Hybrid Systems/ Time Automata II

01266212
CYBER PHYSICAL SYSTEM DESIGN

SEMESTER 1-2021

Original contents from
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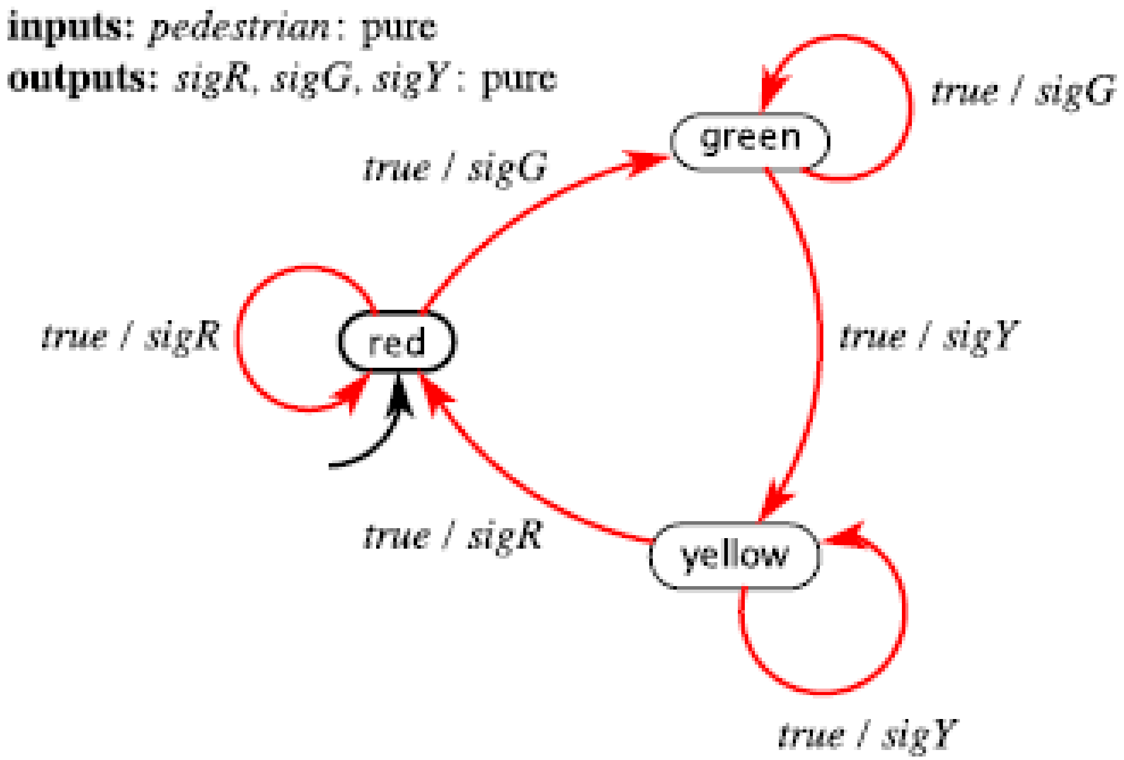
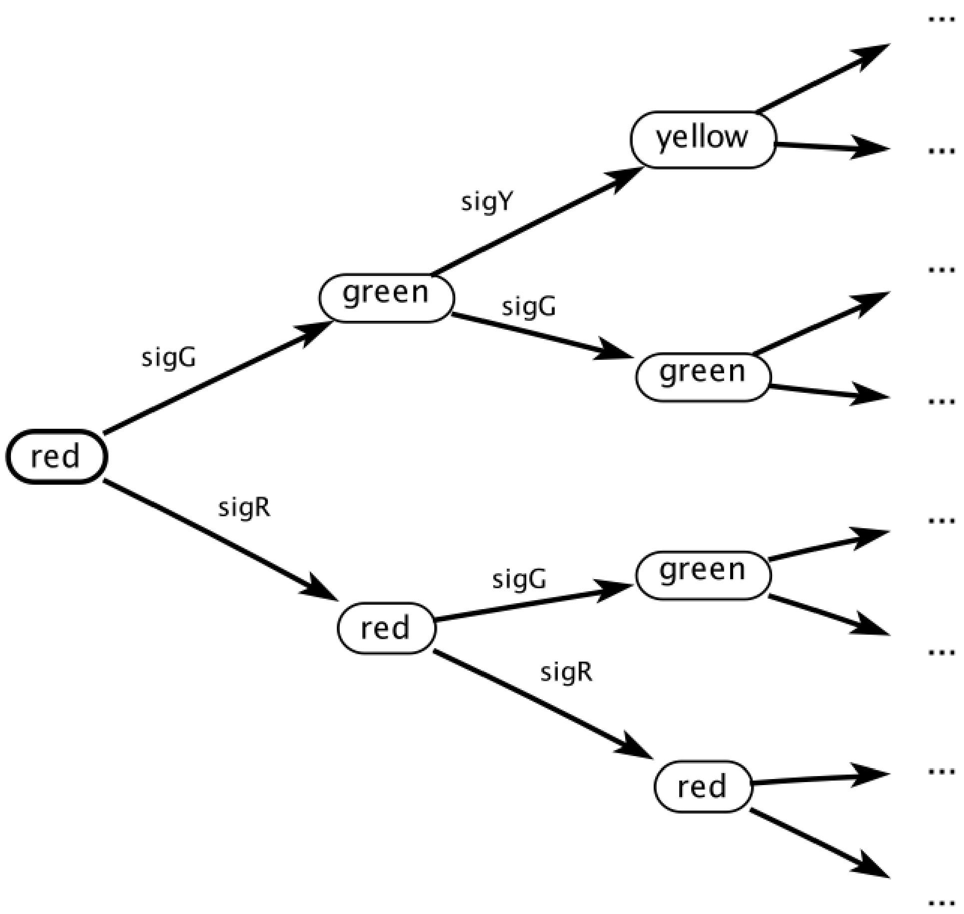
Nondeterministic FSM specifying order of signal lights



