The History of Analytic Philosophy

Part 1: The Birth of Analytic Philosophy

Mathematics has always served as the cornerstone of rationality and exactitude in the academic community. Scientific laws are modeled by equations, experimental data is gathered through quantifiable measurements, and even visual and auditory art adhere to mathematical patterns and principles to enhance their aesthetic value. When there is intellectual doubt in one's field, one turns to mathematics to provide answers. This is because math is precise, objective, and infallible. When a conclusion is reached using math, it can be believed with absolute certainty. But how do we know this? What justification do we have to know that the logic used by mathematicians can be trusted, and more importantly, how do we know the premises (axioms) deduced from are true? Questions like these were presupposed as true until the development of Non-Euclidean geometry and transfinite numbers in the late 19th century. These discoveries, which seemed to contradict the foundations of mathematics, led many to question which theory to believe, and subsequently question the validity of math entirely.

When truth itself is under question, philosophers take it upon themselves to find answers. As a result, philosophers and mathematicians alike had to solve the problem of the axioms by reevaluating and justifying the fundamental propositions of mathematics, as well as solve the problem of proof, that is, provide an unobjectionable defense for the rules of the system.

Rigor in mathematics begins with the Greek mathematician Euclid's work *Elements*, in which he is able to axiomatize geometry, beginning with only twenty-three definitions, five postulates, and five common notions. Of course there were important discoveries before this, and math was employed by the masses in daily activities, but this is the first documented place where every new finding is strictly deduced from a given axiom, or from a previous proven proposition. The proofs in the *Elements* were so airtight that there was no possible doubt about the legitimacy of the claims. With this standard in place, mathematicians attempted to emulate this rigor in all future endeavors.

The fifth of Euclid's postulates, nicknamed the "Parallel Postulate" because it indirectly gives a definition for parallel lines, was slightly different from the other four. It seemed obvious, but its lengthy formulation compared to the others caused suspicion in academics for centuries, including al-Khwarizmi, Descartes, and even Newton (Greenberg). Eventually, in the early 19th century, Hungarian János Bolyai and Russian Nikolai Lobachevsky were able to (independently) formulate a new system of geometry, with equally sound logic as Euclid's, but without the Parallel Postulate. They realized that seemingly parallel lines could eventually cross or bend away from each other if the plane they were on was thought of as not flat, but curved. With this new system introduced, nobody was sure which was true. Further complicating matters, Bernhard Reimann confirmed shortly after an infinitude of such Non-Euclidean geometries. With infinite valid interpretations of geometry, there in turn seemed to be infinite truths, a serious epistemological problem.

Several decades later, Georg Cantor unveiled another counter-intuitive, yet groundbreaking discovery. With his very clever diagonal argument, he showed that there is no bijection between the real numbers, itself is an infinite set, and the natural numbers, also an infinite set. In other words, he showed that there are different degrees of infinity, and some are "larger" than others. Much like Bolyai and Lobachevsky's work, this made mathematicians question even their most fundamental assumptions.

Hilbert's speech in 1900 outlined 23 essential problems that he urged mathematicians to solve within the next century, one of which challenged mathematicians "To prove that they [the axioms of mathematics] are not contradictory, that is, that a finite number of logical steps based upon them can never lead to contradictory results" (Hilbert 414). Axiomatizing mathematics and ensuring it's consistency was an extraordinary task to undertake, but with the uncertainty surrounding Non-Euclidean geometry and Cantor's theory of transfinite numbers, mathematicians were desperate to find an answer to Hilbert's problem and save mathematics from the emerging crisis. They needed reasons to believe the fundamental truths they were working from were in fact true.

The problem of the axioms, as opposed to the problem of the proof, seems to be the more difficult one to provide a convincing answer for. As evidenced by Euclid, axioms are the basis for any mathematical system, so how can you justify something that is self-evidently or definitionally true? There are four different approaches to this answer: logicism, intuitionism, conventionalism, and formalism.

