

Exercise sheet 2

Max Wisniewski, Alexander Steen

Task 1

Let P and Q be two convex polygons with n and m vertices respectively, each polygon is given as a list of its vertices sorted in counter-clockwise (or clock-wise) order. Give a sweep-line algorithm that computes all intersections between P and Q in $O(n + m)$ times.

Solution:

tbd

Task 2

Let P be a polygon with n vertices and h holes.

(a)

Give a reasonable definition for a triangulation of P .

Solution:

tbd

(b)

Show that P has a triangulation.

Proof:

tbd

(c)

Find a formula for the number of triangles in any triangulation of T , and proof that it is correct.

Solution:

tbd

Task 3

Let P be a simple polygon with n vertices and let T be a triangulation of P . The *dual graph* of T , named T^* , is the graph whose vertices are the triangles of T in which two triangles are adjacent if and only if they share a diagonal.

(a)

Show that T^* is a tree.

Proof:

tbd

(b)

Use T^* to give an alternative proof that T is 3-colorable.

Proof:

tbd

(c)

Suppose $n \geq 4$. An *ear* of T is a triangle in T that has two polygon edges as sides. Show that T contains at least two ears.

Proof:

tbd

(d)

Let $n \geq 4$. Show that P has a diagonal that partitions P into two simple polygons with at least $\frac{n-3}{3} + 2$ vertices.

Proof:

tbd