Theory of Computation   HW4 CFL Pumping Lemma   Ryzeson Maravich

1

a

Claim: The class of context-free languages is closed under union.
Proof: For any two context-free languages L_1 and L_2, the union of these two languages can be achieved with the additional rule

$$S \to S_1 | S_2$$

followed by the rules for both L_1 and L_2, where S_1 and S_2 are the languages' respective start variable, and S is the new start variable.

b

Claim: The class of context-free languages is closed under concatenation.
Proof: For any two context-free languages L_1 and L_2, the concatenation of these two languages can be achieved with the additional rule

$$S \to S_1 S_2$$

followed by the rules for both L_1 and L_2, where S_1 and S_2 are the languages' respective start variable, and S is the new start variable.

c

Claim: The class of context-free languages is closed under star.
Proof: For any context-free language L_1, L_1^* can be achieved by appending the start variable to the end of each part of the first rule, and adding ϵ as a terminal in the start rule (if it does not already exist).

2

Claim: The language $L_1 = \{0^n 1^n 0^n 1^n | n \geq 0\}$ is not context-free.
Proof: Suppose L_1 is context-free. Let p be the pumping length for L_1 given by the pumping lemma. Select the string $s = 0^p 1^p 0^p 1^p$. We will show that no matter how we divide s into $uvxyz$, the pumping lemma is violated. We will proceed in 3 cases.

1. v and y are in the same section (collection of the same characters). Pumping uv^2xy^2z causes one section to get bigger than the others.
2. v and y are in different sections. Because $|vxy| \leq p$ by the pumping lemma, v and y must be in adjacent sections. Pumping uv^2xy^2z causes not all sections to be equal in size.
3. v or y span different sections. Because $|vxy| \leq p$, if v spans two sections, y cannot, and vice versa. Pumping uv^2xy^2z causes not all sections to be equal in size.

Each case leads to a violation of the pumping lemma, and thus a contradiction of our assumption, therefore L_1 is not context-free.

3

Claim: The language $L_2 = \{t_1\#t_2\#\cdots\#t_k | k \geq 2$, each $t_i \in \{a, b\}$, and $t_i = t_j$, for some $i \neq j\}$ is not context-free.
Proof: Suppose L_2 is context-free. Let p be the pumping length for L_2 given by the pumping lemma. Select the string $s = a^p b^p \# a^p b^p$. We will show that no matter how we divide s into $uvxyz$, the pumping lemma is violated. We will proceed in 3 cases.

1. v or y contains a $\#$. Pumping uv^2xy^2z gives more than one consecutive $\#$, so $s \notin L_2$.
2. v and y both appear before or both appear after the $\#$. Pumping uv^2xy^2z causes $t_1 \neq t_2$ so $s \notin L_2$.
3. x contains the $\#$. Because $|vxy| \leq p$, v must consist only of b's and y must consist only of a's. Pumping uv^2xy^2z causes $t_1 \neq t_2$, so $s \notin L_2$.

Each case leads to a violation of the pumping lemma, and thus a contradiction of our assumption, therefore L_2 is not context-free.

4

Let B be the language of all palindromes over $\{0, 1\}$ containing equal numbers of 0s and 1s.
Claim: B is not context-free.
Proof: Suppose B is context-free. Let p be the pumping length for B given by the pumping lemma. Select the string $s = 1^p 0^{2p} 1^p$. We will show that no matter how we divide s into $uvxyz$, the pumping lemma is violated. We will proceed in 3 cases.

1. v and y are both contained in the same section (collection of the same characters). Pumping uv^2xy^2z either gives more 0s than 1s or more 1s than 0s.
2. v and y are in different sections. Because $|vxy| \leq p$, v and y must be in adjacent sections. Pumping uv^2xy^2z gives a string that is no longer a palindrome, so $s \notin B$.
3. v or y span different sections. Because $|vxy| \leq p$, if v spans two sections, y cannot, and vice versa. Pumping uv^2xy^2z gives a string that is no longer a palindrome, so $s \notin B$.

Each case leads to a violation of the pumping lemma, and thus a contradiction of our assumption, therefore B is not context-free.

I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.