

代数学I宿題(10)

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Problem 1.

1. Let π be a plane and F be a regular n -sided polygon. Then define $D_n = \{\phi : \pi \rightarrow \pi \mid \phi : \text{motion with } \phi(F) = F\}$.
Let $\{a_1, \dots, a_n\}$ denote the vertices of the regular n -sided polygon. Since any elements in D_n is a motion, for any $\sigma \in D_n$, it will be regarded as a permutation of the vertices. Therefore D_n is a subgroup of S_n .
2. (a) Since $\#D_n = 2n$, $\#D_4 = 8$.
(b) D_4 is isomorphic to $[(1234), (24)]$. Since $(1234)(24) = (14)(23)$ and $(24)(1234) = (12)(34)$, D_4 is not a non-commutative group.
3. D_3 is isomorphic to $[(123), (13)]$. And since $S_3 = [(123), (13)]$, D_3 is isomorphic to S_3 .