Problem 3: Prove that h1 is consistent.

We have,

(Initial state)

7	2	4
5		6
8	3	1

Goal

1	2	3
4	5	6
7	8	0

With cost from one state n to the next state n' be $c(n,n')$.

For the heuristic function h1 to be consistent,

$$h(n) \leq c(n,n') + h(n')$$

Consider 3 posibile successor states:

1.) In n', the number of misplaced tiles stays the same as in n.

Here, $c(n,n') = 1$ and $h1(n') = h1(n)$

$\Rightarrow h1(n) \leq 1 + h1(n)$

$\boxed{\therefore h1(n) \leq 1 + h1(n)} \longrightarrow h(n) \leq c(n,n') + h(n')$

2.) In n', the number of misplaced tiles is 1 less than in n.

Here, $c(n,n') = 1$ and $h1(n') = h1(n) - 1$

$\Rightarrow h1(n') \leq 1 + h1(n) - 1$

$\boxed{\therefore h1(n) \leq h1(n)} \longrightarrow h(n) \leq c(n,n') + h(n')$

3.) In n', the number of misplaced tiles is 1 more than in n.

Here, $c(n,n') = 1$ and $h1(n') = h1(n) + 1$

$\Rightarrow h1(n) \leq 1 + h1(n) + 1$

$\boxed{\therefore h1(n) \leq 2 + h1(n)} \longrightarrow h(n) \leq c(n,n') + h(n')$

In all 3 cases, we have $h(n) \leq c(n,n') + h(n')$, which means the value of $f(n)$ does not decrease in any case. Therefore, $h1(n)$ is consistent.