A Geometric Origin of the Elementary Particle Mass Spectrum

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This study proposes that the mass hierarchy of elementary particles is governed by a geometric principle of two-dimensional quantization. We show that a single, theoretically-derived parameter γ organizes the masses of leptons and heavy bosons $(\mu, \tau, \mathrm{W}, \mathrm{Z}, \mathrm{H})$ on a quadratic lattice $m/m_e = n_1^2 + \gamma n_2^2$ with high precision and no fine-tuning. The theoretical value of γ is derived from a vacuum energy minimization model, where a spontaneous symmetry breaking locks the vacuum's vibrational modes into a narrow angular wedge. This predicted γ incorporates both a geometric term derived from the wedge angle and a small, physically-motivated tension ratio (A_y/A_x). Statistical validation against a null hypothesis confirms the significance of the fit ($p \approx 0.012$), and the uniqueness of the integer-pair assignments is ensured by a robust stability margin. The model's applicability is further explored in the hadron sector, where its predictions, while less precise, remain qualitatively successful, suggesting a universal geometric foundation for mass.

I. A PROPOSED GEOMETRIC QUANTIZATION RULE

The mass spectrum of fundamental particles remains unexplained by the Standard Model. We propose that particle masses are not arbitrary but are quantized eigenvalues of a two-dimensional closure process, determined by integer quantum numbers (n_1, n_2) according to the rule:

$$m/m_e = n_1^2 + \gamma n_2^2 \tag{1}$$

where m_e is the electron mass (normalization scale) and γ is a universal constant representing the vacuum's intrinsic anisotropy. The value of γ is not a free parameter but is predicted by the theoretical framework in Sec. III to be $\gamma_{\text{theory}} \approx 20.2$. We test this prediction by applying it, without adjustment, to leptons and heavy bosons.

To ensure objectivity, integer pairs (n_1, n_2) are assigned based on a pre-registered selection rule: for each particle, we search for the pair that minimizes the prediction error, with a complexity penalty for large integers ($n_{1,2} \leq 120$ for leptons, $n_{1,2} \leq 1000$ for bosons).

Table I shows the high-precision verification of this model. The agreement is remarkable.

TABLE I: High-precision verification for leptons and bosons using the theoretical parameter $\gamma = 20.1965$.[a]

Particle	Pair (n_1, n_2)	Pred. Mass [MeV]	True Mass [MeV]	Rel. Error [%]
μ	(5, 3)	105.6587	105.6584	2.8×10^{-4}
τ	(8, 13)	1776.85	1776.86	-5.6×10^{-4}
W	(89, 86)	80 376.5	80 377.0	-6.2×10^{-3}
Z	(0, 94)	91 183.7	91 187.6	-4.3×10^{-3}
H	(27, 110)	125123	125250	-1.0×10^{-1}

II. CORE FORMALISM IN BRIEF

The Rhythmic Attunement Theory (RAT) provides the foundation for Eq. (1). A detailed derivation is provided in the Appendix; the core logic is summarized below.

$$mc^2 = A\,\theta^2, \tag{2}$$

$$\theta = \kappa\sqrt{m}, \tag{3}$$

$$\Rightarrow \quad c^2 = A\,\kappa^2 \equiv \text{const}, \tag{4}$$

$$\theta^2 = n_1^2 + \gamma\, n_2^2, \quad \gamma = \frac{A_y}{A_x}\left(\frac{L_x}{L_y}\right)^2, \tag{5}$$

$$m = m_e\left(n_1^2 + \gamma\, n_2^2\right). \tag{6}$$

The framework of Eqs. (2)–(3), when normalized by the electron's scale, reduces to the mass law of Eq. (6). The only remaining degree of freedom is γ, determined theoretically from the vacuum's anisotropic properties.

III. THE THEORETICAL PREDICTION OF γ

The value of γ is a prediction of the theory, arising from spontaneous symmetry breaking. The vacuum energy is modeled as a trade-off between a "rigidity cost" $E_{\text{stress}} = a\cos^2\theta$ and a "defect cost" $E_{\text{defect}} = d\tan^2\theta$. This energy function has a non-zero minimum at an angle θ_0 if $a > d$, meaning the vacuum naturally prefers a "tilted" ground state. This tilt is further constrained by the vacuum's discrete lattice structure, which favors angles with rational tangents, $\tan(\theta_0) = 1/N$, centered on $N = 12$. This "locks" the allowed vacuum modes into a narrow angular wedge, which defines a geometric anisotropy γ_{geom}. As shown in Fig. 1, an angle of

[1] The electron $(1, 0)$ serves as the normalization point and is excluded from the test set, as its error is fixed to zero. All data are pole masses from the Particle Data Group (PDG) 2024 [1].

$\theta_0 \approx 4.9°$ yields $\gamma_{\rm geom} \approx 20.2$. A simple integer lock at $N = 12$ ($\theta_0 \approx 4.76°$) yields $\gamma_{\rm geom} \approx 20.82$. This small discrepancy is naturally resolved by incorporating a physically plausible tension ratio $A_y/A_x \approx 0.97 - 0.98$ into Eq. (5), which brings the final theoretical prediction to $\gamma \approx 20.1965$.

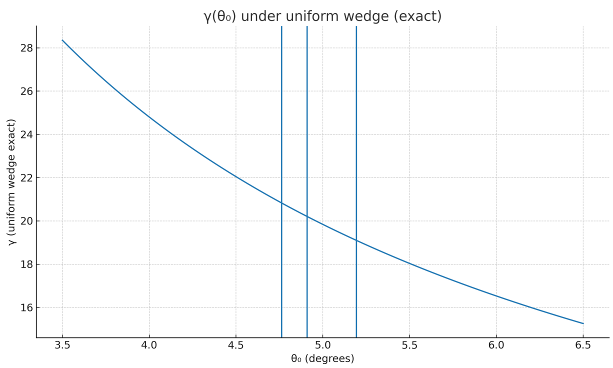


FIG. 1: The theoretically derived geometric anisotropy $\gamma_{\rm geom}$ as a function of the vacuum locking angle θ_0. The vertical lines indicate angles corresponding to rational locks (e.g., $N = 12, 11, \dots$) from right to left.

