Problem Chosen	2021 MCM/ICM Summary Sheet	Team Control Number
F		**2120203**

Research on the Evaluation of Higher Education Health and Sustainability Based on FA-DEA Model

Summary

In this paper, we construct a higher education market based on the country. In this market, we have selected seven countries, including the United States, China, Argentina, Hungary, Czechoslovakia, the Baltic, and Mexico, as representatives of the education market.

We selected six indicators of Intellectual property input, Knowledge acquisition, Number of patents, Number of scientific, High-tech export, and Innovation capacity from 2000 to 2018 as output indicators (Y_i), and selected Education investment proportion and Higher education Five indicators of enrollment rate, National instability, Intellectual property payment, and Number of teachers are used as input indicators (X_i).

For Task 1, we need to evaluate the health of seven countries and classify them. Health is more concerned about the current national education output. Therefore, it is only necessary to select six educational outputs (X_i) as the criterion for judgment and use Model 1 factor analysis (FA) to calculate the comprehensive score of six X_i. The results are shown in Table 1.

Table 1: Education Health Ranking

Country-code	USA	CHN	CEB	MEX	CZE	HUN	ARG
Comprehensive score	1.593	-0.136	-0.164	-0.245	-0.274	-0.364	-0.41
Ranking	1	2	3	4	5	6	7

The bottom two countries are Hungary and Argentina, with scores of -0.364 and -0.410. In the subsequent k-means cluster analysis, these two countries were classified as unhealthy countries. However, using Model 1 to evaluate the country's overall education level lacks a judgment on sustainability, which is far from adequate. Therefore, we proposed Model 2 to facilitate our comprehensive evaluation.

For Task 2, we need to evaluate the sustainability of education. Educational sustainability refers to the ability of the education industry to maintain its health, which can be calibrated by the overall efficiency (TE) of national education. TE is the ability of the education market to transform its input into the output. Therefore, we chose to use Model 2 Data Envelopment Analysis (FA-DEA) to process X_i and Y_i data to obtain TE values. Presented in Figure a.

In the sustainability evaluation of Model 2 in Hungary among the seven countries, the TE curve shows a decreasing trend. Based on the results of Model 1 and Model 2, Hungary has the lowest comprehensive level of health and sustainability in education. Therefore, it is urgent to implement a reasonable policy in Hungary to change this status quo.

For task 3, in the redundancy of the Hungarian inputs indicator (X_i) obtained in Model 2, we found that the number of teachers (X_3) in Hungary in 2018 is relatively large, with the

redundancy of 900 people per million people. That is, having more teachers is not helpful for efficiency (TE). Based on the above research and actual situation, we have formulated a policy for Hungary to reduce 50 people per million people per year according to the actual situation. The timetable is as follows:

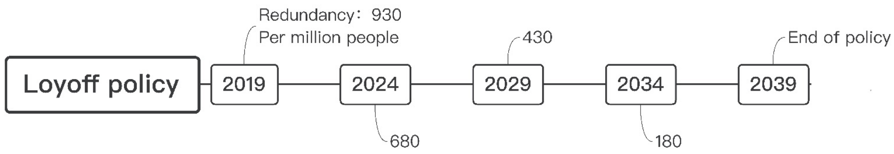
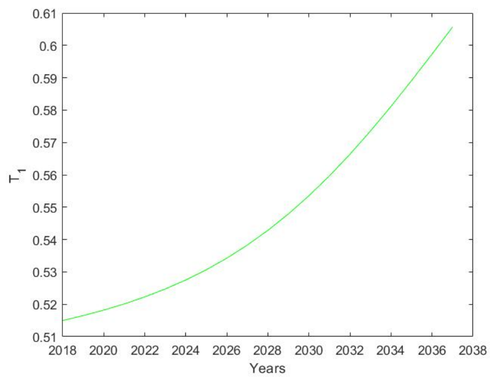


Figure 1: Policy implementation timetable

For task 4, we need to effectively measure the effect of the policy, that is, the change of TE value. The quantitative relationship between efficiency (TE) and each input index (Xi) cannot be determined. There is only a method based on machine learning, namely neural network prediction. Use the method of supervised learning to obtain the change curve of TE value after the implementation of the forecast policy, figure 2(b).

Twenty years after the implementation of the policy, Hungary's TE forecast value has changed from 0.515 in 2019 to 0.662, a significant increase, but it still does not meet the sustainable standard of TE>1. The TE value prediction error rate is only 9.56%. At this time, the teacher redundancy has been zero. In other words, after the implementation of the difficult two decades of policies, national education has shown a trend of improvement, but it has not yet shown a high level of education. In conclusion, the reform of education is a difficult and hindered process.

(a) TE curve.



(b) Change curve of TE value after policy implementation.

Keywords: Higher education; Super-efficiency dea; Factor analysis; Neural network;

Contents

1 Introduction

1.1 Background

Higher education is a key link in cultivating high-level talents in the country, and it is a major event related to the foundation and future of the country. According to statistics from the World Bank, in 2019, the enrollment rate of higher education institutions worldwide has reached 38%. It is not difficult to find that people's desire for education is increasing year by year. At the same time, this puts forward higher requirements for the health and sustainability of higher education in various countries.

1.2 Problems Restatement

In order to clarify the tasks, the 4 tasks can be simplified as follows:

Task 1: Establish a model to evaluate the health of national education and classify the health of each country.

Task 2: Establish a model to evaluate the health and sustainable development of national education.

Task 3: Choose a country that is not healthy and sustainable, propose a target policy for it, and predict future changes in this country.

Task 4: Analyze the prediction results in Task 3 .

