电动力学

0. 矢量分析

矢量：分量形式

- Levi-Civita符号：ε_{ijk}

$$(\vec{A} \times \vec{B})_i = -(\vec{B} \times \vec{A})_i = \varepsilon_{ijk} A_j B_k$$

- $\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} = A_i B_i$

- $\vec{A} \cdot (\vec{B} \times \vec{C}) = \vec{B} \cdot (\vec{C} \times \vec{A}) = \vec{C} \cdot (\vec{A} \times \vec{B}) = \varepsilon_{ijk} A_i B_j C_k$

- $\vec{A} \times (\vec{B} \times \vec{C}) = \vec{B}(\vec{A} \cdot \vec{C}) - \vec{C}(\vec{A} \cdot \vec{B})$

- 旋度(Green定理，Stokes定理)

$$\int_S (\nabla \times \vec{A}) \cdot d\vec{\sigma} = \oint_l \vec{A} \cdot dl$$

- 散度 (高斯定理)

$$\int_V (\nabla \cdot \vec{A}) dV = \oint_S \vec{A} \cdot d\vec{\sigma}$$

- 求边界与求微分 $(\partial M, \omega) = (M, d\omega)$

$$\int_{\partial M} \omega = \int_M d\omega$$

- δ 函数常用表达式

$$\nabla^2 \left(\frac{1}{r}\right) = -4\pi\delta(\vec{r}), \qquad \nabla \cdot \frac{\vec{r}}{r^3} = 4\pi\delta(\vec{r})$$

1. 电磁现象普遍规律

静电场

- 电荷守恒

$$\nabla \cdot \vec{j} + \frac{\partial \rho}{\partial t} = 0 \quad \Leftrightarrow \quad \oiint_S \vec{j} \cdot d\vec{s} = -\frac{dQ}{dt}$$

- 库伦定理

$$\vec{F} = q\,\vec{E}$$

- 静止点电荷的电场强度

$$\vec{E} = \frac{q\vec{r}}{4\pi\varepsilon_0 r^3}$$

- 给定电荷密度 $\rho(x)$ 的电场强度 $\left(\nabla^2\left(\frac{1}{r}\right) = \nabla\cdot\left(-\frac{\vec{r}}{r^3}\right) = -4\pi\delta(r)\right)$

$$\vec{E}(\vec{x}) = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{x}')\vec{r}}{r^3}dV' \quad \Rightarrow \quad \nabla\cdot\vec{E} = \frac{\rho}{\varepsilon_0}\ (\text{高斯定理}), \quad \nabla\times\vec{E} = 0\ (\text{静电场无旋})$$

稳恒电流

- **安培定律：电流在磁场中受力**

$$d\vec{F} = I d\vec{l} \times \vec{B} \quad \Leftrightarrow \quad \vec{F} = q\vec{v} \times \vec{B}$$

- **毕奥-萨伐尔定律：稳恒电流产生磁场**

$$\vec{B}(\vec{x}) = \frac{\mu_0 I}{4\pi} \oint \frac{d\vec{l} \times \vec{r}}{r^3}$$

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int_V \vec{J}(\vec{x}') \times \frac{\vec{r}}{r^3} dV' \quad \Longrightarrow \quad \nabla \times \vec{B} = \mu_0 \vec{J}, \quad \nabla \cdot \vec{B} = 0$$

- **法拉第定律：变化的磁场产生电场**

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \int_S \vec{B} \cdot d\vec{S} \quad \Rightarrow \quad \nabla \times \vec{E} = -\frac{\partial B}{\partial t}$$

位移电流

- 稳恒电流（安培定律）

$$\nabla \times \vec{B} = \mu_0 \vec{j} \quad \Longrightarrow \quad \nabla \cdot \vec{j} = \nabla \cdot \left(\nabla \times \vec{B}\right) = 0$$

- 电荷守恒

$$\nabla \cdot \vec{j} = -\frac{\partial \rho}{\partial t}$$

- 电荷：

$$\nabla \cdot \vec{E} = \rho / \varepsilon_0$$

- 电荷守恒：

$$\nabla \cdot \vec{j} = -\frac{\partial}{\partial t}\left(\varepsilon_0 \nabla \cdot \vec{E}\right) = -\nabla \cdot \left(\varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) \quad \Longleftrightarrow \quad \nabla \cdot \left(\vec{j} + \varepsilon_0 \frac{\partial \vec{E}}{\partial t}\right) = 0$$

- 位移电流 （$\vec{j}_D = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$）

$$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

Maxwell方程

- Maxwell方程

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- 无源（真空）Mawell方程 $\rho = 0$, $\vec{J} = 0$

$$\nabla \cdot \vec{E} = 0, \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

- 电磁波 ($c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$)

$$\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0, \qquad \nabla^2 \vec{B} - \frac{1}{c^2} \frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

- 洛伦兹力

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

Maxwell方程组

微分形式		积分形式
$\nabla\cdot\vec{E}=\rho/\varepsilon_0$	$\Longrightarrow$	$\oiint_{\partial V}\vec{E}\cdot d\vec{s}=\frac{1}{\varepsilon_0}\iiint_V \rho d^3x$
$\nabla\cdot\vec{B}=0$	$\Longrightarrow$	$\oiint_{\partial V}\vec{B}\cdot d\vec{s}=0$
$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$	$\Longrightarrow$	$\oint_{\partial S}\vec{E}\cdot d\vec{l}=-\frac{d}{dt}\iint_S\vec{B}\cdot d\vec{s}$
$\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$	$\Longrightarrow$	$\oint_{\partial S}\vec{B}\cdot d\vec{l}=\mu_0\iint_S\vec{J}\cdot d\vec{s}+\mu_0\varepsilon_0\frac{d}{dt}\iint_S\vec{E}\cdot d\vec{s}$

Maxwell方程组

- (1) 定域规律;
- (2) 线性（与力的叠加性一致）；
- (3) 电荷定域守恒;
- (4) 无磁荷
- (5) ρ, J 为源项，激发电磁场
- (6) 无源电磁场存在
- (7) 预言电磁波

$$\nabla \times \vec{E} = -\frac{\partial}{\partial t}\vec{B}$$

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial}{\partial t}\vec{E}$$

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0, \qquad \nabla^2 \vec{B} - \frac{1}{c^2}\frac{\partial^2 \vec{B}}{\partial t^2} = 0$$

介质中Maxwell方程

- 介质中Maxwell方程

Maxwell方程

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$ 电磁感应 $$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$ 无磁单极 $$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$ 位移电流 $$\nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$ 高斯定律 $$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

- 电位移矢量和磁场强度

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \qquad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

电磁场中受力

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- 本构关系

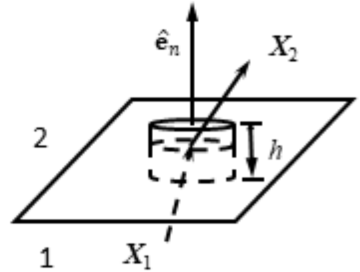
$$\vec{D} = \varepsilon \vec{E}, \qquad \vec{B} = \mu \vec{H}, \qquad \vec{j} = \sigma \vec{E}$$

Maxwell方程积分形式

- **Maxwell**

$$\oint_{\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt}\iint_S \vec{B} \cdot d\vec{s}, \qquad \oint_{\partial S} \vec{H} \cdot d\vec{l} = \iint_S \vec{J} \cdot d\vec{s} + \frac{d}{dt}\iint_S \vec{D} \cdot d\vec{s}$$

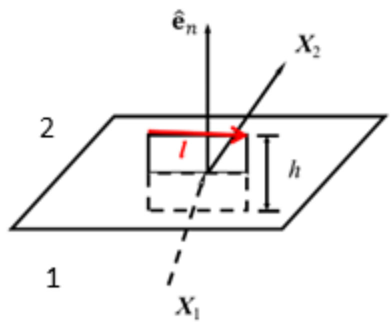
$$\oiint_{\partial V} \vec{D} \cdot d\vec{s} = \iiint_V \rho d^3 x, \qquad \oiint_{\partial V} \vec{B} \cdot d\vec{s} = 0$$



- **切向边值关系**

$\vec{H} \perp \vec{n}$

$$\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0, \qquad \vec{n} \times (\vec{H}_2 - \vec{H}_1) = \alpha_f$$



- **法向边值关系**

$\vec{D} \parallel \vec{n}$

$$\vec{n} \cdot (\vec{D}_2 - \vec{D}_1) = \sigma_f, \qquad \vec{n} \cdot (\vec{B}_2 - \vec{B}_1) = 0$$

*X*代表*D*或*B*

边值关系

- 法向

$$\vec{n}\cdot\left(\vec{D}_2-\vec{D}_1\right)=\sigma_f,\qquad \vec{n}\cdot\left(\vec{B}_2-\vec{B}_1\right)=0$$

- 切向

$$\vec{n}\times\left(\vec{E}_2-\vec{E}_1\right)=0,\qquad \vec{n}\times\left(\vec{H}_2-\vec{H}_1\right)=\alpha_f$$

能量和能流

- 能流密度：玻印廷矢量

$$\vec{S} = \vec{E} \times \vec{H}$$

- 能量变化

$$\delta w = \vec{H} \cdot \delta\vec{B} + \vec{E} \cdot \delta\vec{D}$$

- 真空：$\vec{D} = \varepsilon_0\vec{E}$, $\vec{B} = \mu_0\vec{H}$

$$\vec{S} = \frac{1}{\mu_0}\vec{E} \times \vec{B}$$

$$w = \frac{1}{2}\left(\varepsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)$$

- 各向同性线性介质：$\vec{D} = \varepsilon\vec{E}$, $\vec{B} = \mu\vec{H}$

$$w = \frac{1}{2}(\vec{E} \cdot \vec{D} + \vec{H} \cdot \vec{B})$$

电磁场

- Maxwell方程组

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \qquad \nabla \cdot \vec{D} = \rho$$

$$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{H} = \vec{j} + \frac{\partial \vec{D}}{\partial t}$$