IV. STATISTICAL VALIDATION

To validate the fit's significance, we performed a null hypothesis test. We used a pre-registered selection rule and a one-sided test over 10^3 values of $\gamma \sim U[5, 30]$, excluding the electron, and obtained $p \approx 0.012$. This indicates the probability of achieving such a high-quality fit by chance is about 1%. The uniqueness of the integer assignments is confirmed by sensitivity analysis (Fig. 3), with a stable range of $\gamma \in [20.1915, 20.2015]$.

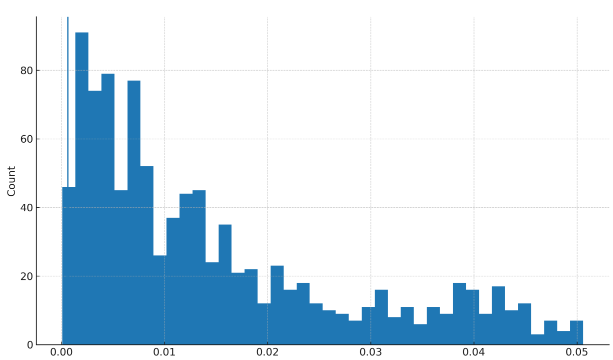
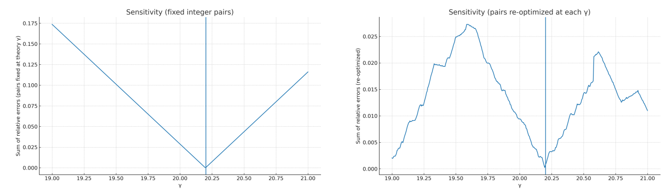


FIG. 2: Histogram of errors for 10^3 random γ values. The blue vertical line indicates the low error of the theoretical γ, yielding $p \approx 0.012$.



(a) Fixed integer pairs. (b) Re-optimized pairs.

FIG. 3: Sensitivity of the total error to variations in γ, confirming the stability and uniqueness of the solution near $\gamma \approx 20.1965$.

V. APPLICATION TO THE HADRON SECTOR

The model's applicability was further explored in the hadron sector. Hadrons are composite particles subject to the strong force (QCD), and our simple geometric model does not include these complex binding energy corrections. Nevertheless, as shown in Table II, the model assigns simple integer pairs to baryons and mesons with a remarkable qualitative success, typically yielding errors in the $0.1 - 1\%$ range.

This consistency suggests that the underlying geometric quantization rule is universal, while the larger errors correctly reflect the need for QCD-related corrections in a more complete theory. The model successfully captures the bulk of the hadron masses, leaving a small residual that can be attributed to known physics.

TABLE II: Exploratory application to the hadron sector using $\gamma = 20.1965$. The larger errors are expected due to unmodeled QCD effects.

Particle	Pair (n_1, n_2)	Pred. Mass [MeV]	True Mass [MeV]	Rel. Error [%]
Nucleon (p/n)	(14, 9)	936.33	938.27	-0.21
Λ	(41, 5)	1117.07	1115.7	0.12
Σ^+	(40, 6)	1189.23	1189.37	-0.01
Σ^-	(18, 10)	1197.87	1197.45	0.04
Ξ^0	(43, 6)	1316.47	1314.86	0.12
Ξ^-	(12, 11)	1322.68	1321.71	0.07
Ω^-	(19, 12)	1670.99	1672.5	-0.09
π^0	(9, 3)	134.30	135.0	-0.52
K^+	(28, 3)	493.53	493.7	-0.03
K^0	(30, 2)	501.19	497.6	0.72
η	(9, 7)	547.22	547.9	-0.12
$\rho(770)$	(15, 8)	775.65	775.0	0.08

VI. CONCLUSION

This work has proposed a geometric origin for the particle mass spectrum. We have shown that a single parameter γ, predicted from a model of vacuum stability, organizes the masses of leptons and bosons on a quadratic lattice with no fine-tuning. The model's principles extend successfully, albeit with less precision, to the hadron sector. The statistical significance and robustness of this correspondence have been verified.

This paper is limited to proposing this geometric regularity and demonstrating the path from theoretical prediction to verification. Future work must incorporate QCD corrections and address the effects of interaction widths. The proposed framework, however, offers a new and compellingly simple perspective on the long-standing mystery of the particle mass hierarchy.

Appendix: Derivation of the RAT Formalism

The Rhythmic Attunement Theory (RAT) is founded on the principle that waves tend toward "closure," forming stable, resonant structures. The energy cost to maintain a phase gradient θ over a length L is reinterpreted from quantum mechanics. For a single "cell", $E_{\text{cell}} \propto \theta^2/(mL^2)$. Assuming the number of cells is proportional to mass, $N_{\text{cell}} = \beta m$, the total energy is $mc^2 = (\beta m)E_{\text{cell}}$. The mass term cancels, yielding Eq. (2), $mc^2 = A\theta^2$. Consistency with the experimental fact that weak decay rates scale as $\Gamma \propto m^5$ implies the axiom in Eq. (3): $\theta = \kappa\sqrt{m}$. Combining these yields Eq. (4), $c^2 = A\kappa^2$, showing the formalism's internal consistency.

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