

Entanglement catalysis for quantum states and noisy channels

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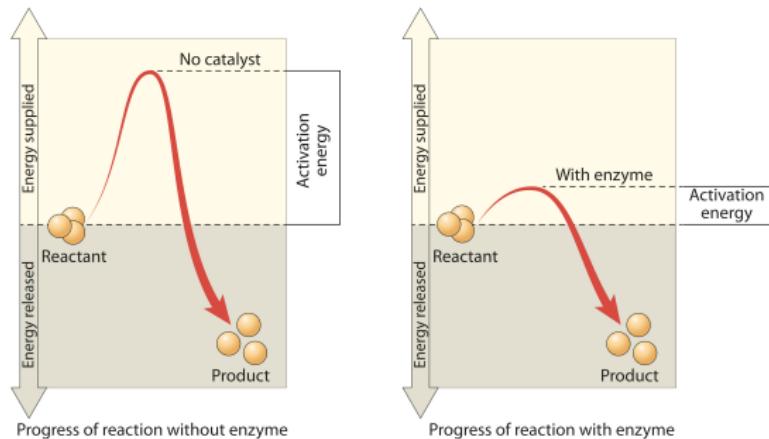


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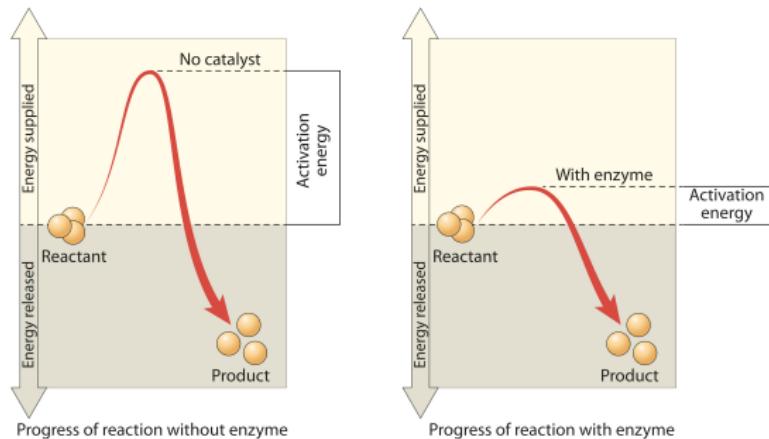
Entangled state conversion and catalysis^a



- **Catalysis in chemistry:** process for increasing the rate of a chemical reaction

^aJonathan and Plenio, Phys. Rev. Lett. **83**, 3566 (1999)

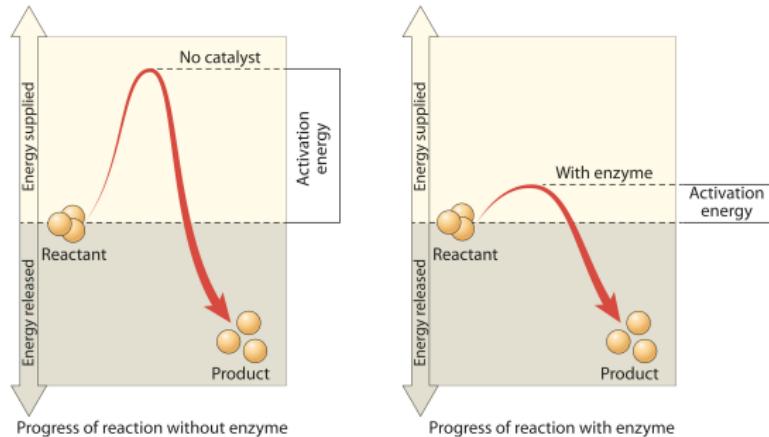
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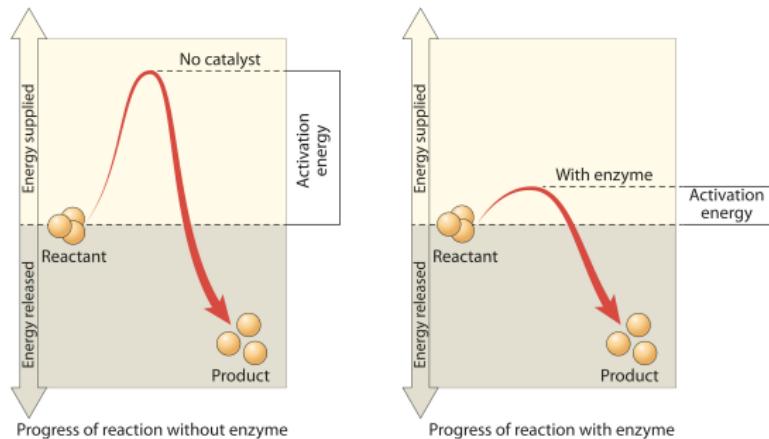
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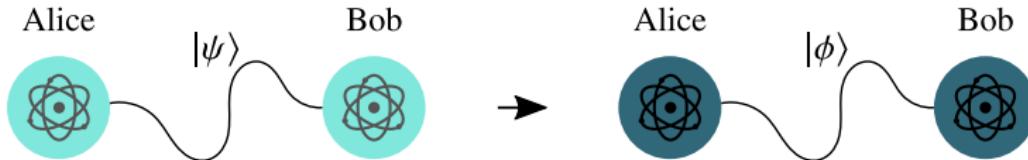
Outline