- 静电场：静电势+泊松方程

$$\nabla \times \vec{E} = 0 \quad \Longrightarrow \quad \vec{E} = -\nabla\phi;$$

$$\nabla \cdot \vec{D} = \rho \quad \Longrightarrow \quad \nabla^2\phi = -\rho/\varepsilon$$

- 静磁场

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{j}$$

2. 静电场

$$\nabla \times \vec{E} = 0, \qquad \nabla \cdot \vec{D} = \rho$$

静电势的Laplace方程，多级展开

静电势

- 静电势 ϕ （$\nabla \times \vec{E} = 0$）

$$\vec{E} = -\nabla\phi, \qquad \phi(P) = \int_P^\infty \vec{E} \cdot d\vec{l}$$

- 连续分布电荷静电势

$$\phi(x) = \int \frac{\rho(x')}{4\pi\varepsilon r} dV'$$

- 泊松方程 （$\nabla \cdot \vec{D} = \rho$）

$$\nabla^2\phi = -\frac{\rho}{\varepsilon}$$

- 边值关系

$$\phi_1 = \phi_2, \qquad \varepsilon_2 \frac{\partial\phi_2}{\partial n} - \varepsilon_1 \frac{\partial\phi_1}{\partial n} = -\sigma$$

- 静电场能量

$$W = \frac{1}{2}\int \vec{E} \cdot \vec{D} dV = \frac{1}{8\pi\varepsilon}\int dV dV' \frac{\rho(x)\rho(x')}{r}$$

标量势的边值条件

- 静电场旋度积分形式

$$\oint_L \vec{E}\cdot d\vec{l} = 0 \quad \Rightarrow \quad \hat{e}_n \times (\vec{E}_2 - \vec{E}_1) = 0$$

- 静电场散度积分形式

$$\oiint_S \vec{D}\cdot d\vec{S} = \iiint_{V_0} \rho\, dV \quad \Rightarrow \quad \hat{e}_n \cdot (\vec{D}_2 - \vec{D}_1) = \sigma$$

- 法向分量

$$\varepsilon_2 \frac{\partial \phi_2}{\partial n} - \varepsilon_1 \frac{\partial \phi_1}{\partial n} = -\sigma$$

- 切向分量（忽略界面厚度）

$$-\frac{\partial \phi_2}{\partial x^i} + \frac{\partial \phi_1}{\partial x^i} = -\frac{\partial}{\partial x^i}(\phi_2 - \phi_1) = 0 \quad \Rightarrow \quad \phi_1 = \phi_2$$

静电问题的唯一性定理

- 唯一性定理：区域 V 内的静电场唯一的两个条件：

  （1）V内自由电荷分布 $\rho(x)$ 给定；

  （2）在V的边界S上给定电势($\phi|_S$)，或者给定电势沿法线方向的导数($\frac{\partial\phi}{\partial n}|_S$)

$$\phi\Big|_S \qquad or \qquad \frac{\partial\phi}{\partial n}\Big|_S$$

- 有导体时的唯一性定理条件：

  （1）给定导体的电势 ϕ_0，或者

  （2）给定导体总电荷 Q_0

导体外， 区域 V' 内条件一样：电荷分布 $\rho(x)$，和边界 S 处 $\phi|_S$ 或者 $\partial\phi/\partial n|_S$

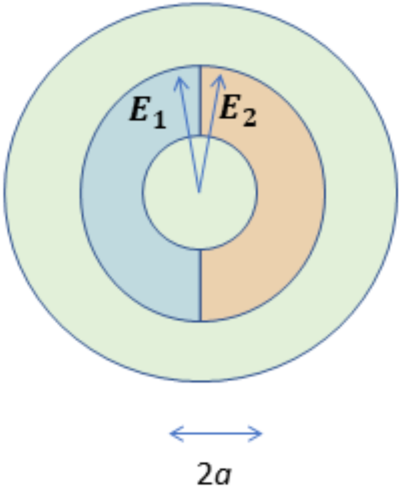
导体周围电场问题

- 内球壳带总电荷Q，外球壳接地，求介质中电场
- 两种介质界面的边值关系：介质界面不带电

$$E_{2t} = E_{1t}, \qquad D_{2n} = D_{1n}$$

- 电场垂直导体表面，球对称的径向电场：

$$\vec{E}_1 = \frac{\alpha}{r^3}\vec{r}, \qquad \vec{E}_2 = \frac{\alpha}{r^3}\vec{r}, \qquad \alpha = \frac{Q}{2\pi(\varepsilon_1 + \varepsilon_2)}$$



- 内球壳外表面的自由电荷密度 ($D_{2n} - D_{1n} = \sigma_f \Leftrightarrow \nabla \cdot \vec{D} = \rho$)

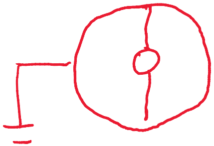
$$\sigma_1 = \frac{\varepsilon_1 Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}, \qquad \sigma_2 = \frac{\varepsilon_2 Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}$$

- 内球壳外表面的总电荷密度 ($\varepsilon_0 E_{2n} - \varepsilon_0 E_{1n} = \sigma_{tot} \Leftrightarrow \nabla \cdot \vec{E} = \rho/\varepsilon_0$)

$$\sigma_1 + \sigma_{P1} = \frac{\varepsilon_0 Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}, \qquad \sigma_2 + \sigma_{P2} = \frac{\varepsilon_0 Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}$$

- 内球壳外表面的束缚电荷密度

$$\sigma_{P1} = \frac{(\varepsilon_0 - \varepsilon_1)Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}, \qquad \sigma_{P2} = \frac{(\varepsilon_0 - \varepsilon_2)Q}{2\pi(\varepsilon_1 + \varepsilon_2)a^2}$$



Laplace方程

- 无自由电荷分布：Laplace方程

$$\nabla^2\phi = 0$$

- 分离变换法解拉普拉斯方程
- 柱对称情形，用勒让德函数展开得到静电势，通式

$$\phi(r,\theta) = \sum_{l=0}^{\infty}\left(a_l r^l + \frac{b_l}{r^{l+1}}\right)P_l(\cos\theta)$$

- 自然边界条件

含 $r = 0$ 区域，$b_l = 0$；

含 $r \to \infty$ 区域，$a_{l\geq 1} = 0$

分离变量法：球坐标通解

- 球坐标下通解

$$\phi(r,\theta,\varphi) = \sum_{l=0}^{\infty}\sum_{m=0}^{l}\left(a_{lm}r^l + \frac{b_{lm}}{r^{l+1}}\right)P_l^m(\cos\theta)\cos(m\varphi) + \sum_{l=0}^{\infty}\sum_{m=1}^{l}\left(c_{lm}r^l + \frac{d_{lm}}{r^{l+1}}\right)P_l^m(\cos\theta)\sin(m\varphi)$$

没有假设电场具有任何对称性

- 柱对称系统，取对称轴为球坐标极轴

$$\phi(r,\theta,\varphi) = \boldsymbol{\phi}(r,\theta) \quad \Longrightarrow \quad m = 0$$

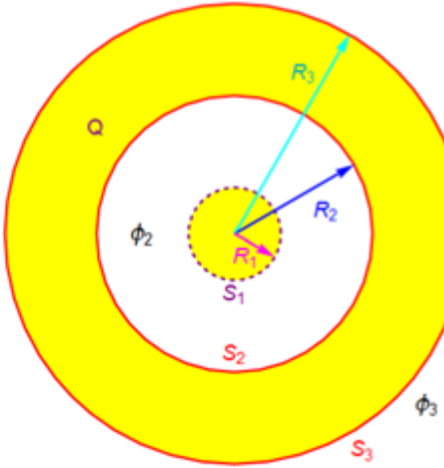
- 柱对称解（$m = 0$）

$$\underbrace{\phi(r,\theta) = \sum_{l=0}^{\infty}\left(a_l r^l + \frac{b_l}{r^{l+1}}\right)P_l(\cos\theta)}$$

- 球对称系统，球对称解与 θ 无关，只有 $l = 0$ 项 $\phi(r,\theta,\phi) = \phi(r)$，（$m = l = 0$）

$$\phi(r) = a_0 + b_0/r$$

分离变量法



- **根据球对称通解 $(\phi_3(\infty) = 0)$**

$$\phi_2 = a_0 + \frac{b_0}{r}, \qquad \phi_3 = \frac{b_0'}{r}$$

- **中央导体球接地**

$$\phi_2(R_1) = a_0 + \frac{b_0}{R_1} = 0 \;\Rightarrow\; b_0 = -R_1 a_0$$

- **导体球壳电势表面相同 $\phi_2(R_2) = \phi_3(R_3)$**

$$\phi_2(R_2) = a_0 + \frac{b_0}{R_2} = \phi_3(R_3) = \frac{b_0'}{R_3} \;\Rightarrow\; b_0' = a_0 R_3\,(R_2 - R_1)/R_2$$

- **球壳带电 Q，利用 ($\nabla \cdot \vec{E} = \rho/\varepsilon_0$)**

$$-\oint_{S_3} \frac{\partial \phi_3}{\partial r} r^2 d\Omega + \oint_{S_2} \frac{\partial \phi_2}{\partial r} r^2 d\Omega = \frac{Q}{\varepsilon_0} \;\Rightarrow\; b_0' - b_0 = \frac{Q}{4\pi\varepsilon_0}$$

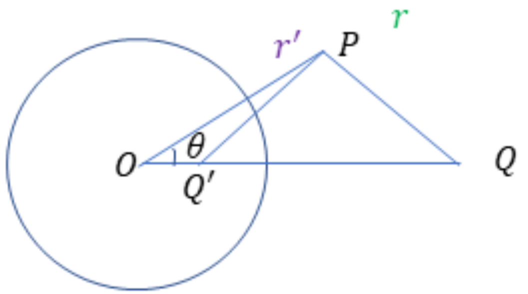
镜像法：唯一性定理

- 真空中有半径为R_0 的导体球，带有电荷Q_0。距球心a ($a > R_0$)处有一点电荷Q，求球外电势
- 根据对称性，把镜像电荷 Q' 放置在OQ连线 Q' 处，设$OQ' = b.\ Q'P = r'$， Q'和Q产生的电势在球面上恒为0，球面上任意点

$$\frac{Q}{r} + \frac{Q'}{r'} = 0 \quad \Rightarrow \quad \frac{r'}{r} = -\frac{Q'}{Q} \quad \Rightarrow \quad b = \frac{R_0^2}{a} \quad \Rightarrow \quad Q' = -\frac{R_0}{a} Q$$

