \cdot e^c} \\ &= \frac{e^{z_i} \cdot e^c}{e^c \cdot \sum_{j=1}^n e^{z_j}} = \frac{e^{z_i}}{\sum_{j=1}^n e^{z_j}} \\ &= \text{softmax}(z)_i \end{aligned}$$