1.3 Our works

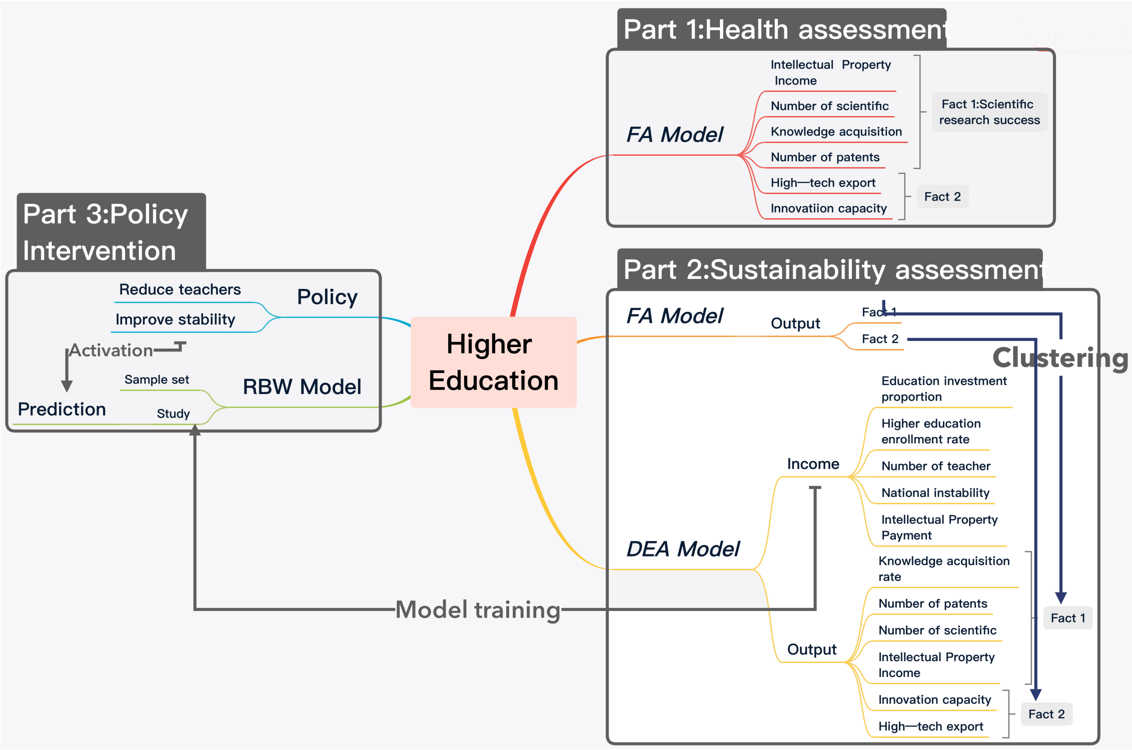
This paper established three models to solve the above problems.

Because the evaluation of the health status of higher education in seven countries requires objective and quantitative methods, comprehensive evaluation of the health status, such as each countrys national knowledge acquisition capacity, intellectual property income, number of patents, number of scientific journals, and innovation capabilities And high-tech export value to establish a health scoring model. At the same time, analyze the status of these seven countries/regions. Introduce K-Means cluster analysis to evaluate the health status of seven countries, and analyze the status of each of these seven countries. K-means cluster analysis is introduced to assess the health status of seven countries.

Consider how to judge whether a countrys education is sustainable. Since this sustainability is a long-term sustainable state, the ratio of sustainable development and economic output to input (TE) is compared. Because the efficient state can be considered sustainable. Therefore, the DEA model in economics was introduced, and the FA-DEA model was established to assess whether a countrys education is sustainable.

Taking into account the need to analyze the situation of the country after the implementation of the policy, we need to establish a predictive model. However, because the data we analyze is interrelated, it is difficult for us to sort out. Because neural network prediction has generalization ability, it can be used in complex environmental information. Based on the neural network prediction model.

Our main thoughts are shown in the mind map.



2 Notations

Symbols	Description
FA	Factor Analysis
$FA - DEA$	Factor Analysis-Date Envelopment Analysis
BP	Back Propagation
RBF	Radial Basis Function
α_i	Output-Income Correlation Coefficient
λ_i	Variance Ratio
X_i	Output
Y_i	Input
Z	Factor of Porosity Score
TE	Combined Efficiency
PTE	Pure Technical Efficiency
F	Policy Effect Function
F_j	Output Main Factor
$S-, S+$	Slack variables
ρ	Peanson similarity coefficient
V	Original data matrix
L	Correlation coefficient matrix

where we define the main parameters while specific value of those parameters will be given later.

3 Simplifying assumptions

1. Each higher education school can be regarded as an independent autonomous region in the national administrative unit. [**1**]
    - There is a global trading market formed by all production factors such as "finance", "knowledge" and "technology" among schools and countries.
    - Various countries compete fairly in the education market, obtain favorable production factors, and achieve system health and system sustainability.
2. Every school meets the description of "positional good". [2] That is, every school with a higher "position" is easier to maintain its own advantages and become a sustainable system. Similarly, every country also meets the above conditions.
3. The data we obtain from the Internet is reliable and authoritative. And all come from the public databases of major international organizations.
4. Each country or university maintains the current development trend, and does not consider the impact of emergencies such as earthquakes, tsunamis, wars, etc. on the health and sustainability of education.

4 Model 1 : Factor Analysis of Higher Education Health Evaluation (FA)

4.1 Overview

Based on the definition of "system health": a collective ability to realize a common vision. Compared with the definition of "sustainable system", it is shorter-term and more focused on results.Therefore, when evaluating the health of national education, we chose a static evaluation method: factor analysis.

Since the comprehensive score Z obtained by the factor analysis method reflects the relative level of higher education in each country, this score Z can be used to judge the health of the country. The higher the Z score, the higher the health of higher education.

The main steps of this model are as follows:

- **Check whether the data can be Factor Analysis.**
- **Analyze the contribution of each factor to national health, select appropriate public factors to replace the original variables, and explain the meaning of public factors.[4]**
- **The factor analysis method is used to obtain the comprehensive score Z of education health in each country.**
- **Using the K-Means clustering analysis, seven countries educational health is divided into three levels.**

4.2 Data preprocessing

In order to assess the health status of higher education in various countries, data from seven countries from 2000 to 2017 were collected and compiled from the World Bank.[]Select some data related to national education, usually used to measure the education level of a country.These include the national knowledge acquisition ability of each country (F1) intellectual property income (F2) the number of patents (F3) number of scientific journals (F4) innovation ability (F5) and high-tech export value (F6). Since not every data is complete, we made predictions and supplements for samples that lack a small amount of data. The selected country names and selected factor data are shown in Table 2.