- 导体总电荷为Q_0, 在球心放等效电荷$Q_0 - Q'$， 它在球面产生均匀电势

$$\phi_{Q_0 - Q'}\Big|_S = \frac{1}{4\pi\varepsilon_0} \frac{Q_0 - Q'}{R_0}$$



- 导体球外任意一点电势

$$\phi = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{r} - \frac{\frac{R_0}{a}Q}{r'} + \frac{Q_0 + \frac{R_0}{a}Q}{R_0}\right)$$

格林函数

- 格林函数 $G(\vec{x},\vec{x}')$定义：位于x'点的单位点电荷激发的电势$G(\vec{x},\vec{x}')$ （$\rho(\vec{x})=\delta(\vec{x}-\vec{x}')$）

$$\nabla^2 G(\vec{x},\vec{x}')=-\frac{1}{\varepsilon_0}\delta(\vec{x}-\vec{x}')$$

x 是场点，∇^2 微分是对 x

- 第一类格林函数：边界S上的条件为

$$G(x,x')\Big|_{x\in S}=0$$

- 第二类格林函数：边界条件为（S为边界的面积）

$$\frac{\partial G(x,x')}{\partial n}\Bigg|_{x\in S}=-\frac{1}{\varepsilon_0 S}$$

- 唯一性定理 $\Rightarrow$ 两类格林函数都是唯一确定的
- 采用镜像势求出的不同边界条件下的点电荷电势就是相应边值问题的泊松方程的格林函数（只差一个常数因子）

孤立点电荷的格林函数

- 格林函数：位于x'点的单位点电荷激发的电势

$$\nabla^2 G(\vec{x},\vec{x}') = -\frac{1}{\varepsilon_0}\delta(\vec{x}-\vec{x}')$$

- 最简单的点电荷电势

$$\phi = \frac{1}{4\pi\varepsilon_0}\frac{1}{r} \quad \Longrightarrow \quad \nabla^2\phi = -\frac{1}{\varepsilon_0}\delta(\vec{r})$$

- 无穷大边界

$$\phi(r\to\infty) = 0$$

- 无穷大边界（无界空间）的格林函数

$$G(\vec{x},\vec{x}') = \frac{1}{4\pi\varepsilon_0}\frac{1}{r} = \frac{1}{4\pi\varepsilon_0}\frac{1}{|\vec{x}-\vec{x}'|} = \frac{1}{4\pi\varepsilon_0}\frac{1}{\sqrt{(x-x')^2+(y-y')^2+(z-z')^2}}$$

多级展开

- 电荷分布 $\rho(x')$，在远离源 x 处的电势 （$r=|\vec{x}-\vec{x}'|$）

$$\phi(\vec{x})=\iiint\frac{\rho(x')}{4\pi\varepsilon_0 r}dV'$$

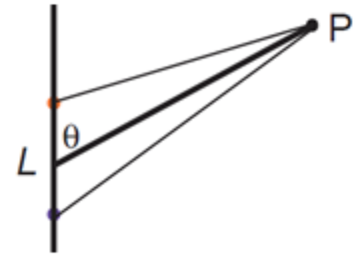
- 利用 $\frac{1}{r}=\frac{1}{R}-\sum_{i=1}^{3}x_i'\frac{\partial}{\partial x_i}\frac{1}{R}+\frac{1}{2!}\sum_{i,j=1}^{3}x_i'x_j'\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}+\cdots$

$$\phi(\mathbf{x})=\frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R}-\sum_{i=1}^{3}p_i\frac{\partial}{\partial x_i}\frac{1}{R}+\frac{1}{6}\sum_{i,j=1}^{3}D_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R}+\cdots\right)$$

- 电荷： $Q=\int\rho(x')dV'$
- 电偶极矩： $p_i=\iiint_V x_i'\rho(x')dV'$
- 电四极矩： $D_{ij}=\iiint_V(3x_i'x_j'-r'^2\delta_{ij})\rho(x')dV'$

电偶极子

- 电偶极矩激发的电势与R^2成反比（次级贡献）



$$\phi^{(1)} = \frac{1}{4\pi\epsilon_0}\vec{p}\cdot\nabla\frac{1}{R} = \frac{1}{4\pi\epsilon_0}\vec{p}\cdot\frac{\vec{R}}{R^3}$$

- 电偶极子：总电荷为零、电偶极矩不为零

$$\vec{p} = \int \vec{x}'\,[Q\delta(\vec{x}'-\vec{x}'_+) - Q\delta(\vec{x}'-\vec{x}'_-)]dV' = Q(\vec{x}'_+ - \vec{x}'_-) = Qd$$

- 电偶极子电势

$$\phi = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) = \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{R}-\vec{x}_+|} - \frac{1}{|\vec{R}-\vec{x}_-|}\right)$$

$$\approx \frac{Q}{4\pi\varepsilon_0}\left(\frac{1}{|\vec{R}| - \hat{e}_r\cdot\vec{x}_+} - \frac{1}{|\vec{R}| - \hat{e}_r\cdot\vec{x}_-}\right) \approx \frac{Q}{4\pi\varepsilon_0}\,\hat{e}_r\cdot\frac{\vec{l}}{R^2}$$

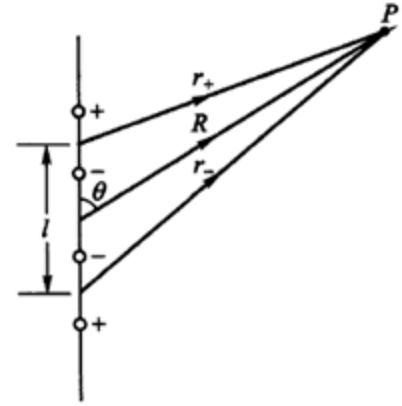
$$\approx \frac{Q}{4\pi\varepsilon_0}\frac{l\cos\theta}{R^2} = \frac{Q}{4\pi\varepsilon_0}\frac{l}{R^2}\frac{z}{R} = -\frac{Ql}{4\pi\varepsilon_0}\frac{\partial}{\partial z}\frac{1}{R} = -\frac{1}{4\pi\varepsilon_0}\left(p_z\frac{\partial}{\partial z}\right)\frac{1}{R}$$

- 电偶极子在远处产生的电势即为其电偶极矩产生的电势

四极矩

- 无迹电四极矩（$D_{11}+D_{22}+D_{33}=0$）：

$$D_{ij}=\iiint_V (3x'_i x'_j - r'^2\delta_{ij})\rho(x')dV'$$



- 考虑一对电偶极子：负电荷位于 $(0,0,-a),(0,0,a)$;

  正电荷位于$(0,0,-b),(0,0,b)$, 电四极矩?

- 电荷密度分布

$$\rho(\vec{x}')=\left(Q\delta(z'-b)+Q\delta(z'+b)-Q\delta(z'-a)-Q\delta(z'+a)\right)\delta(x')\delta(y')$$

- 根据无迹电四极矩公式

$$D_{33}=\iiint(3z'z'-r'^2)\rho(x')dV'=4Q(b^2-a^2),\qquad D_{11}=D_{22}=-2Q(b^2-a^2)$$

- 根据原始公式

$$\bar{D}_{33}=\iiint 3x'_i x'_j\rho(x')dV'=\iiint 3z'z'\rho(x')dV'=6Q(b^2-a^2)$$

电磁场

- Maxwell方程组

$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},\qquad \nabla\cdot\vec{D}=\rho$$

$$\nabla\cdot\vec{B}=0,\qquad \nabla\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

- 静电场：静电势+泊松方程

$$\nabla\times\vec{E}=0\ \Longrightarrow\ \vec{E}=-\nabla\phi;$$

$$\nabla\cdot\vec{D}=\rho\ \Longrightarrow\ \nabla^2\phi=-\frac{\rho}{\varepsilon}\ \Longrightarrow\ \phi(x)=\iiint\frac{\rho(x')}{4\pi\varepsilon_0 r}dV'$$

- 静磁场：矢势/磁标势

$$\nabla\cdot\vec{B}=0\ \Longrightarrow\ \vec{B}=\nabla\times\vec{A}$$

$$\nabla\times\vec{H}=\vec{J}\ \xRightarrow{\nabla\cdot\vec{A}=0}\ \nabla^2\vec{A}=-\mu\vec{J}\ \Longrightarrow\ \vec{A}(x)=\frac{\mu}{4\pi}\iiint\frac{\vec{J}(x')}{r}dV'$$

静电场电势

- 给定电荷分布 $\rho(\vec{x})$，静电势满足泊松（Possion）方程

$$\nabla^2\phi = -\frac{\rho}{\varepsilon}$$

- 求解泊松方程

$$\phi(x) = \int \frac{\rho(x')}{4\pi\varepsilon r} dV', \qquad r = |x - x'|$$

- 点电荷 $\rho(\vec{x}) = Q\delta(\vec{x} - \vec{x}_0)$

$$\phi(x) = \int \frac{\rho(x')}{4\pi\varepsilon r} dV' = \frac{Q}{4\pi\varepsilon r}$$

- 静电势多极展开

$$\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \sum_{i=1}^{3} p_i \frac{\partial}{\partial x_i}\frac{1}{R} + \frac{1}{6}\sum_{i,j=1}^{3} D_{ij}\frac{\partial^2}{\partial x_i \partial x_j}\frac{1}{R} + \cdots\right)$$

- 电荷 $Q = \int \rho(x')dV'$；电偶极矩 $p_i = \iiint_V x_i'\rho(x')dV'$；电四极矩 $\bar{D}_{ij} = \iiint_V 3x_i' x_j' \rho(x')dV'$

