Laboratory Sessions 3:

Topological invariant calculation

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Preparation

INSTALL DGTAL

You need Digital Geometry Tools and Algorithms Library, DGtal for the laboratory sessions. On the PCs in the laboratory room, the library is installed by default. Thus you do not need to settle the working environment.

If you would like to install DGtal on your own PC, for which either Linux or Mac OS is recommended:

1. download the package here, and
2. install DGtal following the instruction.

Please also see here for quick install. As 3D images are treated for this laboratory session, it is recommended to add the install option QGLVIEWER, which may need the QT5 option to be on.

MAKE YOUR PROJECT USING CMAKE TOOLS

It is recommended to use cmake tools for your project as your program needs to be associated to DGtal with all its dependencies. In your project directory, please make a CMakeLists.txt file, following the instruction here. You can also download an example here.

If you can compile a simple program such as the example *helloworld.cpp* (see here) with cmake, for example:

```
mkdir build
cd build
cmake ..
make
```

then, you must be ready to start the following experiments.

OTHER USEFUL TOOLS

You can also use

DGtalTools
G'MIC
Pink
Fiji

to visualize 3D images and verify if your results are correct.

Experiments

STEP 1: DOWNLOAD 3D BINARY IMAGES

Download 3D binary images from

https://github.com/dcoeurjo/VolGallery

Read some images and visualize them by using the DGtal modules:

1. IO
2. Viewer3D

You can download this zip file containing a sample code with CMakeLists.txt and fertility.vol, which allow you to read and visualize the vol file. You can also use 3DvolViewer in DGtalTools.

Observe their topological invariants (the numbers of connected components, cavities and tunnels).

STEP 2: CALCULATE THE NUMBERS OF CONNECTED COMPONENTS AND CAVITIES

Read each binary image and calculate the number of connected components of the foreground (use WriteComponents) as well as the number of the cavities in each image by calculating the number of connected components of the background. Keep in mind which pair of adjacencies is chosen for the foreground and background, for example, $(26,6)$ and $(6,26)$. Please calculate those numbers for both pairs.

STEP 3: CALCULATE THE EULER CHARACTERISTIC

If we choose the 26-connectivity for the foreground, then

1. Make a cubical complex from the foreground. For that, we first create a topological space made from grid cells, which is called a Khalimsky space in DGtal, and initialize the space from the foreground. This DGtal code example may help you.
2. Obtain the numbers of 0-, 1-, 2- and 3-cells in the cubical complex.
3. Calculate the Euler characteristic of the foreground from those numbers.

If we choose the 6-connectivity for the foreground, then we can calculate the Euler characteristic of the background, which is considered with 26-adjacency. Think about why you cannot calculate the Euler characteristic of the background (resp. foreground) when we choose the adjacency pair (26,6) (resp. (6,26)).

STEP 4: CALCULATE THE NUMBER OF TUNNELS

From the number of connected components, the number of cavities, and the Euler characteristic, calculate the number of tunnels in the 3D binary image. Consider both adjacency pairs, (26,6) and (6,26).

STEP 5: VERIFY THE RESULTS

The script "invariants3d" of Pink gives you the topological invariants for the (26,6). Verify if your results are correct, and consider how you can obtain the results of (6,26) from those of (26,6).

Attention: in order to do such verification, you need to convert the image file format from "vol" (DGtal format) to "pgm" (Pink format). You can do it via "raw" format using the DGtal-Tools command "vol2raw" and the Pink command "raw2pgm".

With G'MIC, you also need to convert vol files to raw files (image.raw, for example) and obtain the image size (X, Y, Z). You then obtain betti numbers with the following command:

```
gmic image.raw,uint8,X,Y,Z betti e "\${}" q
```

Here the (26,6)-adjacency pair is considered.