Combinatorics HW1 Pigeonhole Principle Ryzeson Maravich

1 (10 points)

Claim: There are at least two people in Los Angeles with the same number of hairs on their head.
Proof: The most hairs on a human head is 150,000 (baumanmedical.com). The population of Los Angeles is estimated to be just over 4,000,000 (worldpopulationreview.com). Because $4,000,000 > 150,000$, that is there are more people in Los Angeles than hairs on the human head, then there will be at least two people with the same number of hairs on their head due to the Pigeonhole Principle.

2 (10 points)

Claim: The smallest number of people in a group that guarantees that five will share the same birthday month is 49 people.
Proof: We know by the Generalized Pigeon-Hole Principle that if $n > rm$ and we try to place n items (people) into m containers (months), then at least one container will have $r+1$ items. If we want to guarantee 5 people share the same month, then $r + 1 = 5 \Rightarrow r = 4$. There are 12 months so $n > 4(12) \Rightarrow n > 48$. The minimum value for n would therefore be 49 people.

3 (10 points)

In a drawer, you have four pairs of socks colored red, blue, white, and yellow respectively. Without looking at them, you take out one sock at a time.

Claim: You must take out 5 socks to ensure that you have a matching pair.
Proof: We know by the Generalized Pigeon-Hole Principle that if $n > rm$ and we try to place n items (socks) into m containers (different colors), then at least one container will have $r+1$ items. If we want to guarantee that we pull 2 socks of the same color, then $r + 1 = 2 \Rightarrow r = 1$. There are 4 colors so $n > 1(4) \Rightarrow n > 4$. The minimum value for n would therefore be 5 socks.

5 (10 points)

Claim: Given a group of 250 people, at least 3 people will be the same age.
Proof: Jeanne Calment, the oldest person to ever live, lived to be 122 years old. Seeing as how Jeanne holds this record by at least 3 years, let's assume that nobody will ever live longer than her. We know by the Generalized Pigeon-Hole Principle that if $n > rm$ and we try to place n items (people) into m containers (ages), then at least one container will have $r + 1$ items. $250 > r(122) \Rightarrow 250/122 > r \Rightarrow 2.049 > r \Rightarrow r = 2$. Therefore there will be at least 3 $(r + 1)$ people who are the same age.

I affirm that I have upheld the highest principles of honesty and integrity in my academic work and have not witnessed a violation of the Honor Code.