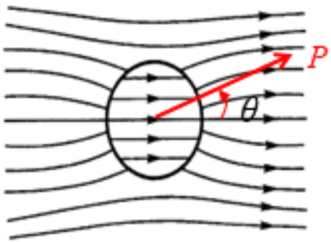
静电场（$\nabla^2\phi = -\rho/\varepsilon$）

1. 静电势： $\nabla\times\vec{E} = 0 \implies \vec{E} = -\nabla\phi$

2. 泊松方程： $\nabla\cdot\vec{D} = \rho \implies \nabla^2\phi = -\rho/\varepsilon$ $+\ \phi|_S$ 或者 $\frac{\partial\phi}{\partial n}|_S \implies$ 唯一性定理

边值条件： $\phi_1 = \phi_2$， $\varepsilon_2\frac{\partial\phi_2}{\partial n} - \varepsilon_1\frac{\partial\phi_1}{\partial n} = -\sigma$

3. Laplace方程： $\nabla^2\phi = 0$

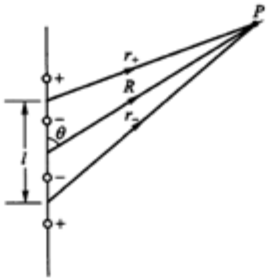


柱对称解（$m=0$）： $\phi(r,\theta) = \sum_{l=0}^{\infty}\left(a_l r^l + \frac{b_l}{r^{l+1}}\right)P_l(\cos\theta)$

球对称解（$m=l=0$）： $\phi(r) = a_0 + \frac{b_0}{r}$

4. 镜像法

5. 格林函数： $\nabla^2 G(\vec{x},\vec{x}') = -\frac{1}{\varepsilon_0}\delta(\vec{x}-\vec{x}')$



6. 多级展开： $\phi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q}{R} - \sum_{i=1}^{3} p_i\frac{\partial}{\partial x_i}\frac{1}{R} + \frac{1}{6}\sum_{i,j=1}^{3} D_{ij}\frac{\partial^2}{\partial x_i\partial x_j}\frac{1}{R} + \cdots\right)$

3. 静磁场

矢势、规范场、AB效应

静磁场 vs 静电场

	静电场	静磁场
场方程	$\nabla\times\vec{E}=0$	$\nabla\cdot\vec{B}=0$
源方程	$\nabla\cdot\vec{E}=\rho/\varepsilon$	$\nabla\times\vec{B}=\mu\vec{J}$
规范势	$\vec{\boldsymbol{E}}=-\boldsymbol{\nabla}\boldsymbol{\phi}$	$\vec{\boldsymbol{B}}=\boldsymbol{\nabla}\times\vec{\boldsymbol{A}}$
势方程	$\boldsymbol{\nabla}^2\boldsymbol{\phi}=-\boldsymbol{\rho}/\boldsymbol{\varepsilon}$	$\boldsymbol{\nabla}^2\vec{\boldsymbol{A}}=-\boldsymbol{\mu}\vec{\boldsymbol{J}}$
通解	$\phi(x)=\frac{1}{4\pi\varepsilon_0}\iiint\frac{\boldsymbol{\rho}(\boldsymbol{x}')}{\boldsymbol{r}}dV'$	$\vec{A}(x)=\frac{\mu}{4\pi}\iiint\frac{\vec{\boldsymbol{J}}(\boldsymbol{x}')}{\boldsymbol{r}}dV'$

静磁场：矢势

- 磁场无源 → 矢势

$$\nabla\cdot\vec{B}=0 \quad\Longrightarrow\quad \vec{B}=\nabla\times\vec{A}$$

- 方程 $\nabla\times\vec{B}=\mu\vec{J} \quad\Longrightarrow$

$$\nabla^2\vec{A}=-\mu\vec{J}$$

- 磁场边值关系

$$\vec{n}\cdot\left(\vec{B}_2-\vec{B}_1\right)=0, \qquad \vec{n}\times\left(\vec{H}_2-\vec{H}_1\right)=\vec{\alpha}_f$$

- 矢势

$$\vec{n}\cdot\left(\nabla\times\vec{A}_2-\nabla\times\vec{A}_1\right)=0 \quad\Longrightarrow\quad \vec{A}_2=\vec{A}_1$$

$$\vec{n}\times\left[\frac{1}{\mu_2}\left(\nabla\times\vec{A}_2\right)-\frac{1}{\mu_1}\left(\nabla\times\vec{A}_1\right)\right]=\vec{\alpha}_f$$

磁矢势

- 磁矢势

$$\vec{A}(x)=\frac{\mu_0}{4\pi}\iiint\frac{\vec{J}(x')}{r}dV'=\frac{\mu_0}{4\pi}\int\vec{J}(x')\left[\frac{1}{R}-\sum_{j=1}^{3}x'_j\frac{\partial}{\partial x_j}\frac{1}{R}+\frac{1}{2!}\sum_{j,k=1}^{3}x'_jx'_k\frac{\partial^2}{\partial x_j\partial x_k}\frac{1}{R}-\cdots\right]dV'$$

- 多级展开

$$\vec{A}(x)=\vec{A}^{(0)}+\vec{A}^{(1)}+\vec{A}^{(2)}+\cdots$$

- 其中

$$\vec{A}^{(0)}(x)=\frac{\mu_0}{4\pi R}\int\vec{J}(x')dV'\xrightarrow{\nabla\cdot\vec{J}=0}\vec{A}^{(0)}=0$$

$$\vec{A}^{(1)}(x)=-\frac{\mu_0}{4\pi}\int\vec{J}(x')x'_j\frac{\partial}{\partial x_j}\frac{1}{R}dV'\xrightarrow{\vec{m}=\frac{1}{2}\iiint\vec{x}'\times\vec{J}(x')dV'}\vec{A}^{(1)}(x)=\frac{\mu_0}{4\pi R^3}\vec{m}\times\vec{R}$$

AB效应：规范势的物理意义

- 磁场无源 → 矢势

$$\nabla \cdot \vec{B} = 0 \quad \Longrightarrow \quad \vec{B} = \nabla \times \vec{A}$$

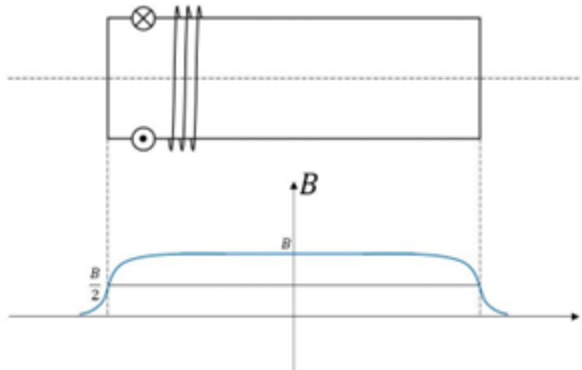
- 经典电磁学，规范势 $\vec{A}$ 的线积分有意义

$$\Phi = \iint \vec{B} \cdot d\vec{S} = \iint (\nabla \times \vec{A}) \cdot d\vec{S} = \oint \vec{A} \cdot d\vec{l}$$

- 量子力学：规范势的物理效应

螺线管磁场

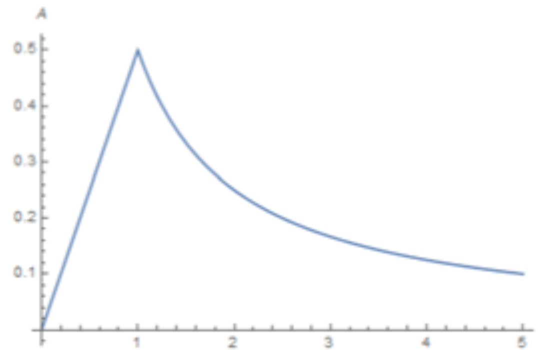
- 考虑一根沿 z 轴放置无穷长螺旋管，半径为 a



$$\vec{B} = \begin{cases} B_0 \hat{e}_z, & r < a \\ 0, & r > a \end{cases}$$

- 螺线管外磁场为零
- 矢势$\vec{A} = A(r)\hat{e}_\varphi$

$$A(r) = \begin{cases} \dfrac{B_0 r}{2}, & r < a \\ \dfrac{B_0 a^2}{2r}, & r > a \end{cases}$$

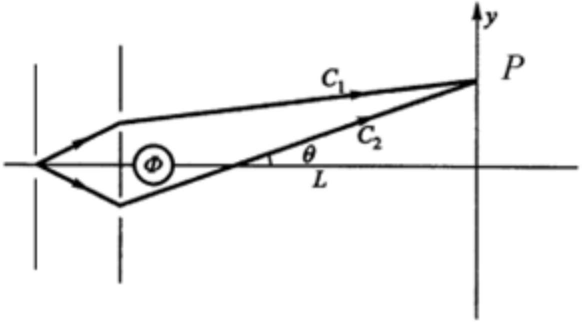


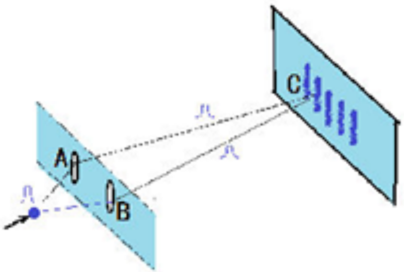
- 矢势在螺线管外不为零

$$B_z = \frac{1}{r}\frac{\partial}{\partial r}(rA_\varphi) - \frac{1}{r}\frac{\partial A_r}{\partial \varphi}$$

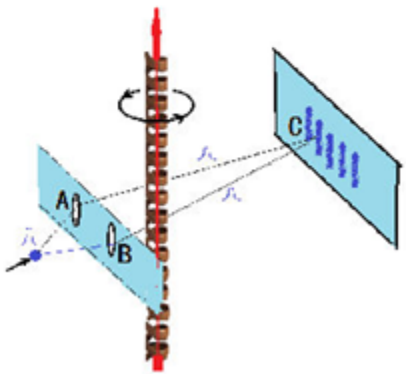
螺线管外电子

- 考虑一根沿 z 轴放置无穷长螺旋管，半径为 a
- 螺线管外磁场为零 $\vec{B}=0$
- 矢势在螺线管外不为零 $\vec{A}\neq 0$
- 电子在螺线管外是否能感受到矢势?