Proponents of logicism (such as Russell, Whitehead, and Frege, all of whom are essential in helping to solve the problem of the proofs) believe that the entirety of mathematics can be derived from basic logic. Intuitionists think that math is in some sense pre-programmed into the human mind, and so math is not really about some real, objective reality. They claim that there is only truth in the mind, and we can use math to express this to others. Conventionalists, such as Henri Poincaré, argue that math is just a language, and as such, it does not matter how you say a certain thing, it still is representing one thing, which is truth. To a conventionalist, to say that Euclidean geometry is true while Non-Euclidean geometry is not, is equivalent to saying that a sentence spoken in German is true while the same sentence spoken in French is not. They are in fact both true, because they represent the same idea, just with a syntactical difference. Formalists, like Hilbert himself, advocate that truth is only relative to the axiomatic set, so the axioms one uses are arbitrary. If you want to use the rules defined by Euclidean geometry you can, and if you want to use the rules defined by Non-Euclidean geometry you can. In both cases you would be correct, because there is no one absolute "truth", just many governed by their respective axiomatic system.

Certainly, there are advantages and disadvantages to each position. For logicists, the infallible nature of logic is reassuring, but using it to define axioms themselves leads to an infinite regress. Intuitionism, while intuitively sensical, is difficult to prove, and lacks overall rigor. Conventionalists and formalists alike account for the coexistence of multiple systems, but because of this, never admit to any universal truth. To this day there is no definitive consensus on which solution is correct, and there are several prominent mathematicians and philosophers that have represented each school of thought.

After the proposal of several solutions to the problem of the axioms, mathematicians had to figure out the problem of the proofs and determine how to logically proceed step by step from the axioms to ensure that the results were just as valid. Without rigorous justification for such a method, any derived theorems and proofs could be questioned, and the truth of mathematics as a whole could be doubted. This process would take considerably more work than the problem of the axioms, with the majority of the groundwork needing to be developed from scratch, and with meticulous care at each moment.

In the late 19th century, Gottlieb Frege, whom many consider to be the father of analytic philosophy, published the *Begriffsschrift*, in which he set forth his project of logicism. By building off of pre-existing logic with the invention of quantifier notation, Frege was able to construct a rigorous system for representing and calculating logical expressions. Although his style and diagrams seem esoteric and obtuse by today's standards, much of modern logic is based on his terminology and methodology. With a logical system ready, the only thing left to do was develop a way to translate language into logical statements that could be dealt with by such a system.

Frege takes this burden upon himself as well in *On Sense and Reference*. Everyday language is slippery, with the meaning of words being hidden, and the core content of what is being said remaining elusive in many cases. The example Frege uses to illustrate this point is our use of the terms "morning star" and "evening star", both of which refer to the planet Venus (Frege 27). The sentence "The morning star is the evening star" conveys new information when said. If we were to replace "morning star" with its synonym "evening star", we get the vacuous tautology "The evening star is the evening star." This causes problems for logic, because substituting y for x when $x = y$ in such a system should not yield different results. Similarly, if the respective definitions for such "synonymous" words are contradictory, such as the case for the evening star and the morning star (Frege 33), these contradictions suggest an even bigger logical problem. Obviously, substituting two identical things that have the same meaning should not affect the overall meaning of the proposition, so our meaning of the term meaning must be faulty.

To get to the underlying cause of this problem, Frege provides three different definitions of how we use the word "meaning." The first he calls the image, which is the subjective idea of the word that appears in one's mind. This can be thought of as the word's connotation, except more personal and emotional, and Frege describes this image as "saturated with feeling" (Frege 29). This almost always varies from person to person, so it is much too subjective for a logician like Frege to use in his theory of language. He instead focuses on a word's sense and reference. The referent of a word is the set of all objects that exist in the world that the word is supposed to signify. The sense of a word consists of the properties and qualities that determine the referent. The sense of a shoe, for example, would be "a covering of the foot with a sturdy sole." This abstract set of conditions then leads us to the referent of shoe, that is the set of all things that match this description.