- ① Exact entanglement catalysis
- ② Approximate entanglement catalysis
- ③ Entanglement distribution with catalysis

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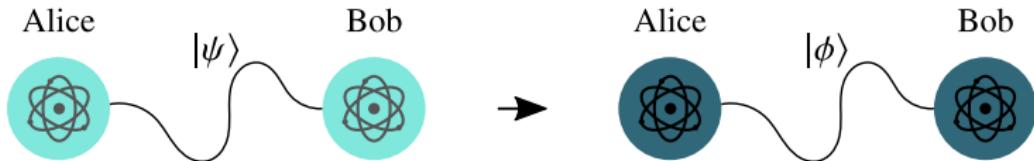


- Setup: Alice and Bob share an entangled state $|\psi\rangle$

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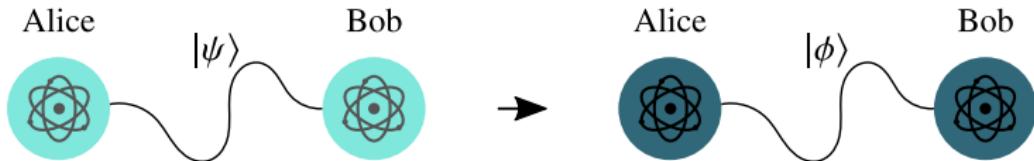


- Setup: Alice and Bob share an entangled state $|\psi\rangle$
- Goal: Convert $|\psi\rangle$ into another state $|\phi\rangle$

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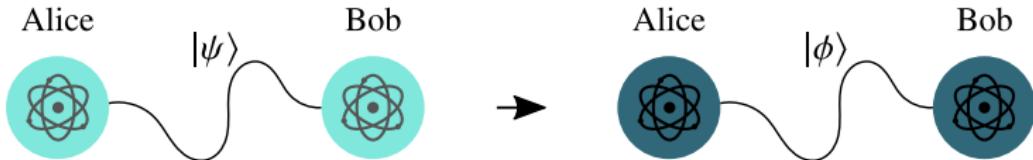


- **Question:** When is a conversion $|\psi\rangle \rightarrow |\phi\rangle$ possible?

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Entangled state conversion and catalysis



- **Question:** When is a conversion $|\psi\rangle \rightarrow |\phi\rangle$ possible?
- **Solution^a:** Conversion possible if and only if