Table 2: Part of the data for each country[10]

Code	F1	F2	F3	F4	F5	F6
USA[1]	2.510	4.35E+10	1.31E+05	3.05E+05	0.021	2.45E+11
CHN[2]	0.002	8.03E+07	2.66E+04	5.31E+04	0.015	1.54E+11
CEB[3]	0.027	2.92E+08	1.78E+04	3.37E+04	0.017	2.57E+10
MEX[4]	0.015	4.31E+07	1.26E+04	5.05E+03	0.017	2.05E+10
CZE[5]	0.024	4.43E+07	4.38E+03	5.37E+03	0.017	6.34E+09
HUN[6]	0.033	1.12E+08	4.13E+03	4.40E+03	0.015	2.27E+10
ARG[7]	0.112	3.68E+07	5.57E+03	4.39E+03	0.014	2.42E+09

1 United States; 2 China; 3 Central Europe and the Baltics;
4 Mexico; 5 Czech Republic; 6 Hungary; 7 Argentina
10 **These data come from the World Bank**

Construct the filtered data into a data matrix. Suppose m is the number of selected countries and n is the number of selected factors. Then $\mathbf{v_{ij}}$ is the raw data of the j-th factor of the i-th country. The constructed data matrix is as follows:

$$V = \begin{pmatrix} v_{11} & v_{12} & \cdots & v_{1m} \\ v_{21} & v_{22} & \cdots & v_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ v_{n1} & v_{n2} & \cdots & v_{nm} \end{pmatrix} \tag{1}$$

Use the above matrix 1 and formula 2 to obtain the standard data.

$$v'_{ij} = \frac{v_{ij} - \overline{x_j}}{\sqrt{\operatorname{Var}(v_j)}} \quad (i = 1, 2, \cdots, n, j = 1, 2, \cdots, m) \tag{2}$$

4.3 Model establishment

4.3.1 Perform KMO and Bartlett tests on standardized data

Calculate the KMO value **(KMO>0.6)**. According to the KMO value test results, there is a correlation between the six factors, and factor analysis can be performed.

Significance test **P<0.01**. According to Bartlett's test result, it is significant at the level, and it is very suitable for factor analysis under the rejection of the null hypothesis.

$$L=\begin{pmatrix} l_{11} & l_{12} & \cdots & l_{1m} \\ l_{21} & l_{22} & \cdots & l_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ l_{m1} & l_{m} & \cdots & l_{mm} \end{pmatrix} \tag{3}$$

Among them, rij is the correlation coefficient between index i and index j, and the calculation formula is as follows:

$$\begin{array}{l} l_{ij}=\operatorname{cov}\left(v_i,v_j\right)=\frac{\sum_{k=1}^{n}(v_{ki}-\bar{v}_i)\left(v_{kj}-\overline{v_j}\right)}{n-1}, n>1, j=1,2,\cdots,m \\ v_j=\left(v_{1j},v_{2j},\cdots,v_{mj}\right)^T, j=1,2,\cdots,m \end{array} \tag{4}$$

4.3.2 Analysis and interpretation of results

The data in Table and Table 3 are calculated by formula 4 as follows.

Calculate the KMO value **(KMO>0.6)**. According to the KMO value test results, there is a correlation between the six factors, and factor analysis can be performed. Significance test **P<0.01**. According to Bartlett's test result, it is significant at the level, and it is very suitable for factor analysis under the rejection of the null hypothesis.

4.3.3 Factor analysis results

Perform factor analysis on the six factors in seven countries, and get the data in Table 3 and Table 4 as follows.

Table 3: Posterior variance interpretation rate

Characteristic Root	Variance Proportion	Accumulate
3.164	0.5274	0.5274
2.718	0.4529	0.9803

Observing Table 2 shows that the six factors are divided into two main factors by factor analysis. Observing the component matrix in Table 3, we can know the score coefficient of each factor. Observe the score coefficient of each factor. F5 and F6 can be referred to as major factor 1 (MF1).F1, F2, F3, and F4 are the main factor 2 (MF2).

Through rotation, the factor has a clearer meaning. [5] Among them, F5 and F6 are mainly practical output and have a larger load with MF1. MF1 can be called the practical output factor (MF1).

F1, F2, F3, and F4 are mainly used for the output of theoretical research and understanding. Therefore, it can be said that MF2 has a large load, so MF2 can be called the scientific research achievement factor (MF2).

According to the data in Table 3 and Table 4, the main factor output score Y and the national education and health comprehensive score Z can be listed. Among them, the larger the comprehensive score Z calculated, the more healthy the national higher education.

$$\mathbf{Y_i}=\sum_{\mathbf{j=1}}^{\mathbf{6}}\gamma_{\mathbf{ij}}\cdot\mathbf{F_j}\quad \mathbf{j}=(\mathbf{1},\mathbf{2}\ldots\mathbf{6}) \tag{5}$$

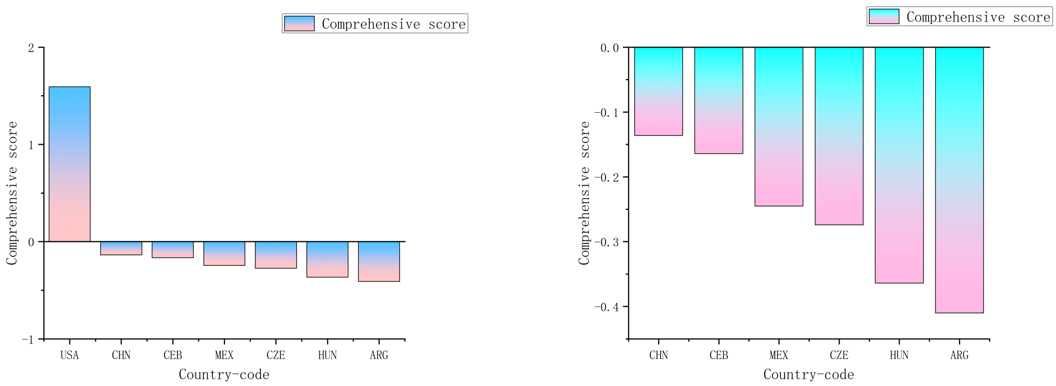
Table 4: Factor loading coefficient

γ_{ij}	F1	F2	F3	F4	F5	F6
MF1	0.27	0.264	0.051	0.039	0.855	-0.73
MF2	-0.031	-0.023	0.212	0.225	-0.693	1.034

$$\mathbf{Z = \sum_{i=1}^{2} k_i \cdot Y_i \quad i = (1, 2)} \tag{6}$$

Y_i is the output score for the two main factors.Which a_{ij} is the score coefficient of each factor in Table 2. F_j is the data of each factor in Table 4. Z is the comprehensive score of national education health.Which k_i is the variance ratio in Table 3.

