代数学 I 宿題 (1)

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Problem 1.

1. Binary operation $\circ$ on G is a mapping from $G \times G$ to G.
2. $(G, \circ)$ is a group if and only if it satisfies the following conditions:
   (a) For all $a, b, c \in G$, $a \circ (b \circ c) = (a \circ b) \circ c$.
   (b) There exists $e \in G$ such that, for every $a \in G$, $a \circ e = e \circ a = a$.
   (c) For each $a \in G$, there exists $b \in G$ such that $a \circ b = b \circ a = e$.
   In correct, the conditions (b) and (c) should be united; there exists $e \in G$ such that, for every $a \in G$, $a \circ e = e \circ a = a$ and there exists $b \in G$ such that $a \circ b = b \circ a = e$.

   $(G, \circ)$ is an abelian group if and only if it satisfies the following conditions:
   (a) $(G, \circ)$ is a group.
   (b) For all $a, b \in G$, $a \circ b = b \circ a$.

Problem 2.

1. Ring of rational integers $\mathbb{Z}$ is an abelian group.
2. General linear group $GL(n, \mathbb{C})$ is a non-abelian group.