$$\sum_{i=1}^k \lambda_i \leq \sum_{i=1}^k \lambda'_i \quad (1)$$

for all $1 \leq k \leq d$

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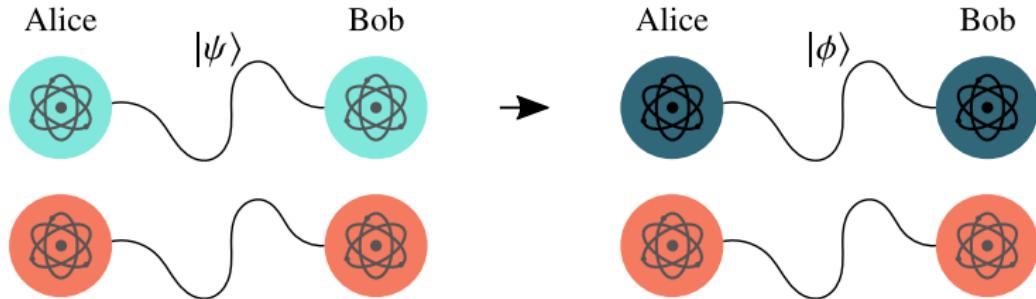


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Entangled state conversion and catalysis



- Transformation $|\psi\rangle \rightarrow |\phi\rangle$ not possible if $\sum_{i=1}^k \lambda_i > \sum_{i=1}^k \lambda'_i$
- Catalytic transformation $|\psi\rangle \otimes |\mu\rangle \rightarrow |\phi\rangle \otimes |\mu\rangle$ possible^b

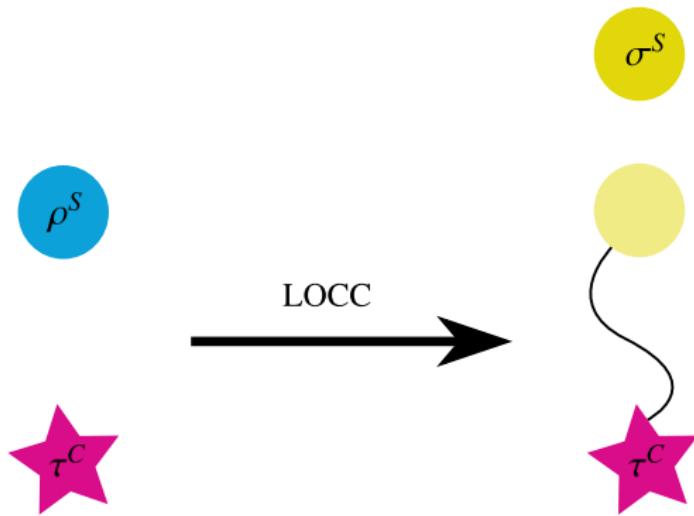
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Approximate catalysis^{abcd}



