Exercise sheet 2

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Problem 1 Convexity

1. Let $\{C_i\}_{i\in I}$ be a set of convex sets. Show that $\bigcap_{i\in I} C_i$ is convex.

   Proof: Since each C_i is convex, it holds that the line $\overline{pq}$, for $p, q \in C_i$, is completely contained in C_i. Since for all $p, q \in \bigcap_{i\in I} C_i$, the segment $\overline{pq}$ is contained in each C_i, it holds that $\overline{pq} \in \bigcap_{i\in I} C_i$.
   $\Rightarrow \bigcap_{i\in I} C_i$ is convex. □

   A similar property can be found for unions of convex sets:
   Claim: For any non-decreasing series of convex sets $(C_i)_{i\in I}$ (with respect to set inclusion), the set $\bigcup_{i\in I} C_i$ is convex.
   Proof: For all $p, q \in \bigcup_{i\in I} C_i$ there exists a $\tilde{i} \in I$ such that $p, q \in C_{\tilde{i}}$, hence $\overline{pq} \in C_{\tilde{i}}$. Since $(C_i)_{i\in I}$ is non-decreasing, $\overline{pq} \in \bigcup_{i\in I} C_i$. □
2. Let P be a finite point set in the plane. Show that the boundary of the convex hull $CH(P)$ of P is a convex polygon whose vertices are points of P.

   TBA
3. Show that the segment between two points $p, q \in P$ is an edge of $CH(P)$ if and only if all points of P lie on the same side of the line through p and q.

   Proof:
   "$\Rightarrow$": By contraposition. Let $\tilde{p}$ a point on the one side of the line through p and q and $\tilde{q}$ a point on the other side. Let $s \in \overline{pq}$. Then, one of $\overline{s\tilde{p}}$ and $\overline{s\tilde{q}}$ is not contained in P. Hence, $\overline{pq}$ cannot be an edge of $CH(P)$.
   "$\Leftarrow$": TBA □

Problem 2 Computing the tangents to a polygon

TBA

Problem 3 Chan's algorithm and superexponential search

1.
2.