Using the established formula to obtain the comprehensive score Z chart of the seven countries is as follows.



(c) Seven countries.

(d) No United State.

Figure 2: Overall ratings

- **The observation diagram (a) shows that the educational health of the USA is much higher than that of the other six countries.** At the same time, considering that the number of patent applications in the United States ranks among the top in the world throughout the year, and patents are of high quality. [**8**] Based on this, it is also credible that the United States is much higher than the other six countries.
- Observing the figure (a) above may mistakenly believe that the education and health levels of the other six countries except the United States are similar. However, observing diagram (b) shows that the health education level of these six countries is quite different, and the multiple of the maximum value and the minimum value has **reached twice as much.**

4.4 Using K-means cluster analysis to judge the national health education level

4.4.1 Model establishment

Because cluster analysis is a method of dividing a large amount of data, the data is classified according to its internal connections. Use formula (0) to find the correlation coefficient and classify the data.

$$\rho_{AB} = \frac{\operatorname{cov}(A,B)}{\sigma_A \sigma_B} = \frac{E\left[(A-\mu_A)(B-\mu_B)\right]}{\sigma_A \sigma_B} = \frac{\sum_{i=1}^{n}(a_i-\mu_B)(b_i-\mu_B)}{\sqrt{\sum_{i=1}^{n}(a_i-\mu_B)^2}\sqrt{\sum_{i=1}^{n}(b_i-\mu_B)^2}} \quad (7)$$

A and B only represent two points.

Use formula 7 to calculate the data in Table 5, and use cluster analysis to classify the health of seven countries. However, the current problem is that a specific classification number needs to be given. Observation table 5 shows that the gap between the United States and the other six countries is too large, and the gap between the largest and the smallest among the six countries other than the United States has reached more than twice. In summary, the seven countries can be vaguely divided into three categories.

Table 5: Education Health Ranking

Country-code	USA	CHN	CEB	MEX	CZE	HUN	ARG
Comprehensive score	1.593	-0.136	-0.164	-0.245	-0.274	-0.364	-0.41
Ranking	1	2	3	4	5	6	7

Table 6: Cluster category

	Category three	Category three	Category one
Z	-0.205± 0.065	-0.387±0.032	1.593

Based on the above considerations, K-Means cluster analysis is used to obtain the chart as shown in the figure below.

The significance test of the sample data results in **P<0.01**; therefore, it is feasible to divide the education health of the seven countries into three categories.

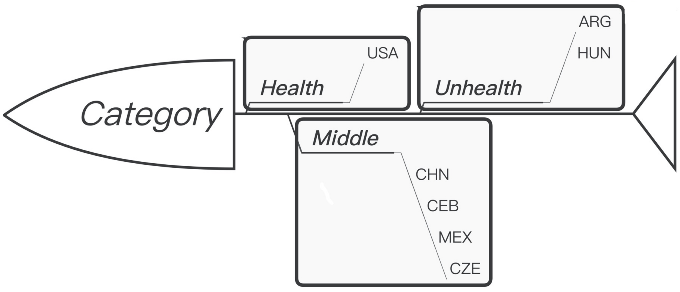
4.4.2 Conclusion

- Looking at Figure 3, we can see that only the USA in the seven countries has achieved health education; most of the countries are at the middle level.
- Among the unhealthy countries are HUN and ARG.

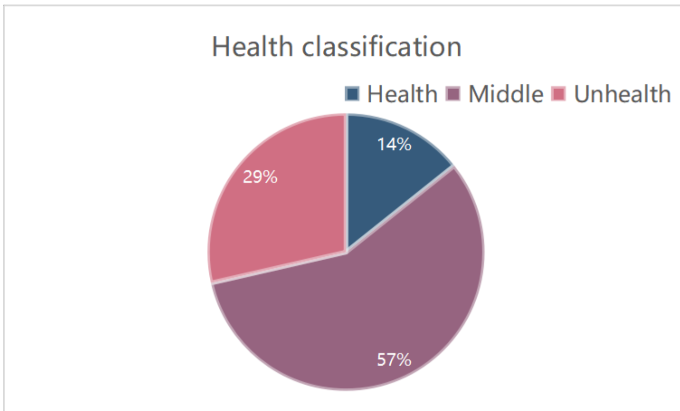
5 Model 2:Factor Analysis-Data Envelopment Analysis of Higher Education Sustainability Evaluation (FA-DEA)

5.1 Overview

Based on the definition of "educational sustainability": a system that maintains its effectiveness over time. It is more process-oriented and long-term. Therefore, when we consider the sustainability of education, if we choose a static evaluation method like Model 1, it will not be able to show the changing trend of a country's education over a period, which is short-term.



(a) Seven countries.



(b) Proportion of each level.

Figure 3: Cluster map and proportion

At the same time, education sustainability assessment is a problem with multiple inputs and outputs, and it needs to be solved from multiple dimensions.