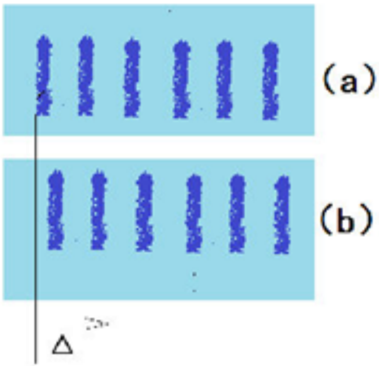




(a) 电子双缝干涉

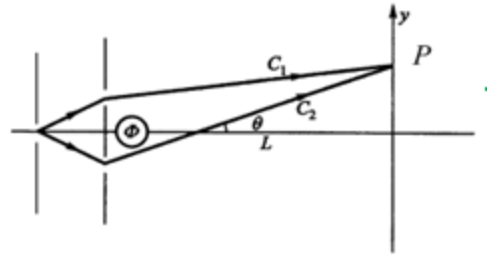


(b) AB效应



(c) 干涉条纹移动△

微观粒子几率



- 若带电荷q的粒子在运动过程中能量守恒，在t_0时刻位于x_0，则按照量子力学，粒子在t时刻出现在x的概率幅为

$$\langle \vec{\mathbf{x}}, t | \vec{\mathbf{x}}_0, t_0 \rangle = \sum_k \exp\left[\frac{i}{\hbar}\int_{C_k} (\vec{\mathbf{p}} + q\vec{\mathbf{A}}) \cdot dx' - \frac{i}{\hbar} E(t - t_0)\right]$$

- 粒子在 t 时刻出现在x的概率

$$|\langle \vec{\mathbf{x}}, t | \vec{\mathbf{x}}_0, t_0 \rangle|^2$$

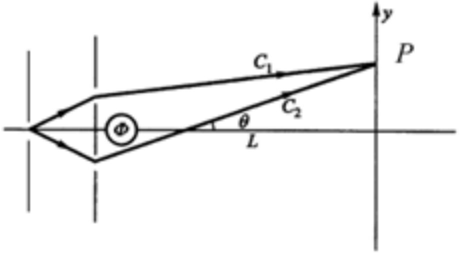
- 通过不同路径 C_1 或者 C_2，电子出现在P点的概率正比于

$$\left|\exp\left[\frac{i}{\hbar}\int_{C_1} (\vec{\mathbf{p}} - e\vec{\mathbf{A}}) \cdot d\vec{x}' - \frac{i}{\hbar} E(t - t_0)\right] + \exp\left[\frac{i}{\hbar}\int_{C_2} (\vec{\mathbf{p}} - e\vec{\mathbf{A}}) \cdot d\vec{x}' - \frac{i}{\hbar} E(t - t_0)\right]\right|^2$$

$$= 2 + \exp\left[\frac{i}{\hbar}\oint_{C_1 - C_2} (\vec{\mathbf{p}} - e\vec{\mathbf{A}}) \cdot d\vec{x}'\right] + \exp\left[-\frac{i}{\hbar}\oint_{C_1 - C_2} (\vec{\mathbf{p}} - e\vec{\mathbf{A}}) \cdot d\vec{x}'\right]$$

$$= 2 + 2\cos(\Delta\varphi_K + \Delta\varphi_{AB})$$

微观粒子几率



- 若带电荷q的粒子在运动过程中能量守恒，在t_0时刻位于x_0，则按照量子力学，粒子在t时刻出现在x的概率幅为

$$\langle \vec{\mathbf{x}}, t | \vec{\mathbf{x}}_0, t_0 \rangle = \sum_k \exp\left[\frac{i}{\hbar} \int_{C_k} (\vec{\mathbf{p}} + q\vec{\mathbf{A}}) \cdot dx' - \frac{i}{\hbar} E(t - t_0)\right]$$

- 通过不同路径 C_1 或者 C_2，电子出现在P点的概率正比于

$$2 + 2\cos(\Delta\varphi_K + \Delta\varphi_{AB})$$

- 动力学相位

$$\Delta\varphi_K = \frac{1}{\hbar} \oint_C \vec{\mathbf{p}} \cdot d\vec{\mathbf{x}}'$$

- A-B效应的相位

$$\Delta\varphi_{AB} = -\frac{e}{\hbar} \oint_C \vec{A} \cdot d\vec{x}' = -\frac{e}{\hbar} \Phi$$

4. 电磁波

电磁场的矢势和标势

- 电场和磁场由矢势和标势（$\vec{A},\varphi$）描写

$$\vec{B}=\nabla\times\vec{A},\qquad \vec{E}=-\nabla\varphi-\frac{\partial\vec{A}}{\partial t}\qquad\Longrightarrow\qquad \nabla\cdot\vec{B}=0,\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

- 满足规范变换

$$\vec{A}'=\vec{A}+\nabla f,\qquad \varphi'=\varphi-\frac{\partial f}{\partial t}\qquad\Longrightarrow\qquad \vec{E}'=\vec{E},\qquad \vec{B}'=\vec{B}$$

- Maxwell方程

$$-\nabla^2\varphi-\frac{\partial}{\partial t}(\nabla\cdot\vec{A})=\frac{\rho}{\varepsilon_0},\qquad \nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}-\nabla\left(\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}\right)=-\mu_0\vec{J}$$

Maxwell方程

- 洛伦兹规范

$$\nabla\cdot\vec{A}+\frac{1}{c^2}\frac{\partial\varphi}{\partial t}=0$$

- 洛伦兹规范下Maxwell方程：达朗贝尔方程

$$\nabla^2\varphi-\frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2}=-\frac{\rho}{\varepsilon_0},\qquad \nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{J}$$

- 库伦规范

$$\nabla\cdot\vec{A}=0$$

- 库伦规范下Maxwell方程

$$\nabla^2\varphi=-\frac{\rho}{\varepsilon_0},\qquad \nabla^2\vec{A}-\frac{1}{c^2}\frac{\partial^2\vec{A}}{\partial t^2}=-\mu_0\vec{J}+\frac{1}{c^2}\frac{\partial\nabla\varphi}{\partial t}$$

无源Maxwell方程 $\Longrightarrow$ 真空/介质中电磁波

- 真空中： $\vec{D}=\varepsilon_0\vec{E},\ \vec{B}=\mu_0\vec{H}$

$$\nabla\cdot\vec{E}=0,\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},\qquad \nabla\cdot\vec{B}=0,\qquad \nabla\times\vec{B}=\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

- 波动方程

$$\nabla^2\vec{E}-\mu_0\varepsilon_0\frac{\partial^2\vec{E}}{\partial t^2}=0,\qquad \nabla^2\vec{B}-\mu_0\varepsilon_0\frac{\partial^2\vec{B}}{\partial t^2}=0$$

- 单色波：$\vec{E}(x,t)=\vec{E}(x)e^{-i\omega t},\quad \vec{B}(x,t)=\vec{B}(x)e^{-i\omega t},\quad k^2=\omega^2\mu\varepsilon$

$$\nabla^2\vec{E}+k^2\vec{E}=0,\qquad \nabla\cdot\vec{E}=0;\qquad \nabla^2\vec{H}+k^2\vec{H}=0,\qquad \nabla\cdot\vec{H}=0$$

$$\vec{B}=\mu\vec{H}=\frac{1}{i\omega}\nabla\times\vec{E},\qquad \vec{E}=-\frac{1}{i\omega\varepsilon}\nabla\times\vec{H}$$

- 平面波：沿 z 方向传播的电磁波的电场 $\vec{E}$

$$\vec{E}(z,t)=\vec{E}_0e^{i(kz-\omega t)}$$

导体内的电磁波

- 导体内部

$$\rho = 0, \qquad \vec{J} = \sigma \vec{E}$$

- 考虑单色波，导体内Maxwell方程

$$\nabla \times \vec{E} = i\omega\mu\vec{H}$$

$$\nabla \times \vec{H} = \sigma\vec{E} - i\omega\varepsilon\vec{E} = -i\omega\varepsilon'\vec{E}$$

复电容率

$$\varepsilon' = \varepsilon + \frac{i\sigma}{\omega}$$

- 电场（复波矢 $\vec{k} = \vec{\beta} + i\vec{\alpha}$)

$$\vec{E}(x) = \vec{E}_0 e^{-\vec{\alpha}\cdot\vec{x}} e^{i(\vec{\beta}\cdot\vec{x} - \omega t)}$$

- **理想导体：穿透深度 $\delta = 0$，导体内 $\vec{E}_c = \vec{B}_c = 0$**

边界条件

- 考虑理想导体边界，电磁波在真空或者介质中传播。导体用角标 c，真空/介质无角标
- 法向 n：导体指向介质/真空
- 理想导体：穿透深度 $\delta = 0$，导体内 $\vec{E}_c = \vec{B}_c = 0$
- 边值关系

$$\vec{n} \times \left(\vec{E} - \vec{E}_c\right) = 0 \quad \Longrightarrow \quad \vec{n} \times \vec{E} = 0$$

$$\vec{n} \times \left(\vec{H} - \vec{H}_c\right) = \vec{\alpha}_f \quad \Longrightarrow \quad \vec{n} \times \vec{H} = \vec{\alpha}_f$$

$$\vec{n} \cdot \left(\vec{D} - \vec{D}_c\right) = \sigma_f \quad \Longrightarrow \quad \vec{n} \cdot \vec{D} = \sigma_f$$

$$\vec{n} \cdot \left(\vec{B} - \vec{B}_c\right) = 0 \quad \Longrightarrow \quad \vec{n} \cdot \vec{B} = 0$$

- 导体表面：$\vec{n} \times \vec{E} = 0 \Longrightarrow E_{\perp} = 0$ 电场 ⊥ 界面；$\vec{n} \cdot \vec{B} = 0 \Longrightarrow B_n = 0$ 磁场 // 界面