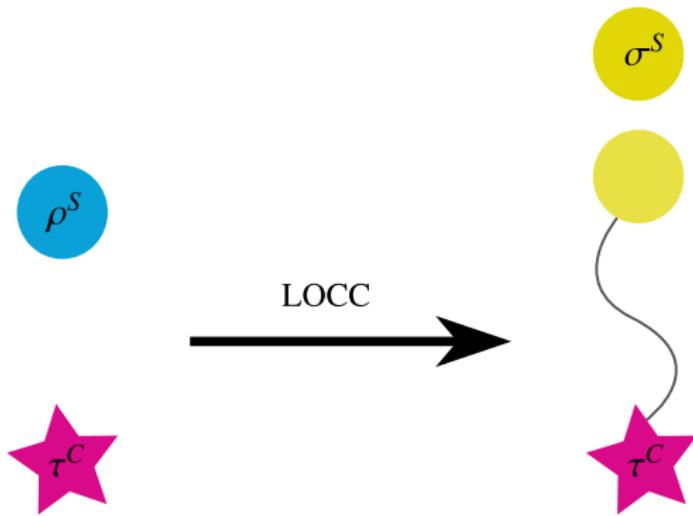
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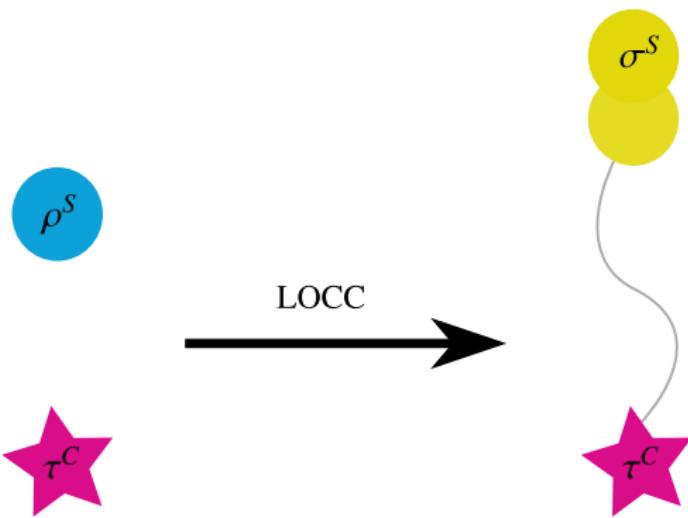
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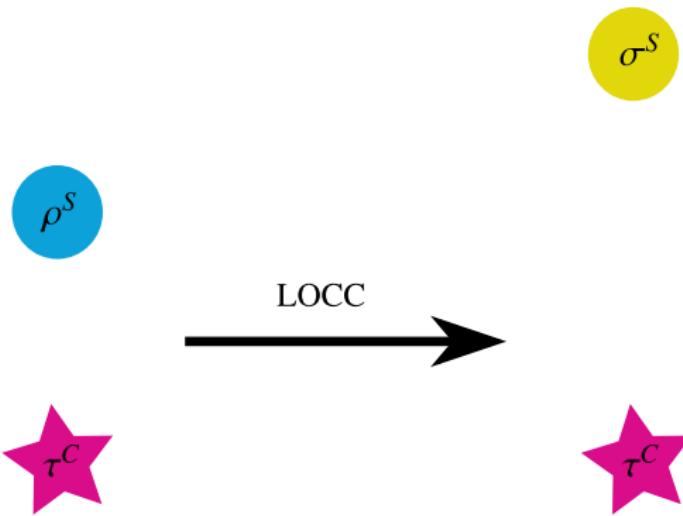
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Approximate catalysis^{abcd}

- Conversion $\rho \rightarrow \sigma$ via LOCC with approximate catalysis possible if for any $\varepsilon > 0$ there is Λ and τ such that

$$\|\Lambda[\rho^{AB} \otimes \tau^{A'B'}] - \sigma^{AB} \otimes \tau^{A'B'}\|_1 < \varepsilon \quad (2)$$

$$\text{Tr}_{AB}(\Lambda[\rho^{AB} \otimes \tau^{A'B'}]) = \tau^{A'B'} \quad (3)$$

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$$\text{Tr}_{AB} (\Lambda[\rho^{AB} \otimes \tau^{A'B'}]) = \tau^{A'B'} \quad (3)$$

- Equivalent formulation: For any $\varepsilon > 0$ and any $\delta > 0$ there is Λ and τ such that

$$\left\| \text{Tr}_{A'B'} (\Lambda[\rho^{AB} \otimes \tau^{A'B'}]) - \sigma^{AB} \right\|_1 \leq \varepsilon \quad (4)$$

$$\text{Tr}_{AB} [\Lambda[\rho^{AB} \otimes \tau^{A'B'}]] = \tau^{A'B'} \quad (5)$$

$$I^{AB:A'B'} (\Lambda[\rho^{AB} \otimes \tau^{A'B'}]) \leq \delta \quad (6)$$

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Approximate catalysis

Theorem^a: For two bipartite quantum states $|\psi\rangle^{AB}$ and $|\phi\rangle^{AB}$ approximate catalytic transformation $|\psi\rangle^{AB} \rightarrow |\phi\rangle^{AB}$ is possible if and only if

$$S(\psi^A) \geq S(\phi^A). \quad (7)$$

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Idea of the proof:

- Catalyst state achieving the transition can be constructed using results from asymptotic entanglement theory and recent results on catalysis in quantum thermodynamics^b

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- Optimality is proven by showing that squashed entanglement^c is monotonic under catalytic LOCC

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Approximate catalysis^{abcde}

Construction of the catalyst for $S(\psi^A) > S(\phi^A)$

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Approximate catalysis^{abcde}

Construction of the catalyst for $S(\psi^A) > S(\phi^A)$

- Take $n, \varepsilon > 0$, and LOCC protocol Λ such that $\|\Gamma - \phi^{\otimes n}\|_1 < \varepsilon$ with $\Gamma = \Lambda(\psi^{\otimes n})$