For the above problems, we choose to use the data envelopment analysis (DEA) method. DEA is a quantitative analysis method that evaluates the relative effectiveness of similar decision-making units (DMU) with multiple inputs and multiple outputs. DEA uses non-parametric linear programming technology to determine a relatively effective production frontier (envelope) by selecting multiple inputs and outputs data of decision-making units. The decision-making unit on the production frontier is DEA effective; at the same time, non-DEA is effective The decision-making unit of DEA is mapped to the effective production frontier of DEA, and the relative efficiency of each decision-making unit is evaluated by comparing the distance of the non-DEA effective decision-making unit from the effective production frontier. It is widely used in schools, hospitals, banks, and libraries. [7] For national education, the DEA model can be explained by such a simple formula

$$TE = \frac{\sum Y_{\text{incomes}}}{\sum X_{\text{inputs}}} \tag{8}$$

In equation 8, the TE value of national education represents the country's ability to convert the input into the output. As stated in the book Social Limits to Growth [8], the phenomenon of What winners win, losers lose is common in positional good, and school is the most typical positional good. [2] Therefore, we have every reason to believe that education in countries with higher TE values and gradually increasing TE curves are easier to maintain long-term effectiveness, which is consistent with the definition of sustainability of education.

5.2 Model assumption

The selected input and output indicators meet the selection requirements. The processing of various indicators, such as flexibility indicators, time lag indicators, and feedback indicators [**9**] is correct.

5.3 Data source and index selection

1. Based on the conclusions of the Model 1 , we selected the six factors of Model 1 as the output indicators of the DEA model. And selected seven countries education investment ratio (X_1), higher education enrollment rate (X_2), number of teacher (X_3), intellectual property investment (X_4) national instability (X_5)as the input indicators X_i.Data are as follows

Table 7: Part of Hungarian outputs data

HUN	2007	2008	2009	2010	2011	2012	2013
X1	4.17	4.11	4.13	4.17	4.16	4.14	4.18
X2	35.94	39.70	44.55	52.24	60.06	64.99	67.54
X3	1.41E+03	1.44E+03	1.47E+03	1.50E+03	1.47E+03	1.57E+03	1.74E+03
X4	2.59E+08	2.69E+08	4.17E+08	4.64E+08	1.05E+09	1.11E+09	1.17E+09
X5	5.12E+02	9.49E+02	3.52E+03	3.39E+03	2.75E+03	3.52E+03	3.12E+03

HUN	2007	2008	2009	2010	2011	2012	2013
X1	4.25	4.53	4.91	4.86	4.90	4.47	4.35
X2	68.28	66.48	64.61	63.72	62.59	61.46	57.07
X3	1.73E+03	1.85E+03	2.01E+03	2.15E+03	2.33E+03	2.42E+03	2.55E+03
X4	1.75E+09	2.60E+09	1.98E+09	1.80E+09	2.03E+09	1.70E+09	1.97E+09
X5	3.38E+03	1.61E+03	1.54E+03	1.44E+03	1.23E+03	1.09E+03	1.22E+03

2. The DEA model requires that there is no strong correlation between output and input indicators themselves [**9**]. For output factors, through Model 1 FA, six factors with strong correlation can be combined into two common factors (Y_i) with weak correlation. Selecting two common factors (Y_i) as output can eliminate the correlation of output. For the input factor, we first perform a correlation test on X_i, and the results show that the correlation between X_i is very weak,The results are shown in Table 8.

Table 8: Add caption

KMO value		0.44
Bartlett sphericity test	Approximate chi-square	14.181
	df	10
	p	0.33

3. Part of the missing data in the DEA model, we use linear interpolation or averaging to make up. Among them, we regard the number of national refugees as national instability (X_5).

5.4 Model building

In order to compare the sustainability of education in various countries more effectively after obtaining the national education TE, we chose the input-oriented super-efficiency DEA model proposed by Andersen and Petersen.[**10**]

Calculated as follows:

$$TE^{*} = \min\left(TE - \varepsilon e S^{+}\right) \tag{9}$$

$$\begin{cases} \sum_{i=1}^{n} X_i\lambda_i + S^{-} = TE \cdot X_k \\ \sum_{i=1}^{n} Y_i\lambda_i - S^{+} = Y_k \\ \lambda_i \geqslant 0 \\ S^{-} \geqslant 0 \\ S^{+} \geqslant 0 \end{cases} \tag{10}$$

In the formula 10:

- TE* is the revised value of the countrys educational efficiency;
- TE is the efficiency value of national education;
- ε is an Archimedes infinitesimal;
- **e** is the natural constant;
- **i** is DMU, here refers to the year, i=1,2,...,k-1,k,k+1,...,n; n is the number of years;
- λ_i is the linear scale factor.
- X_i is the input of the i-th device Index set, X_i=(X_{i1},X_{i2},...,X_{im})T, X_{im} is the m-th input index of the i-th year;
- Y_i's output index set of the i-th year, where Y_i=(Y_{i1},Y_{i2},...,Y_{is})T, yis is the s-th output index of the i-th year;
- S-, S+ are slack variables introduced for the convenience of calculation.

Result judgment: If TE<1, it means that the country's DEA is ineffective and the overall efficiency is low, and optimization management is needed; while TE<1 for many years in a row indicates that the country's sustainability is weak and needs improvement. If TE≥1, it means that the country's DEA is effective, the overall efficiency is good, and it can be maintained. But it is worth mentioning that because we did not choose to use cross-sectional data in the DEA model.[**11**] Therefore, the absolute magnitude of TE values between different countries cannot be compared. The value of TE cannot feed back country rankings and any information.

5.5 Conclusion

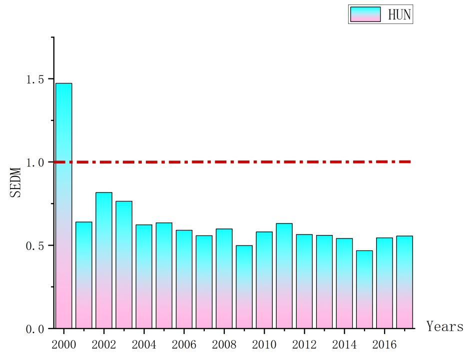
Based on the data of each country from 2000 to 2018, Table 9. We obtained the TE curves of various countries through Matlab and displayed them in figure 4.