Traces

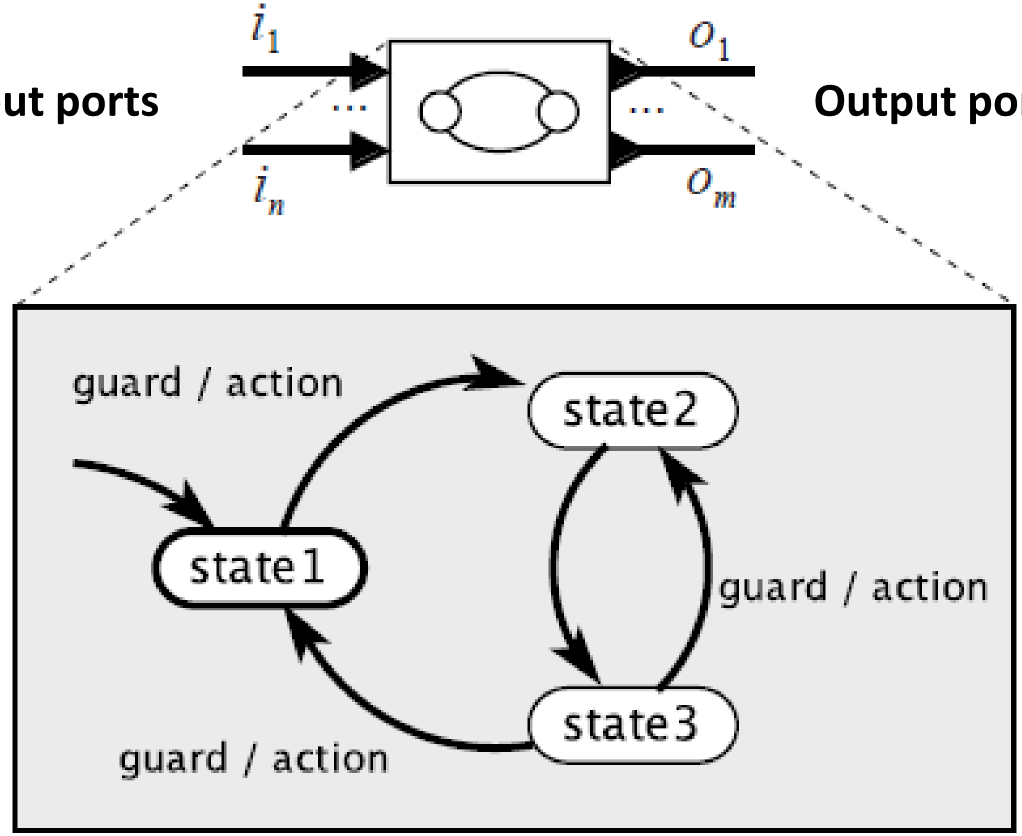
- A **trace** is the record of inputs, states, and outputs in a behavior.
- A **computation tree** is a graphical representation of all possible traces.

FSMs are suitable for formal analysis. For example, **safety** analysis might show that some unsafe state is not reachable.



Actor Model for State Machine

Input ports



Output ports

Both types of ports have a value *present* or *absent* (for a pure signal) or a member of some set of *values* (for valued signal).
The **guards** on the transitions define subsets of possible values on input ports, and the **actions** assign values to output ports.

Assumed that state machines operate in a sequence of discrete reactions and that inputs and outputs are absent between reactions.

Time-based State Machine

- We will generalize state machine model to allow inputs and outputs to be continuous-time signals.
- In order to get state machine models to coexist with time-based models, we need to interpret state transitions to occur on **the same timeline** used for the time-based portion of the system.
- A transition is occurred when a **guard** on an outgoing transition from the current state becomes enabled.
- During the time between reactions, a state machine is understood to not transition between modes.
- But the inputs and outputs are no longer required to be absent during that time.

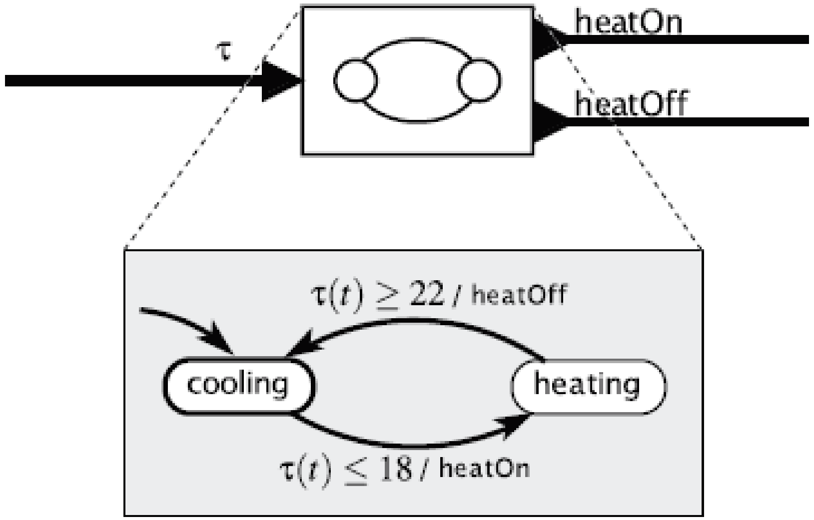
Thermostat with a Continuous-time Input Signal

A state machine with states

$$\Sigma = \{\text{heating},\ \text{cooling}\}$$

and input is a continuous-time temperature signal at <u>time *t*</u>:

$$\tau(t): \mathbb{R} \to \mathbb{R}$$



The initial state is **cooling**, and the transition out of this state is enabled at the earliest time *t* after the start time when $\tau(t) \leq 18$.

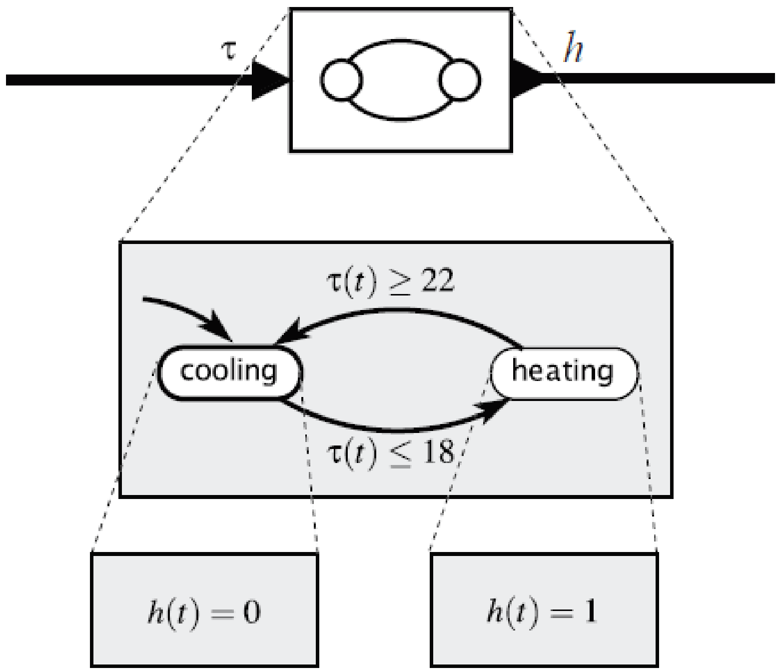
The outputs are present only at the times the transitions are taken.

State Refinements For Continuous-time Ouputs

We wish to produce a control signal whose value is 1 when the **heat** is *on* and 0 when the **heat** is *off*.

Such a control signal could directly drive a heater.

Each state has a refinement that gives the value of the output *h* while the state machine is in that state.