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- Define catalyst state

$$\tau = \frac{1}{n} \sum_{k=1}^n \psi^{\otimes k-1} \otimes \Gamma_{n-k} \otimes |k\rangle\langle k| \quad (8)$$

where Γ_i is the reduced state of Γ

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- Total system-catalyst state:

$$\psi \otimes \tau = \frac{1}{n} \sum_{k=1}^n \psi^{\otimes k} \otimes \Gamma_{n-k} \otimes |k\rangle\langle k| \quad (9)$$

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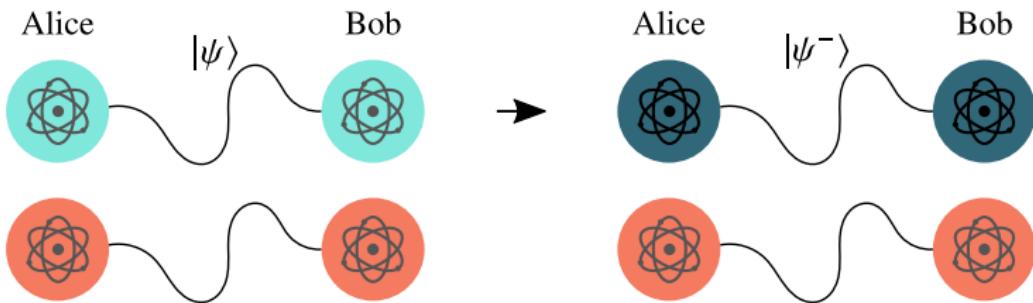
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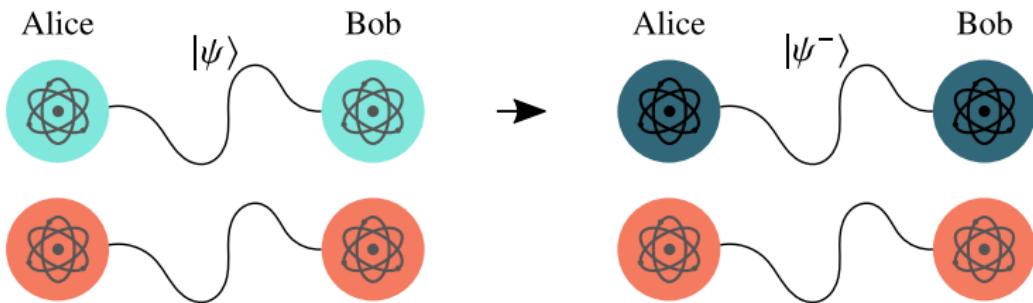


Applications in all singlet-based quantum information protocols:

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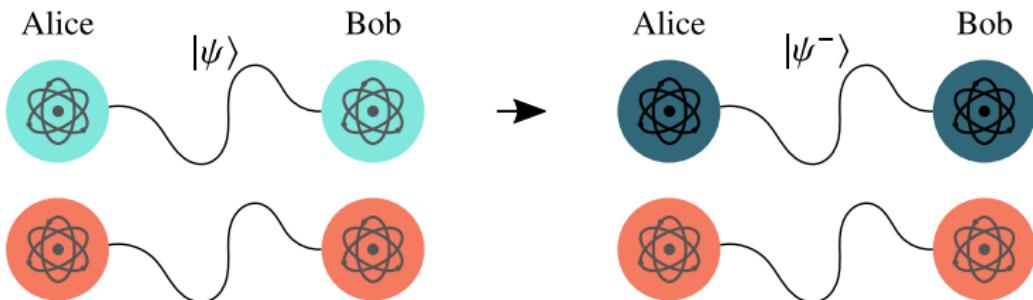
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- $|\psi\rangle$ can be converted into a singlet whenever $S(\psi^A) \geq 1$