With this distinction between sense and reference as two different concepts of "meaning," Frege shows that the sense of the word is the essential definition of meaning. Two words can only be truly synonymous if they have the same sense. The morning star/evening star complication vanishes because the two have different senses. The former is the first star seen before the sunrise in the east, while the latter is the last star before sundown in the west.

Knowing how words function and how we are able to use them is important, but the only way that logic can be employed is with propositions, so it is also necessary to determine what these words mean as a whole. "Every declarative sentence … and its referent, if it exists, is either the true or the false" (Frege 34). By this, Frege is making the obvious point that sentences do not have a reference in the physical world, but can only be true or false. It is through this realization that we can conclude that sense yields meaning, whereas the reference yields truth value.

At this point, Frege has created an extensive logical system for dealing with nearly every kind of proposition, as well as a precise method for converting language into these propositions that can be defined in logical terms. The stage is set for building mathematics from the ground up, but even then there remains a question: What is a number? If there is a definition for a given word, then it is possible to work with it within Frege's logical system. If a word has no sense then this can not be translated. (Or rather in this case, the sense which we conceive matches poorly with what the sense of number actually is, so the translation into formal logic does us no good.)

Frege in his *Foundations of Arithmetic*, insists that "Number is not abstracted from things in the way that colour, weight and hardness are… Number is not anything physical" (Frege 98). When a number is used adjectively to a group of things, it does not describe each individual thing the way any other adjective would. If there are two sturdy shoes, then each shoe is comfy, but it is meaningless to say that each shoe is two. Two refers to the aggregate of items. To reconcile this, Frege claims that the concept of a number is directly related to the existence of an object. There exists a shoe, and there exists a (different) shoe, implying that there are two shoes. "In this respect existence is similar to number," Frege remarks (Frege 103). For Frege, it is the instantiation itself, through the use of quantifiers he devised in the *Begriffsschrift*, that creates the number.

But as Frege said before, a "number" itself is not a physical entity. For example, the two mentioned earlier describes the quantity (repetitions of instantiation) of shoes, but this two is not the same as the two in the equation 1 + 1 = 2. The first describes the quality of two-ness, or having the collective property of encompassing two, whereas the second use is the number two itself. Two is not a thing in the real world to Frege; it "has no location" (Frege 109). To define number, we simply make a bijection to the set of all sets that contain that many members. For example, 0 is equivalent to the set of all sets for which there is a bijection into the empty set. Similarly, 1 is the set of all sets for which there is a bijection into the set containing only the empty set. This lets Frege define numbers in a concrete way that is constructed purely from logic. And with all of the other parts in place, Frege had singlehandedly laid the foundation for totally rigorous and uncompromising true mathematics. That is until Bertrand Russell noticed a single flaw.

While the second volume of Frege's *Foundations on Arithmetic* was being published, he received a letter from Russell. It contained what is now known as Russell's Paradox, which calls for the creation of "the class of all those classes which are not members of themselves." Frege's central axioms should theoretically allow for such a set to be created, but if the set does contain itself, then it should not contain itself, but if it does not, then it should. At first he is unsure whether this portends any major implications to his theories, but as he continues to think about it, he realizes that this simple paradox is devastating to his logicism project, and nullifies the majority of his work on the matter. Despite this, Russell recounts, "I realize there is nothing in my knowledge to compare with Frege's dedication to truth, … and upon finding that his fundamental assumption was in error, he responded with intellectual pleasure…" (Irvine and Deutsch). Thankfully, Frege was not insulted or depressed by the matter, and with his remarkable action of intellectual humility, perfectly embodied the principles of the analytic tradition that he helped create.

