

## Max Wisniewski

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### Task 3: (Makespan - Problem)

In this exercise we should prove the following Lemma on the Makespan-Problem.

**Lemma 2.8:** *For any input to the problem of minimizing the makespan on identical parallel machines for which the processing requirements of each job is more than one-third the optimal makespan, the longest processing time rule computes an optimal schedule.*

**Proof:**

Let  $m$  be the number of machines, and  $J = \{1, \dots, n\}$  the jobs, where  $p_i, i \in J$  is the processing-time.  $c_i, i \in J$  is the time at which Job  $i$  is finished. Let  $C_{\max} = \max_{j \in J} c_j$  the value of the optimal solution and  $C_{\max}^*$  the computationtime of the longest processing rule.

First observe, that in the optimal solution each machine can execute at most two jobs. Next we can assume, that in the optimal solution the machines that executes two jobs, the longest one will always be executed first.

Now we show, that we can modify the optimal solution s.t. we have a solution, that satisfies the longest processing time rule.