1 (10 points)

Claim: For all non-negative integers n, $\binom{2n}{n} = \sum_{k=0}^{n} \binom{n}{k}^2$.
Proof: To prove this, we can use the identity $\sum_{j=0}^{n} \binom{n}{j}\binom{m}{k+j} = \binom{n+m}{n+k}$. If we set $m = n, j = k$, and $k = 0$, our identity becomes $\sum_{k=0}^{n} \binom{n}{k}\binom{n}{k} = \binom{2n}{n} \Rightarrow \sum_{k=0}^{n} \binom{n}{k}^2 = \binom{2n}{n}$

2 (10 points)

Claim: The value of the constant term in the expansion of $(x^2 + \frac{1}{x^2} - 2)$ is 184756.
Proof: We can use the multinomial theorem to compute our multinomial. When substituted in, we get $(x^2 + x^{-2} - 2) = \sum_{a_1+a_2+a_3=10} \binom{10}{a_1,a_2,a_3}(x^2)^{a_1}(x^{-2})^{a_2}(-2)^{a_3}$. We only want to find the possible solutions to $a_1 + a_2 + a_3 = 10$ that give us a constant term. This would only happen when there are the same number of x^2 and x^{-2} terms, so when $a_1 = a_2$. This gives us the solution set of (a_1, a_2, a_3) to be $(0,0,10), (1,1,8), (2,2,6), (3,3,4), (4,4,2)$, and $(5,5,0)$. After substituting these solutions sets into the right side of the multinomial theorem, we get

$$\binom{10}{0,0,10}(x^2)^0(x^{-2})^0(-2)^{10} + \binom{10}{1,1,8}(x^2)^1(x^{-2})^1(-2)^8 + \cdots + \binom{10}{5,5,0}(x^2)^5(x^{-2})^5(-2)^0 =$$

$$\frac{10!}{0!0!10!}(-2)^{10} + \frac{10!}{1!1!8!}(-2)^8 + \cdots + \frac{10!}{5!5!0!}(-2)^0 =$$

$$(1 * 1024) + (90 * 256) + \cdots + (252 * 1) = 184756$$

I affirm that I have upheld the highest standards of honesty and integrity in my academic work and have not witnessed a violation of the honor code.