In "What There Is" and "'Characteristics of Mental Phenomenon", Russel explicates his position that he deems neutral monism**.** Russell believes there to be only one kind of substance that comprises the world, and the only thing that changes is our way of viewing it. When we look at the world through the lens of psychology, we characterize the world as consisting of mental elements, but when viewing through the lens of physics, we characterize the world as consisting of physical elements. Russell compares this to organizing Londoners alphabetically or geographically (Ammerman 32). The categorizations do not create two different groups of people, it just creates two different interpretations of the same people. This is obviously different from the mind-body dualists, but also differs from the materialists, who reject the mind entirely, and the idealists, who reject matter entirely. This metaphysical insight is important for understanding his method of translating reality into language, which he details in one of his most famous works, "On Denoting."

"On Denoting" is Russell's attempt to fix the flaws of Frege's theory of translation, as well as contemporary philosopher Alexus Meinong's ontology. In Frege's case, Russell took issue with his commitment to the Principle of Compositionality and his failure to confront the Law of Excluded Middle. The Principle of Compositionality claims that the meaning of the sentence is the sum of the meaning of its words, and is the reason Frege was so worried about the problems of substituting "synonymous" words in *On Sense and Reference*. The Law of Excluded Middle is a logical rule that simply states that either the given proposition is true, or the negation of that proposition is true. Russell demonstrates that Frege's theory fails on both ends.

Russell provides the sentence "The present King of France is bald" (Russell 490), which he uses to raise some objections to Frege. (To immediately dispel any uncertainty, there is in fact no King of France.) For the Principle of Compositionality to hold, every word must contain meaning in isolation. So what does "the" mean? It is a definite article, and so by itself is meaningless. Perhaps a more convincing example is the segment "the present King." This could possibly refer to many different people, and then refers to nothing when used in the full context. It changes its meaning when in the context of other words, and therefore does not carry its original, isolated meaning.

"The present King of France is bald" also seems to defy the Law of Excluded Middle, at least when following Frege's rules. "The present King of France is bald" is false, but "The present King of France is not bald" is also false. Russell carefully deconstructs this to show how normal grammar deceives us by hiding parts of the complete meaning. For Russell, this sentence actually asserts three different things: There is a thing x such that x is the present King of France, there is only one such x, and x is bald (Russell 482). This is false because the first part is false. For the negation, it would logically read "There is no such thing x such that x is the present King of France, there is only one such x, and x is bald." This is true, because there is no such thing. By breaking the proposition into its fundamental pieces and negating it in its entirety, rather than just the last part, Russell saves sentences of this form from contradicting the Law of Excluded Middle.

Finally, Russell addresses the problem he sees with Meinong's theory, which contains an important distinction between existence and subsistence. If a thing exists, it belongs in the set of all things that are in reality. It is the typical meaning that “exists” possesses. A thing subsists if an idea can be formulated of it or it can be talked about in a sentence. It does not necessarily have to be a physical object in the real world. This categorization lets Meinong talk about things that do not actually exist, such as unicorns or numbers, but this also causes some problems. According to Meinong, sentences like "The round square is round" are true, because the round square subsists and can therefore be talked about, and a round thing being round is an a priori truth. This statement should be false, however; there is no such thing as a round square. This is why Meinong's theory becomes too metaphysically expensive, as it claims false things are true. Conveniently, Russell's solution to the Law of Excluded Middle solves this too. By dissecting "The round square is round" it becomes clear that the sentence is false from the initial assertion that "there exists an *x* such that *x* is a round square" (Russell 491). With that, Russell provides a preferable method of translation that eliminates the flaws of past philosophers' theories.

There is still one slight problem with Russell's theory at this point. With the way that he sets up his theory in "On Denoting", it is still susceptible to his very own paradox: the set of all sets that do not contain themselves. Russell explores the nature of this paradox and its solution in his "Mathematical Logic as based on the Theory of Types." He begins by analyzing similar paradoxes such as "I am lying" and the clever "the least integer not nameable in fewer than nineteen syllables" (Russell 223) and comes to the realization that the root issue of each paradox is that they are all self-referential. By self-referencing, they deceitfully abuse the vagueness of language to appear paradoxical. Russell therefore proposes the rule that "no totality can contain members defined in terms of itself" (237). To avoid this from occurring, different levels or types of propositions must be created such that "the n + 1th logical type will consist of propositions of order n, which will be such as contain propositions of order n - 1, but of no higher order, as apparent variables" (238). First order propositions are those about the world, second order propositions are those about first order propositions, third order are those about the second, ad infinitum. With the stipulation that propositions can only talk about those with a lower type, such "paradoxes" become meaningless, and so Russell's system is now immune to such contradictions.

