1

Claim: The language $A_2 = \{www | w \in \{a,b\}^*\}$ is not regular.
Proof: Suppose A_2 is regular. Let p be the pumping length given by the pumping lemma. Let $s = a^p ba^p ba^p b$. Because $s \in A_2$ and $|s| \geq p$, by the pumping lemma we can split s into three pieces $s = xyz$ such that for $i \geq 0$ $xy^i z \in A_2$.

Because $|xy| \leq p$ by the pumping lemma, y must consist of all a's. The string $xyyz$ would have too many a's, as there would be no way to divide the new string into three equal w's, and thus $s \notin A_2$. This violates the pumping lemma, therefore A_2 is not regular.

2

Claim: The language $B = \{w | w \in \{0,1\}^*$ is not a palindrome$\}$ is not regular.
Proof: First we will prove that $B_c = \{w | w \in \{0,1\}^*$ is a palindrome$\}$ is not regular. Suppose B_c is regular. Let p be the pumping length given by the pumping lemma. Let $s = 0^p 1 0^p$. Because $s \in B_c$ and $|s| \geq p$, by the pumping lemma we can split s into three pieces $s = xyz$ such that for $i \geq 0$ $xy^i z \in B_c$.

Because $|xy| \leq p$ by the pumping lemma, y must consist of all 0s. The string $xyyz$ would have more 0s before the 1 than after the 1, and therefore $s \notin B_c$. This violates the pumping lemma, therefore B_c is not regular.

Languages are closed under complement, so if B is regular, then B_c would also be regular. Because we proved B_c is not regular, B is not regular.

3

Claim: The language $B_2 = \{wtw | w, t \in \{0,1\}^+\}$ is not regular.
Proof: Suppose B_2 is regular. Let p be the pumping length given by the pumping lemma. Let $s = 1^p 001^p 0$. Because $s \in B_2$ and $|s| \geq p$, by the pumping lemma we can split s into three pieces $s = xyz$ such that for $i \geq 0$ $xy^i z \in B_2$.

Because $|xy| \leq p$ by the pumping lemma, y must consist of all 1s. The string $xyyz$ adds 1's to the first part of the string, so that for any t, there is no possible w that could exist for s to be in the language, therefore $s \notin B_2$. This violates the pumping lemma, therefore B_2 is not regular.

4

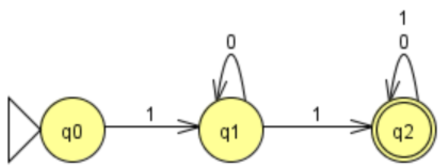
Let $\Sigma = \{1, \#\}$.
Claim: The language $Y = \{w | w = x_1 \# x_2 \# \cdots \# x_k$ for $k \geq 0$, each $x \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$ is not regular.
Proof: Suppose Y is regular. Let p be the pumping length given by the pumping lemma. Let $s = 1^p \# 1^{p+1} \# \cdots \# 1^{2p}$. Because $s \in Y$ and $|s| \geq p$, by the pumping lemma we can split s into three pieces $s = xyz$ such that for $i \geq 0$ $xy^i z \in Y$.

Because $|xy| \leq p$ and $|y| \geq 0$ by the pumping lemma, y must consist of between 1 and p ones. The string $xyyz$ would generate between $p+1$ and $2p$ 1s before the first $\#$. In each case, $s \notin Y$ because some $x_i = x_j$ for $i \neq j$. This violates the pumping lemma, therefore Y is not regular.

5

a

Claim: The language $B = \{1^k y | y \in \{0,1\}^*$ and d contains at least k 1s, for $k \geq 1\}$ is regular.
Proof: The following is a DFA that recognizes B.



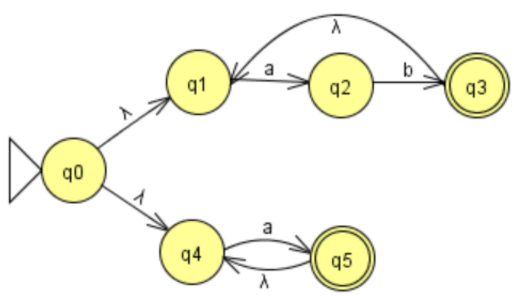
b

Claim: The language $C = \{1^k y | y \in \{0,1\}^*$ and d contains at most k 1s, for $k \geq 1\}$ is not regular.
Proof: Suppose C is a regular language. Let p be the pumping length given by the pumping lemma. Let $s = 1^p 0 1^p$. Because $s \in C$ and $|s| \geq p$, by the pumping lemma we can split s into three pieces $s = xyz$ such that for $i \geq 0$ $xy^i z \in C$.

Because $|xy| \leq p$ and $|y| \geq 0$ by the pumping lemma, y must consist of 1s. The string xz results in fewer 1s before the 0 than after. Thus d will always have more than k 1s, so $s \notin C$. This violates the pumping lemma, therefore C is not regular.

6

The following NFA recognizes the regular expression $a^+ \cup (ab)^+$.



I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.