代数学 I 宿題 (4)

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Problem 1.

1. If $X, Y \in SL(2, \mathbb{R})$, $\det(XY) = 1$ because $\det(XY) = \det(X)\det(Y) = 1 \cdot 1 = 1$. Therefore $XY \in SL(2, \mathbb{R})$. And since $\det(X^{-1}) = (\det(X))^{-1} = 1^{-1} = 1$, $X^{-1} \in SL(2, \mathbb{R})$. Thus $SL(2, \mathbb{R})$ is a subgroup of $GL(2, \mathbb{R})$.

2. Since $\det(XY) = \det(X)\det(Y)$, if $X, Y \in M$, $\det(XY) = \det(X)\det(Y) = (-1) \cdot (-1) = 1$. Therefore $XY \notin M$. Thus M is not a subgroup of $GL(2, \mathbb{R})$.

3. Since $\varepsilon(\tau\sigma) = \varepsilon(\tau)\varepsilon(\sigma)$ and $\varepsilon(\sigma^{-1}) = \varepsilon(\sigma)$, if $\sigma, \tau \in A_4$, then $\tau\sigma \in A_4$ and $\sigma^{-1} \in A_4$ because $\varepsilon(\tau\sigma) = 1$ and $\varepsilon(\sigma^{-1}) = \varepsilon(\sigma) = 1$. Thus A_4 is a subgroup of S_4.

4. If $\sigma, \tau \in H$, then $\tau\sigma \notin H$ because $\varepsilon(\tau\sigma) = \varepsilon(\tau) \cdot \varepsilon(\sigma) = (-1) \cdot (-1) = 1$.

Problem 2.

1. $[M_1] = [4]$,

2. $[M_2] = [4] \cap [8] = [4]$, and

3. $[M_3] = [4] \cap [10] = [2]$.