In 1910, Russell and his Cambridge teacher turned colleague, Alfred North Whitehead, published the momunemental *Principia Mathematica*. With the first volume alone totalling 510 pages, they expounded a system of mathematical logic in the same vein as Frege did with the *Begrsiffsschrift*, avoiding his pitfalls and accounting for Russell's new theory of translation. The proof of 1 + 1 = 2 appearing 200 pages in gives an idea of how painstakingly exact and exhausting this project was. But with his own paradox solved, a superior theory of language, and a system of rigorous logic detailed, Russell finally finished the mission that Frege had set out to accomplish. With both the problem of the axioms and the problem of proof solved, mathematics was rectified as the logical and dependable subject it is known to be.

Part 2: Logical Positivism

At the end of the 19th century, things in the academic world were starting to improve. Just as figures like Russell, Whitehead, and Frege had rigorously reconstructed all of mathematics, physics too was on the verge of completion. Newton's laws took care of gravitation and mechanics, while Maxwell's equations succinctly tied together electricity and magnetism, as well as optics and thermodynamics.

While nearly finished, there were still problems with these current theories that could not be ignored. Maxwell's equations assumed that light was a wave, but there are instances, like the vacuum of space, where there exists no conceivable medium for the light to travel through. Physicists tried to cover up this incongruity with the introduction of what was termed the "luminiferous aether," an underlying substance that light was able to move through, but after contradictory results on the nature of the aether, this did not seem to be an adequate solution either.

After some tinkering with the numbers, a new insight caught the eye of Albert Einstein, who shortly after introduced his Special and later General Theory of Relativity. While this was a brilliant discovery that accounted for the many problems of past theories, it introduced its fair share of strange phenomena as well. If it were correct, occurrences like time dilation, relativistic mass, and the curvature of spacetime itself would revolutionize physics, and a review of fundamental principles would be necessary. Just as non-Euclidean geometry had started a crisis in mathematics, Einstein's Theory of Relativity had done the same for physics, challenging the very foundations that the science was built upon. Frege and Russell had been able to rectify mathematics through their development of logicism, so physicists and philosophers alike hoped that there was some equivalent to which they could turn. To come to their rescue, were the logical positivists.

Logical positivism was a philosophical movement whose long list of members included Moritz Schilck, Rudolf Carnap, Carl Hempel, A.J. Ayer and Otto Neurath, as well as prominent mathematicians such as Hans Hahn, Alfred Tarski, and Kurt Gödel (Creath). This scientific crisis in physics provided the motivation for the logical positivists, but to gain some insight into the basis for their philosophy, it is fitting to continue where the story with Bertrand Russell left off, with his student Ludwig Wittgenstein.

Described by Russell as "the most perfect example I have ever known of genius as I have traditionally conceived, passionate, profound, intense and domineering" (Heaton) Wittgenstein was dedicated to the pursuit of knowledge through logic. While on the front lines of the first World War, Wittgenstein would write down thoughts in his spare time, which would later be collected and published as his *Tractatus Logico Philosophicus*. The work contains numbered statements that were vaguely aphoristic, but as one might come to expect from a philosopher mentored by Russell, logically airtight. His mission was to show what was true about the world, and explain how language serves as an invaluable tool in doing so. He famously says that "The limits of my language mean the limits of my world" (Wittgenstein, 74), developing a causal chain that logically connects what exists in the world, to mental processes, and finally to spoken language.