A thermostat with continuous-time output

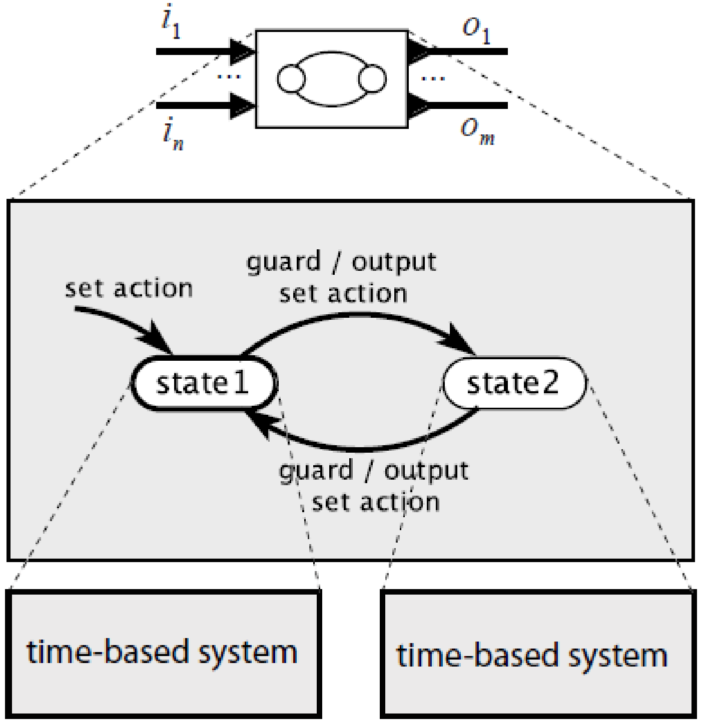
Hybrid Systems

In a hybrid system, **the state refinement** defines dynamic behavior of the outputs and (possibly) additional **continuous state variables.**

In Figure, each state is associated with **a state refinement** labeled in the figure as a **“time-based system.”**

Each transition can optionally specify set actions, which set the values of such additional **state variables** when a transition is taken.

The states of the FSM may be referred to as **modes** rather than states.

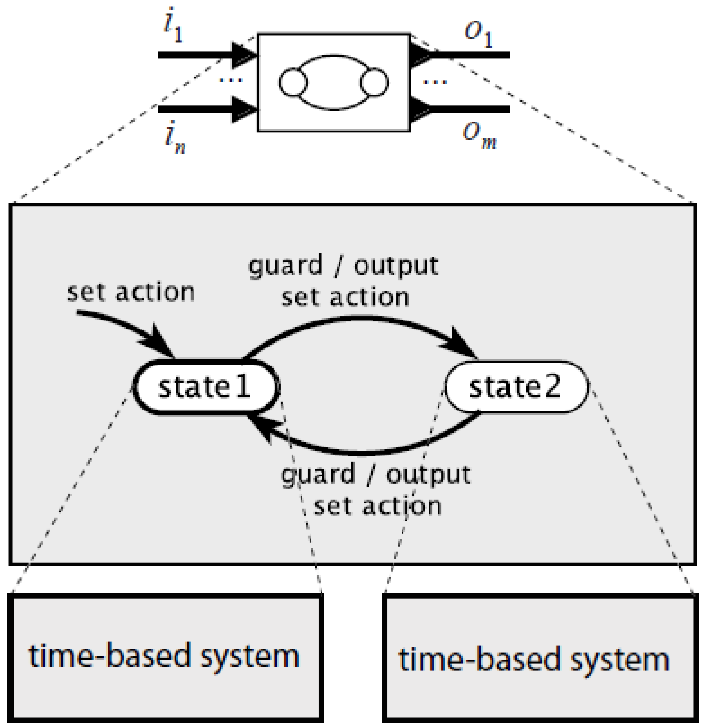


Hybrid Systems

A hybrid system is sometimes called a **modal model**

because it has a finite number of **modes**, one for each state of the FSM.

When it is in a **mode**, it has dynamics specified by **the state refinement** $S_1(t)$.



Timed Automata/Finite State Machine

- Most cyber-physical systems require measuring the passage of time and performing actions at specific times.
- A device that measures the passage of time, a **clock**, that has a particularly simple dynamics: its state progresses linearly in time.
- **Timed automata**, a formalism introduced by Alur and Dill (1994), enable the construction of more complicated systems from such simple clocks.
- **Timed automata** are modal models where the time-based refinements have very simple dynamics; all they do is measure the passage of time.

Clock in Timed Automata

A clock is modeled by a first-order differential equation,

$$\forall\, t \in T_m, \quad \dot{s}(t) = a,$$

where $s\colon \mathbb{R} \to \mathbb{R}$ is a continuous-time signal, $s(t)$ is the value of the clock at time t, and $T_m \in \mathbb{R}$ is the subset of time during which the hybrid system is in mode m.

The rate of the clock, a, is a constant while the system is in this mode.

If a = 1 and T_m = 1, s(t = T_m) = ?

Example: Timed Automaton to Prevent Thermostat Chattering

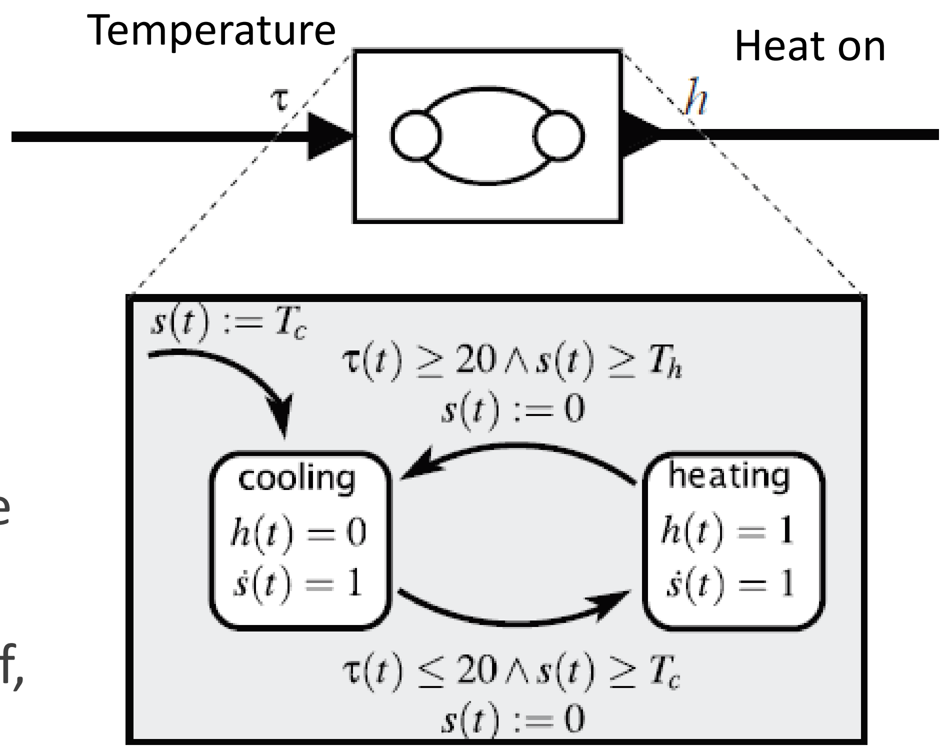
Each state refinement has a clock, which is a continuous-time signal ***s*** with dynamics given by $\dot{s}(t) = 1$.

The initial state cooling has a set action on the dangling transition indicating the initial state, $s(t) := T_c$

The other two transitions each have set actions that reset the clock s to zero. $s(t) := 0$

The guard $(s(t) \geq T_h)$ ensures that the heater will always be on for at least time T_h.

