

Give the context-free grammars that generate the following languages and construct state diagrams of push-down automata for each. In all parts, the alphabet  $\Sigma$  is  $\{0, 1\}$ .

**1**

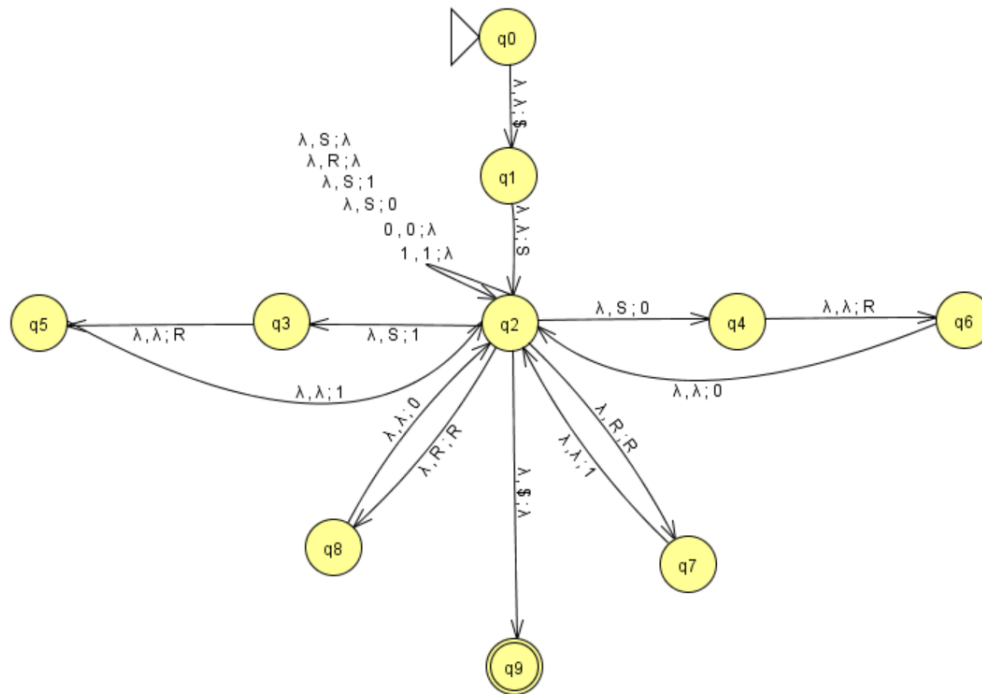
$\{w \mid w \text{ starts and ends with the same symbol}\}$ .

CFG:

$$S \rightarrow 0R0 \mid 1R1 \mid 0 \mid 1 \mid \epsilon$$

$$R \rightarrow 0R \mid 1R \mid \epsilon$$

PDA:



Note that this PDA is a formal construction, so appears much more complicated than the others.

**2**

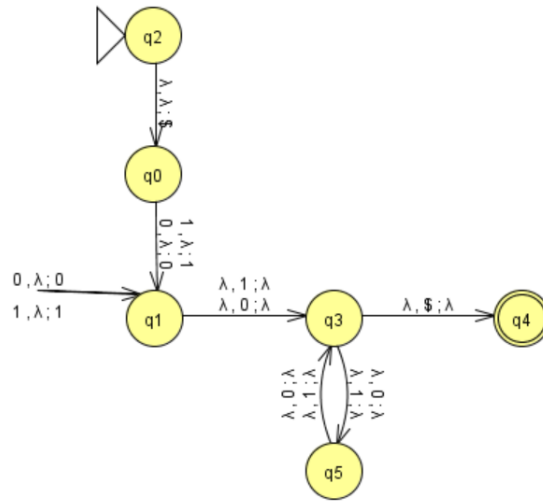
$\{w \mid \text{the length of } w \text{ is odd}\}$ .

CFG:

$$S \rightarrow 0R \mid 1R$$

$$R \rightarrow 00R \mid 01R \mid 10R \mid 11R \mid \epsilon$$

PDA:



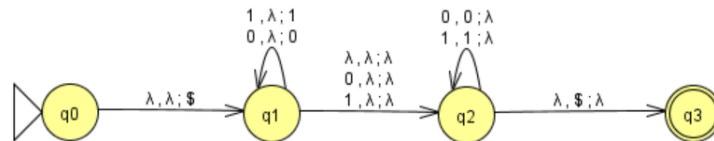
**3**

$\{w | w = w^R\}$ .

CFG:

$$S \rightarrow 0S0 | 1S1 | 1|0|\epsilon$$

PDA:



I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the honor code.