1d 约束：平行板

- 导体表面：$\vec{n}\times\vec{E}=0$ 电场 $\perp$ 界面；$\vec{n}\cdot\vec{B}=0$ 磁场 // 界面

- TEM波：横向电磁波（transverse electromagnetic mode），电场/磁场横向振荡

- 两个平行无穷大导体板之间可以传播TEM波

- 考虑两导体板平行于 x-y 平面 $\Rightarrow$

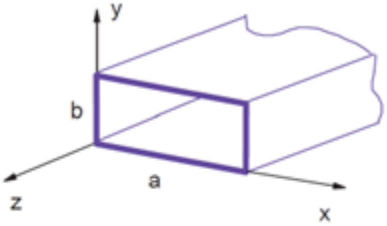
$$\vec{E}=E\,\hat{e}_z,\qquad \vec{B}=B_x\hat{e}_x+B_y\hat{e}_y$$

- 若电磁波沿 x 方向传播，电场

$$\vec{E}=E\,\hat{e}_z,\qquad \vec{B}=B_y\hat{e}_y$$

- 存在一种偏振的TEM波

矩形波导管中电磁波



- Helmholtz 方程 ($k^2 = \omega^2\mu\varepsilon$)

$$\nabla^2\vec{E} + k^2\vec{E} = 0, \qquad \nabla\cdot\vec{E} = 0$$

$$\vec{B} = -\frac{i}{\omega}\nabla\times\vec{E}$$

- 边值关系

$$\vec{n}\times\vec{E} = 0, \qquad \vec{n}\cdot\vec{B} = 0$$

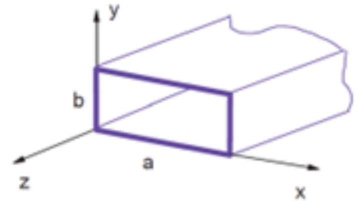
- 考虑沿 z 向传播电磁波

$$\vec{E}(x,y,z) = \vec{E}(x,y)e^{ik_z z}, \qquad \vec{B}(x,y,z) = \vec{B}(x,y)e^{ik_z z}$$

- 横向 Helmholtz 方程

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + k_\perp^2\right)\vec{E}(x,y) = 0\,, \qquad k_\perp^2 = k^2 - k_z^2$$

波导管中电磁波



- 波导管中电场 ($k_x = \frac{m\pi}{a}$, $k_y = \frac{n\pi}{b}$)

$$E_x = A_1 \cos k_x x \sin k_y y\, e^{ik_z z}$$

$$E_y = A_2 \sin k_x x \cos k_y y\, e^{ik_z z}$$

$$E_z = A_3 \sin k_x x \sin k_y y\, e^{ik_z z}$$

- 无源 $\nabla \cdot \vec{E} = 0 \quad \Longrightarrow$

$$k_x A_1 + k_y A_2 - ik_z A_3 = 0$$

- 给定(m,n)，(A_1, A_2, A_3)中两个独立$\Longrightarrow$ 两个独立波模
- 磁场

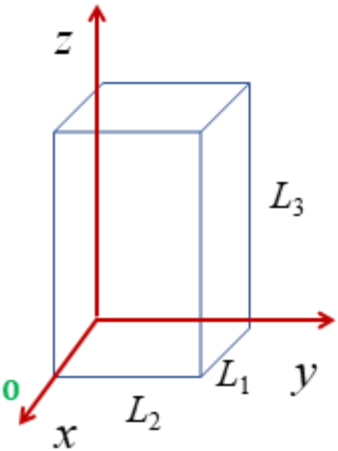
$$\vec{H} = -\frac{i}{\omega\mu}\nabla \times \vec{E}$$

- 例如给定(m,n),

$$1.\ E_z = 0 \ \Longrightarrow\ A_3 = 0 \ \Longrightarrow\ \frac{A_1}{A_2} = -\frac{k_y}{k_x} \ \Longrightarrow\ H_z = -\frac{i(k_x A_2 - k_y A_1)}{\omega\mu}\cos k_x x \cos k_y y\, e^{ik_z z} \neq 0$$

$$2.\ H_z = 0 \ \Longrightarrow\ k_x A_2 - k_y A_1 = 0 \ \Longrightarrow\ A_3 = \frac{A_1(k_x^2 + k_y^2)}{ik_x k_z} \ or\ \frac{A_2(k_x^2+k_y^2)}{ik_y k_z} \Longrightarrow\ E_z \neq 0$$

有限体积内的电磁波



- 电磁波限制在长方体内：直角坐标+分离变量

  (u 表示 $\vec{E}$ 或 $\vec{H}$ 的任意一个直角坐标分量)

$$u(x,y,z) = X(x)Y(y)Z(z)$$

- Helmholtz方程 $\nabla^2 u + k^2 u = 0$

$$X''(x) + k_x^2 X(x) = 0, \qquad Y''(y) + k_y^2 Y(y) = 0, \qquad Z''(z) + k_z^2 Z(z) = 0$$

- 波数

$$k^2 = k_x^2 + k_y^2 + k_z^2 = \omega^2 \mu \varepsilon$$

- 线性独立解

$$\sin k_j x_j, \qquad \cos k_j x_j, \qquad e^{\pm i k_j x_j}$$

- 边值关系

$$\vec{n} \times \vec{E} = 0, \qquad \vec{n} \cdot \vec{B} = 0$$

长方体内电磁波

- 束缚解 $\left(k_x = \frac{m\pi}{L_1}, k_y = \frac{n\pi}{L_2}, k_z = \frac{p\pi}{L_3}\right)$

$$E_x = A_1 \cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y = A_2 \sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z = A_3 \sin(k_x x)\sin(k_y y)\cos(k_z z)$$

- 无源条件 $\nabla \cdot \vec{E} = 0$

$$k_x A_1 + k_y A_2 + k_z A_3 = 0$$

- $\vec{k}$ 不能有两个分量等于零。(m,n,p)谐振波

$$\frac{m}{L_1}A_1 + \frac{n}{L_2}A_2 + \frac{p}{L_3}A_3 = 0$$

谐振腔

- 谐振腔的本征频率只有离散值
- 谐振腔中电磁波具有本征振荡模式
- 谐振腔（$L_1 > L_2 > L_3$）允许的最低本征频率为 (1,1,0)

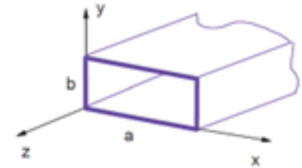
$$f_{110} = \frac{1}{2\sqrt{\mu\varepsilon}}\sqrt{\frac{1}{L_1^2}+\frac{1}{L_2^2}}$$

- 谐振腔允许通过的电磁波最大波长

$$\lambda_{110} = 2\left(\frac{1}{L_1^2}+\frac{1}{L_2^2}\right)^{-\frac{1}{2}}$$

有界空间的电磁波总结

- 波导管（2d约束），横向 Helmholtz 方程



$$\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+k_\perp^2\right)\vec{E}(x,y)=0\,,\qquad k_\perp^2=k^2-k_z^2$$

- 电场（$k_x=\frac{m\pi}{a},\quad k_y=\frac{n\pi}{b}$）

$$E_x=A_1\cos k_x x\,\sin k_y y\,e^{ik_z z}$$

$$E_y=A_2\sin k_x x\,\cos k_y y\,e^{ik_z z}$$

$$E_z=A_3\sin k_x x\,\sin k_y y\,e^{ik_z z}$$

- TE模：$E_z=0$；TM模：$H_z=0$

- 谐振腔（3d约束），电场或磁场分量的Helmholtz方程（$k^2=k_x^2+k_y^2+k_z^2=\omega^2\mu\varepsilon$）

$$X''(x)+k_x^2X(x)=0,\qquad Y''(y)+k_y^2Y(y)=0,\qquad Z''(z)+k_z^2Z(z)=0$$

- 束缚解（$k_x=\frac{m\pi}{L_1},k_y=\frac{n\pi}{L_2},k_z=\frac{p\pi}{L_3}$）

$$E_x=A_1\cos(k_x x)\sin(k_y y)\sin(k_z z)$$

$$E_y=A_2\sin(k_x x)\cos(k_y y)\sin(k_z z)$$

$$E_z=A_3\sin(k_x x)\sin(k_y y)\cos(k_z z)$$

5. 电磁辐射

含时电磁场

- 矢势和标势

$$\vec{B} = \nabla \times \vec{A}, \qquad \vec{E} = -\nabla\varphi - \frac{\partial \vec{A}}{\partial t}$$

- 洛伦兹规范（$\nabla \cdot \vec{A} + \frac{1}{c^2}\frac{\partial \varphi}{\partial t} = 0$）$\Rightarrow$ 达朗贝尔方程

$$\nabla^2\varphi - \frac{1}{c^2}\frac{\partial^2 \varphi}{\partial t^2} = -\frac{\rho}{\varepsilon_0}, \qquad \nabla^2\vec{A} - \frac{1}{c^2}\frac{\partial^2 \vec{A}}{\partial t^2} = -\mu_0\vec{J}$$

- 无源解：平面电磁波 （$\varphi_0 = c\hat{e}_k \cdot \vec{A}_0$）

$$\vec{A} = \vec{A}_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}, \qquad \varphi = \varphi_0 e^{i(\vec{k}\cdot\vec{x}-\omega t)}$$

- 有源解：推迟势

$$\varphi(\vec{x},t) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\vec{x}',t-r/c)}{r}dV', \qquad \vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x}',t-r/c)}{r}dV'$$

交变源

- 考虑频率为 ω 的交变电流

$$\vec{J}(\vec{x},t)=\vec{J}_\omega(\vec{x})e^{-i\omega t}$$

- 交变推迟势，矢势

$$\vec{A}(\vec{x},t)=\vec{A}_\omega(\vec{x})e^{-i\omega t}=\left(\frac{\mu_0}{4\pi}\int\frac{\vec{J}_\omega(\vec{x}')e^{ikr}}{r}dV'\right)e^{-i\omega t}=\frac{\mu_0}{4\pi}\int\frac{\vec{J}(\vec{x}',t)}{r}e^{ikr}dV'$$

