ノート

2020/03/25

- $q^{(t)} = \begin{bmatrix} q_0^{(t)} \\ q_1^{(t)} \end{bmatrix} \in (0,1)^2, q_0^{(t)} + q_1^{(t)} = 1$: 確率変数
- $W(y|x)$: channel の prob. distribution.

$q^{(t)}$ は次で更新する。

$$q^{(t)} \overset{f}{\mapsto} q^{(t+1)} := \frac{1}{Z} \begin{bmatrix} q_0^{(t)} \times \dfrac{1}{\exp \mathbb{KL}[W(*|0) \parallel r]} \\ q_1^{(t)} \times \dfrac{1}{\exp \mathbb{KL}[W(*|1) \parallel r]} \end{bmatrix}, \tag{1}$$

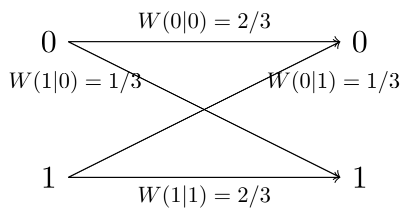
$$r_j := \sum_{i \in \{0,1\}} q_i^{(t)} \cdot W(j|i), \quad r = \begin{bmatrix} r_0 \\ r_1 \end{bmatrix} \tag{2}$$

Z は規格化定数。

f は全単射。f の逆像を求める。

例 1. $q^{(t)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$

$W(*|0) = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, W(*|1) = \begin{bmatrix} \frac{1}{3} \\ \frac{2}{3} \end{bmatrix}$



よって、

$$
\begin{aligned}
r_0 =& q_0^{(t)} \cdot W(0|0) + q_1^{(t)} \cdot W(0|1) = \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{2}, \\
r_1 =& q_0^{(t)} \cdot W(1|0) + q_1^{(t)} \cdot W(1|1) = \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{2}{3} = \frac{1}{2}, \\
\therefore \quad r =& \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}.
\end{aligned}
$$

従って、

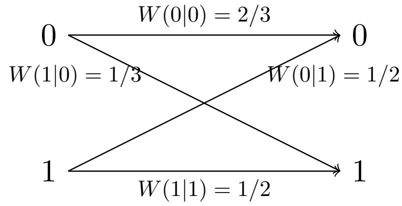
$$
\begin{aligned}
\mathbb{KL}[W(*|0) \parallel r] =& \frac{2}{3} \log \frac{\frac{2}{3}}{\frac{1}{2}} + \frac{1}{3} \log \frac{\frac{1}{3}}{\frac{1}{2}} = \frac{2}{3} \log \frac{4}{3} + \frac{1}{3} \log \frac{2}{3} = \frac{5}{3} \log 2 - \log 3, \\
\mathbb{KL}[W(*|1) \parallel r] =& \frac{1}{3} \log \frac{\frac{1}{3}}{\frac{1}{2}} + \frac{2}{3} \log \frac{\frac{2}{3}}{\frac{1}{2}} = \frac{1}{3} \log \frac{2}{3} + \frac{2}{3} \log \frac{4}{3} = \frac{5}{3} \log 2 - \log 3.
\end{aligned}
$$

以上から

$$
\begin{aligned}
q_0^{(t+1)} =& \frac{1}{Z} \times \frac{1}{2 \exp\left(\frac{5}{3} \log 2 - \log 3\right)}, \\
q_1^{(t+1)} =& \frac{1}{Z} \times \frac{1}{2 \exp\left(\frac{5}{3} \log 2 - \log 3\right)},
\end{aligned}
$$

例 2. $q^{(t)} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}$.

$$
W(*|0) = \begin{bmatrix} \frac{2}{3} \\ \frac{1}{3} \end{bmatrix}, \quad W(*|1) = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix}.
$$



よって、

$$
\begin{aligned}
r_0 =& \frac{1}{2} \cdot \frac{2}{3} + \frac{1}{2} \cdot \frac{1}{2} = \frac{7}{12}, \\
r_1 =& \frac{1}{2} \cdot \frac{1}{3} + \frac{1}{2} \cdot \frac{1}{2} = \frac{5}{12}, \\
\therefore \quad r =& \begin{bmatrix} 7/12 \\ 5/12 \end{bmatrix}.
\end{aligned}
$$

従って、

$$
\begin{aligned}
\mathbb{KL}[W(*|0) \parallel r] =& \frac{2}{3}\log\frac{\frac{2}{3}}{\frac{7}{12}} + \frac{1}{3}\log\frac{\frac{1}{3}}{\frac{5}{12}} = \frac{2}{3}\log\frac{8}{7} + \frac{1}{3}\log\frac{4}{5} = \frac{8}{3}\log 2 - \frac{1}{3}\log 5 - \frac{2}{3}\log 7, \\
\mathbb{KL}[W(*|1) \parallel r] =& \frac{1}{2}\log\frac{\frac{1}{2}}{\frac{7}{12}} + \frac{1}{2}\log\frac{\frac{1}{2}}{\frac{5}{12}} = \frac{1}{2}\log\frac{6}{7} + \frac{1}{2}\log\frac{6}{5} = \log 2 + \log 3 - \frac{1}{2}\log 5 - \frac{1}{2}\log 7.
\end{aligned}
$$

以上から

$$
\begin{aligned}
q_0^{(t+1)} =& \frac{1}{Z} \times \frac{1}{2\exp\left(\frac{8}{3}\log 2 - \frac{1}{3}\log 5 - \frac{2}{3}\log 7\right)} = \frac{1}{Z} \times \frac{5^{1/3} \cdot 7^{2/3}}{2^{11/3}}, \\
q_1^{(t+1)} =& \frac{1}{Z} \times \frac{1}{2\exp\left(\log 2 + \log 3 - \frac{1}{2}\log 5 - \frac{1}{2}\log 7\right)} = \frac{1}{Z} \times \frac{5^{1/2} \cdot 7^{1/2}}{2^2 \cdot 3}
\end{aligned}
$$