This simultaneously provides a convincing argument for outlining what kinds of things can't be talked about, and in turn, can't be properly known or understood with any certainty. For Wittgenstein, most of metaphysics falls into this category. Things like questioning God's existence, while important for spiritual purposes, have no place in the rigorous field of philosophy. The *Tractatus* ends with the statement “Whereof one cannot speak, thereof one must be silent,” (Wittgenstien, 90) emphatically claiming that there are just some things that cannot be discussed meaningfully in a philosophical search for truth.

The logical positivists would read through the *Tractatus* at their meetings, discussing with each other Wittgenstein's ideas. They agreed with him, that untestable, metaphysical claims seemed of little use to philosophers, and as such should not be worried about. But there was still a dire need to figure out what was true about the world, and how they could ascertain this knowledge. The logical positivists advocated an empirical standard for testability, known as the “verification principle” or “criterion of cognitive significance” which states that a statement is only verifiable, and therefore meaningful, "if he [the philosopher] knows what observations would lead him, under certain conditions, to accept the proposition as being true, or reject it as being false" (Ayer, 6). In other words, the only worthwhile claims are those that can be evaluated (supported or discredited) by the use of sense experience. If they can’t be verified in this manner, then there is no way of knowing if they are true or false, and therefore they are of no use to analytic philosophers.

This quickly leads to an important issue, which is how scientific laws are supposed to be validated and believed. Scientific laws universally mandate how things in the world behave, and if general enough, should be applicable to everything. When acting in accordance with the verification principle though, each one of these potential cases must be individually observed as true for the scientific law to properly be accepted. This entails an infinite amount of such observations, which is quite clearly impossible.

Because of this, the logical positivists were forced to flesh out a weaker form of the verification principle. Showing what this newly formed process might look like, Carnap provides the example of claiming to see a white object on the table. "The degree of confirmation, after a few observations have been made, will be so high that we practically cannot help accepting the sentence. But even in this case there remains still the theoretical possibility of denying the sentence. Thus even here it is a matter of decision or convention" (Carnap, 425). This preserves the end goal of gaining information about the world by sacrificing the ability to determine truthhood or falsehood with absolute certainty. Carnap is not saying that statements can't be true or false, they must be, but is simply saying that our observations can only give us probable cause for believing in them. They are either true or false, but the point in which we start to conclude they are true or false is entirely subjective.

The adoption of this view successfully brandishes questions about unobservable phenomena as "pseudo-questions," and casts them out of the realm of philosophy, just as Wittgenstein had hoped. If there are no clearly defined conditions for a proposition to be true, then there is no point in engaging in them, because not only can a definite conclusion fail to be reached, but it is impossible to gain any useful knowledge in the process. The next objective of the logical positivists was to take this knowledge that could be discerned about the world, and properly categorize it.

To do this, they espoused the analytic/synthetic distinction proposed by Immanuel Kant. Analytic statements are those which are definitionally true, while synthetic statements are dependent upon some outside knowledge about the world. To use Kant's own examples, the statement "All bodies are extended," is analytic, because nothing else needs to be known to understand that it is true. The notion of occupying space is “covertly contained” in the concept of physical objects. His example of a synthetic proposition is “All bodies are heavy” (Kant, 130). This statement relies on other contingent facts, which in this case, is the existence of gravity. Just being a physical object alone does not provide enough information to conclude that it is also heavy. This is an empirical claim, and depending on the definition of “heavy” could be false. Every possible proposition can be placed into one of these two disjoint sets. The reason this distinction is so important is that it aids our understanding of each problem and tells us in which ways it can be solved.

For analytic statements, it can be clearly seen that determining their truth value reduces to being able to apply logical and linguistic analyses. Fortunately for the logical positivists, Russell had just formalized and perfected this exact endeavour. Although the original print run for *Principia Mathematica* had been entirely sold out, Russell was able to send Carnap a handwritten account of the important information (Linsky and Irvine). After the group reviewed it, they agreed with his work, and championed logicism as their method for dealing with analytic propositions.