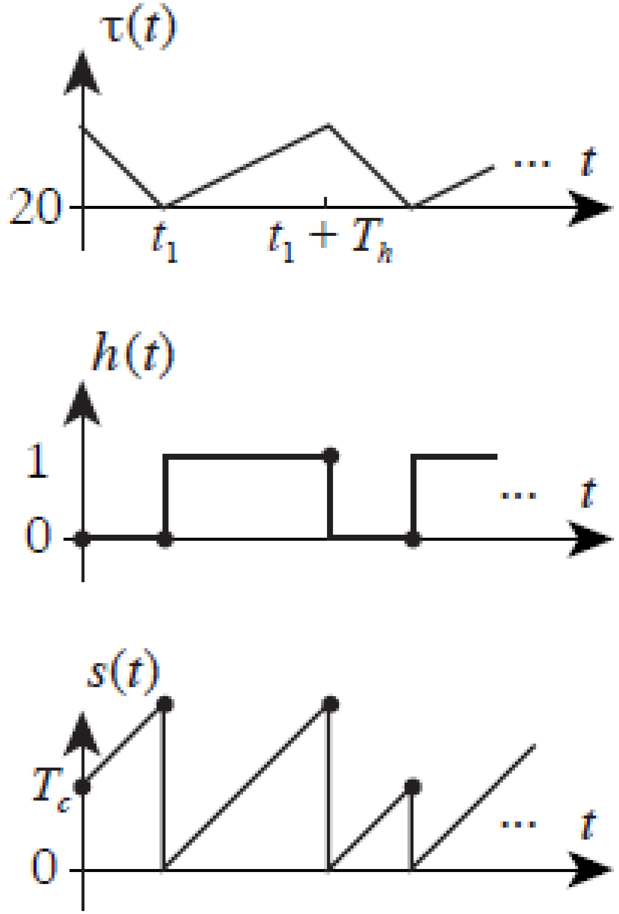
The guard $(s(t) \geq T_c)$ specifies that once the heater goes off, it will remain off for at least time T_c.

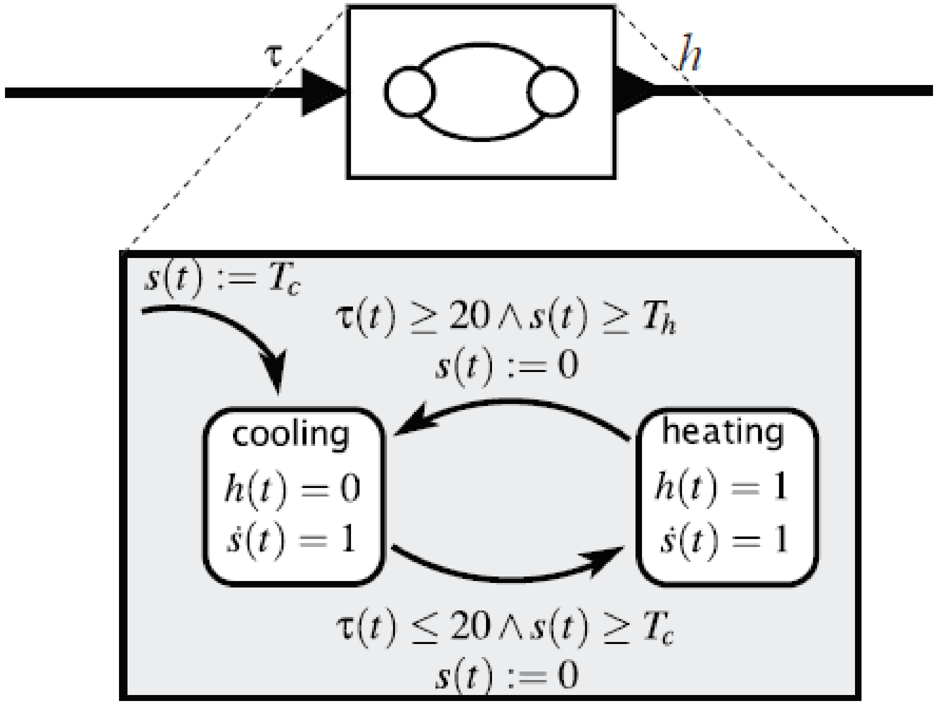


Temperature threshold is 20 with minimum times $\boldsymbol{T_c}$ and $\boldsymbol{T_h}$ in each mode

Reading Assignment Example 4.3 at page 84-85

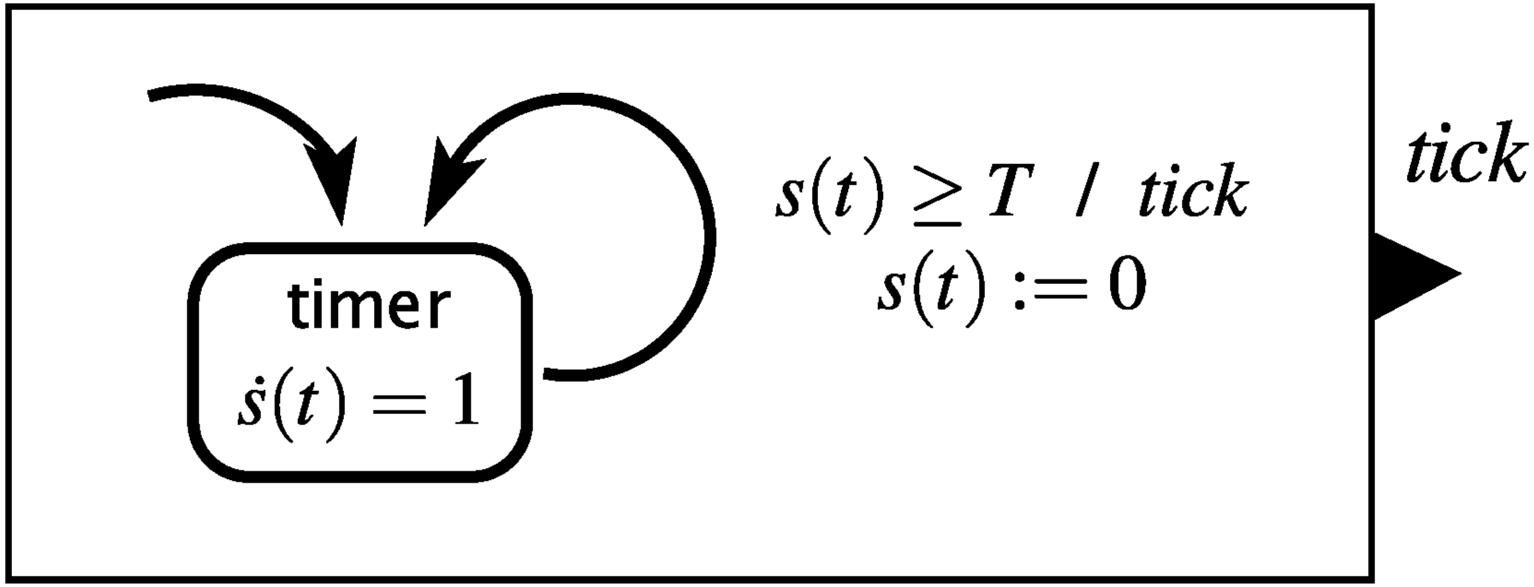
Possible Execution of the Timed Automaton





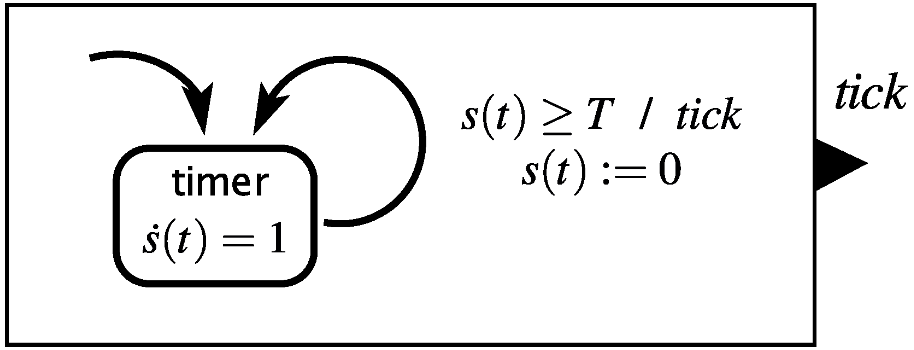
Example: “Tick” Generator (Timer)

How would you model a timer that generates a ‘tick’ each time T time units elapses?

