代数学I宿題(2)

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Problem 1.

1. (a) The number of elements of G is $n!$.
   (b) The identity element of G is $I_E : E \to E; x \mapsto x$.
2. (a) The number of elements of G_0 is n^n.
   (b) G_0 is not a group because the inverse mapping of f doesn't exist when f is not a bijection.

Problem 2.
If $a+b$ equals $a+c$, b equals c by adding $-a$ from left to right

Problem 3.

	identity	element	inverse	associative
$(\mathbb{Q}, +)$	0	x	$-x$	-
$(\mathbb{Z}, -)$	-	-	-	doesn't satisfy
$(\mathbb{R}, \times)$	1	0	doesn't exist	-
$(\mathbb{C}^\times, \div)$	-	-	-	doesn't satisfy
$(\mathbb{R}[X], +)$	0	$\sum_{i=0}^{n} a_i X^i$	$\sum_{i=0}^{n} (-a_i) X^i$	-
$(V, +)$	zero vector $\mathbf{0}$	$\mathbf{v}$	$-\mathbf{v}$	-
$(S_5, \circ)$	$I : \{0, ..., 4\} \to \{0, ..., 4\}; x \mapsto x$	σ	inverse mapping σ^{-1}	-

If $c \neq 0$, for all $a, b \in G$, $a - (b - c)$ doesn't equal $(a - b) - c$, therefore $(\mathbb{Z}, -)$ doesn't satisfy an associative law. Hence $(\mathbb{Z}, -)$ is not a group.
Since $(1 \div 2) \div 2 = 1/4$ and $1 \div (2 \div 2) = 1$, $(\mathbb{C}^\times, \div)$ doesn't satisfy an associative law. Hence $(\mathbb{C}^\times, \div)$ is not a group.