代数学I宿題(4)

中野竜之介 8310141H

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Problem 1.

1. Since $S_3 = \{(1),(12),(13),(23),(123),(132)\} = \{(1),(12),(13),(23),(13)(12),(12)(13)\}$, $A_3 = \{(1),(123),(132)\}$.

-	(1)	(123)	(132)
(1)	(1)	(123)	(132)
(123)	(123)	(132)	(1)
(132)	(132)	(1)	(123)

2.

Problem 2.

1. For all $i, j \in \{1, ..., n\}$ with $i \neq j$, $\zeta_n^i \neq \zeta_n^j$ because if $0 < x \leq \pi$, $\sin x$ and $\cos x$ are injection. And $(\zeta_n^i)^n = (\zeta_n^n)^i = (\cos 2\pi + i \sin 2\pi)^i = 1^i = 1$. If $k < n$, $\zeta_n^k = \cos \frac{2k\pi}{n} + i \sin \frac{2k\pi}{n} \neq 1$. Therefore the order of U_n is n.

2. $U_5 = \{1, \zeta_5, \zeta_5^2, \zeta_5^3, \zeta_5^4\} = \{1, \cos\frac{\pi}{5} + i\sin\frac{\pi}{5}, \cos\frac{2\pi}{5} + i\sin\frac{2\pi}{5}, \cos\frac{3\pi}{5} + i\sin\frac{3\pi}{5}, \cos\frac{4\pi}{5} + i\sin\frac{4\pi}{5}\}$.
   Subgroups of U_5 are only $\{1\}$ and U_5 because 5 is a prime number.

3. $U_6 = \{1, \zeta_6, \zeta_6^2, \zeta_6^3, \zeta_6^4, \zeta_6^5\} = \{1, \frac{1}{\sqrt{3}} + \frac{1}{2}i, \frac{1}{2} + \frac{1}{\sqrt{3}}i, i, -\frac{1}{2} + \frac{1}{\sqrt{3}}i, -\frac{1}{\sqrt{3}} + \frac{1}{2}i\}$.
   Subgroups of U_6 are only $\{1\}, \{1, \zeta_6^2, \zeta_6^4\}, \{1, \zeta_6^3\}$ and U_6 because if m and n are coprime, then $m\mathbb{Z} + n\mathbb{Z} = \mathbb{Z}$.