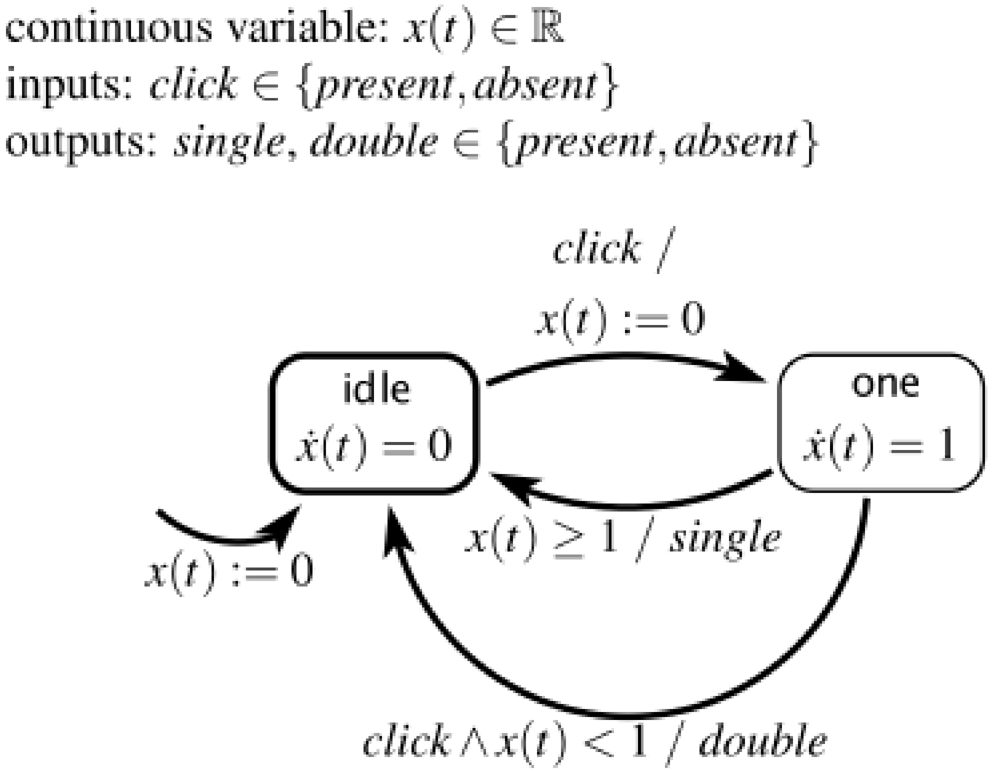


A similar timed automaton can model a generator of a timer interrupt.