Let us take a look at what interesting conclusions we have drawn from the TE curve:

Table 9: TE value of each country

Years	CEB	USA	CHN	CZE	MEX	HUN	ARG
2000	0.98655	1.30811	1.30311	1.10692	0.98657	1.47277	0.99646
2001	1.13149	0.88468	0.98254	1.46812	1.03709	0.64007	0.93353
2002	1.06866	0.87591	0.91417	1.06366	0.76526	0.81663	0.7927
2003	1.02395	0.82177	0.97253	0.94148	0.88158	0.76466	0.99707
2004	0.97084	0.87972	1.06985	1.61869	0.79031	0.62315	0.87765
2005	1.02672	0.8444	1.10322	0.83519	0.7553	0.63485	0.72941
2006	1.01569	0.84707	0.96628	1.05704	0.82475	0.59052	0.6354
2007	0.87425	1.00468	0.87876	1.27666	0.60642	0.55794	0.39667
2008	0.86628	1.00131	0.88267	1.00145	0.76299	0.59801	0.55618
2009	0.81775	0.75369	0.80611	0.77452	0.90872	0.49825	0.53301
2010	0.91933	0.80894	0.96233	0.8749	0.92644	0.57992	0.61855
2011	1.06537	0.84874	0.99851	1.18829	0.97863	0.63045	0.68584
2012	0.97023	0.83539	0.98663	1.12611	0.99812	0.564	0.64114
2013	0.92831	0.83821	1.0002	0.9087	1.05113	0.55945	0.72795
2014	0.97391	0.85667	0.97399	0.9116	0.95856	0.54074	0.72838
2015	0.93963	0.80536	0.9545	0.86286	0.96206	0.46793	0.93214
2016	0.98526	0.88065	0.85069	0.88178	1.00856	0.54475	1.13734
2017	1.19377	0.82901	0.89939	1.12875	1.15449	0.55511	1.34922

(a) TE curve.



(b) HUN TE histogram.

Figure 4: Comparison of Hungary's TE value with other countries

- From 2007 to 2008, the change rate of TE value in Argentina showed a turning point, and this growth trend has been maintained for the next ten years. What is the reason for this phenomenon According to reports, since Cristina became president in 2007, She has inherited and implemented many related policies that help economic reforms.With the help of these policies, even in the 2008 economic crisis, Argentina has shown a relatively stable economic situation,[**12**] which is strikingly similar to the growth trend of the TE value of education in Argentina.It shows that our TE curve is in good agreement with the reality, and it can also reflect a country's educational sustainability over a period of time.

- More importantly, since 2000, Hungary's TE curve has been showing a downward trend and has maintained this trend.

- The TE curves of the remaining countries have remained at a relatively high level and maintained a relatively more stable level. In summary, combined with the results of Model 1 and Model 2, Hungary's national education health and sustainability are at a disadvantage among the seven countries. Therefore, if effective policies are not implemented, the state of Hungarys national education will become worse.

At the same time, we use DEA to obtain the redundancy of Hungary's national education investment Xi , so that we can better formulate reasonable policies for Hungary, as shown in Figure 11.

Table 10: Add caption

Years	X_3	X_5	X_3 redundancy	X_5 redundancy
2000	1409.5191	512	1190.961086	0
2001	1438.7672	949		
2002	1471.878	3515	11.223223	2947.8864
2003	1496.8433	3394		
2004	1473.531	2747	0	1982.0598
2005	1574.2712	3516		
2006	1744.9886	3117	165.2328044	2314.3302
2007	1734.9104	3384	8.292727966	2397.3062
2008	1851.9062	1612	0	2.419E-10
2009	2014.5713	1535		
2010	2149.8141	1435	0	244.01718
2011	2326.1666	1234	502.0904683	0
2012	2416.4776	1085		
2013	2546.0846	1218	429.7517389	109.01212
2014	2673.4344	1264	629.2591907	235.65905
2015	2589.0979	1447	577.5727714	424.99372
2016	2645.7568	2909	723.6785842	1759.6524
2017	2921.5331	3336	980.0066582	2180.4919

Input redundancy: the part of education input that does not contribute to the efficiency TE, that is, the red part in figure 11.

From the above results, we find that Hungarys education investment redundancy is mainly

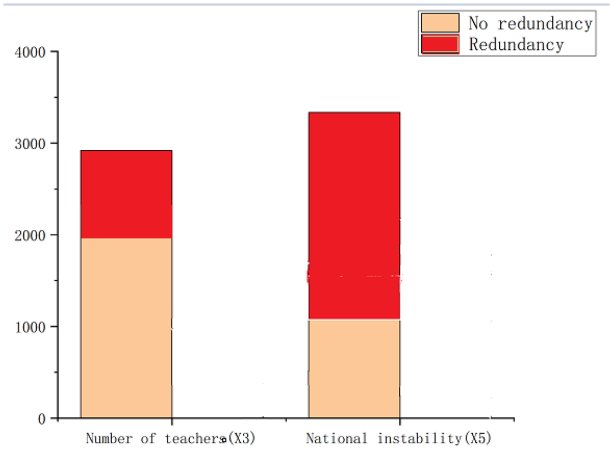


Figure 5: Policy implementation timetable

concentrated in the number of teachers and the countrys instability. Because of the above characteristics, we have formulated the following policies for Hungary:

- **Reduce the number of teachers in the country every year.** Due to the excessive number of teachers, mutual inefficiency is caused.
- **Improve the stability of the country and reduce the number of refugees.** Only when the country obtains a more stable environment, can education be more sustainable.

In the next model 3, we will discuss the changes in the sustainability of education in Hungary after the policy is implemented. In order to facilitate research, we simplified the policy and only considered the impact on TE value after the country implemented the layoff policy. The policy implementation process is described in Timetable 6.

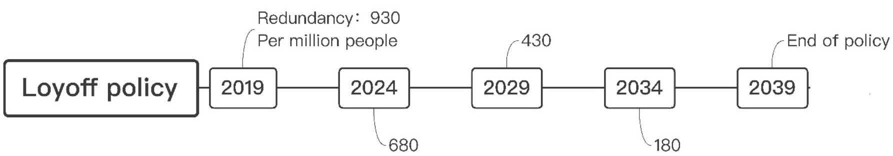


Figure 6: Policy implementation timetable

We will reduce the redundancy of 980 teachers per million people at a rate of 50 per year, and the reduction will be completed in about 20 years.

