代数学I宿題(7)

中野竜之介 8310141H

January 26, 2020

Problem 1.

1. $f : G \to G'$:isomorphism $\Leftrightarrow$ f:bijective homomorphism.
2. G is isomophic to $G' \Leftrightarrow \exists f : G \to G'$ s.t. f:isomorphism.

Problem 2.

1. $f(xy) = |xy| = |x| \cdot |y| = f(x)f(y)$, hence f is a homomorphism.
2. Im $f = \mathbb{R}^+$, Ker $f = \{\pm 1\}$.
3. Since Ker $f = \{\pm 1\}$, $\mathbb{R}^\times/\{\pm 1\} = \mathbb{R}^\times/\text{Ker } f \simeq \text{Im } f = \mathbb{R}^+$ from fundamental theorem on homomorphisms.
4. $\mathbb{R}$ is isomophic to $\mathbb{R}^+$ by a homomorphism $f : \mathbb{R} \to \mathbb{R}^+; x \mapsto \exp x$. Hence $\mathbb{R}^\times/\{\pm 1\} \simeq \mathbb{R}$.