

代数学I宿題(7)

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Problem 1.

1. H is a normal subgroup of $G \Leftrightarrow \forall x \in G, \forall y \in H, xyx^{-1} \in H$.
2. Define a quotient group G/H as follows.
 - (a) set: $G/H = \{aH \mid a \in G\}$
 - (b) operation: $aH \cdot bH = (ab)H$ ($a, b \in G$).
3. Let H be any subgroup of G . Then for all $x \in G, y \in H, xyx^{-1} = xx^{-1}y = ey = y \in H$. Therefore H is a normal subgroup of G . Since G is a commutative group. Hence $aH \cdot bH = (ab)H = (ba)H = bH \cdot aH$. Therefore G/H is a commutative group.

Problem 2.

1. The table for A_4 is as follows.

-	(1)	(12)(34)	(13)(24)	(14)(23)
(1)	(1)	(12)(34)	(13)(24)	(14)(23)
(12)(34)	(12)(34)	(1)	(14)(23)	(13)(24)
(13)(24)	(13)(24)	(14)(23)	(1)	(12)(34)
(14)(23)	(14)(23)	(13)(24)	(12)(34)	(1)

Because of the table, A_4 is a subgroup of V_4 .

2. Since $S_4 = [(12), (13), (14), (23), (24), (34)]$, the table as follow where (i, j) is $\sigma_i \sigma_j \sigma_i^{-1}$.

-	(1)	(12)(34)	(13)(24)	(14)(23)
(12)	(1)	(12)(34)	(14)(23)	(13)(24)
(13)	(1)	(14)(23)	(13)(24)	(12)(34)
(14)	(1)	(13)(24)	(12)(34)	(14)(23)
(23)	(1)	(13)(24)	(12)(34)	(14)(23)
(24)	(1)	(14)(23)	(13)(24)	(12)(34)
(34)	(1)	(12)(34)	(14)(23)	(13)(24)

3. The table for A_4/V_4 as follow.

-	V_4	$(123)V_4$	$(132)V_4$
V_4	V_4	$(123)V_4$	$(132)V_4$
$(123)V_4$	$(123)V_4$	$(132)V_4$	V_4
$(132)V_4$	$(132)V_4$	V_4	$(123)V_4$