Time



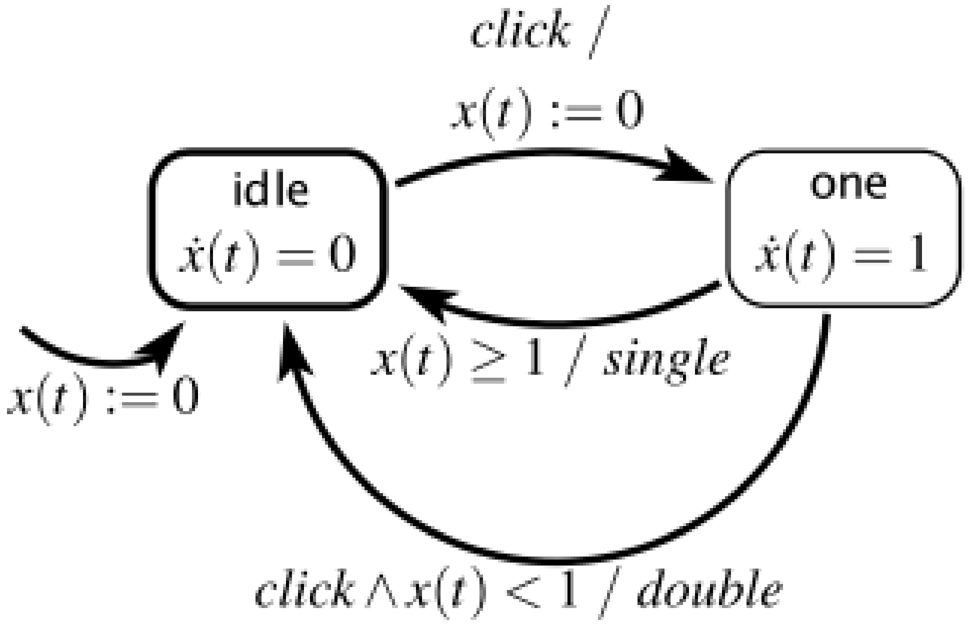
Example: Mouse Double Click Detector



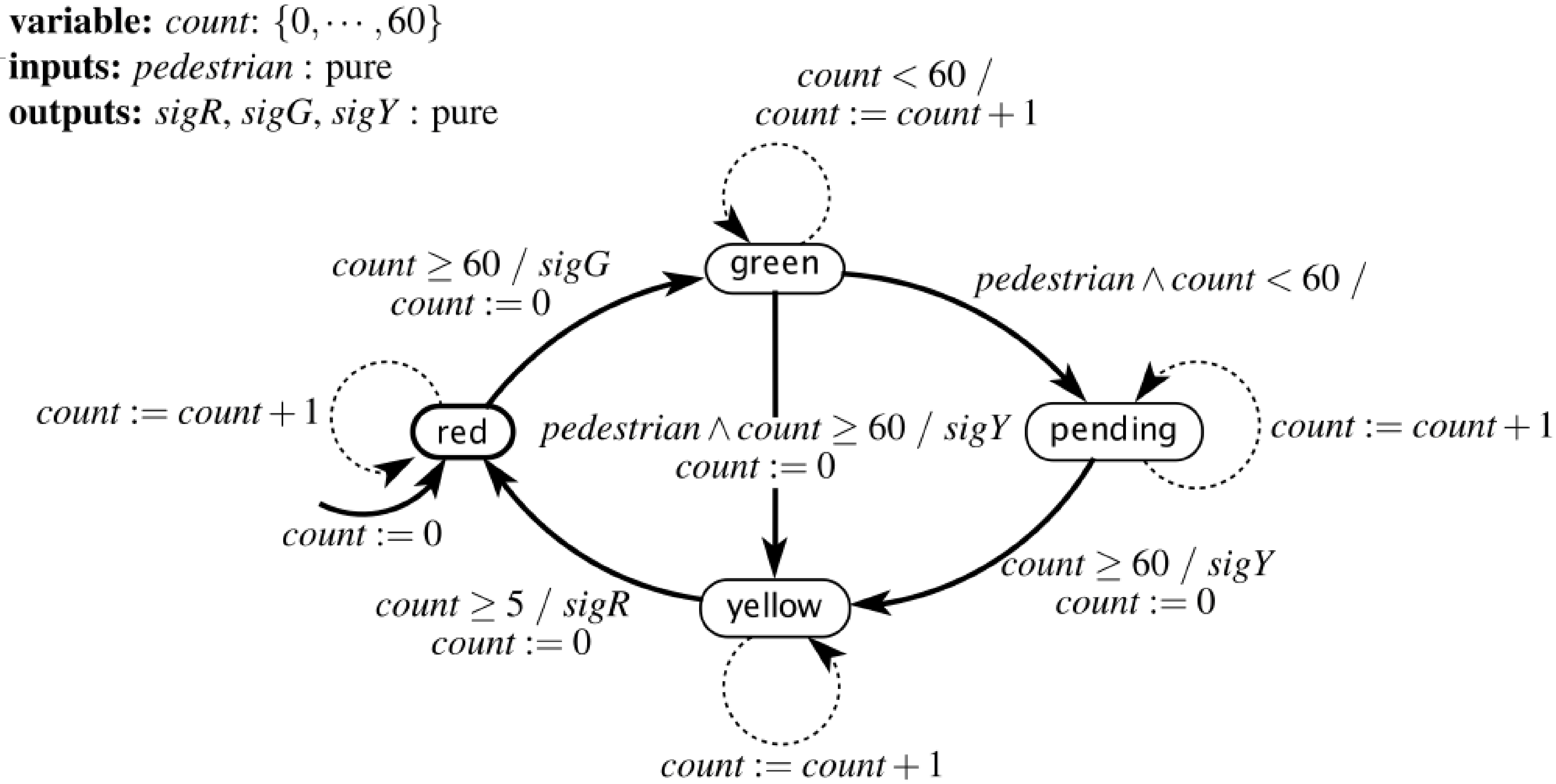
This simple form of hybrid system is called a timed automaton, where the dynamics is just passage of time.

Quiz: How many states does this automaton have?

continuous variable: $x(t) \in \mathbb{R}$
inputs: $click \in \{present, absent\}$
outputs: $single, double \in \{present, absent\}$



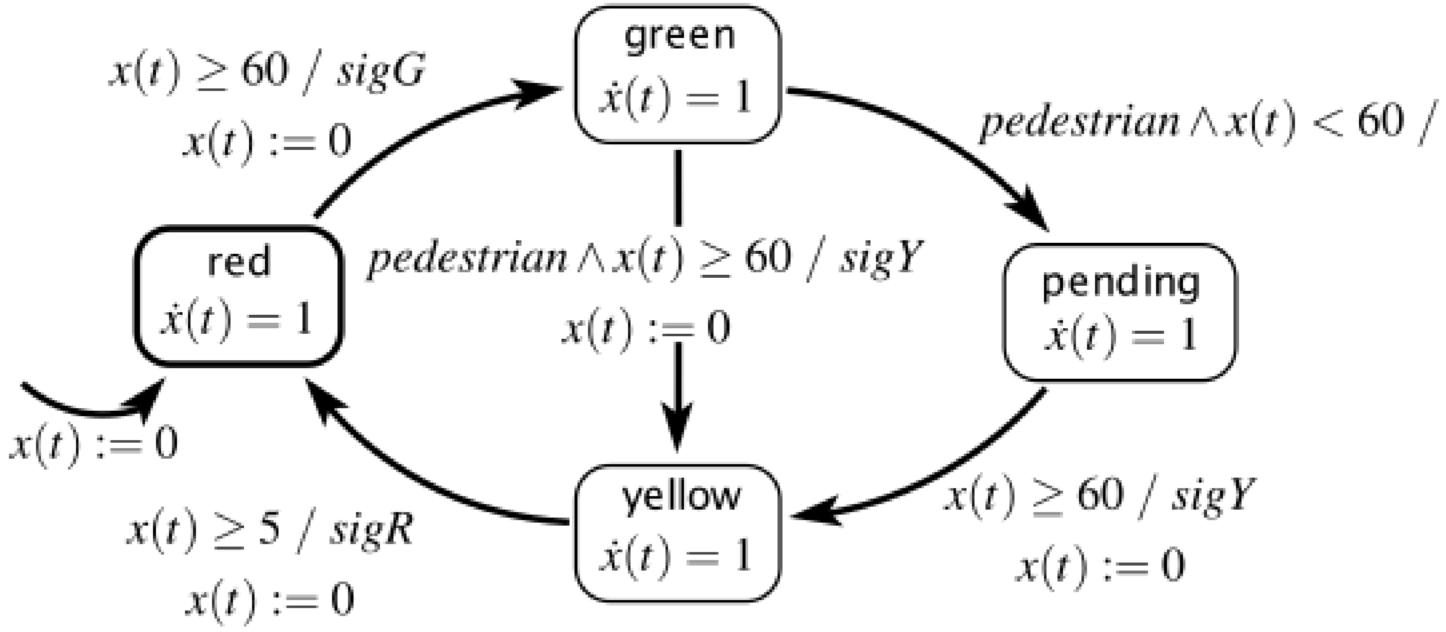
Extended state machine model of a traffic light controller at a pedestrian crossing:



This model assumes one reaction per second (a *time-triggered* model)

Timed automaton model of a traffic light controller

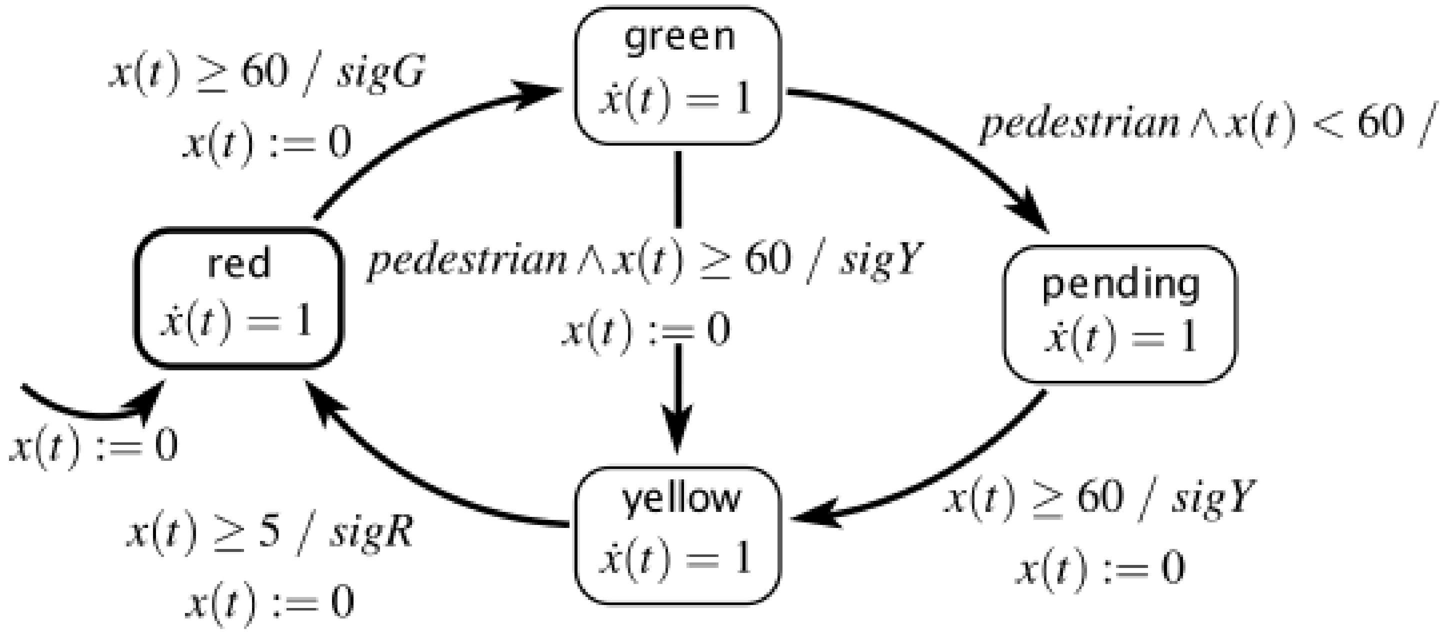
continuous variable: $x(t) : \mathbb{R}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure



This light remains green at least 60 seconds, and then turns yellow if a pedestrian has requested a crossing. It then remains red for 60 seconds.

When do reactions occur in a hybrid automaton?

continuous variable: $x(t) : \mathbb{R}$
inputs: *pedestrian* : pure
outputs: *sigR*, *sigG*, *sigY* : pure



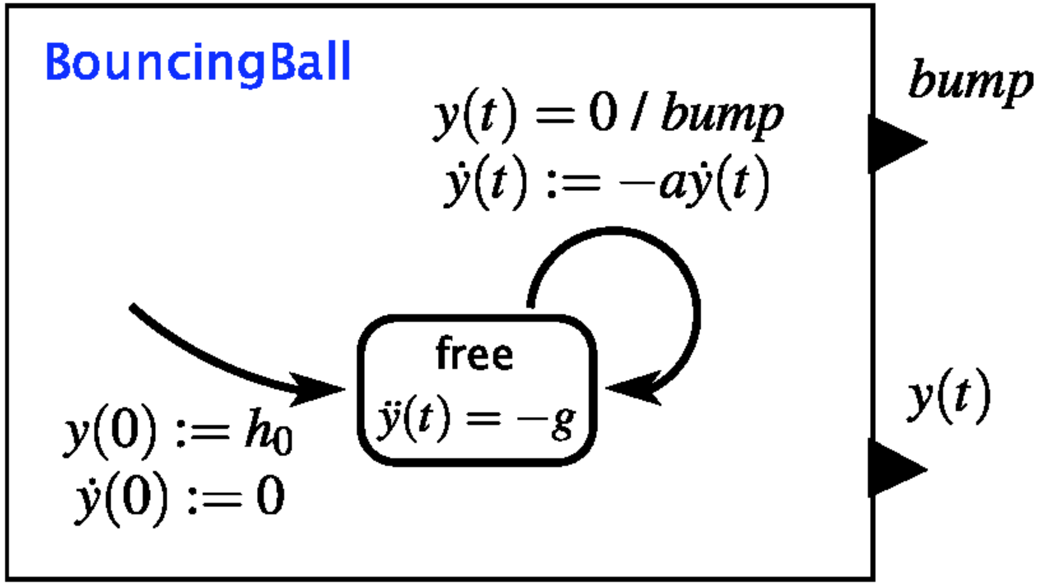
Reactions are occurring continually; with the continuous state variable *x* being continually updated.

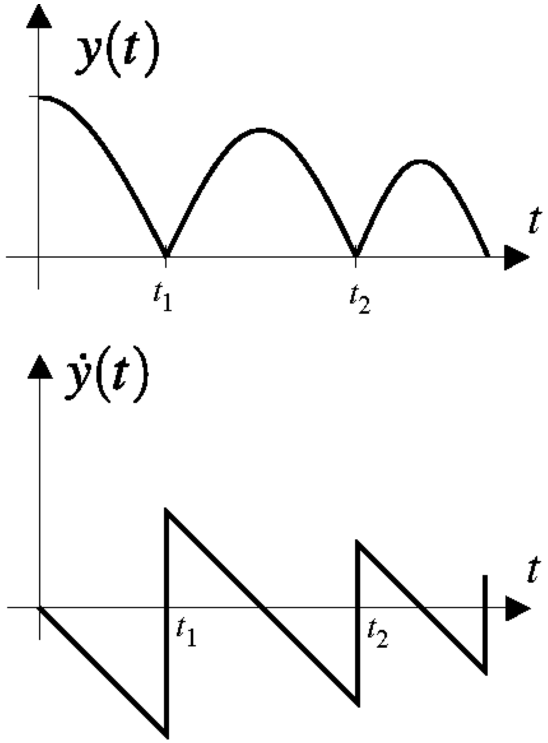
Higher-Order Dynamics

Hybrid systems are much more interesting when the behavior of the refinements is more complex.

The actions on one or more state transitions define the discrete event behavior that combines with the time-based behavior.

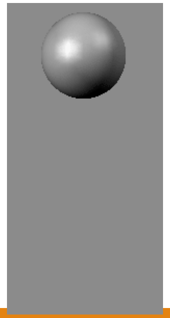
Hybrid Automaton for Bouncing Ball





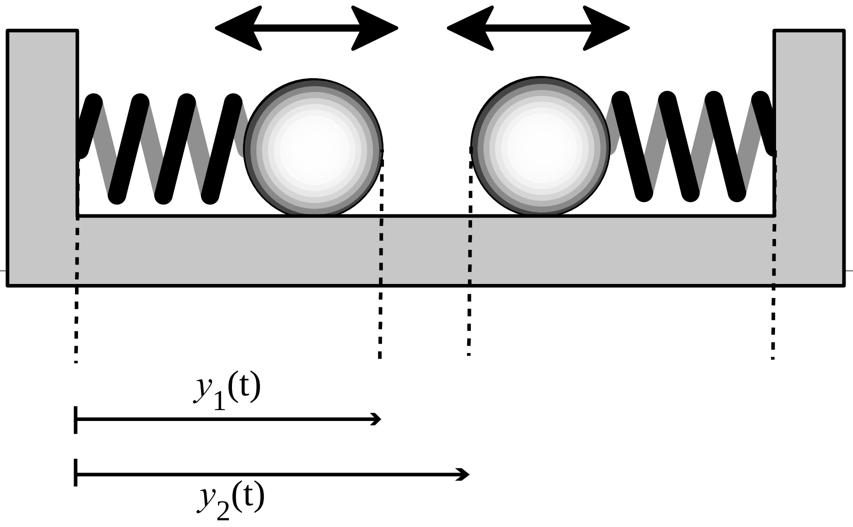
y – vertical distance from ground (position)

