1

For any string $w = w_1w_2 \dots w_n$, $w^R = w_n \dots w_2w_1$.
For any language A, let $A^R = \{w^R | w \in A\}$.

Claim: If A is regular, so is A^R.
Proof: Let $N = \{Q, \Sigma, \delta, q_0, F\}$ recognize A.

Construct $N_R = \{Q_R, \Sigma, \delta_R, q_R, F_R\}$ to recognize A^R.

1. $Q_R = Q \cup \{q_R\}$.
2. q_R is the new start state.
3. $F_R = \{q_0\}$.
4. Define δ_R so that for any $q \in Q_R$ and any $a \in \Sigma_\epsilon$,

$$\delta_R(q, a) = \begin{cases} F & q_R, a = \epsilon \\ \emptyset & q_R, a \neq \epsilon \\ \{q_n | \delta(q_n, a) = q\} & q \in Q. \end{cases}$$

2

Let $B_n = \{a^k | k \text{ is a multiple of } n\}$.

Claim: For each $n \geq 1$ the language B_n is regular.
Proof: Construct $N = \{Q, \Sigma, \delta, q_0, F\}$ to recognize B_n.

1. $Q = \{q_0, q_1, \dots, q_{n-1}\}$.
3. q_0 is the start state.
4. $F = \{q_0\}$.
5. Define δ so that for any $q \in Q$, and any $a \in \Sigma_\epsilon$,

$$\delta(q_r, a) = \begin{cases} q_{r+1} & q_r \neq q_{n-1} \\ q_0 & q_r = q_{n-1}. \end{cases}$$

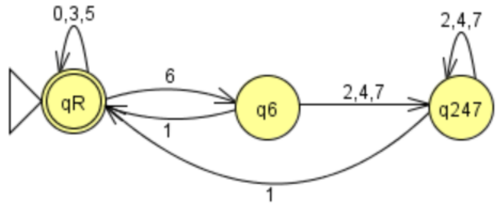
3

Let

$$\Sigma_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}. \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$$

such that $_3$ contains all size 3 columns of 0s and 1s. A string of symbols in Σ_3 gives three rows and columns. Consider each row to be a binary number and let $B = \{w \in \Sigma_3^* |$ the bottom row of w is the sum of the top two rows$\}$.

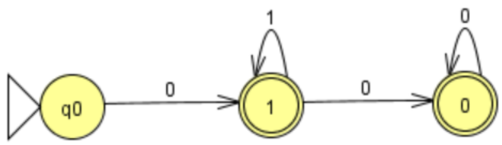
Claim: B is regular.
Proof: Let the elements of of Σ_3 be denoted by their corresponding decimal value as if reading top to bottom were the same as left to right. (e.g. $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = 1$ and $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = 6$). The following is a DFA that recognizes B^R.



Therefore we know that B^R is regular, and because regular languages are closed under reverse, B is regular.

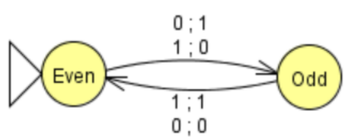
4

The following is an NFA that recognizes the language $0^+1^*0^*$.



5

The following is a Finite State Transducer whose output string is identical to the input string on the even positions but inverted on the odd positions. Assume the alphabet is $\{0, 1\}$.



I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.