Synthetic statements are not as immediately easy to deal with. Due to their inherent structure, they require additional information beyond what one can immediately gather from the terms themselves, usually in the form of an observation. Consequently, strict deduction can not be employed for synthetic statements as it can with analytic statements. The logical positivists used the weak verification principle to deal with these. If a claim could be tested, then the truth value corresponded to how much corroborating evidence it garnered through experiments or other ways of gathering empirical data. The underlying subjectivity of this process did not satisfy the logical positivists, who demanded rigor in their attempt to understand science. They noticed, however, that it could be reduced to a matter of probability, a field of mathematics that could in fact be axiomatized through deductive logic. They decided that the work of 18th century statistician Thomas Bayes encapsulated exactly what they wished to do. Bayes' Theorem outlines "a method by which we might judge concerning the probability that an event has to happen, in given circumstances, upon supposition that we know nothing concerning it but that, under the same circumstances, it has happened a certain number of times, and failed a certain other number of times" (Bayes). Similar to Euclid's Elements, Bayes starts his deduction from nothing more than definitions and postulates, so if reasoned correctly, should yield absolute truth.

With the criterion of cognitive significance, the analytic/synthetic division, logicism, and Bayesianism, the four pillars of logical positivism were erected; the foundation was complete. What initially seemed like a carefully constructed and impervious basis, however, did not last for long. Soon, problems started to emerge, and the cracks in the underlying support became evident.

The first thing to go was the criterion of cognitive significance. Even after the verification principle was relegated to its weaker form, it still contained some inherent problems. In his "Problems and Changes in the Empiricist Criterion of Meaning" Hempel admits that "it is useless to continue to search for an adequate criterion of testability in terms of deductive relationships to observation sentences." Every time a new formulation of this principle was introduced, they found that it was either "too restrictive or too inclusive, or both" (Ammerman, 221). This position was held and compounded by another philosopher, Willard Van Orman Quine.

Quine's work, “Two Dogmas of Empiricism,” is often cited as the greatest piece of philosophy of the 20th century (Godfrey-Smith, 30-33). It is so powerful because it challenges the division of analytic and synthetic statements, an idea that has comprised the groundwork in epistemology since its inception in the 18th century. Quine takes issue with the inability to define what an analytic statement is.

He seems to concede that statements like “No unmarried man is married”' are true by virtue of their structure, but other types of analytic statements like “No bachelor is married” are not as immediately obvious. (Ammerman, 198-199) This, he says, relies on a notion of synonymy, which he then attempts to define (reminiscent of Frege's On Sense and Reference). It does not suffice to reference a dictionary to determine what a synonym is, claims Quine, because dictionaries are based on synonymy, the concept we wish to define. He then inspects the interchangeability to be the definition of synonymy. Here again there arises a problem, as “it is not quite true that the synonyms 'bachelor' and 'unmarried man' are everywhere interchangeable *salva veritate*,” such as the “Bachelor has fewer than 10 letters,” so he dismisses these cases to focus on what he calls “cognitive synonymy” (201-202). This can only be acceptably defined for when two statements are only interchangeable *salva veritate* out of necessity**.** But herein lies the problem: Analyticity reduces to synonymy, which reduces to necessity, which reduces to analyticity. (203-204). Therefore the only justification for analytic statements was circular.

This quite clearly poses a great threat to logical positivism. If propositions could no longer be categorized neatly, then there was no easy way to determine how to approach them. With this blurred distinction, they couldn't just rely on logicism for one type and Baynesian for the other. It was possible that every piece of knowledge could now have to rely on a mixture of both, or some new approach entirely. There was no longer any certainty.