6 Model 3Neural network predicts policy implementation effects

6.1 Overview

From the redundancy values of different input indicators Xi in model 2, we have already clarified the reason for the low TE value of Hungary's education, that is, the number of teachers (X3) and the country's instability (X5) are large in redundancy. The redundancy is the extra part that does not help TE. Based on the above analysis, We have established in Model 2 policies and programs that are most conducive to improving the sustainability of education in Hungary.

For the sake of simplifying the model, we temporarily only implement the policy of reducing the number of teachers and predict the impact of this policy on the national educational sustainability TE value. Although the impact of the policy on X3 and the policy timetable has been determined, it is still far from enough to predict the country's TE value. Because the relationship between the TE value of DEA and the input and output is complex and non-linear, which makes prediction difficult.

In our attempt to find an effective forecasting plan, we found that our model has the following characteristics:

1. The data sample is relatively small, only the TE value of Hungary from 2000 to 2018.
2. Data has multiple inputs (Xi) and outputs (Yi).
3. The relationship between variables is complex and difficult to fit.

It is precisely because of the above characteristics that many commonly used forecasting models are very weak in dealing with this problem. For example time series model, gray forecast, etc.

In the end, we chose to use a neural network algorithm to predict the TE value of Hungary under policy implementation.

The neural network has the following characteristics:

1. The neuron can obtain the hidden functional relationship of the data through the learning and training of the sample.
2. The neural network can obtain the laws hidden in the data by learning and training the input sample data and use the learned laws to predict future data.
3. The neural network is a universally applicable function approximator, which can approximate any continuous function with arbitrary precision.

In the end, due to the small sample size, we chose the RBW neural network with strong learning ability and less sample size.This is a feedforward neural network. The network is seen as an approximation to a certain set of unknown functions. This approximation can be achieved in any form of function (linear or non-linear).At the same time, the RBF network also has good promotion capabilities. Besides, it avoids some tedious calculations, making the learning speed much higher than the traditional BP method.

RBF networks usually consist of a single input layer, a single hidden layer, and a single output layer.The entire network realizes the nonlinear conversion from input space to output space, and this is all due to the hidden layer basis function (the commonly used hidden layer basis function is Gaussian function).[**14**]

6.2 Model building

6.2.1 Problem analysis

The basis of the RBF neural network is the local response of the neuronal cells of the human brain to the outside world. This is a feedforward neural network. The network is regarded as an approximation to a set of unknown functions, and this approximation can be realized in any form of the function (linear or non-linear).At the same time, the RBF network also has good promotion capabilities. Besides, it avoids some tedious calculations, making the learning speed much higher than the traditional BP method.

RBF networks usually consist of a single input layer, a single hidden layer, and a single output layer.The entire network realizes the nonlinear conversion from input space to output space, and this is all due to the hidden layer basis function (the commonly used hidden layer basis function is Gaussian function).

6.2.2 Data collection and processing

The super-efficiency value obtained from task two (and after interpolation), and the input data set (X1, X2, X3, X4, X5) of Hungary during the 18 years from 2001 to 2017. In this model, the input data set is used as input, and the super efficiency value is used as output. A network with five inputs and one output is formed. The data of the first fifteen years is used as the training sample set, and the data of the next three years is used as the test sample set.

Table 11: The output data and TE value of Hungary from 2000 to 2017

Years	X1	X2	X3	X4	X5	TE
2000	4.16605	35.94101	1409.519	2.59E+08	512	1.47277
2001	4.11127	39.69805	1438.767	2.69E+08	949	1.1447
2002	4.13457	44.55069	1471.878	4.17E+08	3515	0.81663
2003	4.17343	52.23563	1496.843	4.64E+08	3394	0.76466
2004	4.15786	60.0567	1473.531	1.05E+09	2747	0.62315
2005	4.14432	64.99212	1574.271	1.11E+09	3516	0.63485
2006	4.18482	67.54416	1744.989	1.17E+09	3117	0.59052
2007	4.25405	68.28498	1734.91	1.75E+09	3384	0.55794
2008	4.52806	66.48384	1851.906	2.6E+09	1612	0.59801
2009	4.912	64.60789	2014.571	1.98E+09	1535	0.49825
2010	4.85608	63.72239	2149.814	1.8E+09	1435	0.57992
2011	4.89692	62.59323	2326.167	2.03E+09	1234	0.63045
2012	4.46934	61.46318	2416.478	1.7E+09	1085	0.564
2013	4.35102	57.06502	2546.085	1.97E+09	1218	0.55945
2014	4.30338	52.01587	2673.434	1.7E+09	1264	0.54074
2015	4.28657	48.96214	2589.098	2.01E+09	1447	0.46793
2016	4.22764	48.03886	2645.757	1.45E+09	2909	0.54475
2017	4.20774	48.50038	2921.533	1.6E+09	3336	0.55511

6.2.3 Calculation processing

The following formulas are used to standardize the input and output data of the sample.[**15**] The algorithm executed by the inverse processing function is:

$$\tilde{t} = \frac{2\left(t - t_{\min}\right)}{t_{\max} - t_{\min}} - 1 \tag{11}$$

In the formula: t_{max} is the variable before normalization; t_{min} and t are the maximum and minimum values of t respectively; $\tilde{t}$ is the variable after normalization.

$$t = 0.5(\tilde{t} + 1) \cdot (t_{\max} - t_{\min}) + t_{\min} \tag{12}$$

This is the structure diagram of our RBF neural network after training.(figure7)

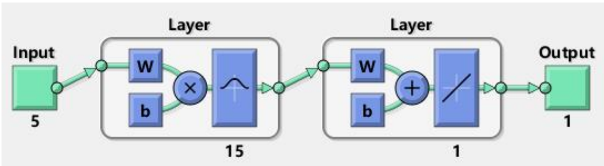


Figure 7: Diagram of neural network structure.