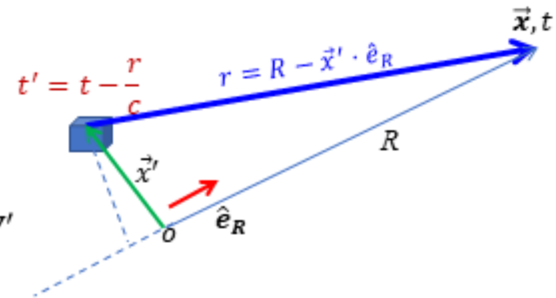
- 标势

$$\varphi(\vec{x},t)=\left(\frac{1}{4\pi\varepsilon_0 i\omega}\int\frac{\nabla'\cdot\vec{J}_\omega(x')}{r}e^{ikr}dV'\right)e^{-i\omega t}=\frac{1}{4\pi\varepsilon_0 i\omega}\int\frac{\nabla'\cdot\vec{J}(x')}{r}e^{ikr}dV'$$

- 多极展开领头阶

$$\vec{A}(\vec{x},t)=\frac{\mu_0 e^{ikR}}{4\pi R}\int_V\vec{J}(\vec{x}',t)dV'$$

多极展开



- 矢势（$\vec{A}(\vec{x},t) = \frac{\mu_0}{4\pi}\int \frac{\vec{J}(\vec{x}',t)}{r} e^{ikr}\, dV'$ ）

$$\vec{A}(\vec{x},t) \approx \frac{\mu_0}{4\pi}\int_V \frac{\vec{J}(\vec{x}',t)e^{ik(R-\hat{e}_R\cdot\vec{x}')}}{R-\hat{e}_R\cdot\vec{x}'}dV'$$

- 远场近似（$R \gg |\vec{x}'|$）+ 长波极限（$\frac{l}{\lambda} \ll 1$）

$$\frac{1}{R-\hat{e}_R\cdot\vec{x}'} \approx \frac{1}{R} + \frac{\hat{e}_R\cdot\vec{x}'}{R^2} \approx \frac{1}{R}, \qquad e^{-ik\hat{e}_R\cdot\vec{x}'} \approx 1 - i2\pi\hat{e}_R\cdot\frac{\vec{x}'}{\lambda} + \cdots$$

- 远场矢势展开

$$\vec{A}(\vec{x},t) \approx \frac{\mu_0 e^{ikR}}{4\pi}\int_V \frac{\vec{J}(\vec{x}',t)}{R}(1 - ik\hat{e}_R\cdot\vec{x}' + \cdots)dV' = \vec{A}^{(0)} + \vec{A}^{(1)} + \cdots$$

- 领头阶和次领头阶

$$\vec{A}^{(0)}(\vec{x},t) = \frac{\mu_0 e^{ikR}}{4\pi R}\int_V \vec{J}(\vec{x}',t)dV', \qquad \vec{A}^{(1)}(x) = -ik\frac{\mu_0 e^{ikR}}{4\pi R}\int_V \vec{J}(x')(\hat{e}_R\cdot\vec{x}')dV'$$

多极辐射

- 电偶极辐射（$p_i = \iiint_V x_i' \rho(x',t) dV' \quad \Rightarrow \quad \dot{\vec{p}} = \int \vec{J}(\vec{x}',t) dV'$）

$$\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \vec{J}(\vec{x}',t) dV' = \frac{\mu_0 e^{i\vec{k}\cdot\vec{R}}}{4\pi R} \dot{\vec{p}}$$

- 磁偶极辐射（$\vec{m} = \int_V \left(\frac{1}{2}\vec{x}' \times \vec{J}'\right) dV'$ ）

$$\vec{A}_m(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \hat{e}_R \cdot (\vec{x}'\vec{J}')_a dV' = ik \frac{\mu_0 e^{ikR}}{4\pi R} \hat{e}_R \times \vec{m}$$

- 电四极辐射（$D_{ij} = \iiint_V (3x_i' x_j' - \vec{x}'^2 \delta_{ij}) \rho(x',t) dV'$）

$$\vec{A}_q(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \hat{e}_R \cdot (\vec{x}'\vec{J}')_S dV' = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \left(\frac{1}{6} \hat{e}_R \cdot D\right)$$

- 一级贡献

$$A^{(1)}(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V J(x')(\hat{e}_R \cdot x') dV' = \vec{A}_m + \vec{A}_q$$

电偶极辐射

- 含时电偶极矩（$p_i = \iiint_V x_i' \rho(x', t) dV'$）

$$\dot{\vec{p}} = \int \vec{J}(\vec{x}', t) dV'$$

- 交变电流矢势领头阶

$$\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \vec{J}(\vec{x}', t) dV' = \frac{\mu_0 e^{ikR}}{4\pi R} \dot{\vec{p}}$$

- 电偶极辐射

$$\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{i\vec{k}\cdot\vec{R}}}{4\pi R} \dot{\vec{p}}$$

矢势领头阶：电偶极辐射

- 利用含时电偶极矩

$$\dot{\vec{p}} = \int \vec{J}(\vec{x}',t)dV' = \left(\int \vec{J}_\omega(\vec{x}')dV'\right)e^{-i\omega t}$$

- 电偶极矢势

$$\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{i\vec{k}\cdot\vec{R}}}{4\pi R}\dot{\vec{p}}$$

- 磁感应强度 ($\ddot{\vec{p}} = -i\omega\dot{\vec{p}}$)

$$\vec{B} = \nabla\times\vec{A} \approx \frac{\mu_0}{4\pi R}\left(\nabla e^{ikR}\right)\times\dot{\vec{p}} = \frac{\mu_0 e^{ikR}}{4\pi R} ik\hat{e}_R\times\dot{\vec{p}}$$

$$= \frac{\mu_0 e^{ikR}}{4\pi R}\frac{ik}{(-i\omega)}\hat{e}_R\times\ddot{\vec{p}} = \frac{1}{4\pi\varepsilon_0 c^3}\frac{e^{ikR}}{R}\ddot{\vec{p}}\times\hat{e}_R$$

- 电场强度

$$\vec{E} = \frac{ic}{k}\nabla\times\vec{B} \approx \frac{1}{4\pi\varepsilon_0 c^2}\frac{e^{ikR}}{R}\left(\ddot{\vec{p}}\times\hat{e}_R\right)\times\hat{e}_R$$

多极辐射

- 电偶极辐射（$p_i = \iiint_V x_i' \rho(x',t) dV' \quad \Rightarrow \quad \dot{\vec{p}} = \int \vec{J}(\vec{x}',t) dV'$）

$$\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \vec{J}(\vec{x}',t) dV' = \frac{\mu_0 e^{i\vec{k}\cdot\vec{R}}}{4\pi R} \dot{\vec{p}}$$

- 磁偶极辐射（$\vec{m} = \int_V \left(\frac{1}{2}\vec{x}' \times \vec{J}'\right) dV'$）

$$\vec{A}_m(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \hat{e}_R \cdot (\vec{x}'\vec{J}')_a dV' = ik \frac{\mu_0 e^{ikR}}{4\pi R} \hat{e}_R \times \vec{m}$$

- 电四极辐射（$D_{ij} = \iiint_V (3x_i' x_j' - \vec{x}'^2 \delta_{ij}) \rho(x',t) dV'$）

$$\vec{A}_q(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V \hat{e}_R \cdot (\vec{x}'\vec{J}')_S dV' = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \left(\frac{1}{6} \hat{e}_R \cdot D\right)$$

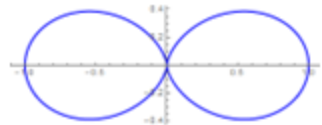
- 一级贡献

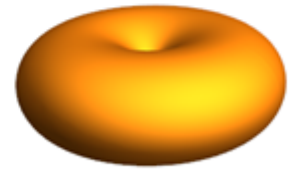
$$A^{(1)}(x) = -ik \frac{\mu_0 e^{ikR}}{4\pi R} \int_V J(x')(\hat{e}_R \cdot x') dV' = \vec{A}_m + \vec{A}_q$$

多极辐射角分布

- 电偶极辐射 $\vec{A}^{(0)}(\vec{x}) = \frac{\mu_0 e^{i\vec{k}\cdot\vec{R}}}{4\pi R}\dot{\vec{p}}$

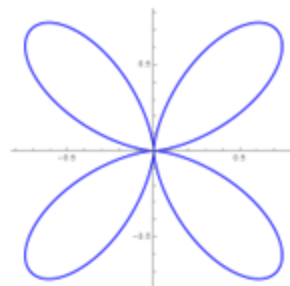
$$\frac{dP}{d\Omega} \propto sin^2\theta, \qquad P \propto \left(\frac{l}{\lambda}\right)^2$$

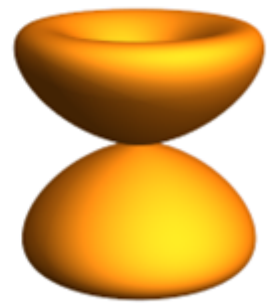




- 磁偶极辐射 $\vec{A}_m(x) = ik\frac{\mu_0 e^{ikR}}{4\pi R}\hat{e}_R \times \vec{m}$

$$\frac{dP}{d\Omega} \propto sin^2\theta, \qquad P \propto \left(\frac{l}{\lambda}\right)^4$$





- 电四极辐射 $\vec{A}_q(x) = -ik\frac{\mu_0 e^{ikR}}{4\pi R}\left(\frac{1}{6}\hat{e}_R \cdot \dot{D}\right)$

$$\frac{dP}{d\Omega} \propto sin^2\theta cos^2\theta, \qquad P \propto \left(\frac{l}{\lambda}\right)^4$$

6. 相对论

洛伦兹变换、四动量、质能关系

伽利略相对论

- **伽利略变换**

$$t' = t$$

$$\vec{x}' = \vec{x} - \vec{b}t$$

- **伽利略相对性原理：物理定律在伽利略群变换下不变**
- **伽利略群：转动+平移+伽利略变换** $\{R, \vec{b}, \vec{c}, e\}, R \in SO(3), \vec{b}, \vec{c} \in \mathbb{R}^3, e \in \mathbb{R}$

$$\begin{cases} t' = t + e \\ \vec{x}' = R \cdot \vec{x} - \vec{b}t + \vec{c} \end{cases}$$