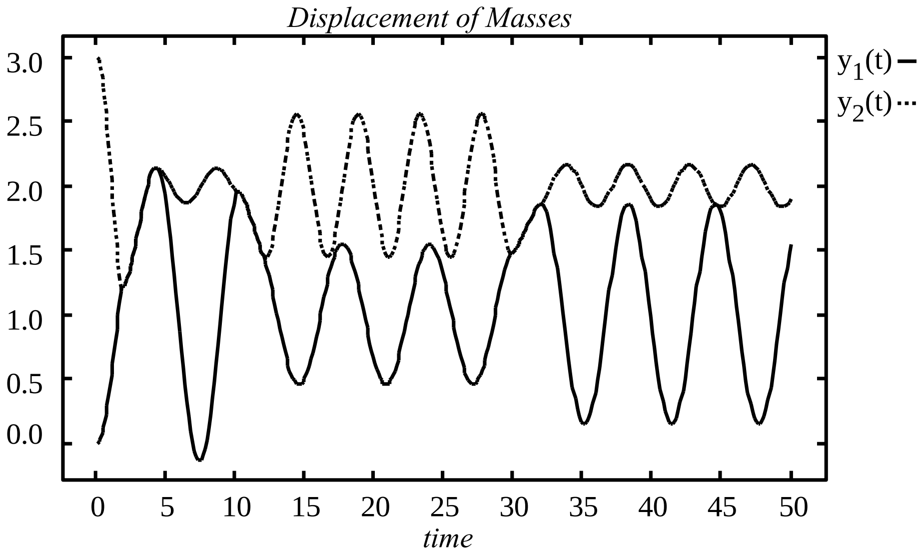
a – coefficient of restitution, $0 \cdot a \cdot 1$



If you plotted y(t), what would it look like?

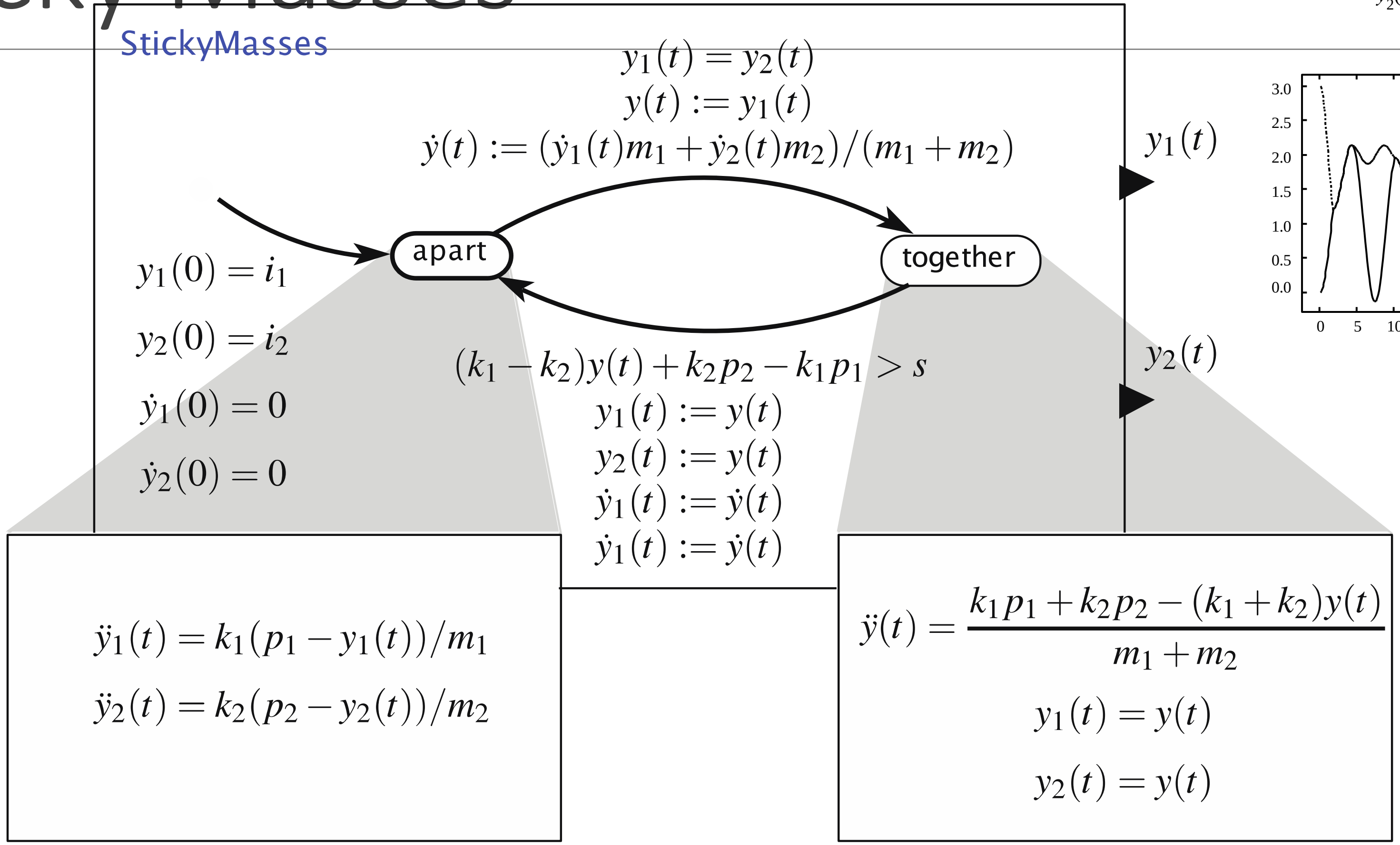
Sticky Masses

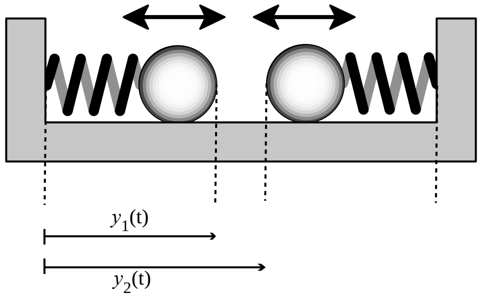


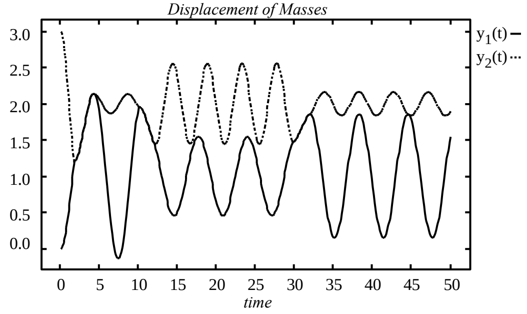


Reading Assignment Example 4.6 at page 88-91

Sticky Masses







Where do Hybrid Systems arise?

- Digital controller of physical “plant”
  - thermostat
  - intelligent cruise/powertrain control in cars
  - aircraft auto pilot
- Phased operation of natural phenomena
  - bouncing ball
  - biological cell growth
- Multi-agent systems
  - ground and air transportation systems
  - interacting robots

Reading Assignment

Section 4.2.1, 4.2.2 and 4.2.3 from the main Textbook

Reference

- Lee, Edward & Seshia, Sanjit. (2011). Introduction to Embedded Systems - A Cyber-Physical Systems Approach.
- Lecture Note Slides from EECS 149/249A: Introduction to Embedded Systems (UC Berkeley) by Prof. Prabal Dutta and Sanjit A. Seshia