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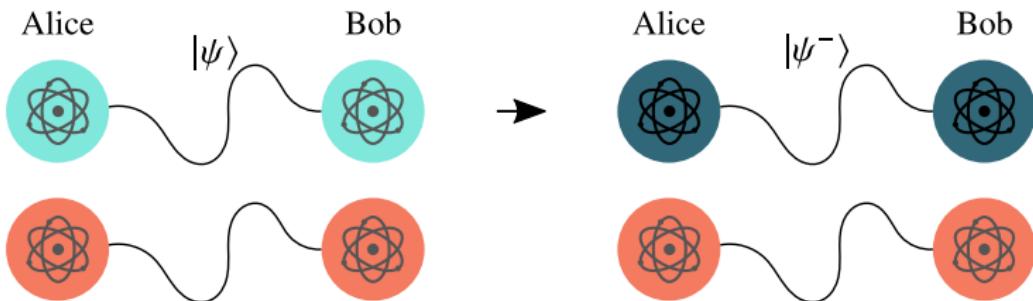
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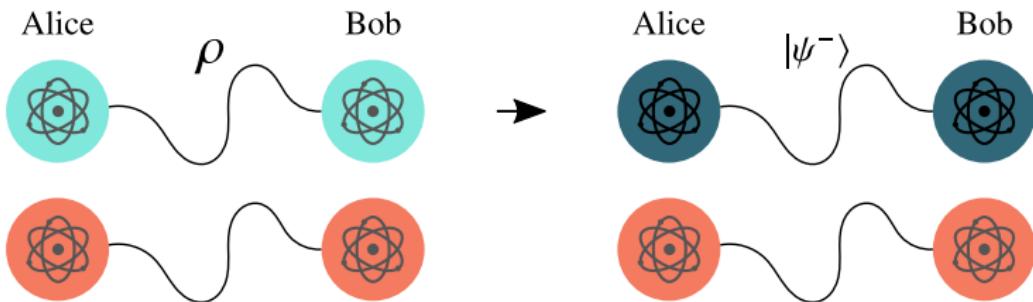
Applications in all singlet-based quantum information protocols:

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Applications in all singlet-based quantum information protocols:

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- Methods allow for **noisy initial states**

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Universal catalysis

- For a catalytic transition $|\psi\rangle^{AB} \rightarrow |\phi\rangle^{AB}$ the catalyst states depend on the initial and the final state

^aDatta, Kondra, Miller, Streltsov, arXiv:2202.05228

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- Question: does there exist a **universal catalyst**, enabling catalytic transition for all bipartite pure states?

Theorem^a: For every $\varepsilon > 0$ there exists a universal catalyst state τ_ε such that for every pair of pure states $|\psi\rangle^{AB}$ and $|\phi\rangle^{AB}$ with $S(\psi^A) \geq S(\phi^A)$ there is an LOCC protocol Λ for which

$$\left\| \Lambda \left(\psi^{AB} \otimes \tau_\varepsilon^{A'B'} \right) - \phi^{AB} \otimes \tau_\varepsilon^{A'B'} \right\|_1 < \varepsilon \quad (10)$$

$$\text{Tr}_{AB} \left[\Lambda \left(\psi^{AB} \otimes \tau_\varepsilon^{A'B'} \right) \right] = \tau_\varepsilon^{A'B'} \quad (11)$$

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Catalysis in settings beyond iid

- Asymptotic iid setting: Alice and Bob have access to n copies of a bipartite state $|\psi\rangle$

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Theorem^a: Given fidelity $f > \frac{1}{2}$ and any $\varepsilon > 0$, there is a sequence of two-qubit states $\{|\psi_i\rangle\}$ such that for any n the probability of converting the state $\otimes_{i=1}^n |\psi_i\rangle$ into a singlet, via LOCC with fidelity f , is smaller than ε . At the same time, an unbounded number of singlets can be extracted from $\otimes_{i=1}^n |\psi_i\rangle$ with the help of catalysis, as $n \rightarrow \infty$.

