

$z \in \mathbb{R}^n$, $c \in \mathbb{R}$

demonstrar $\text{softmax}(z) = \text{softmax}(z + c \cdot 1)$

$$\text{softmax}(z)_i = \frac{e^{z_i}}{\sum_{j=1}^n e^{z_j}}, \text{ así que } \text{softmax}(z + c \cdot 1)_i = \frac{e^{z_i + c}}{\sum_{j=1}^n e^{z_j + c}}$$

utilizando $e^{a+b} = e^a \cdot e^b$ tenemos que:

$$\begin{aligned}\text{softmax}(z + c \cdot 1)_i &= \frac{e^{z_i + c}}{\sum_{j=1}^n e^{z_j + c}} = \frac{e^{z_i} \cdot e^c}{\sum_{j=1}^n e^{z_j} \cdot e^c} \\ &= \frac{e^{z_i} \cdot e^c}{e^c \cdot \sum_{j=1}^n e^{z_j}} = \frac{e^{z_i}}{\sum_{j=1}^n e^{z_j}} \\ &= \text{softmax}(z)_i\end{aligned}$$