At this point, there was no robust way to filter out meaningless statements, and no way to categorize them after that. Although this was a devastating setback, as long as the tools needed to handle possible statements functioned properly, the movement could survive. Unfortunately, this was far from what followed. A year after graduating from the university of Vienna, logician Kurt Gödel published “On Formally Undecidable Propositions of Principia Mathematica and Related Systems,” which contained his famous Incompleteness Theorem (Kennedy). As the title suggests, it is directed toward Principia Mathematica (PM), the cornerstone of the logicism project. Before Russell and Whitehead started to build their formal logical system, Russell took a precautionary measure in his “Theory of Types,” outlawing the formulation of self-referential statements such as “This sentence is false” (Russell 223). With this in place, he saved his system from the irreparable paradoxes that had befell past attempts. What Gödel was cleverly able to do was number each logical proposition in a system like Russell's, and through mathematical manipulation, create a self-referential statement (Kennedy). This bypasses Russell's restriction, because it was constructed entirely within the confines of PM's rules! So just as Russell had destroyed Frege’s *Begriffsschrift*, Gödel too had invalidated all of Russell's PM.

From here, Gödel was able to conclude that any formal logical system must either be inconsistent or incomplete (Kennedy). In other words, there was knowledge that could not be gathered from an existing framework, and if it could, then part of the existing information must be wrong. For logical positivists, this news was disastrous. Even if they were still able to assume analytic truths as existing, there was no way of rigorously dealing with them. Logicism was innately flawed.

Bayesianism, the preferred course of action for handling synthetic statements, had its own drawbacks as well. As it dealt with empirical claims, it fell victim to the many problems of induction. Already known was Hume's problem of induction, which points out the circularity in the argument that the only reason we believe induction is through an inductive call to its past success. This alone was enough to pose a threat to Bayesianism, but Hempel developed another paradoxical thought experiment that dealt directly with the verification process that the logical positivists were employing.

For the logical positivists, the likelihood of a proposition being true is directly tied to its supporting evidence. But what exactly constitutes valid “evidence”? For Hempel, this is not entirely obvious. He considers the claim “All ravens are black”. Every time a black raven is observed, Bayesianism dictates that the probability that the statement is true increases. This makes logical and intuitive sense. But as Hempel notices, a statement and its contrapositive are always equivalent, so if the statement “If something is a raven, then it is black” is true, then “If something is not black, then it is not a raven” should also be true (Hempel, 12). With this reworded but equivalent statement, observing a non-black, non-raven should serve as perfectly good evidence for the original hypothesis. But it is an absurd conclusion to believe that observing a blue sky provides justification for our original claim. This is known as the “paradox of confirmation” (14). With a little more analysis, even stranger implications arise. Just observing the blue sky seems to support an infinite number of possibly contradictory claims, such as “All ravens are yellow” and “All ravens are green”. With so many confounding issues, Bayesianism could not reconcile them all.

And so, pillar by pillar, logical positivism was destroyed. It can be seen that the fall of logical positivism was not the result of criticism from some opposition, but brought upon almost entirely from its own members. Quine, Gödel, and Hempel were all logical positivists (at some point and to differing degrees,) and their own theories and realizations made them come to terms with the reality that their philosophy was flawed. They were not afraid to make this known, being the first to publish these aforementioned problems. Ayer retrospectively claimed that while “nearly all of it was false, it was true in spirit” (Manufacturing Intellect), and the truth of this assertion can be seen to this day.

For physics, it gave no conclusive answer to which theory was correct, but the general attitude and methodology of logical positivism remained intact, and proved useful not only for guiding work in physics, but science as a whole. The focus on testable events gave physicists the peace of mind knowing there was a framework that they could rely on for pursuing truth, provided social sciences with a more rigorous approach to their fields, and perhaps most importantly, yielded a compelling argument to rebute the dangerous, pseudo-sceintific claims of Nazi serology that dominated the era. The logical positivists made it clear that they were “against the German tradition, both intellectually and politically” (Manufacturing Intellect). It was a radical movement that impacted academic life and shaped science throughout the 20th century and beyond.

Logical positivism also restructured the purpose of philosophy itself. A focus on meaningful propositions refined the scope of philosophy to assist in clarifying the contents of problems, not in creating nonsensical ones of their own. Although originally dispelled disciplines like metaphysics have eventually creeped back into philosophy's purview, they are still treated with the logical and linguistic precision that have become the central tenants of the analytic tradition.

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