Theory of Computation HW13 NP-Complete Ryzeson Maravich

1

Let *DOUBLE-SAT* $= \{\langle\phi\rangle | \phi$ has at least two satisfying assignments$\}$.

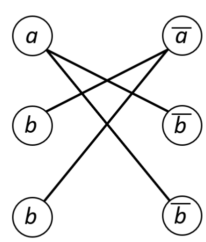
Claim: *DOUBLE-SAT* is NP-Complete.
Proof: We will create a reduction from *SAT* to *DOUBLE-SAT*. To do so, we simply need to AND the clause "$(x \vee \overline{x})$" to any formula ϕ. If ϕ is satisfiable, then we know that our addition will guarantee at least two satisfying arrangements. These would be the original assignments of values to the variables in ϕ, with x being false for one assignment, and x being true for the other. Because the additional clause will always be true, *phi* will remain true. Similarly, if ϕ was originally unsatisfiable, no additional AND statement, regardless of its truth value, would ever make ϕ true. Because *SAT* is NP-Complete and *SAT* $\leq_p$ *DOUBLE-SAT*, *DOUBLE-SAT* is also NP-Complete.

2

Perform the following conversions as described in the textbook.

a. Reduce *SAT* (in CNF) to 3-*SAT* on input $\phi = (a \vee b) \wedge (a \vee \overline{b} \vee c \vee d)$.
In order to reduce ϕ to 3-*SAT*, we need to change it so each clause has exactly 3 literals. By adding a (or b) to the first clause, this will not change the truth value of ϕ. The second clause has 4 literals, so needs to be broken down into two separate clauses, each with 3 literals. This can be done by changing $(a \vee \overline{b} \vee c \vee d)$ to $(a \vee \overline{b} \vee z) \wedge (\overline{z} \vee c \vee d)$, where z is a new variable that can take on a value to satisfy this formula if and only if the formula was satisfiable before the conversion. By constructing a truth table, it can be seen that either formula is only false when a, c, and d are false, and b is true. Thus the entire reduction yields $(a \vee b \vee a) \wedge (a \vee \overline{b} \vee z) \wedge (\overline{z} \vee c \vee d)$.

b. Reduce 3-*SAT* to *CLIQUE* on input $\phi = (a \vee b \vee b) \wedge (\overline{a} \vee \overline{b} \vee \overline{b})$.
Just as ϕ is satisfiable, there exists a 2-clique in the corresponding graph, because there are 2 clauses in ϕ. This occurs twice each for the pairs $(a, \overline{b})$ and $(\overline{a}, b)$ as shown below.



c. Reduce 3-*SAT* to *SUBSET-SUM* on input $\phi = (a \vee b \vee b) \wedge (\overline{a} \vee \overline{b} \vee \overline{b})$.
First let us construct the following table.

	a	b	c_1	c_2
y_1	1	0	1	0
z_1	1	0	0	1
y_2	0	1	1	0
z_2	0	1	0	1
g_1	0	0	1	0
h_1	0	0	1	0
g_2	0	0	0	1
h_2	0	0	0	1
t	1	1	3	3

Using the rows, we can create the set S of numbers that can be chosen as possible addends. Therefore $S = \{1, 1, 10, 10, 101, 110, 1001, 1010\}$. The last row, $t = 1133$, is the number to be summed to. It can be checked that the ϕ is true if and only if the numbers in the rows of the corresponding variables add exactly to t.

I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.