代数学I宿題(3)

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Problem 1.

1. Since

$$\begin{aligned}\sigma :&1 \mapsto 2 \mapsto 3 \mapsto 1,\\ \tau :&1 \mapsto 2 \mapsto 3 \mapsto 4 \mapsto 1,\end{aligned}$$

$$\begin{aligned}&1 \mapsto 2 \mapsto 3\\ \sigma\tau :&2 \mapsto 3 \mapsto 1\\ &3 \mapsto 4 \mapsto 4\\ &4 \mapsto 1 \mapsto 2,\\ &= 1 \mapsto 3 \mapsto 4 \mapsto 2 \mapsto 1 \text{ and}\end{aligned}$$

$$\begin{aligned}\tau\sigma :&1 \mapsto 2 \mapsto 3\\ &2 \mapsto 3 \mapsto 4\\ &3 \mapsto 1 \mapsto 2\\ &4 \mapsto 4 \mapsto 1\\ &= 1 \mapsto 3 \mapsto 2 \mapsto 4 \mapsto 1,\end{aligned}$$

$\sigma\tau = (1342)$ and $\tau\sigma = (1324)$.

2. Since

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 1 & 4 \end{pmatrix}, \tau = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \end{pmatrix}$$

$$\sigma^{-1} = \begin{pmatrix} 2 & 3 & 1 & 4 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 1 & 2 & 4 \end{pmatrix} = (132).$$

$$\tau^{-1} = \begin{pmatrix} 2 & 3 & 4 & 1 \\ 1 & 2 & 3 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 2 & 3 \end{pmatrix} = (1432).$$

3. Since

$$\begin{aligned}\tau^2 :& 1 \mapsto 2 \mapsto 3\\ & 2 \mapsto 3 \mapsto 4\\ & 3 \mapsto 4 \mapsto 1\\ & 4 \mapsto 1 \mapsto 2\\ =& 1 \mapsto 3,\\ & 2 \mapsto 4\\ =& 2 \mapsto 4,\\ & 1 \mapsto 3,\end{aligned}$$

$\tau^2 = (13)(24) = (24)(13).$

4. Since

$$\begin{aligned}\sigma^3 :& 1 \mapsto 2 \mapsto 3\\ & 2 \mapsto 3 \mapsto 1\\ & 3 \mapsto 1 \mapsto 2\\ &= 1 \mapsto 2 \mapsto 3 \mapsto 1,\\ \sigma^3 &= (1)\end{aligned}$$

Since $(13)^2 = (24)^4 = (1)$,

$$\begin{aligned}\tau^4 &= (\tau^2)^2\\ &= (13)(24)(13)(24)\\ &= (13)(13)(24)(24) = (1)(1) = (1).\end{aligned}$$

5. Since $\sigma = (123) = (13)(12)$ and $\tau = (1234) = (14)(13)(12)$, $\sigma \in A_4$ and $\tau \notin A_4$.

Problem 2. Since $(GL(2,\mathbb{R}), \cdot)$ is group isomorphic to $(\hom(\mathbb{R}^2, \mathbb{R}^2), \circ)$, $f_A : \mathbb{R}^2 \to \mathbb{R}^2$ is a bijection and $\hom(\mathbb{R}^2, \mathbb{R}^2)$ is a permutation group, $GL(2,\mathbb{R})$ is a permutation group where $\hom(\mathbb{R}^2, \mathbb{R}^2) = \{f : \mathbb{R}^2 \to \mathbb{R}^2 |\ f$ is a bijection and linear maping$\}$.

Problem 3. $(\mathbb{Q}, +), (\mathbb{Z}, +)$ are subgroups of $(\mathbb{R}, +)$