^aDatta, Kondra, Miller, Streitsov, arXiv:2202.05228

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Entanglement distribution with catalysis^a

- **Setup:** Alice and Bob have access to the quantum channel

$$\Lambda_l[\rho] = e^{-\alpha l} \rho + (1 - e^{-\alpha l}) \frac{1}{2} \quad (12)$$

^aDatta, Kondra, Miller, Streltsov, arXiv:2202.05228

Entanglement distribution with catalysis^a

- **Setup:** Alice and Bob have access to the quantum channel

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- **Goal:** Distribute entanglement through the channel

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- Entanglement distribution possible for $l < \ln 3/\alpha$

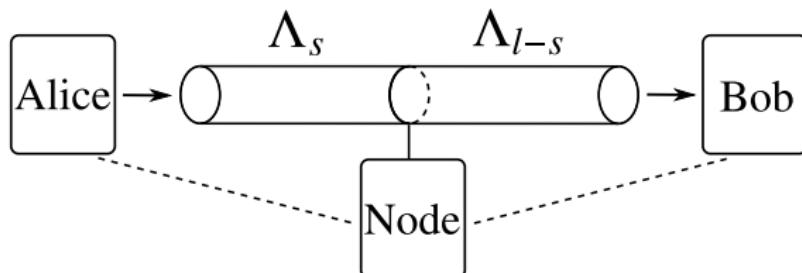
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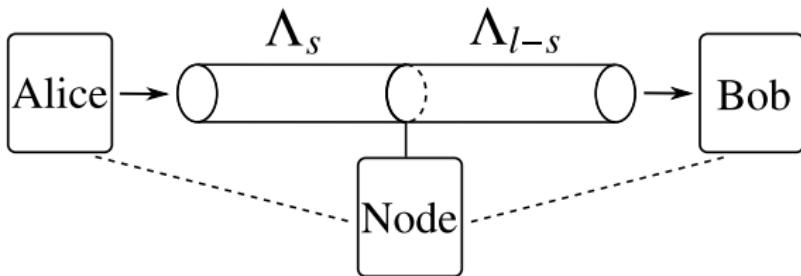
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- Can the length be increased with an intermediate repeater station+catalysis?



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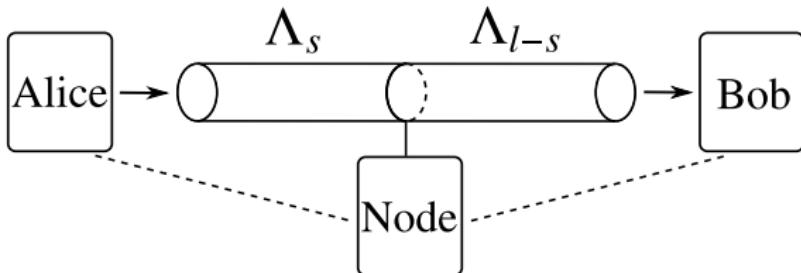
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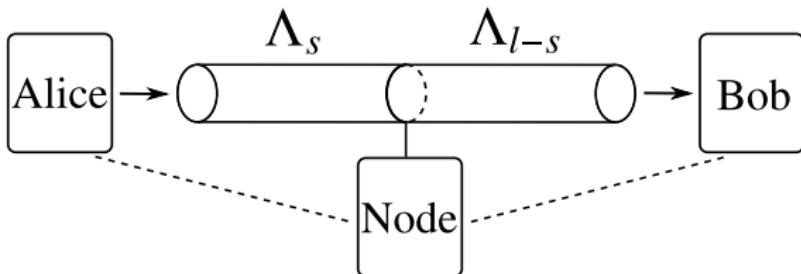
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Entanglement distribution with catalysis^a



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- **Catalysis without repeater:** no improvement, entanglement distribution possible for $l < \ln 3/\alpha$
- **Repeater+catalysis:** entanglement distribution possible for $l < 2 \ln 3/\alpha$

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Summary

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Summary

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Team



For more details see
T. V. Kondra, C. Datta, and A. Streltsov,
Catalytic Transformations of Pure Entangled States,
Phys. Rev. Lett. **127**, 150503 (2021)

C. Datta, T. V. Kondra, M. Miller, A. Streltsov,
Entanglement catalysis for quantum states and noisy channels,
arXiv:2202.05228

C. Datta, T. V. Kondra, M. Miller, A. Streltsov,
Catalysis of entanglement and other quantum resources,
arXiv:2207.05694