洛伦兹变换

- 闵氏时空保持 $ds^2 = -c^2dt^2 + dx^2$ 不变 → 洛伦兹变换（boost）

$$\Lambda(\theta) = \begin{pmatrix} ch\theta & -sh\theta \\ -sh\theta & ch\theta \end{pmatrix}$$

- 令 $\gamma = ch\theta = \frac{1}{\sqrt{1-\beta^2}} \to sh\theta = \beta\gamma \quad \left(\beta = \frac{v}{c}\right)$,　γ-洛伦兹因子

$$\begin{pmatrix} ct' \\ x' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta \\ -\gamma\beta & \gamma \end{pmatrix} \begin{pmatrix} ct \\ x \end{pmatrix} \Rightarrow \begin{cases} t' = \gamma(t - \frac{vx}{c^2}) \\ x' = \gamma(x - vt) \end{cases}$$

固有时

- **固有时 τ：静止粒子（$dx = dy = dz = 0$）的时间**

$$ds^2 = -c^2 dt^2 + dx^2 + dy^2 + dz^2 = -c^2 d\tau^2$$

- **时间延迟效应**

$$\tau_{AB} = \int_A^B d\tau = \int_A^B \sqrt{dt^2 - \frac{dx^2 + dy^2 + dz^2}{c^2}} = \int_{t_A}^{t_B} dt \sqrt{1 - \frac{v^2(t)}{c^2}} < \int_{t_A}^{t_B} dt$$

- **运动时钟变慢**

$$\tau_{AB} < t_B - t_A$$

- **匀速运动**

$$v(t) = v = const \quad \Longrightarrow \quad \Delta t = \gamma \Delta\tau$$

相对论运动学

- 固有时

$$d\tau = \sqrt{1-\frac{v^2}{c^2}}\,dt = \frac{dt}{\gamma}$$

- 有质量粒子的 4-动量（$m \neq 0$）：$p^\mu \equiv m\frac{dx^\mu}{d\tau} = mu^\mu$

$$p^0 = mc\gamma \equiv E/c, \qquad \vec{p} = m\gamma\vec{v}$$

- 不变质量

$$\eta_{\mu\nu}p^\mu p^\nu = -m^2c^2\gamma^2 + m^2\gamma^2\vec{v}^2 = -m^2c^2$$

- 质能关系

$$E^2 = m^2c^4 + \vec{p}^2c^2 \xrightarrow{\vec{p}=0} E = mc^2$$

- 无质量粒子（类光 $d\tau = 0$）的 4-动量（$E = pc$）

$$p^\mu = (E/c, p, 0, 0)$$

- 4-速度，$u^\mu = (\gamma c, \gamma\vec{v})$

$$\eta_{\mu\nu}u^\mu u^\nu = -c^2$$

Maxwell方程

1. 矢量形式

$$\nabla\cdot\vec{E}=\rho/\varepsilon_0,\qquad \nabla\times\vec{B}=\mu_0\vec{j}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{B}=0,\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

2. 4维矢量势和电磁场张量

$$A^\mu=\left(\frac{\phi}{c},\vec{A}\right)\quad\Longrightarrow\quad A_\mu=\left(-\frac{\phi}{c},\vec{A}\right)$$

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu$$

3. 矩阵形式 ($\vec{E}=-\frac{\partial\vec{A}}{\partial t}-\nabla\phi,\ \vec{B}=\nabla\times\vec{A}$)

$$F^{\mu\nu}=\begin{pmatrix}0 & E_1/c & E_2/c & E_3/c\\ -E_1/c & 0 & B_3 & -B_2\\ -E_2/c & -B_3 & 0 & B_1\\ -E_3/c & B_2 & -B_1 & 0\end{pmatrix},\qquad F_{\mu\nu}=\begin{pmatrix}0 & -E_1/c & -E_2/c & -E_3/c\\ E_1/c & 0 & B_3 & -B_2\\ E_2/c & -B_3 & 0 & B_1\\ E_3/c & B_2 & -B_1 & 0\end{pmatrix}$$

Maxwell方程

1. 矢量形式

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}, \qquad \nabla \times \vec{B} = \mu_0 \vec{j} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0, \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

2. 4维流

$$j^\mu = (c\rho, \vec{j}) \quad \Longrightarrow \quad \partial_\mu j^\mu = \frac{\partial \rho}{\partial t} + \nabla \cdot \vec{j} = 0$$

3. 协变形式

$$\boxed{\partial_\mu F^{\mu\nu} = -\mu_0 j^\nu}$$

$$\varepsilon^{\alpha\mu\nu\rho}\, \partial_\mu F_{\nu\rho} = 0$$

Maxwell方程

1. 协变洛伦兹规范 （$A^{\mu}=\left(\frac{\phi}{c},\vec{A}\right)$）

$$\partial_{\mu}A^{\mu}=\frac{1}{c^2}\frac{\partial\phi}{\partial t}+\nabla\cdot\vec{A}=0$$

2. Maxwell方程 $\Rightarrow$ 达朗贝尔方程

$$\partial_{\mu}F^{\mu\nu}=-\mu_0 j^{\nu}\quad\Rightarrow\quad\partial_{\mu}\partial^{\mu}A^{\nu}=-\mu_0 j^{\nu}$$

3. 洛伦兹标量

$$G_{\mu\nu}H^{\mu\nu},\ldots\quad\Leftarrow\quad k^{\mu'}k_{\mu'}=k^{\mu}k_{\mu}\quad\Leftarrow\quad k^{\mu'}=\left(\frac{\partial x'}{\partial x}\right)^{\mu}_{\nu}k^{\nu},\qquad k_{\mu'}=\left(\frac{\partial x}{\partial x'}\right)^{\nu}_{\mu}k_{\nu}$$

4. 例子:

$$F_{\mu\nu}F^{\mu\nu}=2\left(\vec{B}^2-\frac{\vec{E}^2}{c^2}\right)$$

$$\varepsilon^{\alpha\beta\mu\nu}F_{\alpha\beta}F_{\mu\nu}=-\frac{8}{c}\vec{E}\cdot\vec{B}$$

Maxwell方程

1. 4-矢量变换

$$A^0 = \frac{A'^0 - bA'^1/c}{\sqrt{1-b^2/c^2}}, \qquad A^1 = \frac{A'^1 - bA'^0/c}{\sqrt{1-b^2/c^2}}, \qquad A^2 = A'^2, \qquad A^3 = A'^3$$

2. 4维矢量势

$$\phi = \frac{\phi' - bA'_x}{\sqrt{1-b^2/c^2}}, \qquad A_x = \frac{A'_x - b\phi'/c^2}{\sqrt{1-b^2/c^2}}, \qquad A_y = A'_y, \qquad A_z = A_z{}'$$

3. 电磁场

$$E_x = E'_x, \qquad E_y = \gamma(E'_y - bB'_z), \qquad E_z = \gamma(E'_z + bB'_y)$$

$$B_x = B'_x, \qquad B_y = \gamma(B'_y + bE'_z/c^2), \qquad B_z = \gamma(B'_z - bE'_y/c^2)$$

4. 矢量形式

$$E'_{\parallel} = E_{\parallel}, \qquad \vec{E}'_{\perp} = \gamma\left(\vec{E} + \vec{b} \times \vec{B}\right)_{\perp}$$

$$B'_{\parallel} = B_{\parallel}, \qquad \vec{B}'_{\perp} = \gamma\left(\vec{B} - \frac{\vec{b}}{c^2} \times \vec{E}\right)_{\perp}$$

Maxwell方程

Maxwell方程组

微分形式	积分形式
$\nabla \cdot \vec{E} = \rho/\varepsilon_0$	$\oiint_{\partial V} \vec{E} \cdot d\vec{s} = \frac{1}{\varepsilon_0} \iiint_V \rho d^3 x$
$\nabla \cdot \vec{B} = 0$	$\oiint_{\partial V} \vec{B} \cdot d\vec{s} = 0$
$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$	$\oint_{\partial S} \vec{E} \cdot d\vec{l} = -\frac{d}{dt} \iint_S \vec{B} \cdot d\vec{s}$
$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$	$\oint_{\partial S} \vec{B} \cdot d\vec{l} = \mu_0 \iint_S \vec{J} \cdot d\vec{s} + \mu_0 \varepsilon_0 \frac{d}{dt} \iint_S \vec{E} \cdot d\vec{s}$

介质中Maxwell方程

- 介质中Maxwell方程

Maxwell方程

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

电磁感应

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

无磁单极

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}$$

位移电流

$$\nabla \times \vec{B} = \mu_0 \vec{J} + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$$

$$\nabla \cdot \vec{D} = \rho$$

高斯定律

$$\nabla \cdot \vec{E} = \frac{\rho}{\varepsilon_0}$$

- 电位移矢量和磁场强度

$$\vec{D} = \varepsilon_0 \vec{E} + \vec{P}, \qquad \vec{H} = \frac{\vec{B}}{\mu_0} - \vec{M}$$

电磁场中受力

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$$

- 本构关系

$$\vec{D} = \varepsilon \vec{E}, \qquad \vec{B} = \mu \vec{H}, \qquad \vec{J} = \sigma \vec{E}$$

Maxwell方程

1. 矢量形式

$$\nabla\cdot\vec{E}=\frac{\rho}{\varepsilon_0},\qquad \nabla\times\vec{B}=\mu_0\vec{j}+\mu_0\varepsilon_0\frac{\partial\vec{E}}{\partial t}$$

$$\nabla\cdot\vec{B}=0,\qquad \nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

2. 4维流

$$j^\mu=(c\rho,\vec{j})\quad\Longrightarrow\quad \partial_\mu j^\mu=\frac{\partial\rho}{\partial t}+\nabla\cdot\vec{j}=0$$

3. 协变形式

$$\boxed{\partial_\mu F^{\mu\nu}=-\mu_0 j^\nu}$$

$$\varepsilon^{\alpha\mu\nu\rho}\,\partial_\mu F_{\nu\rho}=0$$

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