6.2.4 Result display

The realization of this model is based on the neural network toolbox provided by Matlab. While using the RBF neural network for data prediction, it also adds the code of the conventional BP neural network for comparison with the former.According to our policy guidelines, we have added a cycle code to the original basis to achieve a decrease of X_3 by 50 units year by year. The results are as follows:

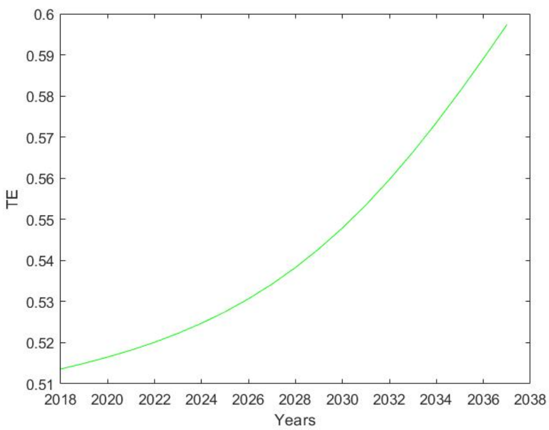


Figure 8: Predictive analysis results

6.3 Conclusion

Not only that, for the sample points of the next three years, the results of the model solution are: 0.5572, 0.5311, 0.5158, and the relative errors of the data in the next three years are calculated, respectively: 19.08%, 2.51%, and 7.08%. Simply calculate the average relative error of 9.56%. At the same time, the mean square error (MSE) of the RBF neural network prediction data is 27.7%.

At the same time, when performing traditional BP neural network prediction, it cannot adaptively determine the number of hidden neurons in the training process like RBF. It can only be determined based on experience. Because we only make a simple comparison, we choose The number of hidden neurons is 2 and 4. In fact, the operation result oscillates greatly, which also shows the rationality of the prediction result of the RBF neural network from the side.

7 Strength and Weakness

7.1 Strength

- **Evaluation of national education from multiple dimensions.** Regarding the national education level, we chose to conduct a comprehensive assessment from two aspects of health and sustainability, weighing the common impact of short-term health and long-term sustainability. This method is efficient and consistent with the facts.

- **The sustainability of national education can be fully evaluated.** In evaluating the sustainability of national education, we have established a multi-input and multi-output FA-DEA model.

- **Using the DEA model, we can formulate the best policy for the country, start from the shortest path, and obtain high efficiency.**The input redundancy obtained by using each input index obtained by the DEA model, that is, the part that each input index does not contribute to efficiency.For example, the number of national teachers in Hungary has a large redundancy. The country only needs to formulate layoffs for teachers to achieve high efficiency.

- **When predicting the effects of national policies after implementation, the actual situation can be fully considered.** Using neural network algorithms, it is possible to learn the impact of multiple input and output indicators on efficiency to a greater extent. Therefore, it can be highly realistic to predict the efficiency change curve after the national policy acts on the input.

7.2 Weakness

Although we try our best to improve our models, but there are still some weaknesses in our models. For the policy of reducing the number of teachers, we only use a certain value for fitting, which is far from the reality.

8 Sensitivity analysis

In this paper, we need to analyze the sensitivity of the teacher reduction rate obtained in Model 3. We respectively substitute the speeds 30, 40, 50, 60, 70, and 90 into the neural network model for prediction. As shown in Figure 9.

From the trend in the figure, it can be seen that as the rate of teacher redundancy increases, the growth trend of TE gradually increases. And when the teacher reduction rate reaches 90, the amount of redundancy is reduced rapidly. After eleven years, there was negative redundancy, that is, excessive policy regulation, which reduced the part of teachers that contributed to

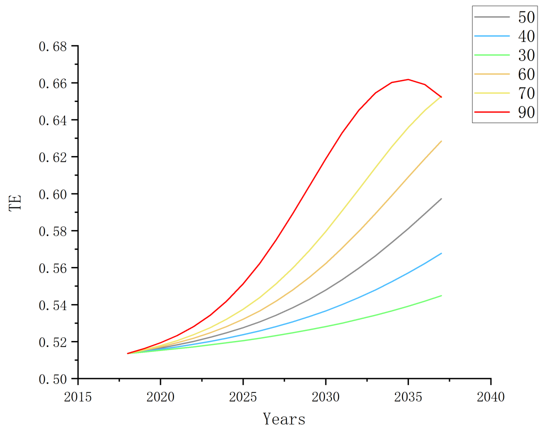


Figure 9: Sensitivity analysis curve

efficiency, and on the contrary appeared the phenomenon of reduced efficiency. In the figure, this trend is very obvious. Obviously, the sensitivity of our prediction function is very good.

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Appendices

Appendix A First appendix

In addition,

Input matlab source:

```
clc, clear
a=xlsread('Hu.xlsx','B2:G19');
a=a';
P=a(1:5,1:end-3); [PN,PS1]=mapminmax(P);
T=a(6,1:end-3);  [TN,PS2]=mapminmax(T);
net_1=newrb(PN,TN)
x=a(1:5,end-2:end); xn=mapminmax('apply',x,PS1);
yn1=sim(net_1,xn); y1=mapminmax('reverse',yn1,PS2)
delta1=abs(a(6,end-2:end)-y1)./a(6,end-2:end)
Hu=xlsread('Hu.xlsx','B19:F19')';
hect=ones(30,1);
for i=1:30
    Hu(3)=3000-(i-1)*50;
    Huu=mapminmax('apply',Hu,PS1);
    Ww=sim(net_1,Huu);
    Wn=mapminmax('reverse',Ww,PS2);
    hect(i)=Wn(1);
end
%xlswrite('try.xlsx',hect(1:20))
plot(2018:2037,hect(1:20),'g')
xlabel('Years');ylabel('TE');
net2=feedforwardnet(2);
%net2=feedforwardnet(4);
net2=train(net2,PN,TN);
yn2=net2(xn);y2=mapminmax('reverse',yn2,PS2)
```

Appendix B Second appendix

some more text **Input matlab source:**

```
clc,clear;
Z=xlsread('Hun.xlsx');
X=Z(1:end-3,2:6);
b=Z(1:18,9:14);
F_1=[-0.73,0.855,0.039,0.051,0.264,0.270];
F_2=[1.034,-0.693,0.225,0.212,-0.023,-0.031];
Yhat=b*F_1';
Yba=b*F_2';
Y=[Yhat,Yba];
super=deasuper(X, Y, 'orient', 'io');
deadisp(super);
```